MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
2, 32, 1994

ROTATORY VIBRATION OF AN AEOLOTROPIC NON-HOMOGENEOUS ANNULAR DISK OF VARIABLE THICKNESS

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The object of this paper is to determine the mean stresses, displacements and especially the frequencies due to rotatory vibrations of an aeolotropic non-homogeneous annular disk of variable thickness. The elastic constants, density of material and thickness of the disk are considered to be a function of position. Interesting particular cases are considered and compared numerically and graphically with the established results.

1. Introduction

Expansion of research into different types of vibrations set up in elastic bodies of different shape has been observed for a long time. Material of the elastic medium was considered to be purely isotropic. But then considering the physical aspect it was found that instead of assuming purely isotropic material it would be justified to consider it as aeolotropic. There is a variety of elastic crystals employed in engineering structures, which reveal not only anistotropic character but also display a non-homogeneity of various types. It is commonly seen in manufacturing industries that rotation of elastic disks, cylinders, spheres etc. is essentially required over the areas where rotatory vibration set up within the bodies.

The problems of rotatory vibration of different bodies were discussed by Love (1944). Chatterjee (1967) studied rotatory vibration of an anisotropic thin circular plate. Mollah (1975) obtained stresses and frequencies due to rotatory vibration of an aeolotropic non-homogeneous annular disk.

In the present contribution the Author has discussed the problem of determinating stresses, displacements and frequencies due to rotatory vibration of an aeolotropic non-homogeneous annular disk of variable thickness. Considering the non-homogeneity of the material it is assumed that:

- Elastic constants vary arbitrarily as the nth power of radial distances
- Elastic constants, together with the material density, vary as the nth power of radial distances.

It is also assumed that the disk thickness $2h$, at a distance r from the axis is given by the formula

$$h = h_0 r^\beta$$

The particular case $n = 2$ has been considered separately and no rotatory vibration was found. It has been also observed that in a thin circular ring the rotatory vibration is not possible. All the results being obtained have ben compared with the results presented by previous researchers. The numerical values of frequency equation roots for different interesting cases have been obtained and finally variation of frequencies for a few cases has been shown graphically versus variation of the disk radius.

2. Formulation of the problem

Let the z-axis be perpendicular and pass through the disk of radius b centre. There is a hole in the disk of radius a. Employing polar coordinates (r,θ) the stress-strain relations for transversely isotropic material were given by Love (1944) as follows

$$\begin{aligned}
\sigma_{rr} &= c'_{11}e_{rr} + c'_{12}e_{\theta\theta} \\
\sigma_{\theta\theta} &= c'_{12}e_{rr} + c'_{11}e_{\theta\theta} \\
\sigma_{r\theta} &= c'_{66}e_{r\theta}
\end{aligned} \tag{2.1}$$

where $c'_{12} = c'_{11} - 2c'_{66}$, and

c'_{ij}	–	elastic constants being functions of r only
σ_{rr}, $\sigma_{\theta\theta}$	–	components of mean radial and tangential stresses, respectively
$\sigma_{r\theta}$	–	component of the mean shearing stress
e_{rr}, $e_{\theta\theta}$	–	components of mean radial and tangential strains, respectively
$e_{r\theta}$	–	component of the mean shearing strain in the disk middle plane.

If u_r and u_θ are the components of mean displacements in r and θ directions, respectively, for the rotatory vibration we assume

$$u_r = 0 \qquad\qquad u_\theta = f(r)\mathrm{e}^{\mathrm{i}\omega t} \tag{2.2}$$

The strain components were given by Love (1944) as follows

$$e_{rr} = e_{\theta\theta} = 0 \qquad\qquad e_{r\theta} = \Big(\frac{df}{dr} - \frac{f}{r}\Big)\mathrm{e}^{\mathrm{i}\omega t} \tag{2.3}$$

Considering the non-homogeneity of the material we assume

$$c'_{11} = c_{11}r^n \qquad c'_{12} = c_{12}r^n \qquad c'_{66} = c_{66}r^n \qquad n \in C \tag{2.4}$$

where c_{ij} are the values of c'_{ij} for the homogeneous case $(n = 0)$.

Substituting Eqs (2.3) and (2.4) into Eq (2.1) we get

$$\sigma_{rr} = \sigma_{\theta\theta} = 0 \qquad\qquad \sigma_{r\theta} = c_{66}r^n\Big(\frac{df}{dr} - \frac{f}{r}\Big)\mathrm{e}^{\mathrm{i}\omega t} \tag{2.5}$$

We assume the disk thickness to be equal to $2h$ at a distance r and

$$h = h_0 r^\beta \qquad\qquad \beta \in R \tag{2.6}$$

where h_0 is a constant.

The only non-vanishing stress equation of equilibrium (cf Timoshenko and Goodier, 1951) is

$$\frac{\partial}{\partial r}(h\sigma_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}(h\sigma_{\theta\theta}) + \frac{2h}{r}\sigma_{r\theta} = \rho h\frac{\partial^2 u_\theta}{\partial t^2} \tag{2.7}$$

where ρ is the mass per unit volume.

3. Solution of the problem at a constant density

Substituting Eqs (2.5) and (2.6) into Eq (2.7) we obtain

$$r^2\frac{d^2f}{dr^2}+(n+\beta+1)r\frac{df}{dr}+\Big[\lambda^2r^{2-n}-(n+\beta+1)\Big]f=0 \tag{3.1}$$

where

$$\lambda^2=\frac{\rho\omega^2}{c_{66}} \tag{3.2}$$

3.1. Solution of the problem when elastic constants do not obey the square law $(n\neq 2)$

The dependent variable f varies according to the following relation

$$f=Ur^{-\frac{n+\beta}{2}} \tag{3.3}$$

and the independent variable r can be written as follows

$$\lambda r^{\frac{2-n}{2}}=\frac{2-n}{2}z \tag{3.4}$$

finally we obtain

$$\frac{d^2U}{dz^2}+\frac{1}{z}\frac{dU}{dz}+\Big[1-\frac{\mu^2}{z^2}\Big]U=0 \tag{3.5}$$

where

$$\mu=\frac{n+\beta+2}{2-n} \tag{3.6}$$

Solution of Eq (3.5) is

$$U=AJ_\mu(z)+BY_\mu(z) \tag{3.7}$$

where

A, B	–	constants
$J_\mu(z)$, $Y_\mu(z)$	–	Bessel functions of order μ and of the first and second kind, respectively.

Substituting Eqs (3.3), (3.4) and (3.7) into Eq (2.5) we get

$$u_\theta=r^{-\frac{n+\beta}{2}}\Big[AJ_\mu(kr^s)+BY_\mu(kr^s)\Big]e^{i\omega t} \tag{3.8}$$

and

$$\begin{aligned}\sigma_{r\theta} &= c_{66} r^{\frac{n-\beta-2}{2}}\Big\{A\Big[ksr^s J'_\mu(kr^s) - \frac{n+\beta+2}{2}J_\mu(kr^s)\Big] + \\ &+ B\Big[ksr^s Y'_\mu(kr^s) - \frac{n+\beta+2}{2}Y_\mu(kr^s)\Big]\Big\}e^{i\omega t}\end{aligned} \tag{3.9}$$

where

$$s = \frac{2-n}{2} \qquad\qquad k = \frac{\lambda}{s} \tag{3.10}$$

The following recurrence relations (cf Watson, 1966)

$$J'_\mu(x) = J_{\mu-1}(x) - \frac{\mu}{x}J_\mu(x) \qquad\qquad Y'_\mu(x) = Y_{\mu-1}(x) - \frac{\mu}{x}Y_\mu(x) \tag{3.11}$$

transfer Eq (3.9) into

$$\sigma_{r\theta} = c_{66} r^{\frac{n-\beta-2}{2}}[AF_1(r) + BF_2(r)]e^{i\omega t} \tag{3.12}$$

where

$$\begin{aligned}F_1(r) &= skr^s J_{\mu-1}(kr^s) - (n+\beta+2)J_\mu(kr^s) \\ F_2(r) &= skr^s Y_{\mu-1}(kr^s) - (n+\beta+2)Y_\mu(kr^s)\end{aligned} \tag{3.13}$$

Introducing the boundary conditions

$$\sigma_{r\theta} = 0 \qquad \text{on} \qquad r = a \quad \text{and} \quad r = b \tag{3.14}$$

into Eq (3.12) and eliminating A and B, we get the frequency equation as

$$\frac{F_1(a)}{F_2(a)} = \frac{F_1(b)}{F_2(b)} \tag{3.15}$$

Then with the help of recurrence relations

$$\begin{aligned}J_{\mu-1}(x) &= \frac{2\mu}{x}J_\mu(x) - J_{\mu+1}(x) \\ Y_{\mu-1}(x) &= \frac{2\mu}{x}Y_\mu(x) - Y_{\mu+1}(x)\end{aligned} \tag{3.16}$$

the frequency equation (3.15) reduces to

$$\frac{J_{\mu+1}(p)}{Y_{\mu+1}(p)} = \frac{J_{\mu+1}(\eta p)}{Y_{\mu+1}(\eta p)} \tag{3.17}$$

where $p = ka^{\frac{2-n}{2}}$ and $\eta p = kb^{\frac{2-n}{2}}$ so that $\eta = (b/a)^{\frac{2-n}{2}} > 1$.

3.2. Particular cases

3.2.1. The aeolotropic non-homogeneous disk of constant thickness

Putting $\beta = 0$, from Eqs (3.6) and (3.13) we get

$$\mu = \frac{2+n}{2-n} = \nu$$

$$F_1(r) = skr^s J_{\nu-1}(kr^s) - (n+2)J_\nu(kr^s) = f_1(r)$$

$$F_2(r) = skr^s Y_{\nu-1}(kr^s) - (n+2)Y_\nu(kr^s) = f_2(r)$$

and the frequency equation can be written as follows

$$\frac{f_1(a)}{f_2(a)} = \frac{f_1(b)}{f_2(b)} \tag{3.18}$$

which was obtained by Mollah (1975)

3.2.2. The aeolotropic homogeneous disk of constant thickness

Putting $n = \beta = 0$, from Eqs (3.6) and (3.10) we get $\mu = 1$ and $s = 1$. The frequency equation is

$$\frac{J_2(ka)}{Y_2(ka)} = \frac{J_2(kb)}{Y_2(kb)} \tag{3.19}$$

which was obtained by Chatterjee (1967).

3.3. Solution of the problem for a thin ring

The frequency equation can be rewritten as follows

$$F(a) = F(b) \tag{3.20}$$

where

$$F(x) = \frac{ksx^s J'_\mu(kx^s) - \frac{n+\beta+2}{2} J_\mu(kx^s)}{ksx^s Y'_\mu(kx^s) - \frac{n+\beta+2}{2} Y_\mu(kx^s)} \tag{3.21}$$

Substituting for $b = a + \delta a$ into Eq (3.20) and then approaching the limit $\delta a \to 0$, we get the frequency equation of a thin ring in the following form

$$\frac{dF(a)}{da} = 0 \tag{3.22}$$

Using the formula $J_\mu(x)Y'_\mu(x) - Y_\mu(x)J'_\mu(x) = \frac{2}{\pi x}$ from Eqs (3.21) and (3.22) we get

$$k^2 = 0 \qquad \text{i.e.} \qquad \lambda^2 = 0 \tag{3.23}$$

Thus the rotatory vibration of a thin ring is not possible.

3.4. Solution of the problem when the elastic constants obey the square law $(n = 2)$

Putting $n = 2$ into Eq (3.1) we get

$$r^2\frac{d^2 f}{dr^2} + (3+\beta)r\frac{df}{dr} + (\lambda^2 - 3 - \beta)f = 0 \tag{3.24}$$

Solution of this ordinary differential equation is

$$f = A_1 r^{p_1} + B_1 r^{q_1} \tag{3.25}$$

where

$$p_1\,,q_1 = \frac{-(2+\beta) \pm \sqrt{(2+\beta)^2 - 4(\lambda^2 - 3 - \beta)}}{2} \tag{3.26}$$

and A_1 and B_1 are constants.

Thus

$$u_\theta = [A_1 r^{p_1} + B_1 r^{q_1}]\mathrm{e}^{\mathrm{i}\omega t} \tag{3.27}$$

and

$$\sigma_{r\theta} = c_{66}r^n[A_1(p_1 - 1)r^{p_1-1} + B_1(q_1 - 1)r^{q_1-1}]\mathrm{e}^{\mathrm{i}\omega t} \tag{3.28}$$

Introducing the boundary conditions $\sigma_{r\theta} = 0$ when $r = a$ and $r = b$, we get the frequency equation $p_1 = 1$ or $q_1 = 1$ and then

$$\lambda = 0 \qquad \text{i.e.} \qquad \omega = 0 \tag{3.29}$$

Thus when $n = 2$, the rotatory vibration is not possible.

4. Solution of the problem when also density obeys the power law

It is assumed in this case that as well as Eqs (2.4) and (2.6) the additional formula for the material density is employed

$$\rho = \rho_0 r^n \qquad n \in C \tag{4.1}$$

where ρ_0 is constant, being the value of ρ for homogeneous material when $n = 0$.

Substituting Eqs (2.5), (2.6) and (4.1) into Eq (2.7) we get

$$\frac{d^2 f}{dr^2} + \frac{n+\beta+1}{r}\frac{df}{dr} + \Big(\frac{\rho_0\omega^2}{c_{66}} - \frac{n+\beta+1}{r^2}\Big) f = 0 \tag{4.2}$$

Substitution for

$$f = ur^{-\frac{n+\beta}{2}} \tag{4.3}$$

where u is a function of r only, reduces Eq (4.2) to

$$\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \Big(k_0^2 - \frac{\nu^2}{r^2}\Big) u = 0 \tag{4.4}$$

where

$$k_o^2 = \frac{\rho_0\omega^2}{c_{66}} \qquad\qquad \nu = \frac{n+\beta+2}{2} \tag{4.5}$$

Solution of Eq (4.4) reads

$$u = A_2 J_\nu(k_0 r) + B_2 Y_\nu(k_0 r) \tag{4.6}$$

Thus

$$u_\theta = r^{-\frac{n+\beta}{2}}[A_2 J_\nu(k_0 r) + B_2 Y_\nu(k_0 r)]\mathrm{e}^{\mathrm{i}\omega t} \tag{4.7}$$

Employing Eqs (4.3), (3.11) and (2.5) we get

$$\sigma_{r\theta} = c_{66} r^{-\frac{n+\beta}{2}}[A_2 \varPhi_1(r) + B_2 \varPhi_2(r)]\mathrm{e}^{\mathrm{i}\omega t} \tag{4.8}$$

where

$$\begin{aligned} \varPhi_1(r) &= k_0 r J_{\nu-1}(k_0 r) - (n+\beta+2) J_\nu(k_0 r) \\ \varPhi_2(r) &= k_0 r Y_{\nu-1}(k_0 r) - (n+\beta+2) Y_\nu(k_0 r) \end{aligned} \tag{4.9}$$

Introducing the boundary conditions (3.14) and using Eq (3.16) one obtains the frequency equation as follows

$$\frac{J_{\nu+1}(p_0)}{Y_{\nu+1}(p_0)} = \frac{J_{\nu+1}(\eta_0 p_0)}{Y_{\nu+1}(\eta_0 p_0)} \tag{4.10}$$

where $p_0 = k_0 a$ and $\eta_0 p_0 = k_0 b$ so $\eta_0 = \frac{b}{a} > 1$.

5. Numerical results and discussion

It is well known (cf Gray and Mathews, 1985) that the qth root $p^{(q)}$ of the equation

$$\frac{J_\mu(p)}{Y_\mu(p)} = \frac{J_\mu(\eta p)}{Y_\mu(\eta p)} \qquad \eta > 1 \tag{5.1}$$

is

$$p^{(q)} = \delta_1 + \frac{\alpha}{\delta_1} + \frac{\beta - \alpha^2}{\delta_1^3} + \frac{\nu - 4\alpha\beta + 2\alpha^3}{\delta_1^5} + \dots \tag{5.2}$$

where

$$\delta_1 = \frac{q\pi}{\eta - 1} \qquad \alpha = \frac{4\mu^2 - 1}{8\eta}$$

$$\beta = \frac{4(4\mu^2 - 1)(4\mu^2 - 25)(\eta^3 - 1)}{3(8\eta)^3(\eta - 1)}$$

$$\gamma = \frac{32(4\mu^2 - 1)(16\mu^4 - 456\mu^2 + 1073)(\eta^5 - 1)}{5(8\eta)^5(\eta - 1)}$$

To calculate the qth natural frequency $\omega^{(q)}$ we assume the disk to be made of zinc, for which the elastic constants together with the density of the boundary layer $(r = a)$ are given by Hearmon (1961) for the homogeneous case

$$c'_{11} = c_{11} = 14.30 \times 10^{11} \text{ dynes/cm}^2$$

$$c'_{12} = c_{12} = 1.70 \times 10^{11} \text{ dynes/cm}^2$$

$$\rho = \rho_0 = 7.1 \text{ gm/cm}^3$$

Tables 1 and 2, respectively, present the qth natural frequencies $\omega^{(q)}$ $(q = 1, 2, 3)$ calculated from the frequency equations (3.17) and (4.10) for different values of δ $(\delta = b/a)$. Different types of thickness and different types of the material non-homogeneity have been considered. In calculations we put $a = 2$ for each case.

Table 1. $\omega^{(q)}$, the qth frequencies calculated from Eq (3.17)

Curve	n	β		$\delta = b/a$					
				1.5	2.0	2.5	3.0	3.5	4.0
F_1	0	0	$\omega^{(1)}$	3.04	1.60	1.12	0.86	0.70	0.54
			$\omega^{(2)}$	5.96	3.02	2.05	1.56	1.27	1.07
			$\omega^{(3)}$	8.91	4.48	3.01	2.28	1.83	1.54
F_2	1	0	$\omega^{(1)}$	4.80	2.75	2.07	1.73	1.52	1.36
			$\omega^{(2)}$	9.38	5.17	3.74	3.02	2.58	2.29
			$\omega^{(3)}$	14.01	7.66	5.50	4.40	3.73	3.27
F_3	1	1	$\omega^{(1)}$	4.89	2.88	2.22	1.88	1.67	1.53
			$\omega^{(2)}$	9.43	9.24	3.83	3.12	2.68	2.39
			$\omega^{(3)}$	14.04	7.70	5.56	4.47	3.80	3.35
F_4	0	-1	$\omega^{(1)}$	3.10	1.67	1.18	0.88	0.60	0.24
			$\omega^{(2)}$	5.99	3.07	2.10	1.61	1.31	1.11
			$\omega^{(3)}$	8.93	4.51	3.04	2.31	1.87	1.58
F_5	1	-1	$\omega^{(1)}$	4.74	2.65	1.96	1.61	1.38	1.24
			$\omega^{(2)}$	9.35	5.12	3.68	2.95	2.50	2.20
			$\omega^{(3)}$	13.99	7.62	5.45	4.35	3.67	3.21
F_6	1	-2	$\omega^{(1)}$	4.69	2.58	1.87	1.51	1.29	1.13
			$\omega^{(2)}$	9.32	5.08	3.64	2.90	2.45	2.14
			$\omega^{(3)}$	13.98	7.60	5.42	4.32	3.63	3.17
F_7	0	-1	$\omega^{(1)}$	3.00	1.54	1.06	0.82	0.69	0.61
			$\omega^{(2)}$	5.94	2.99	2.02	1.52	1.23	1.04
			$\omega^{(3)}$	8.89	4.46	2.99	2.25	1.81	1.51
F_8	-1	-1	$\omega^{(1)}$	1.90	0.89	0.60	0.58	0.88	1.71
			$\omega^{(2)}$	3.76	1.73	1.08	0.77	0.60	0.50
			$\omega^{(3)}$	5.63	2.59	1.61	1.14	0.86	0.69
F_9	0	-2	$\omega^{(1)}$	2.98	1.51	1.02	0.77	0.64	0.56
			$\omega^{(2)}$	5.92	2.97	1.99	1.50	1.20	1.01
			$\omega^{(3)}$	8.88	4.45	2.97	2.23	1.79	1.49

Table 2. $\omega^{(q)}$, the qth frequencies calculated from Eq (4.10)

Curve			$\delta = b/a$					
			1.5	2.0	2.5	3.0	3.5	4.0
L_1	$n = 0$	$\omega^{(1)}$	3.05	1.60	1.12	0.87	0.70	0.54
	$\beta = 0$	$\omega^{(2)}$	5.96	3.02	2.05	1.56	1.27	1.07
	$n + \beta = 0$	$\omega^{(3)}$	8.91	4.48	3.01	2.28	1.84	1.54
L_2	$n + \beta = 1$	$\omega^{(1)}$	3.10	1.67	1.18	0.89	0.60	0.24
		$\omega^{(2)}$	5.99	3.07	2.10	1.61	1.32	1.11
		$\omega^{(3)}$	8.93	4.51	3.04	2.31	1.87	1.58
L_3	$n + \beta = -1$	$\omega^{(1)}$	3.00	1.54	1.06	0.82	0.69	0.61
		$\omega^{(2)}$	5.94	2.99	2.02	1.53	1.23	1.04
		$\omega^{(3)}$	8.89	4.46	2.99	2.25	1.81	1.52
L_4	$n + \beta = -2$	$\omega^{(1)}$	2.98	1.51	1.02	0.77	0.64	0.56
		$\omega^{(2)}$	5.92	2.97	1.99	1.51	1.20	1.01
		$\omega^{(3)}$	8.88	4.44	2.97	2.23	1.79	1.49

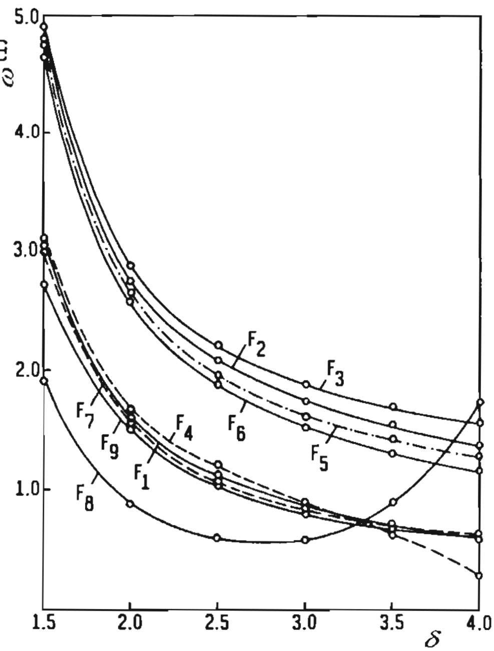

Fig. 1. Curves showing variation of the first frequencies $\omega^{(1)}$ (from Table 1)

In Fig.1 and Fig.2 variation of the first natural frequencies $\omega^{(1)}$ from Tables 1 and 2, respectively, are shown graphically versus δ, for different values of n and β.

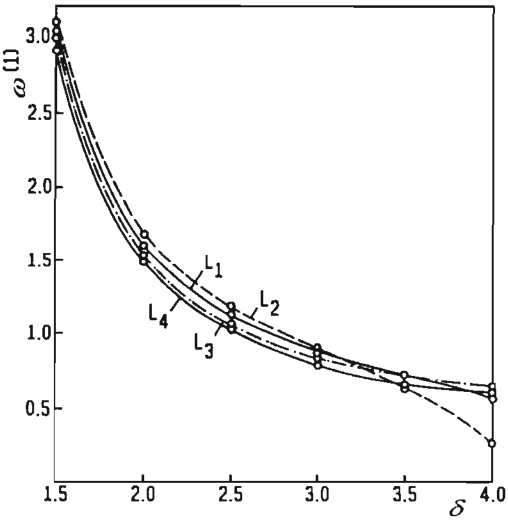

Fig. 2. Curves showing variation of the first frequencies $\omega^{(1)}$ (from Table 2)

It can be seen from Fig.1 (see the curves F_1, F_4, F_7, F_9 and F_2, F_3, F_5, F_6, respectively) that the first natural frequencies for homogeneous cases $(n = 0)$ are always of lower values than those calculated for non-homogeneous cases. Except the curve F_8 $(n = \beta = -1)$, for which one finds that the first natural frequency decreases as δ increases approaching approximately the value of 2.75 and then rapidly increases with the increase of δ. All the curves shown in Fig.1 for the non-homogeneous case (F_2, F_3, F_5 and F_6) display a concave character decreasing slightly and tending to a straight line as δ increases. For the homogeneous case (F_7 and F_9) the aforementioned conclusion holds also true when $\beta < 0$, it can be seen that the first natural frequency becomes constant as δ increases.

It is interesting to note that for the homogeneous case $(n = 0)$ the curves F_1 and F_4 initially display the concave character and then become convex as δ increases, for $\beta \geq 0$. The curves L_1 and L_2 in Fig.2 are of the same nature as the curves F_1 and F_4 in Fig.1.

Fig.3 $\div$ Fig.6 show diagrams of the second and the third frequencies $\omega^{(2)}$ and $\omega^{(3)}$, respectively, from Table 1 and Table 2. The second frequency courses (Fig.3 and Fig.4) are of similar nature to the third frequency courses

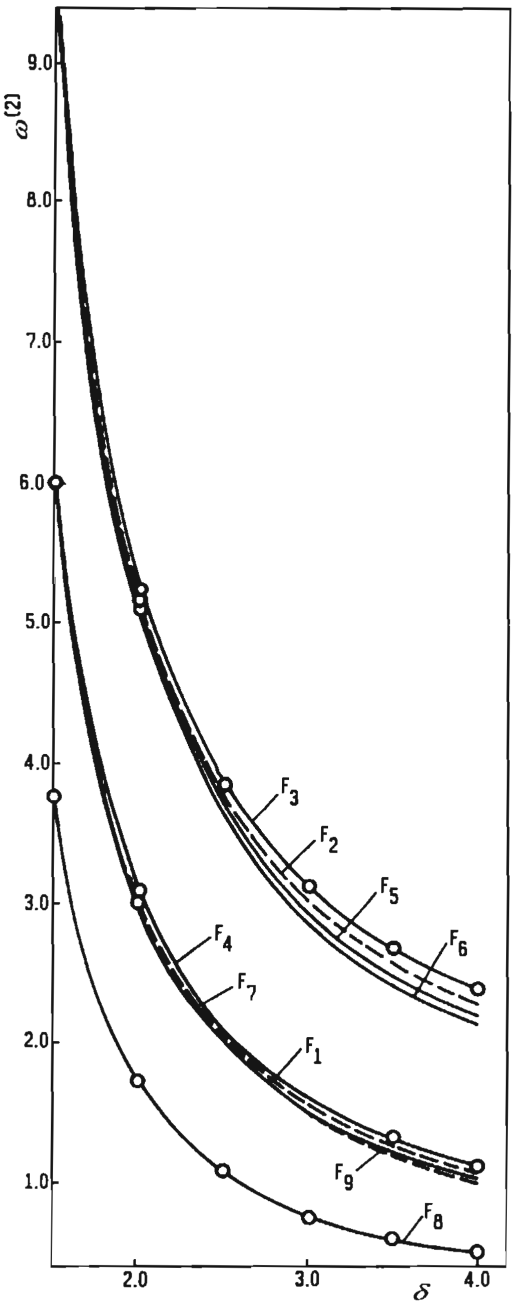

Fig. 3. Curves showing variation of the second frequencies $\omega^{(2)}$ (from Table 1)

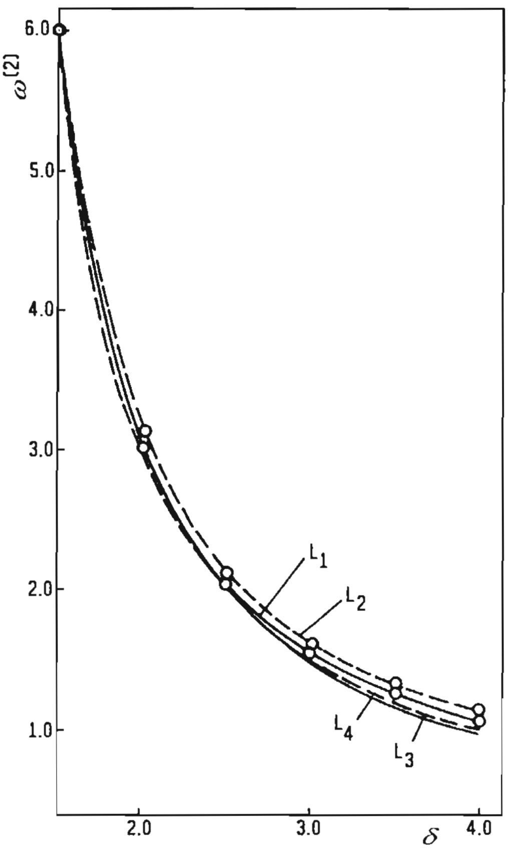

Fig. 4. Curves showing variation of the second frequencies $\omega^{(2)}$ (from Table 2)

(Fig.5 and Fig.6) the corresponding cases. The curves F_8 shown in Fig.3 and Fig.5, respectively, tend to a straight line as δ increases.

General conclusions formulated for the first natural frequency courses hold true for the second and the third natural frequency courses, respectively, except the F_8 course (Fig.3 and Fig.5), natures of which disagree with the F_8 curve shape from Fig.1.

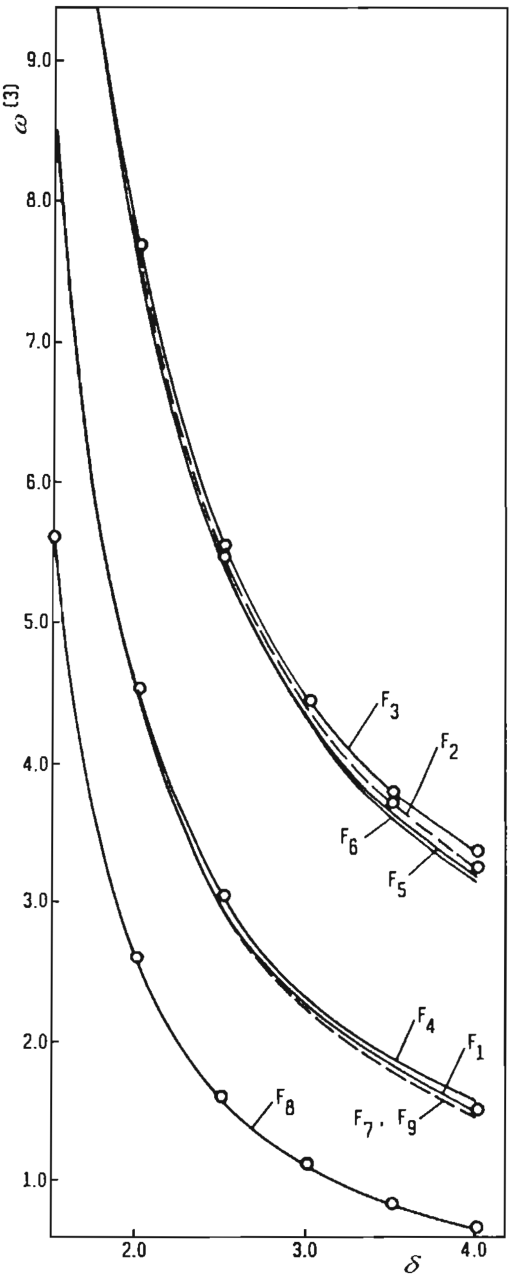

Fig. 5. Curves showing variation of the third frequencies $\omega^{(3)}$ (from Table 1)

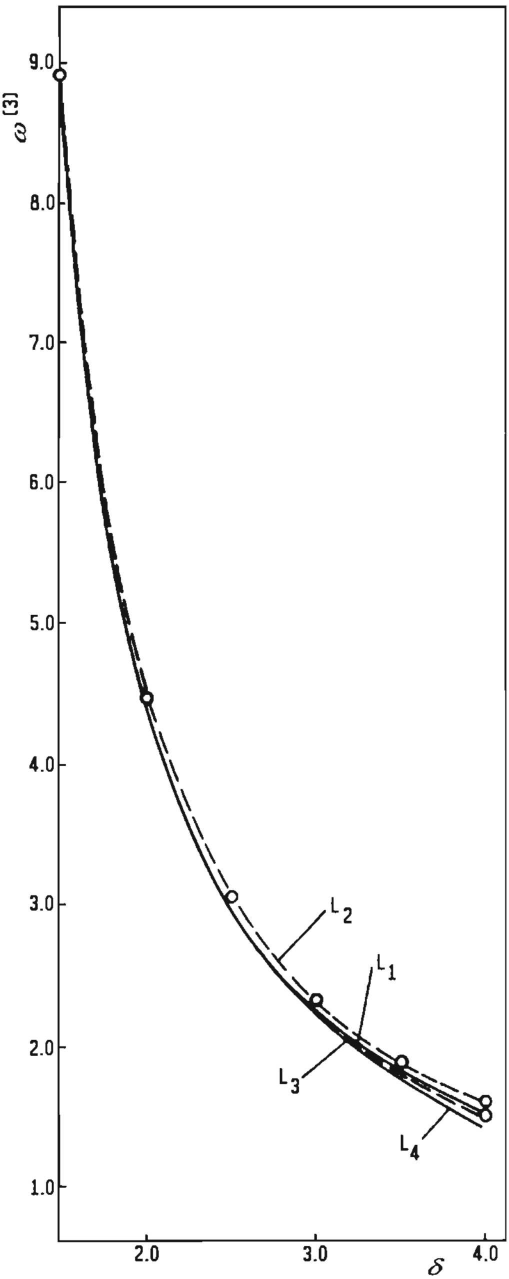

Fig. 6. Curves showing variation of the third frequencies $\omega^{(3)}$ (from Table 2)

6. Conclusions

- For the non-homogeneous material $(n = 1)$ the values of natural frequencies in the case when the disk thickness increases $(\beta > 0)$ with the radius are higher than those calculated in the case when the disk thickness decreases $(\beta < 0)$. The values of natural frequencies in the case of constant thickness disk $(\beta = 0)$ are always lower than those calculated for $\beta > 0$ and higher than those calculated for $\beta < 0$.
- The natural frequencies due to the rotatory vibration of a non-homogeneous disk are always of higher values than those calculated for the homogeneous case due to the disk thickness variation.

Acknowledgement

The Author is most grateful to the referee for his valuable suggestions and comments on improving the paper.

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Drgania rotacyjne aeolotropowego niejednorodnego pierścienia kołowego o zmiennej grubości

Streszczenie

Celem pracy jest wyznaczenie naprężeń, przemieszczeń i częstości drgań rotacyjnych aeolotropowego niejednorodnego pierścienia kołowego zmiennej grubości. Przyjęto, że stałe sprężyste, gęstość materiału i grubość pierścienia są funkcjami położenia. Rozpatrzono różne szczególne przypadki a wyniki w postaci numerycznej i graficznej porównano z wcześniej otrzymanymi rezultatami.

Manuscript received June 4, 1993; accepted for print November 5, 1993