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ENERGY TRANSFER IN TWO-DEGREE-OF-FREEDOM VIBRATING SYSTEMS – A SURVEY

DANUTA SADO

Warsaw University of Technology

The purpose of this paper is the survey and the analysis of modern literature concerning the problems energy transfer in nonlinear two-degree-of-freedom systems. Particular attention is given to the kinds of nonlinearities taken into account by different authors. It has been found that not all kinds of nonlinearities were widely examined. There are not many works taking into account simultaneously the nonlinearities in form of polynominals coupled with a nonlinearity resulting from existance of pendulum in the system. This type of systems has a wide technical application.

1. Introduction

The subject of this work is a phenomenon of energy transfer between modes of vibrations in connection with occurrence of other effects such as different kinds of coupling or a phenomenon of internal and external resonances.

During the investigation of vibrations of non-linear systems with many-degree-of-freedom the analysis of a kind of the coupling between particular degrees-of-freedom is very important. It is essential to find out how the coupling is being realized and how it influences on the motion of the whole system. The resultant motion is often being treated as a combination of the motions of a few separate partial systems of one degree-of-freedom. These partial systems can be coupled together. i.e. vibrations in one system influence on the vibrations in another system, and this phenomenon can be invertible or not.

They say, that in a system there is no coupling if it can move in such a way that each coordinate is totally independent of all the others.

The coupling between partial systems may be of a different character. Ziemba (1959), Osiński (1978) consider that the coupling is elastic (or by a generalized displacement), if between coordinates of the system there are relations of this kind, that during a statical deflection of one coordinate, the elastic forces which cause a statical deflection of the remaining coordinates, appear. For this type of coupling, in an expression for a potential energy, appear mixed products of the generalized coordinates.

Another kind of coupling is co-called inertial coupling (or a mass coupling). It appears when the inertial forces connected with the acceleration of one coordinate generate the accelerations of the remaining coordinates. In case of this type of coupling in expression for the kinetic energy appear mixed products of generalized velocities.

In a system without coupling there will be neither mixed products of generalized coordinates in expression for the potential energy, nor mixed products of generalized velocities in expression for the kinetic energy.

The examples of realization of different types of couplings for two-mass systems are shown in Fig.1.

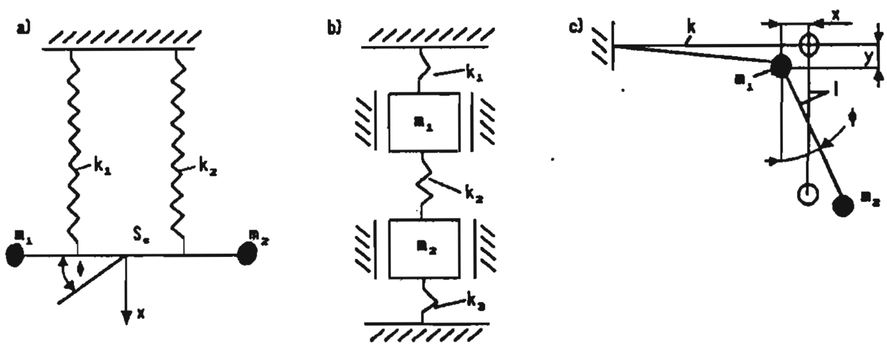

Fig. 1. a) – system without a coupling, b) – system with a spring coupling, c) – system with an inertial coupling

In Fig.1a the system without a coupling is shown. Because of the effect of the symmetry the vertical motion of the centre of mass (coordinate x) and the revolution relative to the centre of mass (coordinate φ) can be examined independently. In Fig.1b is shown the system coupled by the generalized displacement, that is being realized by the spring of stiffness k_3. And in Fig.1c there is shown the example of the inertial coupling. On the elastically suspended body of the mass m_1 that can oscillate in the vertical plane, a pendulum of the lenght l and the mass m_2 is suspended. If the body of the mass m_1 will be forced into the vibration, the inertial force of the components $m_1\ddot{x}$ and $m_1\ddot{y}$ will cause a change of the angle φ of the pendulum as a result of the action of the moments $m_1\ddot{x}y$ and $m_1\ddot{y}x$. Similarly, motion of the pendulum will cause the motion of the body of the mass m_1.

One ought to pay attention to the fact that the further acceptance of the nonlinear elastic characteristics can change the character of the coupling by non-linear elements in the way depending on the kind of taken nonlinearity.

Equations describing the motion of the coupled non-linear systems haven't usually got the exact solutions, and the approximate method application depends on the kind of nonlinearity.

For certain group of the systems of two coupled non-linear differential equations solving Dasarathy (1969) and (1970) as well as Srirangarajan and Dasarathy (1975) used for example the method of the equivalent linearization. By the suitable transformation of variables and time they obtained the equivalent linear system without a coupling. The authors presented the usefulness of this method, but they did not present the explicit results obtained after its application.

The notion of the kind of coupling is important as well as the notion of its intensity, because in the coupled systems can appear very interesting phenomena of the energy transfer from one degree-of-freedom to another. The character of the energy transfer can be different. In systems coupled elastically can appear the beating phenomenon, and in the inertial coupling systems the phenomenon of the autoparametric resonance.

The energy transfer can be partial or total depending on the choice of parameters.

It appears that the total transfer of energy occurs when the ratio of natural frequencies of the linearized system is near to the ratio of integers. In the literature this phenomenon is called "internal resonance".

If the damping appears in the system, then according both to the value of damping and of the kind of coupling, in the internal resonance range can occur the vibrations at one frequency or the vibrations of type of beating.

In the case of vibrations forced harmonically, if the frequency of excitation is near to one of the eigenfrequncies, can occur furthermore the phenomenon of external resonance. It is essential to estimate, how the vibration at the frequency force can be accepted as a predominant over the autoparametric oscillations. Such assumption is adopted in most works on the aforementioned problem.

An assumption, that besides of the harmonic component with the excitation frequency appear other harmonics due to the commensuration of the natural frequencies is valid, but an assumption of their explicit form is usually connected with some restrictions and can involve the incomplete results.

2. Kinds of nonlinearity and applied methods of investigations

2.1. Systems with nonlinearity of duffing-type

Many investigators examined the non-linear two-degree-of-freedom systems composed of two bodies, assuming, that they are connected by a flexible element of Duffing type and one of this bodies is attached also flexibly to the base. It was assumed usually, that the nonlinearity is cubic or quadratic. They investigated the free and the forced vibrations of conservative systems (Arnold, 1955; Rosenberg and Atkinson, 1959; Atkinson, 1961; Tondl, 1963; Szemplińska-Stupnicka, 1963; Půst, 1963) or of damping systems (Yamamoto and Yasuda, 1977; Yamamoto, Yasuda and Nagasaka, 1977; Sethna, 1960; Carter and Liu, 1961).

The analysis of harmonically forced vibrations of a system with cubic nonlinearity was done by Arnold (1955). The author using the Ritz method investigated the behaviour of the system near to the resonance for hard and soft elastic characteristics.

Also the cubic nonlinearity was assumed in works by Rosenberg and Atkinson (1959), Atkinson (1961). They examined the free and forced oscillations and payed attention, to the fact that in the equations appeared terms due to a coupling in a form as in the Mathieu equation. They defined the zones of stability. The results of the asymptotic method were compared with the results obtained from the method of analogic simulation.

The quadratic nonlinearity was assumed by Tondl (1963). He analysed the phenomenon of the internal resonance. After transformation of equations into the quasinormal form he solved them using the method of small parameter. Author obtained the internal resonance in the first approximation. In successive approximations he obtained ultraharmonic and subharmonic components. He noticed that the resonance curve can have more then one peak.

Szemplińska-Stupnicka (1963) also assumed the quadratic nonlinearity. She investigates the free and the harmonically forced vibrations. The author made an analysis of natural forms derived by the small parameter method. Considering free oscillations she made the assumption, that these oscillations differed not much from the sum of normal oscillations. She derived the solution in the form of power series. Investigating the oscillations forced by harmonic force she assumed the steady-state solution at frequency of the excitation.

Půst (1963) presented an analysis of the motion of two-degrees-of-freedom system with non-linear element under harmonic excitation. He defined the resonance curves, but the solution was limited to the first approximation.

Yamamoto and Yasuda (1977), Yamamoto, Yasuda and Nagasaka (1977) studied the oscillations forced by harmonic force systems with cubic and quadratic nonlinearity. These authors investigated the external resonance which occured near to the lower natural frequency (Yamamoto and Yasuda, 1977) and near to the higher natural frequency (Yamamoto, Yasuda and Nagasaka, 1977), respectively. They looked for periodic and almost periodic solutions, respectively. They proved that under frequency of the excitation near to the lower natural frequency existed only two frequency solutions, but when the frequency of the excitation was near to the higher natural frequency, two-frequence and one-frequence solutions could appear. In both cases they proved the existance of almost periodic oscillations.

Sethna (1960) investigated the system with an internal and external resonance generated by two harmonic forces with different amplitudes and the same frequencies. The results obtained using to approximate method were compared with the results obtained from the analog simulation.

Carter and Liu (1961) studied the system with cubic spring nonlinearity with regard to the viscotic damping. The authors studied only the first approximation of the asymptotic method and the amplitudes of the solution were defined using the graphical procedure.

The phenomenon of simple external and combined external resonances in Duffing systems with simultaneous occurrence of internal resonance is described in works by Szemplińska-Stupnicka (1969) and (1975), Bajkowski (1978), Szemplińska-Stupnicka and Bajkowski (1980), Bajkowski and Szemplińska-Stupnicka (1986) and (1987), Asfar, Nayfeh and Barrash (1987), Den Hartog (1962), Masri (1972), Shaw J., Shaw S.W. and Haddow (1989), Nissen, Popp and Schmalhorst (1985), Rice and McCraith (1986). Szemplińska-Stupnicka (1969) and (1975) applied the averaging Ritz method and results were compared with the results obtained using analog simulation method.

The investigation of conditions, under which the non-linear system realizes the steady-state harmonic, ultraharmonic or subharmonic response can be found in work by Bajkowski (1978). The method of Ritz averaging permited to find so-called "zones of attraction", which however, in author's opinion, were only a part of the zone assigned by the analog technic.

Also Szemplińska-Stupnicka and Bajkowski (1980) showed that under harmonic excitation the harmonic components of different kind appeared.

Bajkowski and Stupnicka-Szemplińska (1986) and (1987) showed, that the averaging method generally applied could carry on an essential divergence to the results of computer simulation. The authors suspect that these errors arise due to the simplifying assumption interrelated with this method and concerning the relationship between particular harmonic amplitudes. The authors advised to use the Ritz method, which permited to treat all amplitudes as unknown.

Other authors recommend to applay the method of multiple scales in investigation of the differential equations with modulation of the amplitudes and phases (Asfar, Nayfeh and Barrash, 1987; Shaw J., Shaw S.W. and Haddow, 1989; Ertas and Chew, 1990; Nayfeh, 1981, 1983a-d, 1989; Nayfeh and Mook, 1979; Nayfeh, Mook and Sridhar, 1974; Nayfeh and Asfar, 1986; Nayfeh and Zavodney, 1986; Zavodny and Nayfeh, 1988; Nayfeh, Balachandran, Colbert and Nayfeh M.A., 1989; Nayfeh and Balachandran, 1990; Tezak, Nayfeh and Mook, 1982).

Asfar, Nayfeh and Barrash (1987) used this method to investigate the vibration with harmonic excitation in the system in which the non-spring vibration eliminator of type "Lanchester" was used. The authors assumed, that the spring in main system has a cubic non-linearity. Applying the method of multiple scales they plotted the resonance curves for main resonance. The results from this analytic method were compared with the numerical results. The advantges of application of this type of dampers is presented in monograph Den Hartog (1962) and the example of its application can be find in the work by Masri (1972), where the study of motion in the case of the bounds one put on it is presented.

Furthermore Shaw J., Shaw S.W. and Haddow (1989) took into acount the non-linear elasticity of the additional system. The investigations are realized using the method of multiple scales. In this work are found zones of stability and of bifurcations, and there are presented maps of the steady-state solutions in the phase plane.

The cubic and quadratic nonlinearities were studied by Nissen, Popp and Schmalhorst (1985). They modeled the elastic system conformed to Bolleville washers.

In the disputable work Rice and McCraith (1986) presented a different design that enables the regulation of asymmetry degree.

The application of this type of elastic element is presented by Rice (1986), where using a harmonic balance method the authors studied the dynamic damper employing a non-linear Duffing-type coupling spring element. They presented a numerical approach with a result of perturbation procedure.

Similar investigations for cubic non-linearity were presented byRice and McCraith (1987). Also Golhoud, Masri and Anderson (1987) applied the asymptotic method to investigation of the system with cubic nonlinearity under the action of harmonic excitation. They presented the theoretical and the experimental results. The influence of various dimensionless parameters on the system response was determined.

The system revealing the cubic nonlinearity and under inertial excitation was investigated by Ertas and Chew (1990). They solved the equations using the multiple scale method for conditions of internal and external resonances, i.e. when the excitation frequency is near to the one of natural frequencies.

The multiple scale method is applied in many works by Nayfeh (1981), (1983) and (1989), Nayefh and Mook (1979). In monographs they advice its application to a wide scale of non-linear equations, especially where one should take into accaunt the modulation of amplitude and phase. Its different applications can be found in works Nayfeh (1983a-d) and (1989), Nayfeh and Mook (1979), Nayfeh, Mook and Sridhar (1974), Nayfeh and Asfar (1986), Nayfeh and Zavodney (1986), Zavodny and Nayfeh (1988), Nayfeh, Balachandran, Colbert and Nayfeh M.A. (1989), Nayfeh and Balachandran (1990), Tezak, Nayfeh and Mook (1982), Nayfeh and Jebril (1987), Streit, Bajaj and Krausgrill (1988). Nayfeh applies this method investigating the systems under parametrical excitation and the system with harmonic excitation when the system is coupled autoparametrically (Nayfeh, 1983a,d and 1989; Nayfeh and Mook, 1979; Nayfeh, Mook and Sridhar, 1974). Applying this method Nayfeh (Nayfeh and Asfar, 1986; Nayfeh and Zavodney, 1986; Zavodny and Nayfeh, 1988) investigates: simple parametrics and combined resonances, respectively. In Nayfeh, Balachandran, Colbert and Nayfeh M.A. (1989), Nayfeh and Balachandran (1990) also received periodic, almost periodic and chaotic responses. The author applied the multiple scale method also to the systems of equations containing more than one term of parametrical character with different amplitudes and phases (cf Tezak, Nayfeh and Mook, 1982; Nayfeh, 1983b,c; Nayfeh and Jebril, 1987).

Streit, Bajaj and Krousgrill (1988) investigated the effect of coupling also applying multiple scale method.

Very interesting method presented Sprysl (1987). He connected the multiple scale method with the perturbational method and obtained modified version of the Krylov-Bogolyubov-Mitropolsky method. Using this method the author investigated the free vibrations of system displaying quadratic nonlinearity.

Besides the cubic and quadratic elastic nonlinearity other authors assumed different kinds of nonlinearity. Pipes (1953), Wang and Pilkey (1975) assumed that the coupling absorber spring has nonlinear force-displacement characteristics of hyperbolic sine type. Pipes (1953) studied the effect of nonlinearity and presented grafical solution. Wang and Pilkey (1975) presented the method of linear programming used to determination of optimal parameters of investigated system. The authors examined the free oscillations for internal resonance and the forced oscillations furthermore for conditions of external resonance. The authors showed, that the system in the internal resonance range, put in oscillations with the excitation frequence near to one natural frequence, can force to resonance the other form of oscillations.

One should give attention to the fact, that there exist a range, when the coupling systems transfer all the energy from one-degree-of-freedom to the other and vice versa. We have to do with permanent growing or diminishing of vibrations amplitude. It vill be very important to examin the time of these changes in relation to the kind of nonlinearity. Therefore it is important to investigate the time of energy transfer from one-degree-of-freedom to the second for this type of vibration systems.

2.2. Systems of pendulum type

Important technical applications have the dynamical two degres-of-freedom systems with elements of the mathematical or physical pendulum type. In systems of this type usually as a result of inertial coupling occurs the excitation of autoparametric vibrations. This phenomenon was observed for the first time some years ago for mathematical pendulum suspended on a spring. Analysis of this system is presented in monographs by Nayefh, Mook and Sridhar (1974), Minorsky (1967), Evan-Iwanowski (1976).

Minorsky (1967) presented an approximate analysis (for small angles) of this system for the spring of linear characteristic. He showed that in some ranges of natural frequencies ratio the character of vibrations changes qualitatively, as a result of autoparametric excitation. The parametric development of vibrations of one degree of freedom proceeds due to extinction of vibrations of other degree of freedom. This phenomenon was observed when the frequency of longitudinal vibrations of pendulum is the duplex frequency of rotational vibrations. The author draws attention to the fact, that this process is not invertible, i.e. rotational vibrations will develop by cost of longitudinal vibrations as a result of parametric excitation mechanism, but the contrary process proceeds on the base of harmonic excitation. This analysis concernes the conservative system.

Evan-Iwanowski (1976) present conditions of inertial parametric resonance occurrence of this type of pendulum with regard to the viscous damping. Author calls attention to the transfer of energy from one degree of freedom to the second and vice versa. The system of this type was presented also by Nayefh, Mook and Sridhar (1974).

The model of pendulum on spring was assumed by many investigators (cf Van der Burch, 1968; Srinivasan and Sankar, 1974; Sethna, 1963). In these works the asymptotic Krylov-Bogolyubov method was applied.

Van der Burch (1968) studied the application of this system in the case, when the ratio of frequency of longitudinal vibrations to rotational was 2, Srinivasan and Sankar (1974) investigated this system for ratio 1 and 0.5.

Sethna (1963) assumed the model of mathematical pendulum suspended on spring and forced harmonically by the changing force and by the moment at the same frequencies. Taking into account viscous damping of both motions the author investigated the system under internal resonance when the frequency of forcing is near to the frequency of longitudinal or rotational vibrations. He claimed the existance of the bifurcation point for the frequency of forcing moment near to the frequency of longitudinal vibrations. He used the averaging method (Sethna, 1965), where he assumed the cubic or quadratic nonlinearity.

Wilms and Cohen (1990) showed that an elastic cylinder rolling on a curved surface can be described by the same equations as the system consisting of an extensible pendulum. In this work they confined themselves to the derivation of equations of motion.

Schmidt (1990) investigated the rotary flexible pendulum assembling the mass, the spring and the weightless rod, that is connected by joints on one end. The author, using the method of averaging, obtained approximate solutions near to the equilibrium positions.

Many researches payed attention to investigations of system modeled as the double pendulum (cf Nayfeh, 1987; Kane and Djerassi, 1987a,b).

Nayfeh (1987) applying the method of multiple scales investigated the vibrations in the range of internal resonance in the cases, when the frequency of excitation was near to lower or higher natural frequency. He determined steady state responses and their stability. He investigated Poincaré and Hopf bifurcations.

Kane and Djerassi (1987a,b) presented the method of total or partial linearization, applied to nonlinear couplied systems of equations. As an example they analysed the motion of double pendulum with linear torsion spring.

Sevin (1961), Struble and Heinbockel (1962) and (1963) investigated the free motion of mechanical system consisting of platform suspended on hangers. He modeled this system as a beam-pendulum system. The authors analysed the phenomenon of energy transfer between modes of vibrations caused by the commensuration of natural frequencies.

Sevin (1961) showed that as a result of coupling in the system autoparametric vibrations appeared. He presented an approximate analysis of these vibrations and determined the stability diagram. He observed an interesting phenomenon of energy transfer between the beam and the pendulum oscillations, respectively, which occurs when the beam frequency redouble the pendulum frequency.

Struble and Heinbockel (1962) using asymptotic method, presented the first approximate solution to the equations derived by Sevin for conditions of internal resonance. The authors (1963) presented the second approximation of this solution and demonstrated the energy transfer from one degree of freedom to the second and vice versa. They presented the phase trajectories for different parameters of the system.

The asymptotic method applied by Sevin (1961), Struble and Heinbockel (1962) and (1963) is presented by Strouble (1963), (1964) and (1965), where it is used to solve the equations under parametric excitation.

The autoparametric system with pendulum was assumed as a model by Haxton and Barr (1972), Ibrahim and Roberts (1976) and (1977), Hatwal and Mallik (1983a,b), Sado (1984), Schmidt and Tondl (1986), Osiński and Sado (1987).

Osiński and Sado (1987) presented the application of the "delta" method to numerical procedure of the differential equations solving, which described the motion of vibration system with a finite number of degrees of freedom. As an example they applied this method to investigation of the system with inertial coupling. Sado (1984) analysed this system. The author presented the energy transfer from one degree of freedom to the second near to the autoparametric resonance. She analysed the free oscillations of conservative system.

The similar system under harmonic excitation was investigated by Hatwal, Malik and Ghosh (1983a,b). They applied the method of harmonic balance and compared the received results with those of experimental results.

Many investigators assumed the model of autoparametric vibration damper with inverted pendulum (cf Haxton and Barr, 1972; Ibrahim and Roberts, 1976 and 1977; Schmidt and Tondl, 1986), using the method of asymptotic approximation to analytical solution. These authors assumed different kinds of excitation. Haxton and Barr (1972) studied harmonically forced vibrations. They presented the experimental results too. Ibrahim and Roberts (1976) and (1977) investigated the system subject to a random excitation. They obtained (1976) the response of the system to the random excitation under conditions of the internal resonance. They investigated also (1977) the stochastic stability of steady state solution. This model elaborated by above mentioned authors was assumed in monograph by Schmidt and Tondl (1986) as an example of application of the asymptotic approximation method to the autoparametric coupling systems.

The system with pendulum is also assumed by Krasnopolskaya and Švec (1987). They investigated the energy transfer between modes of vibrations under conditions of main parametric resonance. They applied the averaging method in their works.

In driving systems the model of damper with centrifugal pendulum is applied (cf Crossley, 1952 and 1953; Newland, 1964; Sharif-Batkiar and Shaw, 1988; Shaw and Wiggins, 1988; Shaw and Rand, 1989; Shaw and Shaw, 1989; Yoshida, 1989; Moore and Shaw, 1990).

Yoshida (1989) chose the parameters of dynamic dampers of his type for ships and described also results of practical application of different types of structural solutions.

This type of damper is applied to investigation of torsional oscillations in light aircraft engines. Crossley (1952) and (1953) investigated free vibrations and vibrations forced by harmonic variable moment, respectively. He made the investigations for centrifugal pendulum with small and wide angles. Similar model was assumed by Newland (1964). He applied the linear approximation of motion equations for small and high amplitudes. He pays attention to the fact, that the analysis can be extended on practical systems involving several pendulum absorbers. The effects of motion-limiting stops on the dynamic behaviour of a system with centrifugal pendulum are studied by Sharif-Bakhtiar and Shaw (1988). They,taking into account the influence of the stops, analysed the stability of non-linear impacting periodic motions of pendulum. This model of a pendulum type centrifugal vibration absorber was assumed by Shaw and Wiggins (1988). The authors presented analytical, simulation and experimental studies. They indicated that the chaotic motions can appear under bifurcation conditions.

Similar works for an inverted pendulum with motion limiting stops were presented by Shaw and Rand, (1989), Shaw and Shaw (1989), Moore and Shaw (1990). There were investigated the bifurcation conditions too.

Szabelski and Samodulski (1985) studied autoparametric vibrations of two-degree-of-freedom system under kinematic excitation. They investigated the motion of the vehicle equiped with the homogeneous rigid tyre and driving along the wavy sinusoidal ground. The equations of motion contained terms of parametric and external excitation. They assumed a non-linear quadratic type elasticity and a linear damping. Periodic solutions were proceeded by the perturbation method and by an analogue simulation.

As one can see the technical application of two-degree-of-freedom systems is very wide. Their applications can be found in works by Genkin and Ryabov (1988), Karamyškin (1988).

2.3. Continuous models

The problem of autoparametric interaction within a vibrating structure was presented by Barr and McWhannell (1971). They investigated the motion of column bulding structures with many bays. Using the small parameter method they predicted theoretically the boundaries of stability and compared the obtained instability regions with those observed in experiments.

Ibrahim and Barr (1975a,b) investigated the autoparametric coupling of liquid motion in cylindrical flexibly mounted tank with the oscillations of the construction itself. They applied the asymptotic method presented by Struble (1965) and the obtained results compared with the experimental results.

Ibrahim and Barr (1975a) assumed that the tank transforms only vertical vibrations and they investigated the influence of these vibrations on motion of the liquid in conditions of inertial resonance i.e. when the structure natural frequency equals twice the liquid sloshing frequency. They showed that the first-order perturbation solution was not adequate to predict the response of the system properly. The second-order solution showed an essential difference from the first-order one. The second-order solution agreed with the experimental results.

Ibrahim and Barr (1975b) assumed that the tank can also move horizontally. In this case the asymptotic approximation up to the first order shows four possible conditions of internal resonances. They obtained a steady-state response in the neighbourhood of the principal internal resonance, but in conditions of combined resonances the system does not achieve a constant amplitude steady-state response. The analytical study was agreed with experimental investigations.

The autoparametric resonance was observed by Piszczek (1961) and (1963). The author (1961) investigated the steady-state vibrations of a column under follower force. He presented the possibility of internal resonance and also indicated a possibility of a destabilizing effect of viscous damping. He investigated (1963) a system modeled as a 2D-beam supported on ends with the help of the rigid bearning elastically fixed on the base. This beam was forced inertially. The approximate solution was presented for horizontal and vertical components of the excitation separately.

The autoparametric vibrations in a system of coupled beams, where the least stiff planes were mutually perpendicular, were studied by Bux and Roberts (1986), Cartmell and Roberts (1988).

Bux and Roberts (1986) investigated the behaviour of an internally generated combination resonance combined with internal resonance of natural frequencies ratio equal to two. They showed that in conditions of an internal resonance the complex form of response of the system to excitation of one mode vibrations can be obtained. As a result of coupling the excitation is transfered to other degrees of freedom.

Cartmell and Roberts (1988) presented complex responses that can be generated within a system of coupled cantilever beams for two internal combined resonances. They showed that this system is very sensitive to the small untuning. The problem was treated by means of the multiple scales method, and the obtained results were compared with the experimental results.

Zavodney and Nayfeh (1989) investigated theoretically and experimentally the motion of the slender cantilever beam carrying the lumped mass under the principal parametric base excitation. They discretized the equation of motion by the Galerkin method and using the method of multiple scales they determined an approximate solution to the temporal equation for the case of a single mode. The results were compared with the results of experiment.

The analysis of vibrations transfer in a plane system of rods is presented by Foryś and Gajewski (1984), Foryś and Niziol (1984). The authors took into account the non-linear damping and the coupling of system elements by internal longitudinal forces, which appeared due transverse forces on ends of next rods. The autoparametric internal resonance was investigated. The authors discretized the equations of motion by the Galerkin method and applied to investigations the approximate method of harmonic balance.

Foryś and Niziol (1984) investigated the autoparametric resonance in a system of three rods regarding the mass of the pivotal joints.

Foryś and Gajewski (1984) studied the effect of geometric parameters on the resonance amplitudes in a system of three rods of variable cross-section.

The energy transfer between modes of vibrations is presented also by Croll (1975a,b). He investigated the motion of shell and shell-like structures. This system was modeled as a two-degree-of-freedom system with kinematic coupling. To solve the equations he applied digital simulation method. The investigations were presented for different values of natural frequences ratio when the frequency of excitation is near to the first or the second natural frequencies, respectively.

Ohmata (1977), Kojima and Saito (1983) investigated the effect of the dynamic damping on a machine attached to an elastic beam, when the machine is forced by a harmonic variable force.

Ohmata assumed, that the absorber produced a linear spring force while Kojima and Saito accepted the absorber producing a hardening spring force in a cubic curve form. The equations of motion were reduced to the Duffing equations and solved by the harmonic balance method.

Nayfeh and Raouf (1987) investigated the autoparametric vibrations in long cylindrical shells. They applied the method of Galerkin to derive a set of nonlinear ordinary differential equations and these equations solved numerically or using the multiple scale method. They observed a saturation phenomenon when the flexural frequency is duplex of breathing frequency. The authors investigated also Hopf bifurcation conditions when the system is harmonically forced.

Crespo da Silva and Zaretzky (1990) studied nonlinear flexural-torsional coupling for cantilevers and clamped-pinned slinding beams. They also applied a combination of the Galerkin procedure and the multiple scale method. They presented and discussed steady-state amplitude-frequency responses.

3. Classification of motion equations

As one can see from the literature discussed above, the investigators examined the discrete and continuous vibrating systems. They investigated both the conservative systems and the systems with damping. They assumed different kinds of elastic and damping characteristics. Free and forced oscillations were studied under different kinds of excitaton forces.

The vibrations of discrete systems were described by ordinary differential equations. The continuous systems were described by partial differential equations, which usually were transformed to the form of ordinary differential equations. Further reserches can then be confined to the analysis of systems of ordinary differential equations.

Equations of motion describing vibrations of non-linear two-degree-of-freedom systems are presented in literature in different forms, often convenient for specific asymptotic method applied by particular authors. The comparison of these equations in their genuine form is therefore difficult and often impossible.

In purpose to realize a comparative analysis of these equation systems from the point of view of an adequate coupling terms existance the author of this work assumed the system of equations in the following form

$$\begin{aligned} a_0\ddot{x}_1 + a_1\ddot{x}_2 + a_2\dot{x}_1^2 + a_3\dot{x}_1\dot{x}_2 + a_4\dot{x}_2^2 + a_5\dot{x}_1 + a_6\dot{x}_2 + a_7 &= a_8F_1(t) \\ b_0\ddot{x}_1 + b_1\ddot{x}_2 + b_2\dot{x}_1^2 + b_3\dot{x}_1\dot{x}_2 + b_4\dot{x}_2^2 + b_5\dot{x}_1 + b_6\dot{x}_2 + b_7 &= b_8F_2(t) \end{aligned} \tag{3.1}$$

where

a_n, b_n $(n = 0, 1, \ldots, 8)$ – coefficients depending on generalized coordinates x_1, x_2
$F_1(t)$, $F_2(t)$ – excitation forces depending on time.

The coefficients a_n and b_n may be linear or nonlinear functions of coordinates x_1 and x_2. Usually the nonlinearities of type x_1^2, x_2^2, $(x_1 - x_2)^2$, x_1^3, x_2^3, $(x_1 - x_2)^3$, rarely type $\sin x_1$, $\sin x_2$ or $\sin x_1 \sin x_2$ are assumed.

With the system of Eqs (3.1) were compared the systems of equations assumed by precited authors. The results are presented in Table 1.

Table 1

Ref.	Coefficients																	
	a_0	b_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	a_5	b_5	a_6	b_6	a_7	b_7	a_8	b_8
1	2		3		4		5		6		7		8		9		10	
3,67	+			+											+	+		
80	+			+							+			+	+	+		
72	+			+		+	+				+			+	+	+		
16 17 81	+			+	+	+	+	+	+	+					+	+		
1 60 90	+			+											+	+	+	
9	+			+							+				+	+	+	
11	+			+							+			+	+	+	+	
2	+			+							+	+	+	+	+		+	
*	+			+							+	+	+	+	+	+	+	
35 57 89	+			+							+	+	+	+	+	+		+
71	+			+											+	+	+	+
**	+			+							+			+	+	+	+	+
***	+			+							+	+	+	+	+	+	+	+

* – 4-6, 17-19, 37, 56, 64-66, 78, 91-93, 95, 101
** – 12, 13, 41-44, 47, 50, 52, 54, 83, 94
*** – 4-6, 19, 24, 64, 66, 91-93, 99, 100

1	2	3	4	5	6	7	8	9	10
73	+	 +	 +	+		+	 +	+ +	+ +
20,38 82,96 98	+ +	 +		 +	+			+ +	
32,33	+ +	+ +				 +	+	+ +	
59,68 74,87 88	+ +	+ +			+			+ +	
14	+ +	+ +	+	+	 +				
7	+ +	+ +	 +		+			+ +	
21,22	+ +	+ +	+ +		 +			+ +	
77	+ +	+ +				+	 +	+ +	+
69	+ +	 +				+		+ +	+
25,27	+ +	+ +			+	+	 +	+ +	+
27,30 31	+ +	+ +			+ +	+	 +	+ +	+
36	+ +	+ +	 +		+	+	 +		+
14,15	+ +	+ +	+	+	 +				+
75,79	+ +	+ +	+ +	+			+ +		+
28,29	+ +	+ +	 +		+			+ +	+
61,62	+ +	+ +						+	+ +
45 48 49	+ +	+ +	+ +	+ +	+ +	+ +	+ +	+ +	+ +

In this table the terms of equations are classifieed together according to designations assumed in formula (3.1). The plus in the table shows that the equation assumed by author contained this term. The free place means that in this case the equation did not contain the suitable term.

As one can see there are many free places in this table, i.e. the equations of motion do not contain all terms which are responsible for different kind of nonlinearities of system.

Most coefficients were taken into account by Nayfeh (1987), Nayfeh, Balachandran, Colbert and Nayfeh (1989), Nayfeh and Balachandran (1990). But and this author assumed that the functions $a_n(x_1, x_2)$ and $b_n(x_1, x_2)$ are in form of polynomials. Thanks to this assumption he could apply the multiple scale method in its differents versions according to the degree of assumed polynomials.

In the case when nonlinearities are expressed by other functions, the difficulties appear in the application of asymptotic method, since nonlinear functions in this case must be expanded in series taking a few terms of series into account. It results in further errors in description of real systems.

1. Conclusions

As can be seen from the above cited literature the problem of energy transfer from one degree of freedom to other in systems with inertial coupling is an important problem and many investigators were engaged in solving it.

Equations describing the motion of this type of systems are non-linearly coupled and have no exact solutions. The approximate methods are applied. Harmonic solutions, assumed to asymptotic methods do not always render the full character of a response. In all analytic methods of approximation the condition of small parameters take place, what in the case of their modulation for autoparametric vibrations is not true and is difficult to estimation. Therefore it seems, that it is useful to apply numerical methods to investigations since we do not need limit ourself to small oscillations then.

It appears, that the essential influence on the response of the coupling system has the kind of nonlinearities of spring and damping. Not all kinds of nonlinearities are widely examined. There are many works which regard the influence of quadratic and cubic elastic nonlinearities on the behaviour of systems described by equations of Duffing type. There are not many works taking into account the non-linear damping.

Not numerous authors regarded the influence of nonlinearities of spring and dampers of this type connected with the nonlinearity resulting from the existance of pendulum in the system. Usually in systems with pendulum it is assumed, that in main system the spring and the damper display linear characteristics.

Usually are investigated the steady-state solutions to internal and external resonance, but nothing is said about the time of steading of these oscillations. In autoparametric systems the energy is transformed from one degree of freedom to another in a closed cycle. The time of this cycle depends on the kind of nonlinearity.

The examination of influence of spring and dampers nonlinearities on the change of time of the cycle of energy transfer is very important. It plays an essential role in the investigations as well of free oscillations as of forced oscillations.

The knowledge of the cycle of energy transfer enables us to find optimal parameters of work of many mechanical systems, which can be reduced to this type of models.

This problem is therefore very important and demands for further investigations.

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Przenoszenie energii w układach drgających o dwóch stopniach swobody – praca przeglądowa

Streszczenie

Celem niniejszego opracowania był przegląd i analiza omawianych we współczesnej literaturze problemów dotyczących przekazywania energii w nieliniowych układach drgających o dwóch stopniach swobody. Szczególną uwagę zwrócono na rodzaje nieliniowości uwzględniane przez poszczególnych autorów. Stwierdzono, że nie wszystkie rodzaje nieliniowości zostały przebadane w szerokim zakresie, że zwłaszcza mało jest prac uwzględniających jednoczesne występowanie nieliniowości typu wahadła z nieliniowościami typu wielomianów. Tego typu układy mają zaś duże znaczenie techniczne.

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