MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
2, 32, 1994

OPTIMIZATION WITH A PERIODIC CONTROL AND CONSTRAINTS ON THE STATE VARIABLES[1]

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In this paper, on the basis of Pontryagin's maximum principle, the necessary conditions for the optimality of a periodic control, have been found, while taking into account certain constraints imposed on the state variables.

1. Formulation and solution to the problem

The problem of optimization of systems described, among other things, by a periodic control within a limited interval has been presented before Piekarski (1992).

On the basis of Pontryagin's maximum principle, the author presents a theorem specifying necessary conditions under which such a control is optimal. Some technical applications of the theory formulated by Piekarski (1992) have been presented by Gajewski and Piekarski [2].

The aim of this paper is to generalize the afore mentioned theorem, for the case of optimization with constraints imposed on the state variables.

In order to describe the conditions of periodicity and the considered constraints accurately, the whole interval of optimization $[0, L]$ is divided into subintervals, each of the length

$$T = \frac{L}{n} \qquad n = 1, 2, ..., \widetilde{n} < \infty \tag{1.1}$$

[1]This paper was supported by the Grant PB 0269/P4/93/05

A part of the independent variable $t \in [0, L]$ contained by the interval q shall be denoted t_q

$$t_q \in [(q-1)T, qT] \qquad q = 1, 2, ..., n \tag{1.2}$$

If the inequality

$$(r-1)T \leq t_r \leq rT \tag{1.3}$$

is true, where r is an arbitrary fixed value of the index q, then the variable t_q can be described by the following formula

$$t_q = t_r + (q - r)T \tag{1.4}$$

The following assumptions have to be made about the control $u(t)$ specifying the optimization process:

- It is admissible, i.e. it is contained in a fixed set of defined and sectionally continuous functions
$$u(t) \in U \tag{1.5}$$
A the points of discontinuity, the right-hand limit of the control is assumed as its value.

- It is periodic in a limited interval, i.e. we can choose a subset of functions from the set U, for which the following is true
$$u(t_r) = u(t_q) \tag{1.6}$$
for each q and fixed value of r.

Control functions described by assumptions (1.5) and (1.6) are simultaneously admissible and periodic within a limited interval.

Two types of constraints are imposed on the state variable:

1. The continuous type

$$\begin{aligned} &q_m(\boldsymbol{x}(mT)) = 0 \\ &x_i(mT+0) - x_i(mT-0) = 0 \\ &m = 1, 2, ..., (n-1) \qquad i = 1, 2, ..., N \end{aligned} \tag{1.7}$$

The quantity $\boldsymbol{x}(mT)$ represents a vector of state at a boundary point mT of subintervals m and $(m+1)$. We assume, that the scalar function q_m generally assumes a different value for different values of m.

2. The discontinuous type

$$x_i(mT+0) - x_i(mT-0) = h_{mi}(\boldsymbol{x}(mT-0)) \tag{1.8}$$

The quantity h_m represents a vector of N coordinates h_{mi}. We assume that functions q_m and h_{mi} are continuous together with their derivatives with respect to their arguments. No single variable can be limited by both kinds of constraints at the same point, because they rule out each other.

We shall consider physical systems, described by a finite number of real variables $x_i(t)$ appearing in constraints (1.7), (1.8). We assume, that the state of the object is described by a system of N ordinary differential equations of the first degree

$$\frac{d}{dt}x_i(t) = f_i(x(t), u(t), t) \tag{1.9}$$

We assume that functions f_i are defined for all x, u, t, continuous together with their derivatives $\partial f_i/\partial x$ for all x, u and sectionally continuous for t.

To the equation of state (1.9) we add the following boundary conditions

$$\rho_l(x(0), x(L)) = 0 \qquad l = 1, 2, ..., p \leq 2N \tag{1.10}$$

We assume that functions ρ_l are of the same class as functions g_m and h_{mi}.

We shall be searching for the solution to the following optimization problem: let the functions $x_i(t)$ satisfy the equation of state (1.9) with the boundary conditions (1.10) and the constraints (1.7), (1.8), with the control $u(t)$ defined by Eqs (1.5) and (1.6); we aim to find such optimal $x(t)$, $u(t)$, which will make the cost functional

$$J = \int_0^L f_0(x(t), u(t), t)\, dt \tag{1.11}$$

assume its minimal value. We assume, that the function f_0 is of the same class as functions f_i.

To solve the problem, we have to formally eliminate the periodicity (1.6), by reducing the optimization given within the interval $[0, L]$ to an equivalent optimization within the subinterval $[(r-1)T, rT]$. This requires an increase in the dimension of space of the state variables and controls. To obtain such an increase we introduce the following vector functions, dependent on t_q

$$x(t_q) = (x_1(t_q), ..., x_N(t_q)) \tag{1.12}$$

$$u(t_q) = (u_1(t_q), ..., u_K(t_q)) \tag{1.13}$$

for all the values of q. The vector of state has ; nN, and the vector of control has nK coordinates. From Eq (1.12) it is obvious that the variables of state $x_i(t)$ in particular subintervals q, are marked as $x_i(t_q)$. Substitution of Eq (1.6) into Eq (1.13) gives the vector

$$u(t_q) = (u_1(t_r), ..., u_k(t_r)) = u(t_r) \tag{1.14}$$

which is the same in each subinterval q.

Eqs (1.12) and (1.14) together with the change of variables (1.4) allow us to put the equation of state (1.9) in an equivalent form, defined in the subinterval r

$$\frac{d}{dt_r}x_i(t_r + (q-r)T) = f_i\Big(x(t_r + (q-r)T), u(t_r), (t_r + (q-r)T)\Big) \tag{1.15}$$

for all the values of q. To Eqs (1.15) we add the unchanged boundary conditions (1.10) and constraints (1.7), (1.8). The cost functional (1.11), after changing the variables of integration, assumes an equivalent form, also defined in the subinterval r

$$J = \int\limits_{(r-1)T}^{rT} \sum_{q=1}^{n} f_0\Big(x(t_r + (q-r)T), u(t_r), (t_r + (q-r)T)\Big)\, dt_r \tag{1.16}$$

As we can see from the preceding relationships, the problem of optimization with which we started, given in the interval $[0, L]$ with periodic control $u(t)$ is equivalent to the problem defined in the subinterval $[(r-1)T, rT]$, under nonperiodic control $u(t_r)$. The constraints imposed on the variables of state initially given at fixed, internal points of interval $[0, L]$, become boundary conditions for the equivalent optimization problem.

The problem of optimization has been reduced to a typical (without constraints and periodic control) problem of optimal control, the solution to which (precondition of optimality) in the easiest way is found for equivalent continuous systems on the basis of, for example, Pontryagin's maximum principle (cf Gabasov and Kirillova, 1974; Gajewski and Piekarski [2]). Therefore we shall be searching for the solution of an equivalent problem of optimization, given in the subinterval of variable $t_r \in [(r-1)T, rT]$.

If the functions $x_i(t_r + (q-r)T)$ for each q satisfy the equations of state (1.15) with boundary conditions (1.10), (1.7), (1.8), with constraints imposed on control coming from (1.5)

$$u(t_r) \in U \tag{1.17}$$

then we are searching for such optimal $\bar{x}_i$ and $\bar{u}$, which give the cost functional a minimal value.

For this problem, the hamiltonian takes the following form

$$H(t_r) = \sum_{q=1}^{n} H_q(t_r) \tag{1.18}$$

where partial hamiltonians H_q have the form

$$\begin{aligned} H_q(t_r) \quad &= \quad \psi_0 f_0\Big(x(t_r + (q-r)T), u(t_r), (t_r + (q-r)T)\Big) + \\ &+ \quad \sum_{i=1}^{N} \psi_i(t_r + (q-r)T) f_i\Big(x(t_r + (q-r)T), u(t_r), (t_r + (q-r)T)\Big) \end{aligned} \tag{1.19}$$

Adjoint variables ψ_i dependent on arguments $(t_r + (q-r)T)$ must satisfy the following system of differential equations

$$\frac{d}{dt_r}\psi_i(t_r + (q-r)T) = -\frac{\partial H}{\partial x_i(t_r + (q-r)T)} \tag{1.20}$$

which can be written using Eq (1.18) in the form

$$\frac{d}{dt_r}\psi_i(t_r + (q-r)T) = -\frac{\partial H_q(t_r)}{\partial x_i(t_r + (q-r)T)} \tag{1.21}$$

true for every q, for a given value of r. To the system of adjoint equations (1.21) we have to add suitable boundary conditions. As emerges from theory, they take the form of

— for the right-hand limit (in point 0)

$$\psi_i(0) = \frac{\partial \varphi}{\partial x_i(0)} \tag{1.22}$$

— for the left hand-limit (in point L)

$$\psi_i(L) = -\frac{\partial \varphi}{\partial x_i(L)} \tag{1.23}$$

and adequately at points mT

$$\psi(mT + 0) = \frac{\partial \varphi}{\partial x_i(mT + 0)} \qquad \psi(mT - 0) = -\frac{\partial \varphi}{\partial x_i(mT - 0)} \tag{1.24}$$

In the above formulae, the boundary function φ is a linear function of all boundary conditions. For constraints of the continuous type (1.7) it takes the form

$$\begin{aligned}\varphi_1 &= \sum_{l=1}^{p} \rho_l \varphi_l(x(0), x(L)) + \sum_{s=1}^{n-1} \mu_s g_s(x(sT-0)) + \\ &+ \sum_{s=1}^{n-1}\sum_{k=1}^{N} \mu_{sk}[x_k(sT+0) - x_k(sT-0)]\end{aligned} \tag{1.25}$$

whereas for constraints of the discontinuous type (1.8) it takes the following form

$$\begin{aligned}\varphi_2 &= \sum_{l=1}^{p} \rho_l \varphi_l(x(0), x(L)) + \\ &+ \sum_{s=1}^{n-1}\sum_{k=1}^{N} \mu_{sk}[x_k(sT+0) - x_k(sT-0) - h_{sk}(x(sT-0))]\end{aligned} \tag{1.26}$$

After substituting for φ_1, φ_2 into Eqs (1.22) and (1.23) we get the boundary conditions at points 0, L for adjoint variables and both types of constraints, in the form

$$\begin{aligned}\psi_i(0) &= \sum_{l=1}^{p} \rho_l \frac{\partial \varphi_l(x(0), x(L))}{\partial x_i(0)} \\ \psi_i(L) &= -\sum_{l=1}^{p} \rho_l \frac{\partial \varphi_l(x(0), x(L))}{\partial x_i(L)}\end{aligned} \tag{1.27}$$

To get relationships between $\psi(0)$ and $\psi(L)$ in a form resembling Eq (1.10), we have to eliminate constants ρ_l from expressions (1.27).

For constraints (1.7) after substituting (1.25) into (1.24) we get

$$\begin{aligned}\psi_i(mT-0) &= \sum_{s=1}^{n-1}\sum_{k=1}^{N} \mu_{sk} \frac{\partial x_k(sT-0)}{\partial x_i(mT-0)} - \sum_{s=1}^{n-1} \mu_s \frac{\partial q_s(x(sT-0))}{\partial x_i(mT-0)} \\ \psi_i(mT+0) &= \sum_{s=1}^{n-1}\sum_{k=1}^{N} \mu_{sk} \frac{\partial x_k(sT+0)}{\partial x_i(mT+0)}\end{aligned} \tag{1.28}$$

Because derivatives of functions x_k given at points sT with respect to functions x_i at points mT are equal to zero for $m \neq s$, so after eliminating the constants μ_{mi}, we get

$$\psi_i(mT-0) = \psi(mT+0) - \mu_m \frac{\partial q_m(x(mT-0))}{\partial x_i(mT-0)} \tag{1.29}$$

The formulae (1.29) represent the jump conditions, which have to be fulfilled by adjont variables ψ_i, when we impose constraints (1.7). Additional $(n-1)$ constraints can be determined from first $(n-1)$ relations of constraints (1.7).

We use the same approach in the case of constraints (1.8).

Substitution of Eq (1.26) into (1.24) gives

$$\begin{aligned} \psi_i(mT-0) &= \sum_{s=1}^{n-1} \sum_{k=1}^{N} \mu_{sk} \left[\frac{\partial x_k(sT-0)}{\partial x_i(mT-0)} + \frac{\partial h_{sk}(x(sT-0))}{\partial x_i(mT-0)} \right] \\ \psi_i(mT+0) &= \sum_{s=1}^{n-1} \sum_{k=1}^{N} \mu_{sk} \frac{\partial x_k(sT+0)}{\partial x_i(mT+0)} \end{aligned} \tag{1.30}$$

from which, after eliminating the constants μ_{mi}, we get

$$\psi_i(mT-0) = \psi_i(mT+0) + \sum_{k=1}^{N} \psi_k(mT+0) \frac{\partial h_{mk}(x(sT-0))}{\partial x_i(mT-0)} \tag{1.31}$$

Condition (1.31) defines jumps at points mT, which are experienced by functions ψ_i, when the variables of state meet the constraints (1.8). In Eq (1.31) summation over k appears, because different functions h_{mk} for different values of k, can be dependent on the same function x_i, in a fixed point mT.

From the maximum principle (cf Gajewski and Piekarski, [2]) we conclude, that the control we find in the initial problem, and consequentially in the equivalent problem, can be determined from the condition of optimality

$$H_{opt}(t) = \max_{u(t) \in U} \sum_{q=1}^{N} H_q(t) \tag{1.32}$$

After we utilize the relationship (1.18) we get the formula allowing us to determine the control vector $u(t_r)$, for an equivalent optimization, which is the same in each subinterval q

$$H_{opt}(t_r) = \max_{u(t_r) \in U} \sum_{q=1}^{N} H_q(t_r) \tag{1.33}$$

where partial hamiltonians are given by the formulae (1.19).

On the basis of the previously obtained results, we can formulate a theorem concerning optimization under a periodic control within a finite interval with constraints of types given in Eqs (1.7), (1.8).

Theorem

In order to find the necessary conditions of optimality in a system ruled by the equations of state (1.9) with the boundary conditions (1.10) and the constraints imposed on state variables (1.7), (1.8), with a cost functional (1.11) with control constraints (1.5) and additionally (1.6) we have to:

a) Find an optimal control from the condition of optimality (1.33)

b) Solve the equation of state (1.15) with conditions (1.10) and constraints (1.7) or (1.8)

c) Solve the adjoint equation (1.21) with (1.19) with conditions (1.27) and constraints (1.29) or (1.31), respectively.

In the above considerations, we have to assume the following conditions:

d) Adjoint variables $\psi_i(t_r + (q-r)T)$, and the constant ψ_0 cannot be simultaneously equal to zero

e) generally, the hamiltonian $H_{opt}(t_r)$ is continuous in subintervals q; on a particular condition when T is given, we get $H_{opt}(t_r) = \text{const} \neq 0$, whereas when T is unknown, $H_{opt}(t_r) = 0$, from which we can calculate the value of T.

Note:

In order to eliminate the continuous constraints we have to omit the first condition in Eq (1.7) during calculation and assume, that $\mu_m \equiv 0$ in Eq (1.29). To eliminate the discontinuos constraints, we have to assume $h_{mi} \equiv 0$ in Eqs (1.7) and (1.31). After eliminating the constraints of both types, the above theorem takes the form presented by Piekarski (1992).

2. Example

This example, though very straightforward, illustrates all possible problems that can be encountered while using constraints (1.7) and (1.8). We shall consider an object described by the equation of state

$$\frac{d}{dt}x_1(t) = bu(t) \qquad 0 \le t \le L \tag{2.1}$$

with the boundary condition

$$\varphi_1 = x_1(0) - x_0 = 0 \tag{2.2}$$

Interval $[0.L]$ is divided into three subintervals, that is we assume $n = 3$, $T = L/3$, $q = 1,2,3$. Furthermore, we assume that at points T and $2T$, the constraints imposed on variable x_1 are

— at the point T

$$x_1(T+0) - Bx_1(T-0) = 0 \tag{2.3}$$

which can also be presented in a form appropriate for Eq (1.8)

$$x_1(T+0) - x_1(T-0) = (B-1)x_1(T-0) \equiv h_{11} \tag{2.4}$$

— at the point $2T$

$$\begin{aligned} q_2 &\equiv x_1(2T-0) - A = 0 \\ x_1(2T+0) &- x_1(2T-0) = 0 \end{aligned} \tag{2.5}$$

where constants x_0, A, B are given.

We assume that the control $u(t)$ is defined, sectionally continuous and periodic within the interval $[0, L]$

$$u(t_1) = u(t_q) \qquad q = 1,2,3 \tag{2.6}$$

where t_q is given by the formula (1.4), in which we put $r = 1$

$$t_q = t_1 + (q-1)T \qquad 0 \le t_1 \le T \tag{2.7}$$

Fpr thos problem we shall be searching for the minimum of functional

$$J = \int_0^L \Big(a_1 x_1(t) + \frac{1}{6}u^2(t)\Big)\, dt \tag{2.8}$$

After changing integration variables, using Eq (2.7) and Eq (2.6), the functional takes the following form

$$J = \int\limits_0^T \Big[a_1 \sum_{q=1}^{3} x_1(t_1 + (q-1)T) + \frac{1}{3}u^2(t_1)\Big]\, dt_1 \tag{2.9}$$

After taking into consideration expressions (2.6) and (2.7), the equation of state (2.1) can be presented in subinterval $[0, L]$ in a form appropriate for Eq (1.15)

$$\frac{d}{dt}x_1(t_1 + (q-1)T) = bu(t_1) \tag{2.10}$$

Partial hamiltonians (1.19) for the tested expression take the form $\psi_0 = -1$

$$H_q(t_1) = -\frac{1}{6}u^2(t_1) - a_1 x_1(t_1 + (q-1)T) + b\psi_1(t_1 + (q-1)T)u(t_1) \tag{2.11}$$

The relationships (1.21) together with (2.11) form a differential equation in adjoint variable ψ_1 dependent on arguments $(t_1 + (q-1)T)$

$$\frac{d}{dt_1}\psi_1(t_1 + (q-1)T) = a_1 \tag{2.12}$$

Solutions to these equations are the functions

$$\psi_1(t_1 + (q-1)T) = a_1 t_1 + c_q \tag{2.13}$$

for each q. Three constants of integration c_q can be calculated from the conditions (1.29) and (1.31) and the boundary condition (2.2). From Eqs (2.4) and (2.5) we have

$$\begin{aligned} q_2 &= x_1(2T - 0) - A \\ h_{11} &= (B - 1)x_1(T - 0) \end{aligned} \tag{2.14}$$

After substituting expressions (2.14) into Eqs (1.29) and (1.31), we get

$$\psi_1(2T - 0) = \psi_1(2T + 0) - \mu_2 \tag{2.15}$$

$$\psi_1(T - 0) = B\psi_1(T + 0) \tag{2.16}$$

From the boundary conditions (2.17) and (2.2), we conclude that

$$\psi_1(L) = 0 \tag{2.17}$$

The formulae (2.13), (2.15) and (2.16) allow us to evaluate constants c_q

$$\begin{aligned} c_1 &= -a_1T(2B+1) - \mu_2 B \\ c_2 &= -2a_1T - \mu_2 \\ c_3 &= -a_1T \end{aligned} \tag{2.18}$$

which are defined using a yet unknown constant μ_2.

The optimal control $u(t_1)$ should be calculated from the optimality condition (1.33) in which we have a hamiltonian in the form

$$\begin{aligned} H(t_1) &= \sum_{q=1}^{3} H_q(t_1) = -\frac{1}{2}u^2(t_1) - a_1\sum_{q=1}^{3} x_1(t_1+(q-1)T) + \\ &+ bu(t_1)\sum_{q=1}^{3}\psi_1(t_1+(q-1)T) \end{aligned} \tag{2.19}$$

Seeing as in Eq (2.19) there is a minus sign in front of $u^2(t_1)$, the condition (1.33) takes the form

$$\frac{\partial H(t_1)}{\partial u(t_1)} = 0 \tag{2.20}$$

from where we get

$$u(t_1) = b\sum_{q=1}^{3}\psi_1(t_1+(q-1)T) \tag{2.21}$$

After substituting Eqs (2.13) and (2.18) into Eq (2.21) we get

$$u(t_1) = 3a_1bt_1 - \mu_2 b(B+1) - 2a_1bT(B+2) \tag{2.22}$$

The equations of state (2.10) together with Eq (2.22) have solutions in the form

$$x_1(t_1+(q-1)T) = \frac{2}{3}a_1b^2t_1^2 - \mu_2b^2(B+1)t_1 - 2a_1b^2T(B+2)t_1 + c_{q+3} \tag{2.23}$$

The quantity $\mu_2 b(B+1)$ and constants c_{q+3} are calculated from the boundary condition (2.2) and the constraints (2.4) and (2.5). After certain transformations, we get

$$\mu_2 b(B+1) = \frac{3}{2}a_1bT - 2a_1b(B+2)T + \frac{(x_0B - A)}{b(B+1)T} \tag{2.24}$$

and

$$c_4 = x_0 \qquad c_5 = \frac{B(x_0 + A)}{(B+1)} \qquad c_6 = A \tag{2.25}$$

The first formula substituted into Eq (2.22) allows us to define the optimal control with known constants

$$u(t_1) = 3a_1 b t_1 - \frac{3}{2} a_1 b T - \frac{(x_0 B - A)}{b(B+1)T} \tag{2.26}$$

The solutions (2.23) after substituting from (2.25), take the form

$$x_1(t_1 + (q-1)T) = \frac{3}{2} a_1 b^2 t_1^2 - \frac{3}{2} a_1 b^2 t_1 T - \frac{(x_0 B - A)}{(B+1)T} t_1 + c_{q+3} \tag{2.27}$$

where c_{q+3} are given by the formulae (2.24).

In order to get numerical values of the solutions, we assume that

$$\begin{array}{lll} & L = 6 & x_0 = a_1 = b = 1 \\ & T = 2 & A = 3 \\ \text{and} & & \\ & B = 3 \quad \text{or} & B = 1 \end{array} \tag{2.28}$$

On the basis of Eq (1.8), we can see, that for $B = 3$ the step of the state variable at the point T has the value $2x_1(T-0)$, Fig.1. From Eq (2.26) we get the equation of optimal control

$$u(t_1) = 3t_1 - 3 \tag{2.29}$$

State variables in subintervals (2.27), (2.24) and (2.28) are given by the functions

$$\begin{aligned} x_1(t_1) &= 1.5t_1^2 - 3t_1 + 1 \\ x_1(t_1 + T) &= x_1(t_1 + 2T) = 1.5t_1^2 - 3t_1 + 3 \end{aligned} \tag{2.30}$$

The graphical representation of control (2.29) and of the state variables (2.30) is shown in Fig.1.

In the case of $B = 1$, we conclude from Eq (1.8), that the state variable at the point T is continuous, while at the point $2T$ it is continuous and has the value of 3, Fig.2. The control in each of the subintervals is defined by the expression

$$u(t_1) = 3t_1 - 2.5 \tag{2.31}$$

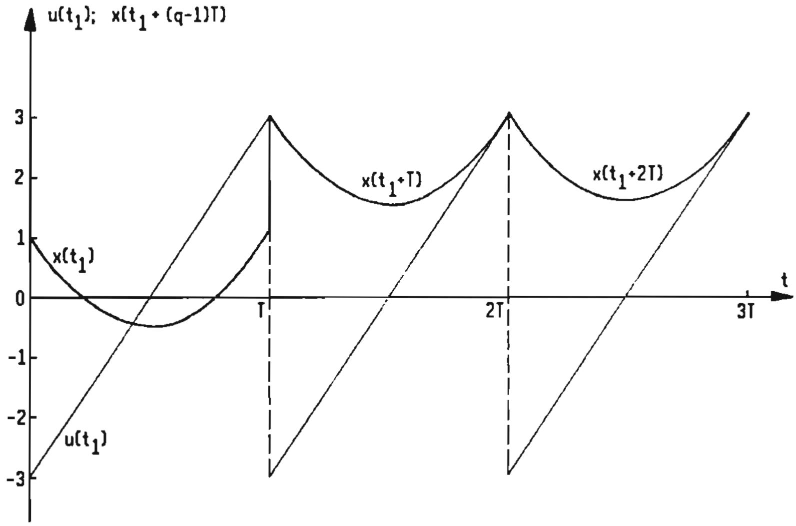
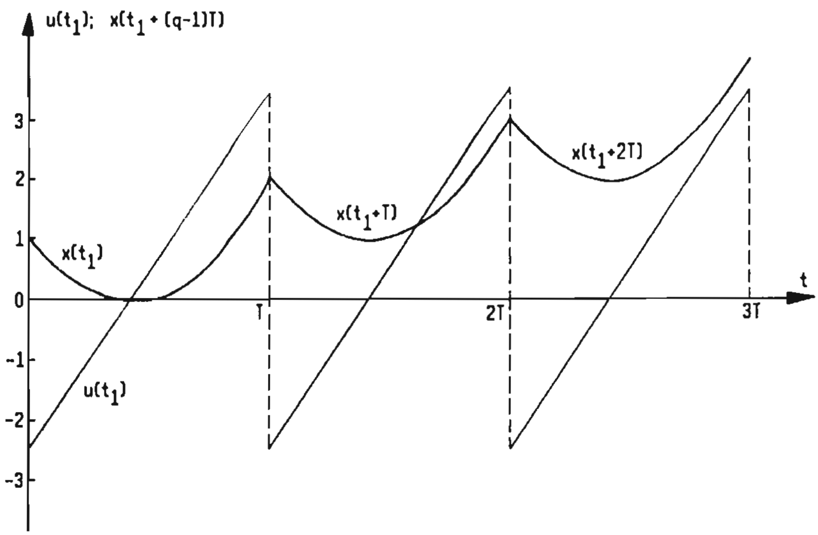

u(t1); x(t1 + (q-1)T)
3
2
1
0
-1
-2
-3
x(t1)
u(t1)
x(t1+T)
x(t1+2T)
t
T
2T
3T

Fig. 1.


u(t1); x(t1 + (q-1)T)
3
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-1
-2
-3
x(t1)
u(t1)
x(t1+T)
x(t1+2T)
t
T
2T
3T

Fig. 2.

Variables of state from Eqs (2.27), (2.24) and (2.28) are given by the formulae

$$
\begin{aligned}
x_1(t_1) &= 1.5t_1^2 - 2.5t_1 + 1 \\
x_1(t_1 + T) &= 1.5t_1^2 - 2.5t_1 + 2 \\
x_1(t_1 + 2T) &= 1.5t_1^2 - 2.5t_1 + 3
\end{aligned}
\tag{2.32}
$$

Graphical representation of the functions (2.31) and(2.32) is shown in Fig.2.

From point e) of the theorem, we conclude that for the optimal $u(t_1)$ and $x_1(t_1 + (q-1)T)$ the hamiltonian (2.19) has a constant value.

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Optymalizacja przy sterowaniu okresowym i ograniczeniach na zmienne stanu

Streszczenie

W pracy, na gruncie zasady maksimum Pontriagina, znalezione zostały warunki konieczne optymalności sterowania okresowego w skończonym przedziale przy uwzględnieniu wybranych ograniczeń na zmienne stanu.

Manuscript received March 12, 1993; accepted for print September 28, 1993