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ON SOME NEW ASPECTS OF CONTACT DYNAMICS WITH APPLICATION IN RAILWAY ENGINEERING[1]

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The paper is devoted to the study of several dynamical phenomena of contact problems, where the wheel/rail motion is connected with an oscillating load caused by corrugation, poligonalisation and other irregularities of rails or wheels. It will be shown that even if the wheel center moves with constant speed, the load applied by contact forces moves with variable speed, thus crossing critical resonance boundaries. Some problems studied by the authors in previous works are revisited.

Key words: dynamics, traveling load, rail-wheel contact, corrugation

1. Introduction

The development of the modern technology of tracked high-speed transportation systems becomes more and more important. This entrails a strong need for simplified but reliable models of contact and models of continuous or hybrid systems in order to study various dynamical effects of the wheel/rail interaction. There is still no general model valid in the whole range of velocities. The model we are proposing here covers the gap in the medium and high speed range at the presence of corrugations. The contact loading influences the durability of infrastructure, the damage to the environment and the comfort of transportation. The beginning of research in this direction was connected

[1]The paper is dedicated to Prof. Dr. Dr. h.c. mult. Oskar Mahrenholtz on the occasion of his 80th birthday.

with the study of moving loads acting on beams. Those were supported by a Winkler foundation (Winkler, 1867) and subjected to a single concentrated force moving at constant speed. The problem was mentioned by Timoshenko (1926). The first stationary solution in the case of a Bernoulli-Euler beam on an elastic foundation was obtained in a proper way by Ludwig (1938), but only for the sub-critical speed range. The case of a moving and oscillating force was formulated, and partly solved, by Mathews (1958). The first fully correct solution of Mathews' problem was given by Bogacz and Krzyżyński (1986). There are various extensions of the classic problem towards more complicated but also more realistic models of contact loads. A great deal of new effects were discovered by Bogacz *et al.* (1998) and Bogacz and Kowalska (2001), who studied problems of oscillating loads, moving along some periodic, i.e. variable in space, structures. The dynamical effects for two-dimensional problems with moving loads have important practical engineering applications. Some problems connected with a system of plates subjected to traveling loads can be found in a recent paper by Bogacz and Frischmuth (2008). The aim of the present paper is the systematization of previous results as well as the generalization towards a discussion of some new effects connected with a moving load along a corrugated rail or rolling surface of the wheel, which lead to oscillation of the speed of the travelling load.

2. Contact problems

Some investigations were devoted to the analysis of the development of corrugations using linear and nonlinear wheel/rail interaction models. A rigid and an elastic contact model were discussed by Frischmuth (2001), elastic and perfect plastic models were studied by Frischmuth and Langemann (2002), and a discussion of frictional wear as the source of surface pattern development in rolling contact is given by Bogacz *et al.* (2005). Similarly as in the approach made by Kowalska (2008), we will study the two-dimensional problem with jumps of wheel/rail contact and inelastic impacts.

Typical corrugations, also called slip-waves (Fig. 1), have a much smaller wavelength, and are believed to be related to percussional effects leading to permanent (plastic) deformations of the surface layer. Analysis of the contact geometry of rigid bodies with harmonic sinusoidal corrugations shows that already at comparatively low speeds two effects may be observed. First, the point of geometrical contact moves forth and back around the actual position of the wheel. Thus we have to deal with a force traveling at variable speed.

Due to large amplitudes of the speed, critical regions of speeds are crossed (see Bogacz and Krzyżyński, 1986; Bogacz *et al.*, 1998). Second, also the value of the normal force oscillates considerably around its mean value, and at a certain velocity the contact is frequently lost and reestablished. At the same time, we observe the onset of a process of rapid plastic deformation due to the increase of contact forces (Bogacz and Kowalska, 2008).

Fig. 1. View of a track segment with the corrugated rail (slip waves)

As an academic example, one can assume a sinusoidal shape of the rail surface and a constant wheel radius. However, the observed wavelengths on worn rails are quite different.

In general, we assume sufficient regularity of the geometry, to ensure the solvability of dynamic equations of motion. As it turns out, however, smoothness is not sufficient. We need also bounds on the departure from the ideal forms, straight respectively circular. We assume the domains occupied by the wheel and rail to intersect in a nonempty set of zero area. The wheel and the rail touch, but do not yet penetrate. In the classic case, the intersection consists of just one point, which is called the *point of contact*. In this case, there exists also a unique angle ϕ, such that the value parametrizes the point of contact on the wheel circumference. It has to be added that during motion the contact may be lost, in which case neither position x_g nor ϕ_g may be defined (Fig. 3). Another non-classic case occurs for heavily corrugated surfaces, e.g. when the wheel convexity is lost. Then several different or even a continuum of points may be in contact. The same is true for models with compliant contact partners. For a rigid contact model, we introduce a unilateral constraint. In this case, for relatively great amplitudes of corrugation, a two point contact is possible (Fig. 2).

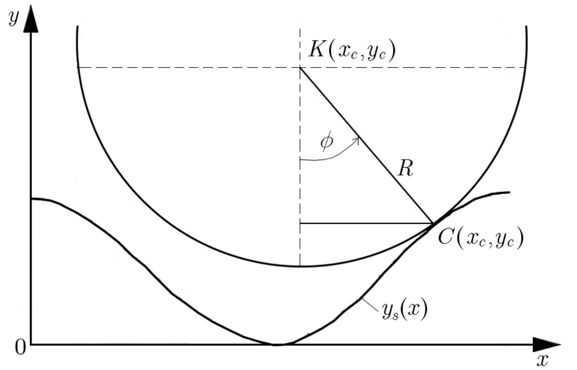

Fig. 2. The case of corrugation curvature greater than the wheel curvature

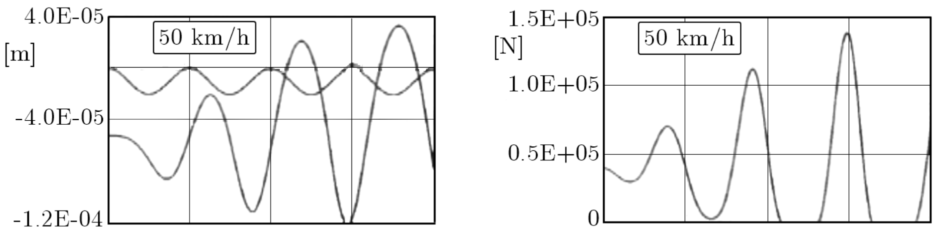

Fig. 3. Profile of the corrugated rail and trajectory of wheel center (left) and the corresponding contact force (right) for the case of amplitude of corrugation $10\,\mu$m, and wave length $\lambda = 50$ mm at speed 50 km/h (Bogacz and Kowalska, 2001)

The case when during rolling motion of a wheel the contact is lost is possible for a corrugation amplitude equal to $10\,\mu$m and a wave length $\lambda = 50$ mm at the speed $V = 50$ km/h. Such a situation is visible on the right-hand side of Fig. 3, where the contact force is equal to zero. It ought to be noticed that the amplitude $10\,\mu$m is relatively small, which is why the contact is lost only in the resonance case. For higher speeds the wheel is rolling without losing the contact. As for the speed at which the lift-off occurs, and where hence the contact is reestablished by an impact, there are several ways of determination. The normal force which keeps the wheel at the actual height of the rail enters the equations of motion in the rigid model via a Lagrange multiplier. As such, it is not determined by the wheel position coordinates x, y. As an alternative, we may allow the domains of the non-deformed – but rigidly moved – bodies to penetrate. We stress that, of course, the real bodies to not penetrate – they avoid this by deforming, mainly in the contact area. The correct solution to the elastic contact problem, i.e. the form of deformed bodies, is hard to calculate. The integral equation formulation of this problem turns out to be illposed. For quick simulations, we use a unilateral nonlinear (Hertzian) spring. This approach gives a normal contact force depending on the elastic approach of the bodies. We have to mention that this characteristic depends on the material, but also on the curvatures of surfaces. Hence, it may vary during motion. Furthermore, the direction of the contact force is perpendicular to the surface at the contact point, which is not exactly in the vertical direction. The modifications necessary to take this into account are straightforward, we leave these technicalities out for the sake of readability of the paper. More crucial is another enhancement of the simple Hertzian contact spring, which is a damping component. We assume the same form of the dependence, just a different coefficient connecting the damping force to the speed of penetration. What remains is the relative velocity in the contact point, which in turn results in a frictional tangential force. By Coulomb's law, this force is directed opposite to the velocity of the tangential motion. The details can be found by Frischmuth (2002). The computer simulation of slip-wave generation was discussed by Bogacz and Frischmuth (2001), see also Fig. 1.

In the majority of studies devoted to dynamically applied loads in civil and railway engineering, a simplification was used that a dynamically applied load can be approximated by addition of 15-30% of the static load value. This amounts to a big mistake even in the case that only the vertical dynamics is taken into account. The results shown on the right side of Fig. 3 confirm this statement. Another simplification is connected with the assumption that the speed of contact loading is constant. This aspect will be discussed in the next part of the paper.

3. Real velocity of moving load

We start with a simple kinematic consideration assuming that the center of the wheel moves at a constant prescribed speed. Further, we assume that both bodies are rigid, and they are in contact all the time in a uniquely defined point of geometrical contact. Finally, at the point of contact there is no relative motion between the wheel and rail, which amounts to the condition that the velocity at the contact point on the wheel circumference is zero. The above conditions define the trajectory of the wheel center and the revolution of the wheel uniquely. Hence, we obtain the vertical acceleration and angular moment and thus the normal force as well as the force in the tangential direction. Under such idealized conditions of rolling contact, we may calculate the position and vertical force in the contact point without solving the equations of motion. We derive from this a new type of the traveling force problem approximated by the formula

$$F_y(t;x) = (F_0 + F_1\sin(\omega_1 t))\delta[x - V_0 t + \varepsilon\sin(\omega_2 t)] \tag{3.1}$$

where F_0 is the value of constant force contribution, F_1 is the amplitude of the oscillating force with frequency ω_1, V_0 is the average speed of motion which oscillates with amplitude ε and with frequency $\omega_2 = k_0 V$ dependent on the wavelength of the corrugation.

It is new here that the force position oscillates around its mean position which moves steadily forward. The critical speeds and frequencies have been derived for the case $\varepsilon = 0$. The oscillating position may exceed the critical boundaries of speed-frequency regions, cf. Bogacz and Krzyżyński (1986), but only on small intervals. The effects of this need further considerations.

Let us assume now an ideal wheel and a rail with a wavy surface. We will try to create a parametrization of the wheel center locus in terms of the horizontal coordinate of the contact point as the parameter. We have to perform vector addition of the contact point $C(x_c; y_c)$ on the rail with the horizontal coordinate s and the normal vector multiplied by the wheel radius R_0. The resulting curve may be non-smooth or even contain loops. True rolling – with continuous one-point contact – is only possible as long as the curve is free of double-points. In Bogacz and Frischmuth (2009) we derived an inequality connecting wavelengths of corrugations and amplitudes on both contact partners, which ensure the applicability of the procedure.

For a realistic wheel radius and wave amplitude the effect is hardly visible in Fig. 4. Thus we show the same plot for a much smaller wheel of ten times reduced radius.

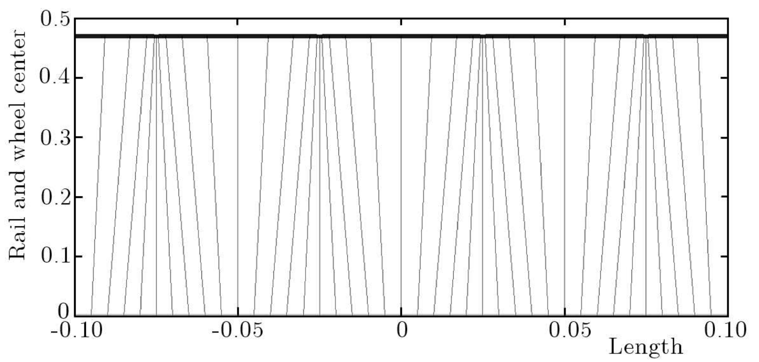

Fig. 4. Locus of the wheel center for an ideal wheel [m] versus rail length [m]

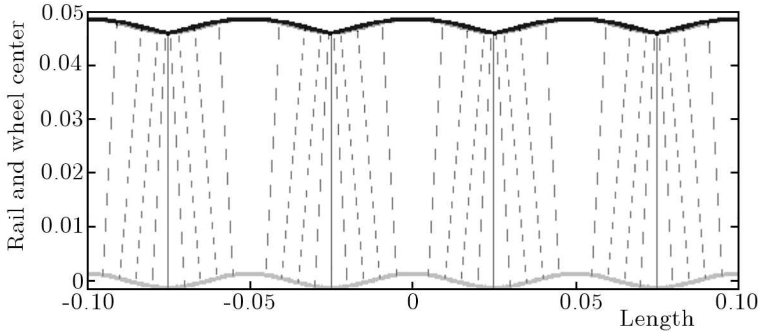

Fig. 5. Locus of the wheel center for a small ideal disk wheel

Figures 4, 5 and 6 show the consequences of the geometric contact condition. Ideal rolling at a constant horizontal speed leads to high variation in the rotational speed, constant revolutions require heavy variation of the horizontal speed. The vertical position of the wheel center versus time is shown in Fig. 6.

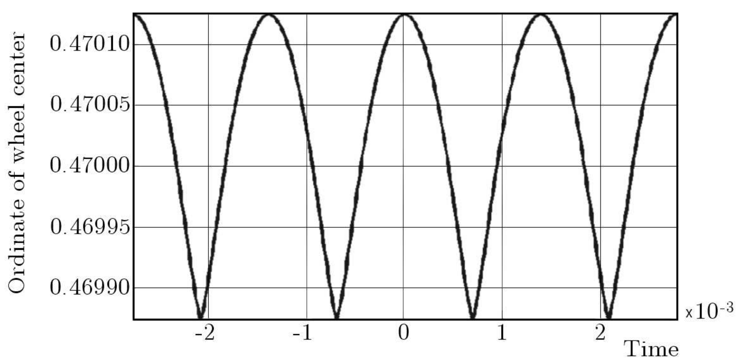

Fig. 6. Vertical position of the wheel center [m] versus time [s]

The horizontal position of the contact point versus the position of the wheel center is shown in Fig. 7.

For the case of a constant horizontal speed of moderate value, i.e. 36 m/s, extreme vertical or horizontal accelerations and very high velocities of contact point motion can be observed. It is interesting and important, particularly from the point of view of wave generation, that the horizontal speed of the moving contact point is so high, cf. Fig. 8. In Fig. 9, we can see the resonance behavior for the very simple case – Bernoulli-Euler beam model on an elastic foundation subjected to a harmonically oscillating load moving with a constant speed.

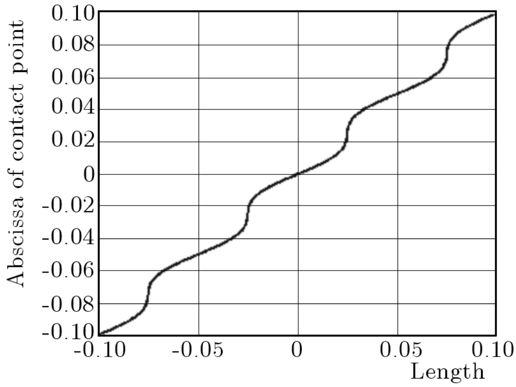

Fig. 7. Horizontal position of the contact point [m] versus the wheel center (steady forward motion)

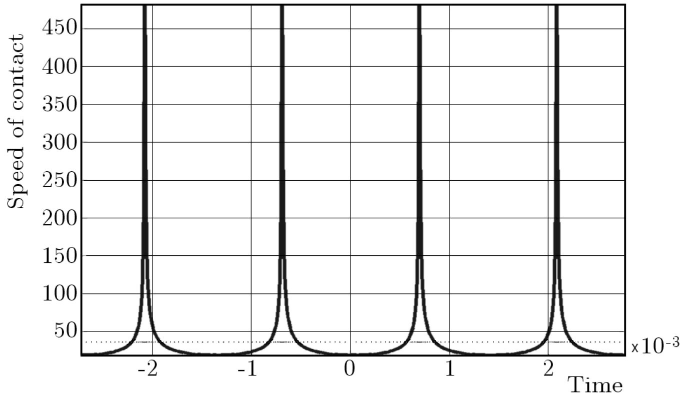

Fig. 8. Horizontal speed of the contact point [m/s] versus time [s]

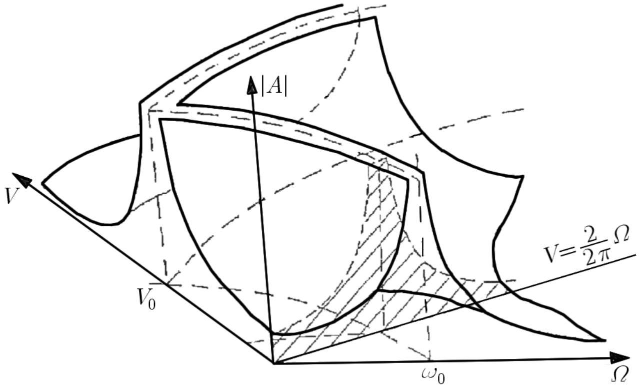

Fig. 9. Response of the Bernoulli-Euler beam subjected to a force moving with the speed V and oscillating with the frequency Ω

In the case of load described by expression (3.1), some resonance boundaries will be crossed. The response of a beam subjected to such a periodically moving load is not easy to determine, even numerically. The difficulties are connected with non-stationary motion due to varying speed. Some approximate solutions of this problem will be given in a forthcoming paper.

The investigated case was devoted to an ideal wheel interacting with the corrugated rail. But the case of a corrugated wheel or a wheel with poligonalisation is also studied, and the results are also very interesting. The study gives an explanation of some phenomena existing in real engineering practice. Part of the investigations can be found in other papers, e.g. in Bogacz and Frischmuth (2009).

4. Summary

In the paper, an overview of the questions connected with the problem of contact loading was presented. Some new results of the investigations were obtained for the case of wheel and rail corrugations.

The critical speed – from the point of view of the theory of traveling loads on an elastically supported beam – may be exceeded for very moderate speeds of the wheelset motion on a corrugated track. Above certain levels of expression of the corrugation patterns the lift-off may occur, which is followed by an impact. In general, there is also a considerable influence of the suspension and control motion on the resulting trajectories. This problem will be studied in next papers.

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O pewnych nowych aspektach dynamiki kontaktu z zastosowaniem w inżynierii kolejowej

Streszczenie

Praca została poświęcona przeglądowemu studium zjawisk dynamicznego kontaktu tocznego, w którym ruch układu koło-szyna jest związany z oscylującym obciążeniem spowodowanym korugacją, poligonalizacją i innymi nierównościami szyny lub koła kolejowego. Wykazano, że nawet w przypadku toczenia się koła z umiarkowaną i stałą prędkością obciążenie kontaktowe szyny jest zmienne i przekracza krytyczne granice rezonansów. Przytoczone zostały niektóre wyniki wcześniejszych badań.

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