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CREEP WITH RUPTURE ANALYSIS OF TRANSVERSELY HETEROGENEOUS CIRCULAR PLATES

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A theoretical analysis of non-linear viscoelastic plates with transverse heterogeneity is presented. Brittle creep rupture processes are described by steady-state creep theory coupled with the Kachanov-Rabotnov damage law. The solutions are based on parametrization of the bending moments. Two numerical examples illustrate the creep and the brittle creep rupture of an annular plate with different kinds of transverse heterogeneity.

1. Introduction

In designing of engineering structures which operate at sufficiently high temperatures, consideration must be given to the possibilities of failure due to creep or creep rupture. The present paper deals with the analysis of circular-symmetrical plates. The solutions presented are related to isotropic plates with symmetric transverse heterogeneity as regards the middle surface (cf Kączkowski, 1980). In particular, the plates with three different layers – sandwich plates – are considered.

The non-linear viscoelastic Norton model (cf Odqvist, 1966) describes the material properties both in secondary and tertiary creep. Here in this model is extrapolated on the net-stress tensor, being the modification of the Cauchy stress tensor accounting for the damaging net area reduction (cf Leckie and Hayhurst, 1974; Białkiewicz, 1980).

A proposed method for the solution of plates allows us to carry out a practical analysis for plates with small transverse heterogeneity and this enables the influence of the shearing forces on the strains to be omitted. The solutions are based on the parametrization of the bending moments introduced by Białkiewicz (1984b) and (1986). In the present paper that parametrization is generalized by the steady-state creep theory coupled with the Kachanov damage growth law (cf Kachanov, 1980) for heterogenous plates. As a result of parametrization the solution of the heterogeneous plate boundary problem is reduced to a set of differential-integral equations which have separated derivatives of unknown functions. This transformation allows us to apply a standard software procedure in the final solution.

The analysis of secondary creep of plates with consideration of shearing forces had been performed by Shably et al. (1982) and the solutions of transient creep with experimental verification by Pospelov and Naumov (1982) and by Odqvist (1953). In general, all of their considerations were limited to analyses of homogeneous plates. In particular some problems of optimization (cf Faktorovitch, 1978; Dulman, 1982; Lepik, 1982; Tchapligina, 1982) and the creep of plates on elastic foundations (cf Hamza and Muggeridge, 1982) were discussed. Finally Ospanova (1982) deals with some aspects of dynamic strains.

Following this introduction we formulate firstly the problem and then at a later stage propose an algorithm of its solution. Two illustrating examples are presented: creep and creep rupture of an annular plate with different kinds of transverse heterogeneity. Some final remarks are also included.

2. Formulation of the problem

The governing set of equations will be formulated in dimensionless variables. For this purpose the following magnitudes can be defined: independent variables (cylindrical coordinates and time)

$$\rho = \frac{r}{h_0} \qquad \zeta = \frac{z}{h_0} \qquad \tau = \frac{t}{t_0} \tag{2.1}$$

stresses and rate of strains

$$s_{ij} = \frac{\sigma_{ij}}{\sigma_c} t_0^{\frac{\nu}{n}} \qquad \bar{\varepsilon}_{ij} = \varepsilon_{ij} t_0 \tag{2.2}$$

deflection and thickness

$$\bar{w} = \frac{w}{h_0} \qquad \chi = \frac{h}{h_0} \tag{2.3}$$

and parameters (material constants)

$$\bar{A} = A\sigma_c^{\nu} t_0^{1-\frac{\nu}{n}} \qquad s_c = \frac{\sigma_c}{k_0} t_0^{-\frac{1}{n}} \tag{2.4}$$

where k_0, t_0 and h_0 are constants which have been introduced in order to secure the non-dimensionality of the relevant formulae.

The forms of the damage and strain rate equations which will be used to describe the law of three-dimensional behaviour of stress rupture are given below

$$\bar{\varepsilon}_{ij} = \Big(\frac{s_e}{S_c(\zeta)(1-\omega)}\Big)^{n(\zeta)-1}\frac{s_{ij}}{S_c(\zeta)(1-\omega)} \tag{2.5}$$

$$D_\tau\omega = \bar{A}(\zeta)\Big(\frac{\sigma_s}{1-\omega}\Big)^{\nu(\zeta)} \tag{2.6}$$

where

$$s_e = \sqrt{\frac{3}{2}s_{ij}s_{ij}} \tag{2.7}$$

$$\sigma_s = \delta(\zeta)\max(s_1,s_2) + [1-\delta(\zeta)]s_e \tag{2.8}$$

Here the non-linear constitutive equations (2.5) together with the damage evolution equation (2.6) (where the rate of damage $D_\tau\omega$ is a power function of the Sdobiryev-Rabotnov reduced stress σ_s, cf Rabotnov, 1966) constitute the coupled damage theory.

The governing set of equations (2.5) and (2.6) given above allows us to work out the solutions for transversely heterogeneous plates. The heterogeneity state is described by the material functions $n(\zeta)$, $\nu(\zeta)$, $s_c(\zeta)$, $\delta(\zeta)$ and $\bar{A}(\zeta)$ with respect to the ζ coordinate which is perpendicular to the middle-surface of the plate $\zeta = 0$. The reduced stress σ_s (2.8) is a function of the principal stresses $\{s_1, s_2\}$ according to the assumption of a plane state of stress in the constitutive formulation of thin plates.

The solutions to the circular-symmetric plates will be shown by a cylindrical system of coordinates (ρ,ϑ,ζ), where the directions: radial ρ and circumferential ϑ are simultaneously principal directions. The constitutive equations (2.5) have the following form

$$-\zeta\partial_\tau\bar{w}_{,\rho\rho} = \frac{1}{2}\Big(\frac{s_e}{S_c(\zeta)(1-\omega)}\Big)^{n(\zeta)-1}\frac{2s_\rho - s_\vartheta}{S_c(\zeta)(1-\omega)} \tag{2.9}$$

$$-\frac{\zeta}{\rho}\partial_\tau\bar{w}_{,\rho} = \frac{1}{2}\Big(\frac{s_e}{S_c(\zeta)(1-\omega)}\Big)^{n(\zeta)-1}\frac{2s_\vartheta - s_\rho}{S_c(\zeta)(1-\omega)} \tag{2.10}$$

where

$$s_e = \sqrt{s_\rho^2 + s_\vartheta^2 - s_\vartheta s_\rho} \tag{2.11}$$

The evolution equation (2.6) has the same form but the principal stresses s_1, s_2 must be replaced by the radial stress s_ρ and the circumferential stress s_ϑ, respectively in expresion (2.8). In further formulating of the boundary value problem the constitutive equations (2.9) and (2.10) will be used in the form reduced to the middle surface of the plate

$$\partial_\tau \varphi_{,\rho} = \frac{1}{2}\Big(1 + \frac{1}{2n(\zeta)}\Big)^{n(\zeta)} \chi^{-1-2n(\zeta)} \Big(\frac{m_e}{S_c(\zeta)(1-\omega)}\Big)^{n(\zeta)-1} \frac{2m_\rho - m_\vartheta}{S_c(\zeta)(1-\omega)} \tag{2.12}$$

$$\frac{1}{\rho}\partial_\tau \varphi = \frac{1}{2}\Big(1 + \frac{1}{2n(\zeta)}\Big)^{n(\zeta)} \chi^{-1-2n(\zeta)} \Big(\frac{m_e}{S_c(\zeta)(1-\omega)}\Big)^{n(\zeta)-1} \frac{2m_\vartheta - m_\rho}{S_c(\zeta)(1-\omega)} \tag{2.13}$$

where

$$\varphi(\rho) = -\bar{w}_{,\rho} \tag{2.14}$$

is the slope of the deflection.

Dimensionless bending moments are assumed in accordance with the following definitions

$$m_\rho = \int_{-\chi(\rho)}^{\chi(\rho)} s_\rho \zeta d\zeta \qquad m_\vartheta = \int_{-\chi(\rho)}^{\chi(\rho)} s_\vartheta \zeta d\zeta \tag{2.15}$$

By analogy to the stress intensity (2.11) the bending moment intensity is introduced

$$m_e = \sqrt{m_\rho^2 + m_\vartheta^2 - m_\vartheta m_\rho} \tag{2.16}$$

For the purpose of complete formulation of the boundary problem of a plate in tertiary creep the constitutive equations (2.12) and (2.13) should be completed by the following equilibrium equations

$$m_{\rho,\rho} + \frac{1}{\rho}(m_\rho - m_\vartheta) + q_\rho + \int_{-\chi(\rho)}^{\chi(\rho)} \frac{s_\rho}{1-\omega} w_{;\rho} \zeta d\zeta = 0 \tag{2.17}$$

$$\rho q_{\rho,\rho} + q_\rho - \bar{q}(\rho)\rho = 0 \tag{2.18}$$

where q_ρ is the shearing force and $\bar{q}$ denotes the load normal to the middle-surface of the plate. In the local formulation of the equilibrium equations true stress has been assumed as

$$\bar{s}_{ij} = \frac{s_{ij}}{1-\omega} \qquad \text{for} \qquad i = j \tag{2.19}$$

except for the shear stress $\tau_{\rho\zeta}$ where the damage influence is negligible.

3. Steady rate creep of a sandwich plate

We will begin with analysing the steady state creep of a sandwich plate (the damage parameter $\omega = 0$ and the evolution equation (2.6) is disregarded).

The middle surface is a plane of symmetry of the assumed heterogeneity. The internal and external layers are characterized by the material constants

$$\Big\{\zeta \in \big(-\chi_i(\rho), \chi_i(\rho)\big) \ : \ s_c(\zeta) = s_{ci}\ , \ \ n(\zeta) = n_i\Big\} \tag{3.1}$$

$$\Big\{\zeta \in \big(-\chi_e(\rho), -\chi_i(\rho)\big) \cup \big(\chi_i(\rho), \chi_e(\rho)\big) \ : \ s_c(\zeta) = s_{ce}\ , \ \ n(\zeta) = n_e\Big\} \tag{3.2}$$

The constitutive equation (2.13) is automatically satisfied when the moments m_ρ and m_ϑ are expressed in terms of the function ψ

$$\begin{aligned}\begin{bmatrix} m_\rho \\ m_\vartheta \end{bmatrix} &= \Big[B\Big(\frac{4\partial_\tau\varphi}{\rho(\sqrt{3}\cos\psi - 3\sin\psi)}\Big)^{\frac{1}{n_e}}\Big(\chi_e^{\frac{2n_e+1}{n_e}} - \chi_i^{\frac{2n_e+1}{n_e}}\Big) + \\ &+ \ C\Big(\frac{4\partial_\tau\varphi}{\rho(\sqrt{3}\cos\psi - 3\sin\psi)}\Big)^{\frac{1}{n_i}}\chi_i^{\frac{2n_i+1}{n_i}}\Big]\begin{bmatrix} \cos(\psi - \frac{\pi}{6}) \\ \cos(\psi + \frac{\pi}{6}) \end{bmatrix}\end{aligned} \tag{3.3}$$

where

$$C = \frac{2n_i s_{ci}}{2n_i + 1}\Big(\frac{4}{3}\Big)^{\frac{n_i-1}{2n_i}} \tag{3.4}$$

$$B = \frac{2n_e s_{ce}}{2n_e + 1}\Big(\frac{4}{3}\Big)^{\frac{n_e-1}{2n_e}} \tag{3.5}$$

The equilibrium equation (2.17) and the constitutive relation (2.12) after parametrization can be written as follows

$$\begin{aligned}&\psi_{,\rho}\Big[(1 + Qg)\sin(\psi - \frac{\pi}{6}) + \frac{P}{n_i}\Big(1 - \frac{n_i}{n_e}Qg\Big)\Big] = \\ &= \frac{1}{n_i}\cos(\psi - \frac{\pi}{6})\Big[S\Big(1 + \frac{n_i}{n_e}Qg\Big) + f_2\Big(1 + \frac{n_i}{n_e}Qf_1\Big) + \frac{1}{\rho}\sin\psi(1 + Qg) + R\end{aligned} \tag{3.6}$$

$$\partial_\tau\varphi_{,\rho} = \partial_\tau\varphi S(\psi) \tag{3.7}$$

where

$$\begin{aligned}P(\psi) &= \cos(\psi - \frac{\pi}{6})\frac{\sqrt{3}\sin\psi + 3\cos\psi}{\sqrt{3}\sin\psi + 3\cos\psi} \\ Q(\psi, \partial_\tau\varphi) &= \frac{C}{B}\Big(\frac{\chi_i}{m}\Big)^{\frac{n_i-n_e}{n_i n_e}} \\ R(\psi, \partial_\tau\varphi) &= \frac{q_\rho}{B}\Big(\frac{m}{\chi_i^{2n_i+1}}\Big)^{\frac{1}{n_i}} \\ S(\psi) &= \frac{1}{\rho}\frac{2\cos(\psi - \frac{\pi}{6}) - \cos(\psi + \frac{\pi}{6})}{2\cos(\psi + \frac{\pi}{6}) - \cos(\psi - \frac{\pi}{6})}\end{aligned} \tag{3.8}$$

$$m(\psi, \partial_\tau \varphi) = \frac{\rho(\sqrt{3}\cos\psi - 3\sin\psi)}{4\partial_\tau \varphi}$$

$$f_1(\rho) = \Big[\Big(\frac{\chi_e}{\chi_i}\Big)^{\frac{2n_e+1}{n_e}} \frac{(2n_e+1)\rho\chi_{e,\rho} - \chi_e}{(2n_e+1)\rho\chi_{i,\rho} - \chi_i} - 1\Big] \frac{(2n_e+1)\rho\chi_{i,\rho} - \chi_i}{(2n_i+1)\rho\chi_{i,\rho} - \chi_i}$$

$$f_2(\rho) = \frac{(2n_i+1)\rho\chi_{i,\rho} - \chi_i}{\rho\chi_i}$$

$$g(\rho) = \Big(\frac{\chi_e}{\chi_i}\Big)^{\frac{2n_e+1}{n_e}} - 1$$

For a given load $\bar{q}(\rho)$, the integral of the equilibrium equation (2.18) is the transverse force. Thus the numerical solution of the steady state creep of a sandwich plate constitutes: equilibrium equations (3.6), a physical relation (3.7) and also a geometrical relation (2.14) with respect to the unknown functions ψ, $\partial_\tau \bar{w}$ and $\partial_\tau \varphi$ given above.

The separated derivatives (with respect to the set of equations of the unknown functions) allow us to apply in the numerical solution the standard procedures for ordinary differential equations. The known form of the parametrizing function ψ means that the values of the bending moments in Eq (3.3) can be calculated.

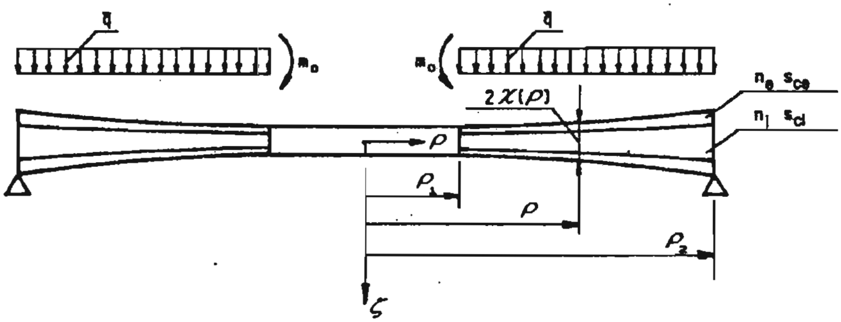

Fig. 1.

The boundary conditions will be formulated for an annular plate simply supported (see Fig.1) along the external edge

$$\partial_\tau \bar{w}(\rho_2) = 0 \tag{3.9}$$

The plate is loaded uniformly with a radial moment m_0 along the internal edge

$$m_\rho(\rho_1) = m_0 \tag{3.10}$$

For the given load and kinematic boundary condition (3.9) the integral of Eq (2.18) (with boundary condition $\bar{q}_\rho(\rho_1) = 0$) is the function of the transverse force

$$q_\rho = \frac{\bar{q}}{2\rho}(\rho^2 - \rho_1^2) \tag{3.11}$$

Since the outer rim ρ_2 is assumed to be load-free ($m_\rho(\rho_2) = 0$), the boundary condition for the parametrizing function (3.3) can be written as follows

$$\psi(\rho_2) = \frac{2}{3}\pi \tag{3.12}$$

By introducing Eq (3.10) into Eq (3.3) we reach the following relation between $\partial_\tau\varphi(\rho_1)$ and $\psi(\rho_1)$

$$\begin{aligned}&\Big[B\Big(\frac{4\partial_\tau\varphi(\rho_1)}{\rho_1[\sqrt{3}\cos\psi(\rho_1) - 3\sin\psi(\rho_1)]}\Big)^{\frac{1}{n_e}}\Big(\chi_e^{\frac{2n_e+1}{n_e}}(\rho_1) - \chi_i^{\frac{2n_e+1}{n_e}}(\rho_1)\Big) + \\ &+C\Big(\frac{4\partial_\tau\varphi(\rho_1)}{\rho_1[\sqrt{3}\cos\psi(\rho_1) - 3\sin\psi(\rho_1)]}\Big)^{\frac{1}{n_i}}\chi_i^{\frac{2n_i+1}{n_i}}(\rho_1)\Big]\cos\Big(\psi(\rho_1) - \frac{\pi}{6}\Big) = m_0\end{aligned} \tag{3.13}$$

Conditions (3.12) and (3.13) formulate a two-point boundary-value problem for Eqs (3.6) and (3.7). This consists in a choice of such magnitudes $\psi(\rho_1)$ for the function $\psi(\rho)$ that the boundary condition (3.13) is satisfied after integrating Eqs (3.6) and (3.7). The method of linear interpolation discussed by Białkiewicz (1984b) and (1986) has been applied to the numerical calculations.

The numerical analysis has been performed for a plate with dimensions $\rho_1 = 5$, $\rho_2 = 20$. The external load was assumed to be uniform and equal to $\bar{q} = 1.1\times10^{-4}$. The material constants $n_i = 3.7$, $s_{ci} = 1$ and $n_e = 6$, $s_{ce} = 0.57$ which are taken for our calculations correspond to the creep behaviour of an aluminium alloy (n_i, s_{ci} – middle layer) and carbon steel (n_e, s_{ce} – topcoats) at a temperature of about 300° C (Odqvist, 1966). The thickness of the layers is taken as the power function (for $\chi_{e0}/\chi_{i0} = 2$)

$$\chi_i(\rho) = \chi_{i0}\Big(\frac{\rho}{\rho_2}\Big)^s \qquad \chi_e(\rho) = \chi_{e0}\Big(\frac{\rho}{\rho_2}\Big)^s \tag{3.14}$$

The distributions of the radial and circumferential moments, m_ρ and m_ϑ, are presented in Fig.2 and 3. Separated lines represent the distribution along the radius for a given thickness (where $s = 0,\ 0.5, 0.75$) and the load of the inner rim is $m_\rho(\rho_1)$, see Fig.2. Corresponding with the above solutions the rates of deflections are shown in Fig.4.

Our investigations of the influence of the thickness ratio (for $s = 1$) on the values of bending moments m_ρ and m_ϑ and rate of deflection $\partial_\tau\bar{w}$ are presented in Fig.5 ÷ 7. The calculations have been carried out for the geometrical dimensions ρ_1, ρ_2 and load $\bar{q}$ assumed in the previous examples.

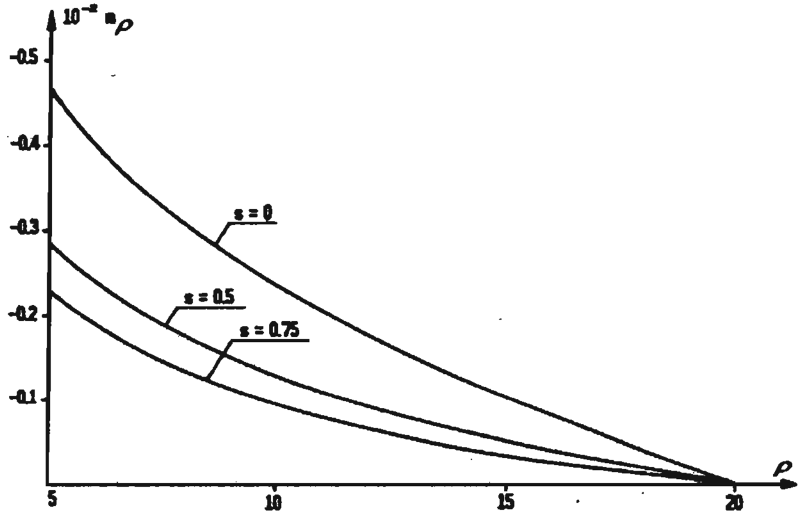

Fig. 2.

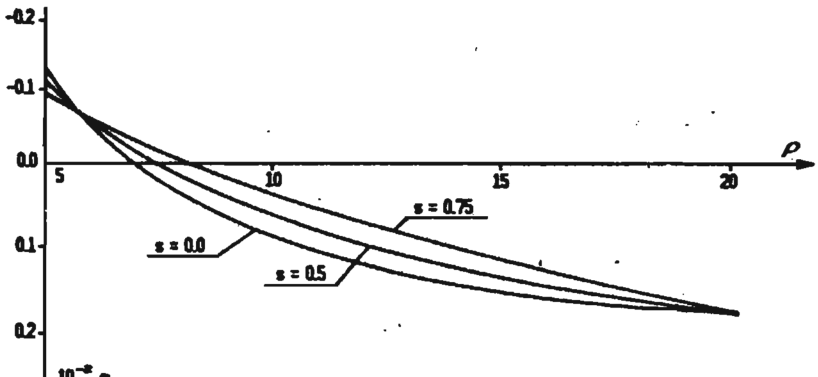

Fig. 3.

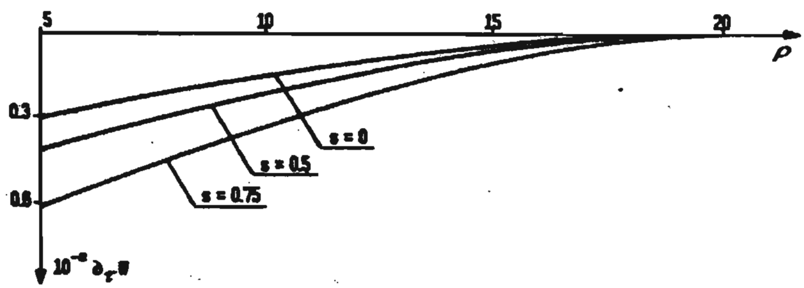

Fig. 4.

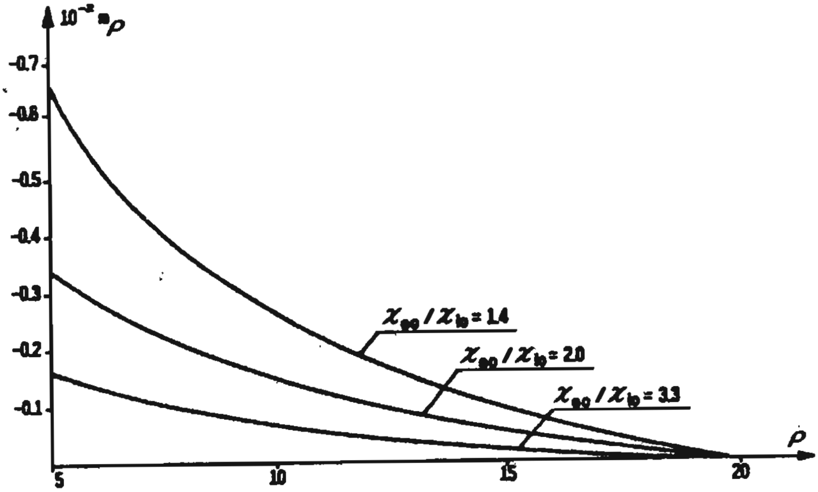

Fig. 5.

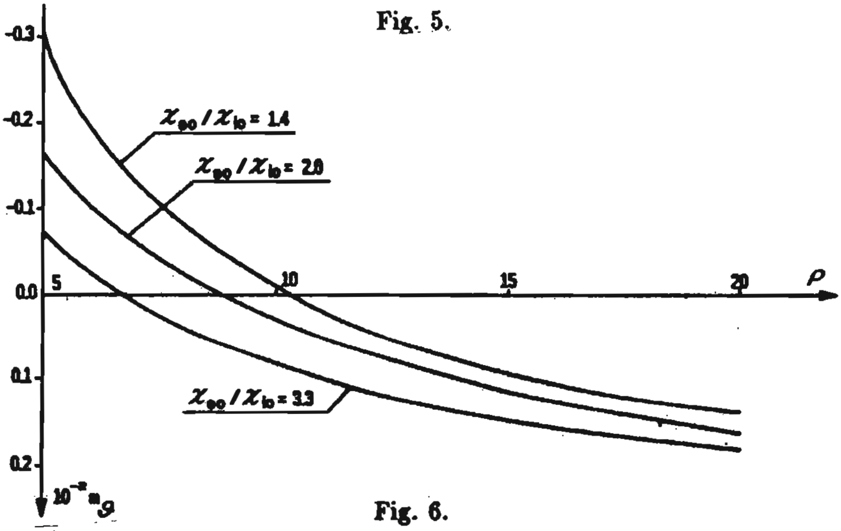

Fig. 6.

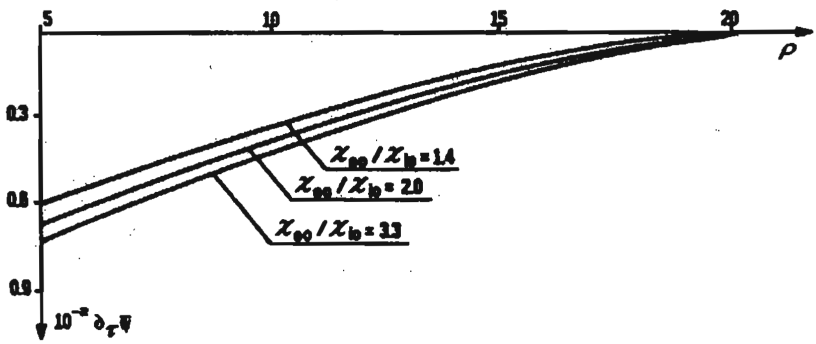

Fig. 7.

4. Tertiary creep

The steady state creep of heterogeneous plates solved in the preceding section is now associated with a brittle rupture. The development of the rupture process will be analysed in time up to the moment of the first cracks. The correctness of such solutions is justified by the results of the papers by Białkiewicz (1984a), Piechnik and Chrzanowski (1970) where it could be seen that the proportion of the total rupture time τ_k to the time of the first crack τ_0 was approximately equal to unity, $\tau_k/\tau_0 \cong 1$. This means that after the first crack appears the rupture process displays an avalanche effect. To simplify the numerical analysis the present solution will be carried out for a sandwich plate with constant thickness. We also assume that only the topcoats (external layers) are the load supporting ones while the internal layer plays a distance role only.

The physical equations (2.13) or (2.10) are now automatically satisfied when the moments m_ρ and m_ϑ or the stresses s_ρ and s_ϑ are expressed in terms of the parametrizing function ψ

$$\begin{bmatrix} m_\rho \\ m_\vartheta \end{bmatrix} = D \int_{\chi_{i0}}^{\chi_{e0}} (1-\omega)\zeta^{1+\frac{1}{n_e}} d\zeta \begin{bmatrix} \cos\left(\psi - \frac{\pi}{6}\right) \\ \cos\left(\psi + \frac{\pi}{6}\right) \end{bmatrix} \tag{4.1}$$

$$\begin{bmatrix} s_\rho \\ s_\vartheta \end{bmatrix} = D(1-\omega) \begin{bmatrix} \cos\left(\psi - \frac{\pi}{6}\right) \\ \cos\left(\psi + \frac{\pi}{6}\right) \end{bmatrix} \tag{4.2}$$

where

$$D = 2s_{ce}\left(\frac{4}{3}\right)^{\frac{n_e-1}{2n_e}} \left(\frac{4\partial_\tau \varphi}{\rho(\sqrt{3}\cos\psi - 3\sin\psi)}\right)^{\frac{1}{n_e}}$$

The equilibrium equation (2.17) after parametrization Eqs (4.1) and (4.2) has the following form

$$\begin{aligned} \psi_{,\rho}\Big[\sin\left(\psi - \frac{\pi}{6}\right) - \frac{1}{n_e}P\Big] &= \frac{1}{n_e}\cos\left(\psi - \frac{\pi}{6}\right)\left(S - \frac{1}{\rho}\right) + \frac{1}{\rho}\sin\psi + \\ &+ \frac{q_\rho m^{\frac{1}{n_e}}}{2s_{ce}}\left(\frac{4}{3}\right)^{\frac{1-n_e}{2n_e}} \int_{\chi_{i0}}^{\chi_{e0}} (1-\omega)\zeta^{\frac{1+n_e}{n_e}} d\zeta \end{aligned} \tag{4.3}$$

Further equations of the parametrizing boundary problem (2.14) and (3.7) are taken in an unchanged form.

The formulation of a governing set of equations is based upon the true stresses (2.19). The progress of the rupture process is determined by the scalar function ω. This function is the integral of the differential evolution equation (2.6) with the following initial condition

$$\omega(\rho,\zeta,0) = 0 \tag{4.4}$$

The function of reduced stress (2.8) for the circular-symmetrical plane problem has the following form

$$\sigma_s = \delta \max(s_\rho, s_\vartheta) + (1-\delta)s_e \tag{4.5}$$

The time of rupture τ_0 in the material particle is identified by the condition for the damage parameter $\omega(\rho, \zeta, \tau_0) = 1$.

Ilustrative solutions are performed for the simple supported annular plate presented in the preceding chapter. According to the present parametrization (4.1) the boundary condition (3.13) of the two-point problem can be written as

$$\begin{aligned} m_0 &= 2s_{ce}\Big(\frac{4}{3}\Big)^{\frac{n_e-1}{2n_e}}\Big(\frac{4\partial_\tau\varphi(\rho_1)}{\rho_1[\sqrt{3}\cos\psi(\rho_1) - 3\sin\psi(\rho_1)]}\Big)^{\frac{1}{n_e}} \cdot \\ &\quad \cdot \int\limits_{\chi_{i0}}^{\chi_{e0}} [1-\omega(\rho_1,\zeta,\tau)]\zeta^{1+\frac{1}{n_e}}d\zeta \cos\Big(\psi(\rho_1) - \frac{\pi}{6}\Big) \end{aligned} \tag{4.6}$$

Further boundary conditions (3.9) and (3.12) together with the function of shearing force have the same form.

A numerical analysis of the brittle-creep rupture process requires the discretization of the solution along the time axis. The solution of the set of equations (2.14), (3.7) and (4.3) for each of the moments of time τ_i will be preceded by the solution of the equation (2.6) with the initial condition $\omega(\rho, \zeta, \tau_{i-1})$. The known form of the function $\omega(\rho, \zeta, \tau_i)$ will make it possible to calculate the integral expressions in Eqs (4.3) and (4.6). Following this we can apply standard integral procedures to the solution of the ordinary differential equations with the unknown functions: ψ, $\partial_\tau \bar{w}$ and $\partial_\tau \varphi$.

Numerical examples have been computed for the following data: $\rho_1 = 6.6$, $\rho_2 = 26.6$, $\chi_{eo} = 1$, $n_e = 1.9$, $s_{ce} = 1$, $\bar{A} = 1$, $\nu = 2$, $\delta = 0$. The assumed physical constants correspond to the creep and rupture behaviour of copper at a temperature of 230° C (cf Odqvist, 1966).

The results of the numerical solutions presented in Fig.8, 9 and 10 correspond to the load $m_0 = -2.0 \cdot 10^{-3}$, $\bar{q} = 0.29 \cdot 10^{-4}$ and the proportion of the thickness of layers $\chi_{e0}/\chi_{i0} = 3.75$. Fig.8 shows the distribution of the damage parameter in a cross-section of the plate at a moment when the first crack appears $\omega(\rho = 6.6, \zeta = 1, \tau_0) = 1$. The distributions of radial and circumferential stresses, s_ρ, s_ϑ, in a cross-section of the plate at the initial moment $\tau = 0$ $(\omega = 0)$ are shown by Fig.9 and 10.

Synthetic curves which show the influence of load: m_0 (for $\bar{q} = 0.29 \cdot 10^{-4}$ and $\chi_{e0}/\chi_{i0} = 3.75$) and $\bar{q}$ (for $m_0 = -0.2 \cdot 10^{-3}$ and $\chi_{e0}/\chi_{i0} = 3.75$) and the proportions of thickness of the layers χ_{e0}/χ_{i0} (for $m_0 = -0.2 \cdot 10^{-3}$ and $\bar{q} = 0.29 \cdot 10^{-4}$) up to the time of the first crack τ_0 are illustrated in Fig.11,

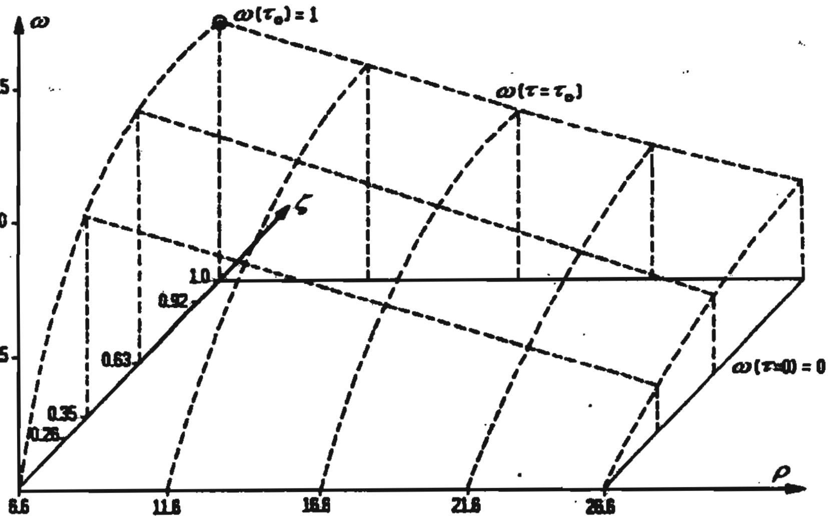
ω
ω(τ₀) = 1
1.5
ω(τ = τ₀)
1.0
ζ
1.0
0.92
0.5
0.63
ω(τ=0) = 0
0.35
0.26
ρ
6.6
11.6
16.6
21.6
26.6

Fig. 8.

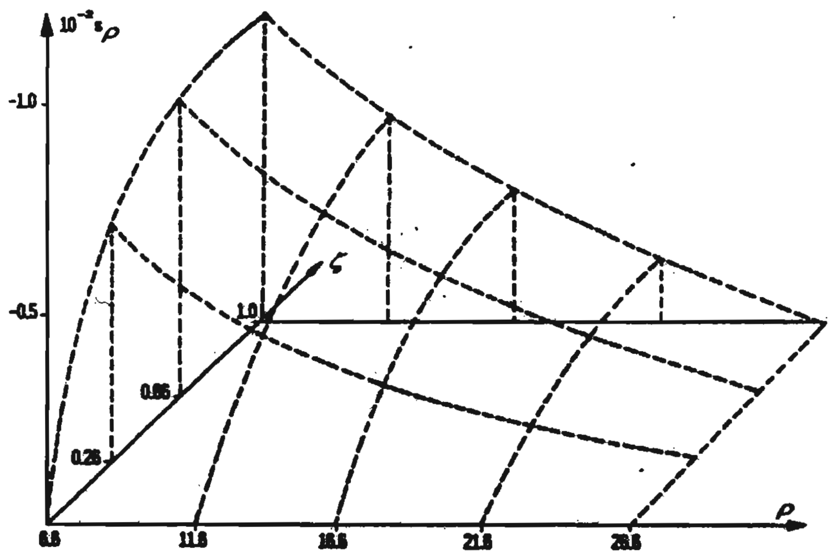
10⁻² s_ρ
-1.0
ζ
-0.5
1.0
0.66
0.26
ρ
6.6
11.6
16.6
21.6
26.6

Fig. 9.

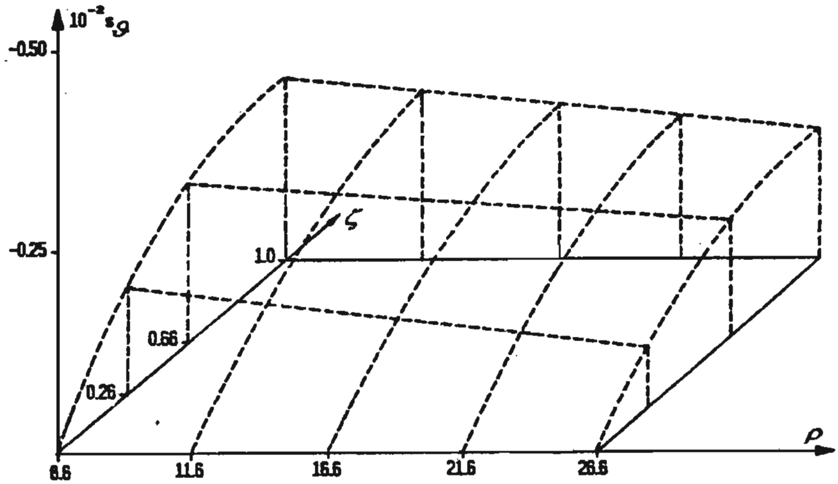

Fig. 10.

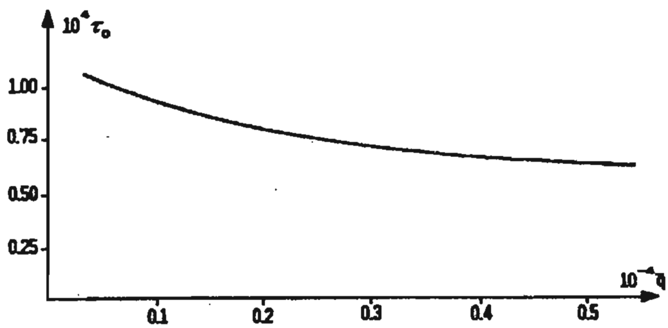

Fig. 11.

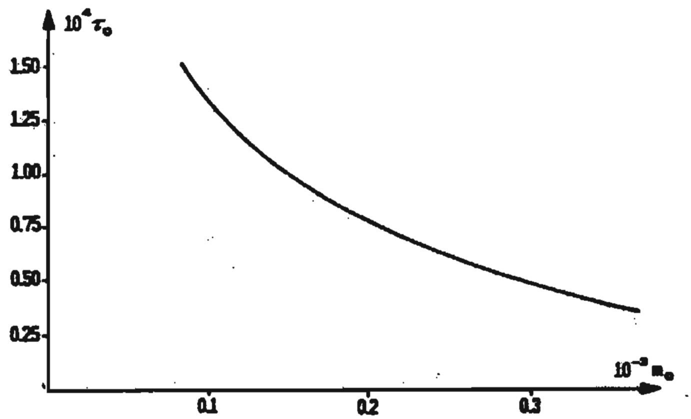

Fig. 12.

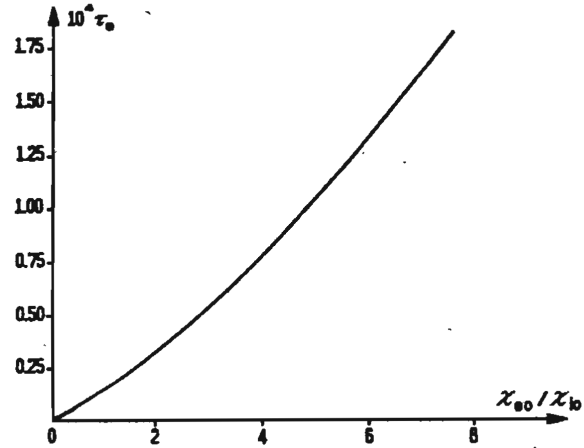

Fig. 13.

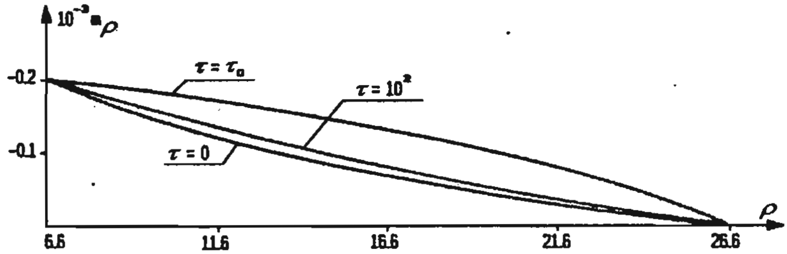

Fig. 14.

12 and 13. Additionally Fig.14 shows the changes in the radial bending moment distribution during the progress of the rupture process for the chosen instants.

5. Final remarks

The examples presented show the efficiency of the proposed algorithm of calculation. The numerical investigations were focused on creep and creep rupture of sandwich plates with an arbitrarily variable layer thickness. The solutions had to be limited to moderate transverse heterogeneity in accordance with the assumption of thin plate theory.

The distribution of the damage parameter (see Fig.8) modeling current state of rupture process can also be understood as a variable in time plate heterogeneity. The response of the assumed mathematical model on that heterogeneity are the true stresses or the true bending moments, Fig.14. For the solved examples (Chapter 4, Fig.11 and 12), the time to the first cracks is approximately 20 to 30 per cent shorter in comparison with the results of the nominal stress analysis ($\omega = 0$).

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Pełzanie z analizą zniszczenia niejednorodnych poprzecznie płyt kołowych

Streszczenie

W pracy przedstawiono teoretyczną analizę płyt nieliniowo-lepkosprężystych poprzecznie niejednorodnych. Model procesu kruchego zniszczenia został przyjęty na podstawie teorii pełzania ustalonego z wykorzystaniem prawa Kaczanowa-Rabotnowa. W rozwiązaniach zastosowano parametryzację momentów płytowych. Dwa przykłady numeryczne ilustrują pełzanie i kruche zniszczenie płyty pierścieniowej przy różnych rodzajach niejednorodności poprzecznej.

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