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A BAR FINITE ELEMENT FOR VIBRATION AND BUCKLING ANALYSIS OF CRACKED TRUSS CONSTRUCTIONS

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The paper presents a method of forming an inertia matrix and linear and geometrical stiffness matrices of a bar finite element with a single, non-propagating transverse one-edge open crack located in its mid-length. The presented method is based on the displacement formulation of FEM and laws of fracture mechanics. It has been found that the crack modified the inertia matrix and the linear stiffness matrix of the element, whereas the geometrical stiffness matrix remained unchanged. Taking advantage of the presented element there were done exemplary numerical calculations illustrating variations of longitudinal natural frequencies of the one sided fixed rod and variations of the values of global buckling load in a simple truss caused by the crack. The effect of inertia matrix form upon the values of longitudinal natural frequencies of the one sided fixed rod were analyzed.

1. Introduction

Cracks occurring in structural elements of machines are responsible for local stiffness variations (cf Irwin, 1956), which in consequence affect their dynamic characteristics. This problem has been a subject of many papers, the review of which is given by Wauer (cf Wauer, 1991). First attempts were devoted to the analysis of simple cracked structures such as beams, shafts and frames with a constant cross-section (cf Okamura et al., 1969; Henry and Okah-Avae, 1976; Mayes and Davies, 1976; Anifantis and Dimarogonas, 1983; Dimarogonas and Papadopoulos, 1983; Christidis and Barr, 1984; Papadopoulos and Dimarogonas, 1987a,b; Ostachowicz and Krawczuk, 1991; Rajab and

Al-Sabeeh, 1991). Real engineering constructions are more complicated and the analytical methods of cracks modelling described in the papers cited above are useless. For this reason some of researchers have started to employ the FEM for modelling the damaged complex structures.

Dirr and Schmalhorst (1987), Ostachowicz and Krawczuk (1990a,b) applied classical 2-D or 3-D finite elements to modelling of cracked structures. A crack was modelled by separating nodes of the elements on both sides of the crack. 2-D isoparametric finite elements with the singular shape function for the analysis of natural vibrations of the beams with double-edge cracks were used by Shen and Pierre (1990). The crack modelling method mentioned above requires using a dense grid of finite elements around the crack edge due to a singular character of stress fields and deformations occuring there.

Other authors use the special finite elements with cracks (cf Gounaris and Dimarogonas, 1988; Haisty and Springer, 1988; Qian et al., 1990 and 1991; Krawczuk, 1992 and 1993; Krawczuk and Ostachowicz, 1993 and 1994) for static and dynamic analysis of cracked structures. The characteristic matrices of these elements can be formulated by means of the flexibility method (cf Haisty and Springer, 1988; Qian et al., 1990 and 1991; Krawczuk, 1992; Krawczuk and Ostachowicz, 1993 and 1994) or FEM (cf Gounaris and Dimarogonas, 1988; Krawczuk, 1993). In the case of flexibility method crack affects only the form of linear stiffness matrix, while for the displacement formulation of FEM the inertia matrix, and the linear and geometrical stiffness matrices change their forms.

In the present paper there has been made an attempt to elaborate a bar finite element with the transverse one-edge open crack. The main objectives are:

- Determination of the characteristic matrices of the bar finite element with the transverse one-edge open crack applying, in contrast to Krawczuk (1992), the displacement formulation of FEM

- Carrying out an analysis of the influence of the magnitude and location of the crack upon the variations of longitudinal natural vibrations of the clamped-free rod

- Investigation of the influence of inertia matrix form upon the longitudinal natural frequencies of the clamped-free rod

- Analysis of the influence of the magnitude of the crack upon the variations of global buckling load of the simple truss.

2. Bar finite element with the transverse one-edge open crack

The bar finite element with the non-propagating, transverse, one-edge, open crack located in the mid-length of the element is shown in Fig.1.

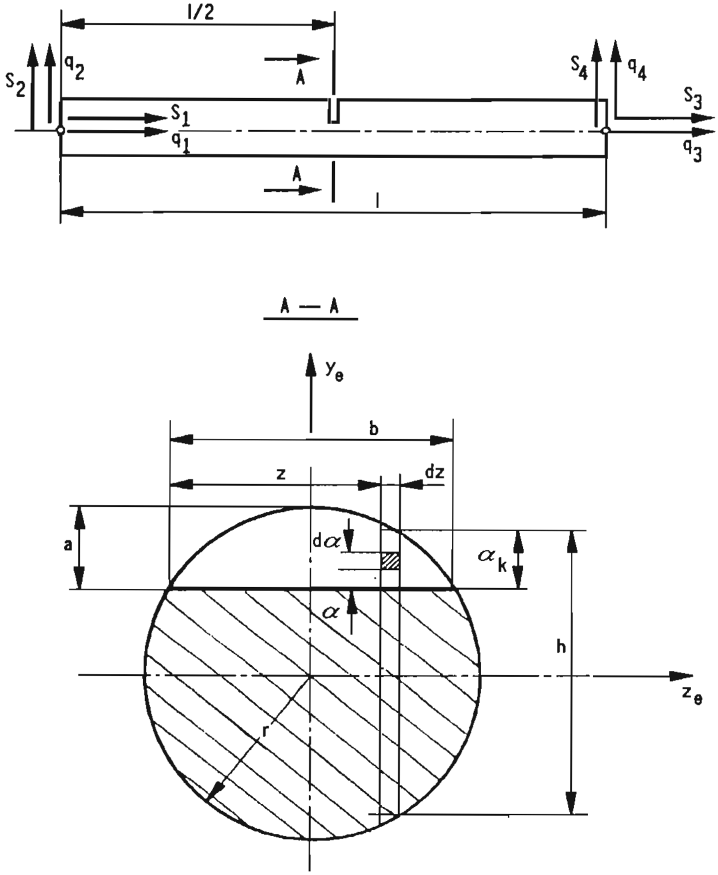

Fig. 1. (a) bar finite element with crack, (b) cross-section of the element at the crack area

Since the crack is responsible for discontinuities within the displacement field of the element (cf Papadopoulos and Dimarogonas, 1987b) there have been assumed the following shape functions

$$\begin{aligned} u_{1x} &= a_1 + a_2 x \qquad & u_{1y} &= a_5 + a_6 x \\ u_{2x} &= a_3 + a_4 x \qquad & u_{2y} &= a_7 + a_8 x \end{aligned} \tag{2.1}$$

together with the following boundary conditions at both ends of the element (Fig.1)

$$
\begin{aligned}
u_{1x}\Big|_{x=0} &= q_1 & \qquad u_{1y}\Big|_{x=0} &= q_2 \\
u_{2x}\Big|_{x=l} &= q_3 & \qquad u_{2y}\Big|_{x=l} &= q_4
\end{aligned}
\tag{2.2}
$$

and continuity conditions at the crack location

$$
\begin{aligned}
u_{1x} &= u_{2x} - c_{11}^1 u_{1x}' & \qquad u_{1y} &\equiv u_{2y} \\
u_{1x}' &= u_{2x}' & \qquad u_{1y}' &\equiv u_{2y}'
\end{aligned}
\tag{2.3}
$$

where c_{11}^1 is the additional longitudinal flexibility of the element due to the crack, form of which is given in the Appendix, indices 1 or 2 denote the left or the right part of the bar elemenct, respectively, and l denotes the length of the element.

Making use of conditions (2.2) and (2.3) the constants $a_1 - a_8$ are

$$
\begin{aligned}
a_1 &= q_1 & \qquad a_5 &= q_2 \\
a_2 &= \frac{-q_1 + q_3}{l + c_{11}^1} & \qquad a_6 &= \frac{-q_2 + q_4}{l} \\
a_3 &= \frac{q_1 l - q_3 c_{11}^1}{l + c_{11}^1} & \qquad a_7 &= q_2 \\
a_4 &= \frac{-q_1 + q_3}{l + c_{11}^1} & \qquad a_6 &= \frac{-q_2 + q_4}{l}
\end{aligned}
\tag{2.4}
$$

2.1. Linear and geometrical stiffness matrices of the element

The elastic strain energy of an element under large deformations can be written in the following form

$$
U_e = \frac{1}{2} \int\limits_{V_1} E\varepsilon_{xx1}^2 \, dV_1 + \frac{1}{2} \int\limits_{V_2} E\varepsilon_{xx2}^2 \, dV_2
\tag{2.5}
$$

where

E – Young modulus
V_i – volume of the left and right part of the element, $(i = 1, 2)$
ε_{xxi} – deformation of the element calculated from the relation

$$\varepsilon_{xxi} = \frac{du_{ix}}{dx} + \frac{1}{2}\Big(\frac{du_{iy}}{dx}\Big)^2 \qquad (i = 1, 2) \tag{2.6}$$

Substituting Eq (2.6) into (2.5) the elastic strain energy of the element is

$$\begin{aligned} U_e &= \frac{AE}{2}\int\limits_0^{\frac{l}{2}} \Big[\Big(\frac{du_{1x}}{dx}\Big)^2 + \frac{du_{1x}}{dx}\Big(\frac{du_{1y}}{dx}\Big)^2 + \frac{1}{4}\Big(\frac{du_{1y}}{dx}\Big)^4\Big]\,dx + \\ &+ \frac{AE}{2}\int\limits_{\frac{l}{2}}^{l} \Big[\Big(\frac{du_{2x}}{dx}\Big)^2 + \frac{du_{2x}}{dx}\Big(\frac{du_{2y}}{dx}\Big)^2 + \frac{1}{4}\Big(\frac{du_{2y}}{dx}\Big)^4\Big]\,dx \end{aligned} \tag{2.7}$$

where A denotes the area of the element cross-section.

Neglecting the higher order terms, and taking into account relations (2.1) and (2.4), the strain energy of the element can be rewritten in the following form

$$U_e = \frac{AEl}{2}\Big[\frac{1}{(l + c_{11}^1)^2}(q_1^2 - 2q_1q_3 + q_3^2) + \frac{q_3 - q_1}{(l + c_{11}^1)l^2}(q_2^2 - 2q_2q_4 + q_4^2)\Big] \tag{2.8}$$

Even in the case of relatively large deflections the quantity $AE(q_3 - q_1)/(l + c_{11}^1)$ may be treated as a constant equal to the axial force F in the bar. The final form of the element strain energy is

$$U_e = \frac{AEl}{2(l + c_{11}^1)^2}(q_1^2 - 2q_1q_3 + q_3^2) + \frac{F}{2l}(q_2^2 - 2q_2q_4 + q_4^2) \tag{2.9}$$

Taking advantage of the Castigliano theorem we obtain relations between the nodal forces and displacements

$$\begin{aligned} S_1 &= \frac{\partial U_e}{\partial q_1} = \frac{AEl}{(l + c_{11}^1)^2}(q_1 - q_3) \\ S_2 &= \frac{\partial U_e}{\partial q_2} = \frac{F}{l}(q_2 - q_4) \\ S_3 &= \frac{\partial U_e}{\partial q_3} = \frac{AEl}{(l + c_{11}^1)^2}(-q_1 + q_3) \\ S_4 &= \frac{\partial U_e}{\partial q_4} = \frac{F}{l}(-q_2 + q_4) \end{aligned} \tag{2.10}$$

Relations (2.10) can be presented in the matrix form

$$S = (\mathbf{K}_{le} + \mathbf{K}_{ge})q \tag{2.11}$$

where

$S = \mathrm{col}(S_1, ..., S_4)$ – column matrix of nodal forces
$q = \mathrm{col}(q_1, ..., q_4)$ – column matrix of nodal displacements
$\mathbf{K}_{le}$ – linear stiffness matrix
$\mathbf{K}_{ge}$ – geometrical stiffness matrix of the element.

Forms of the matrices $\mathbf{K}_{le}$ and $\mathbf{K}_{ge}$ are the following

$$\mathbf{K}_{le} = \frac{AEl}{(l + c_{11}^1)^2} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.12}$$

$$\mathbf{K}_{ge} = \frac{F}{l} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{2.13}$$

It follows from Eqs (2.12) and (2.13) that the crack affected the linear stiffness matrix $\mathbf{K}_{le}$ whereas the geometrical stiffness matrix of the element $\mathbf{K}_{ge}$ appears to have the same form as for the non-cracked bar finite element proposed by Przemieniecki (1968). In the case when the additional longitudinal flexibility c_{11}^1 is equal to zero we obtain the form of the linear stiffness matrix $\mathbf{K}_{le}$ identical to the one given by Przemieniecki (1968) for the non-cracked bar finite element.

2.2. Inertia matrix of the element

The inertia matrix of the element can be expressed by the following relation

$$\mathbf{M}_e = \rho A \int\limits_0^{\frac{l}{2}} \mathbf{N}_1^\top \mathbf{N}_1 \; dx + \rho A \int\limits_{\frac{l}{2}}^{l} \mathbf{N}_2^\top \mathbf{N}_2 \; dx \tag{2.14}$$

where

ρ – mass density of the element

$\mathbf{N}_i$ $(i = 1,2)$ – shape function matrix of the element in the form

$$\mathbf{N}_1 = \begin{bmatrix} 1 - \frac{x}{l+c_{11}^1} & 0 & \frac{x}{l+c_{11}^1} & 0 \\ 0 & 1-\frac{x}{l} & 0 & \frac{x}{l} \end{bmatrix} \tag{2.15}$$

$$\mathbf{N}_2 = \begin{bmatrix} \frac{l-x}{l+c_{11}^1} & 0 & \frac{x-c_{11}^1}{l+c_{11}^1} & 0 \\ 0 & 1-\frac{x}{l} & 0\frac{x}{l} & \end{bmatrix} \tag{2.16}$$

Finally the inertia matrix $\mathbf{M}_e$ of the element takes the form

$$\mathbf{M}_e = \rho A \begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ 0 & m_{22} & 0 & m_{24} \\ m_{31} & 0 & m_{33} & 0 \\ 0 & m_{42} & 0 & m_{44} \end{bmatrix} \tag{2.17}$$

where the entries of the matrix $\mathbf{M}_e$ looking as follows

$$m_{11} = \frac{8l^3 + 18l^2c_{11}^1 + 12l(c_{11}^1)^2}{24(l+c_{11}^1)^2}$$

$$m_{22} = m_{44} = \frac{l}{3}$$

$$m_{33} = \frac{8l^3 - 18l^2c_{11}^1 + 12l(c_{11}^1)^2}{24(l+c_{11}^1)^2}$$

$$m_{13} = m_{31} = \frac{4l^3}{24(l+c_{11}^1)^2}$$

$$m_{24} = m_{42} = \frac{l}{6}$$

When the c_{11}^1 is equal to zero, the form of inertia matrix of the element $\mathbf{M}_e$ is identical to the form of inertia matrix of the non-cracked bar finite element (cf Przemieniecki, 1968).

3. Numerical calculations

Exemplary numerical calculations were intended to determine the effect of the depth and the location of non-propagating, transverse, one-edge, open crack upon longitudinal natural frequencies of the clamped-free rod, and upon the global buckling load for a simple truss. Additionally, there was also carried out the analysis of influence of the inertia matrix formulation way upon the longitudinal natural frequencies of the clamped bar.

3.1. Longitudinal natural frequencies of the cracked, clamped-free rod

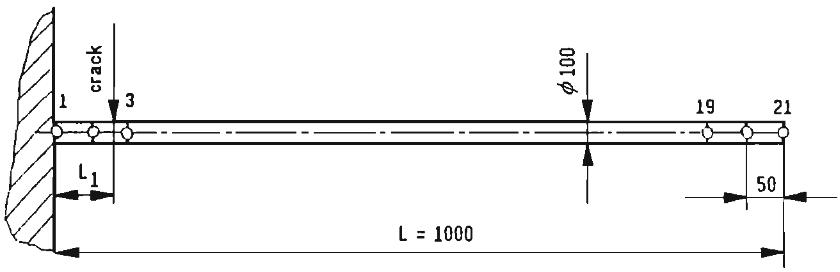

Fig. 2. The cracked clamped-free rod

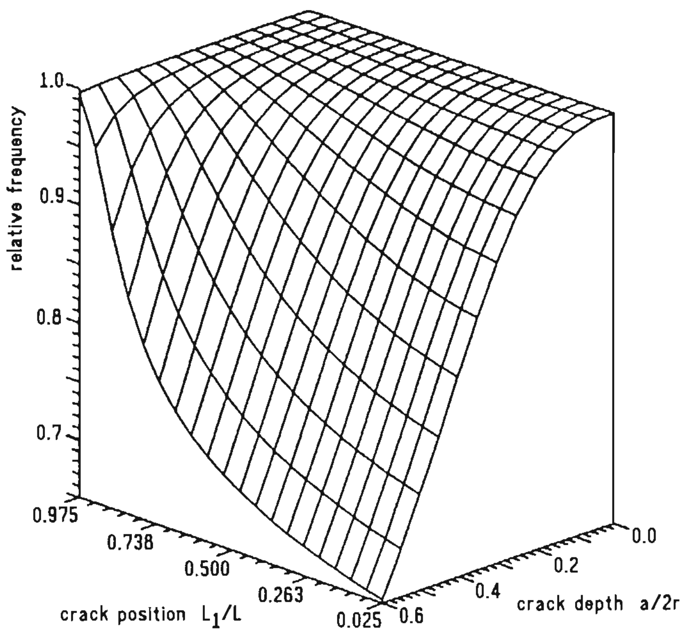

Fig. 3. Effect of the relative depth and location of the crack upon changes in the first longitudinal natural frequency of the clamped-free rod

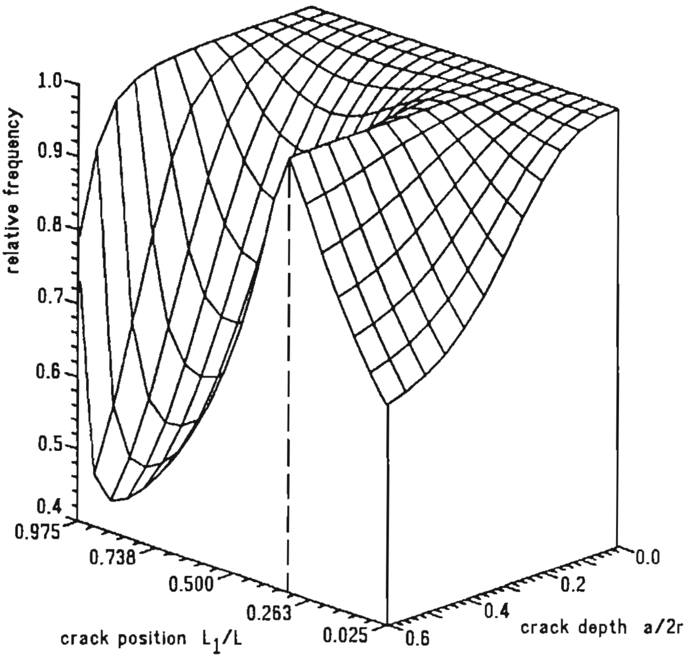

Fig. 4. Effect of the relative depth and location of the crack upon changes in the second longitudinal natural frequency of the clamped-free rod

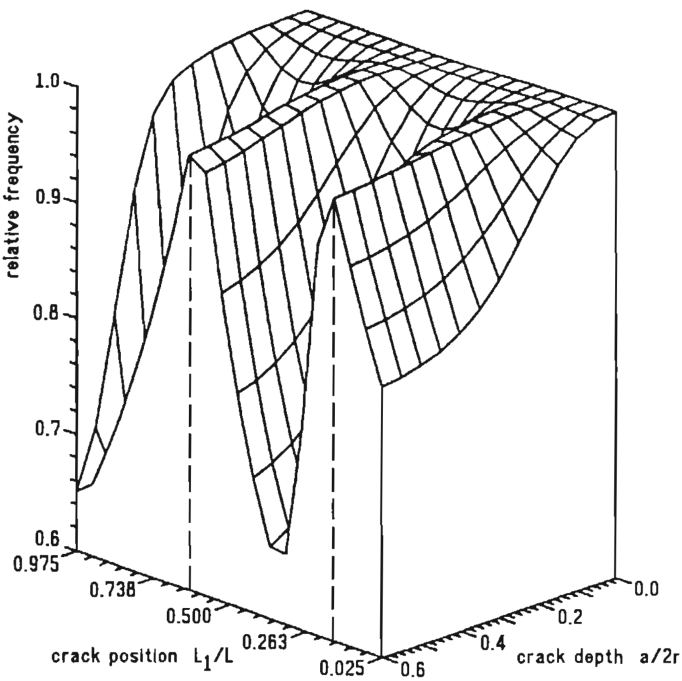

Fig. 5. Effect of the relative depth and location of the crack upon changes in the third longitudinal natural frequency of the clamped-free rod

The clamped-free rod under investigation is shown in Fig.2. For the purpose of discretization there is taken advantage of 20 elements of the same length. One of them contains the crack. The following material constants have been assumed: the Young modulus $2.1 \cdot 10^{11}$ N/m^2, mass density 7860 kg/m^3 and the Poisson ratio $\nu = 0.3$. The results illustrating the effect of the crack depth and location upon the four first longitudinal natural frequencies of the rod are presented in Fig.3. The results are obtained for the modified inertia matrix. The relative frequencies presented in Fig.3 $\div$ Fig.6 are calculated as a quotient of the natural frequency of the cracked rod by the natural frequency of the non-cracked one, for each mode of vibrations respectively.

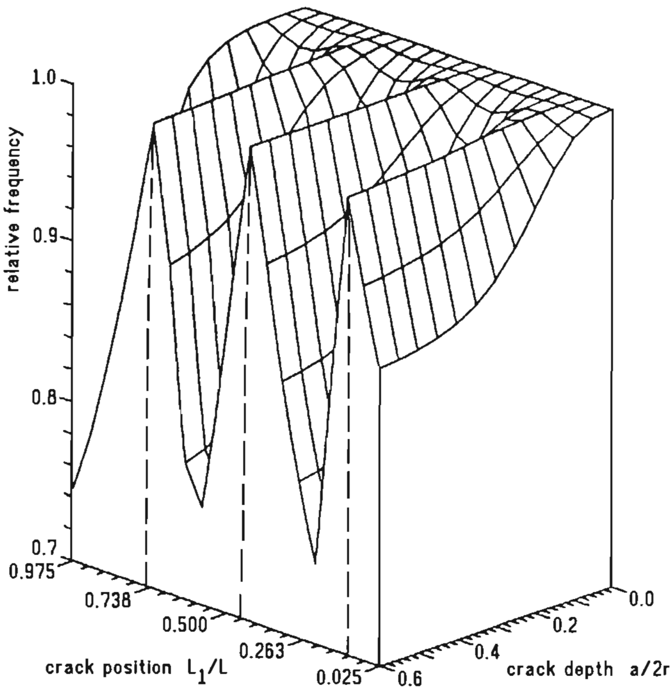

Fig. 6. Effect of the relative depth and location of the crack upon changes in the fourth longitudinal natural frequency of the clamped-free rod

Next there is made an analysis of the effect of the inertia matrix form upon the values of longitudinal natural frequency of the clamped-free rod with the crack of various depth located at a distance of 50 mm from the fixed end. In the first case only the linear stiffness matrix of the cracked element is modified. In the second case there are assumed variations in the inertia matrix and the linear stiffness matrix of the cracked element. The results are presented in Table 1.

Table 1. Influence of the inertia matrix form on longitudinal natural frequencies of the cracked, clamped-free rod (crack location $L_1/L = 0.05$)

	unmodified mass matrix	modified mass matrix
$a/2r$	first mode of vibration [rad/s]	
0.1	8058.44	8060.11
0.2	7864.41	7870.25
0.3	7543.06	7556.83
0.4	7075.50	7099.34
0.5	6440.45	6474.80
0.6	5638.17	5679.25
0.7	4715.42	4754.35
$a/2r$	second mode of vibration [rad/s]	
0.1	24387.08	24420.45
0.2	23854.63	23977.45
0.2	23854.63	23977.45
0.3	23046.87	23304.05
0.4	22033.35	22435.11
0.5	20927.65	21432.12
0.6	19878.41	20395.87
0.7	19014.14	19659.23
$a/2r$	third mode of vibration [rad/s]	
0.1	41350.22	41487.29
0.2	40598.54	41065.26
0.3	39590.27	40449.63
0.4	38523.96	39692.33
0.5	37564.42	38852.87
0.6	36804.38	38010.23
0.7	36261.91	37225.55

4. Buckling of a cracked simple truss

The analysis of the effect of the non-propagating, transverse, one-edge, open crack upon the magnitude of global buckling load is carried out following the example of a simple truss illustrated in Fig.7. The bending deformation and hence the excentricity effect as a consequence of a side crack as well as member buckling are excluded from the analysis. Additionally, there is

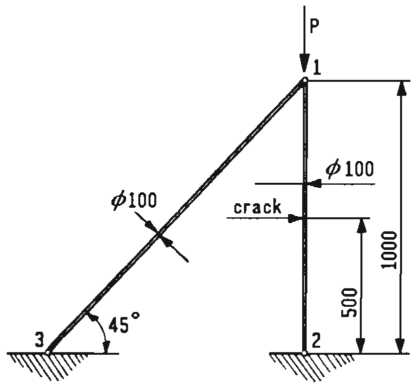

Fig. 7. Method of discretization of the cracked simple truss

assumed that the crack is completely open (cf Anifantis and Dimarogonas, 1983). Material properties are assumed to be the same as in the case of the rod longitudinal natural vibrations analysis. The truss is modelled by 2 finite elements. The crack is located in the vertical element (Fig.7). The results illustrating the effect of crack depth upon the value of global buckling load are shown in Table 2.

Table 2. Influence of the depth of the crack on global buckling load of the cracked simple truss

$a/2r$	buckling load [N]	relative buckling load
0.0	430800.0	1.0000
0.1	430320.0	0.9988
0.2	427260.0	0.9917
0.3	423190.0	0.9823
0.4	417440.0	0.9689
0.5	409500.0	0.9505
0.6	398480.0	0.9249
0.7	383100.0	0.8892

The global buckling load obtained for non-cracked truss agrees with the exact result obtained by Timoshenko and Gere (1961).

5. Conclusions

The paper presents a method of generating a bar finite element with the non-propagating, transverse, one-edge, open crack situated in the mid-length of the element. The presented method is based on the displacement formulation of the FEM and laws of fracture mechanics. The described method makes it possible to construct bar finite elements with various types of crack (double-edge, internal, etc.), if the stress intensity factors for a given type of crack are known. The above element can be used for a statical and dynamical analysis of truss constructions with material defects in the form of cracks.

As a result of the calculations done it was possible to state that:

- The crack reduces the longitudinal natural frequencies of the clamped-free rod (Fig.3 ÷ Fig.6). The decrease in the longitudinal natural frequencies values depends on the depth and location of the crack. An increase in the crack depth reduces natural frequencies depending on the mode shape of vibration. The largest decrease oin natural frequencies is noticed in the case of cracks located in the vibration nodes whereas in the case of the crack located at a loop of the wave the change of natural frequencies is negligible.

- The analysis of the effect of the inertia matrix form upon the values of the longitudinal natural frequencies has proved that the differences in calculated frequencies rise while the modes of vibrations increase (Table 1). For small cracks of depths down to approx. 0.2 of the cross-section diameter of the bar, the differences between longitudinal natural frequencies are insignificant. The inertia matrix modification by taking into account the flexibility coefficients related to the existence of the crack raises the values of the natural frequencies in relation to the unmodified, consistent inertia matrix.

- The value of global buckling load drops together with the increment of crack depth in comparison to the global buckling load for the non-cracked truss (Table 2).

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Appendix

The additional flexibility of the element due to crack c^1_{11} can be calculated by using the Castigliano theorem 2nd part (cf Przemieniecki, 1968)

$$c^1_{ij} = \frac{\partial^2 U^1}{\partial S_i \partial S_j} \qquad (i = j = 1) \tag{A.1}$$

where U^1 denotes the additional elastic strain energy of the element caused by the crack, S_i, S_j are independent nodal forces acting on the element. In the case of the presented element an independent nodal force is the force S_1 – for more details see Krawczuk (1992).

The additional elastic strain energy caused by the crack can be expresed by the following relation (cf Krawczuk, 1992)

$$U^1 = \frac{1-\nu^2}{E} \int\limits_P K_I^2 \, dP \tag{A.2}$$

where

ν – Poisson ratio

P – area of the crack

K_I – stress intensity factor corresponding to the first case of crack evaluation (cf Henry and Okah-Avae, 1976).

The stress intensity factor can be expressed as a function of the independent nodal force S_1

$$K_I = \frac{S_1}{\pi r^2}\sqrt{\pi \alpha_k} f\Big(\frac{\alpha_k}{h}\Big) \tag{A.3}$$

where α_k, h are explained in Fig.1, $f\Big(\frac{\alpha_k}{h}\Big)$ is the correction function taking into account the finite dimensions of element (cf Okamura et al., 1969)

$$f\Big(\frac{\alpha_k}{h}\Big) = \sqrt{\frac{\tan\lambda}{\lambda}}\,\frac{0.752 + 2.02\Big(\frac{\alpha_k}{h}\Big) + 0.37(1-\sin\lambda)^3}{\cos\lambda} \tag{A.4}$$

where $\lambda = \pi\alpha_k/2h$.

Substituting Eqs (A.3) and (A.4) into Eq (A.2) and making use of relation (A.1) we arrive at the additional flexibility of the element caused by the non-propagating, transverse, one-sided, open crack in the form

$$c_{11}^1 = \frac{4(1-\nu^2)}{E\pi r}\int\limits_0^{\bar{a}} \bar{\alpha} f^2(\bar{g})\, d\bar{\alpha} \int\limits_0^{\bar{b}} d\bar{z} \tag{A.5}$$

where r is the radius of the element cross-section, $\bar{a} = a/r$, $\bar{g} = \alpha_k/h$, $\bar{\alpha} = \alpha_k/r$, $\bar{b} = b/r$ are explained in Fig.1.

Prętowy element skończony do analizy drgań i stabilności konstrukcji kratowych z pęknięciami

Streszczenie

W pracy przedstawiono metodę tworzenia macierzy bezwładności oraz sztywności liniowej i geometrycznej prętowego elementu skończonego z pojedyńczym, poprzecznym, niepropagującym, jednostronnym, otwartym pęknięciem zmęczeniowym. Prezentowana metoda opiera się na przemieszczeniowym sformułowaniu MES oraz prawach mechaniki pękania. Wykazano, że pęknięcie występujące w elemencie modyfikuje postacie macierzy mas i sztywności liniowej podczas gdy macierz sztywności geometrycznej elementu pozostaje bez zmian. Wykorzystując opracowany element wykonano przykładowe obliczenia ilustrujące wpływ pęknięcia zmęczeniowego na zmiany częstości własnych drgań wzdłużnych pręta jednostronnie utwierdzonego oraz zmiany wartości siły krytycznej w prostej kratownicy. Przeprowadzono także analizę wpływu postaci macierzy bezwładności na wartości częstości własnych drgań wzdłużnych jednostronnie utwierdzonego pręta z pęknięciem.

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