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SOME REMARKS ON THE SPATIAL RESOLUTION EFFECTS OF A HOT-WIRE TECHNIQUE

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An imperfect spatial resolution of a hot-wire anemometry makes it necessary to introduce a suitable correction procedure into the data processing. To derive the required correction functions based commonly on the assumption of isotropy and homogeneity of turbulence structure there is a need to adopt a certain form of $E(k)$ spectrum, describing the turbulence energy distribution in a high wavenumber range. The paper analyses to what extent the given form of $E(k)$ spectrum affects the numerically computed values of the particular correction functions. The obtained results have been compared with the available experimental data.

1. Introduction

The effect of the finite spatial resolution of hot-wire technique reveals itself especially in the measurements of the fine-grained turbulence when the linear dimension of a hot-wire probe is smaller than the viscous length scale. In order to avoid the systematic error which under these circumstances may be introduced into the experimental results, the proper correction formula must be included in the data processing.

In metrology of turbulent flows this problem is well known since the first work of Dryden et al. (1937). In the later years a great deal of efforts was made to find the best form of the required correction formula. One should mention here the paper by Uberoi and Kovasznay (1953) and by Wyngaard (1968), who derived the correction formula for one-dimensional turbulence energy spectrum $\varphi = F_{11}^m/F_{11}$. The starting point of their approach was the assumption that the hot-wire-sensitivity $s_1(\xi_2)$ to velocity fluctuations u_1 is uniformly distributed along the wire length l. However, as stated in the recent works of Moryń-Kucharczyk (1988), Domagała (1992), Domagała et al. (1992), Elsner et al. (1993), sensitivity $s_1(\xi)$ should be taken proportional to the local hot-wire temperature $\Theta_w(\xi_2)$ which, due to the cooling effect of the probe prongs, is not constant along the wire length. In virtue of this assumption Domagała (1992) and Elsner et al. (1993) formulated the correction function $\Gamma_\varepsilon^m/\varepsilon$ which took into account the influence of the wire length on the measured value of turbulence energy dissipation ε^m.

In all the works mentioned above the correction functions φ and Γ_ε were based on the assumption that the 3D turbulence energy spectrum $E(k)$ in a high wavenumber range could be expressed by the Pao's formula. However, it should be noted here that there were also the critical opinions concerning the restricted applicability of the Pao's spectrum law to the low Reynolds number flows (see e.q. Epik (1980)).On the other hand, as stated by Moryń-Kucharczyk (1988), because of the quotiental character of the correction functions which tends to compensate for the influence of the adopted function $E(K)$, its form should not practically affect the computed values of φ and Γ_ε. The present study is just intended to check the validility of the above hypothesis.

2. Theoretical background

As the full theoretical derivation of the required correction functions has been already given by Domagała (1992) and Elsner et al. (1993), so there is no need to present here this procedure in detail. Thus, only the most fundamental equations will be cited and described.

Correction function for one-dimensional turbulence energy spectrum is defined as

$$\varphi(k_1,l) = \frac{F_{11}^m(k_1,l)}{F_{11}(k_1)} \tag{2.1}$$

where $F_{11}^m(k_1,l)$ is the value expected in experiment performed with a hot-sensor of a given wire length l and $F_{11}(k_1)$ is a "true" 1D energy spectrum, which would be obtained in the ideal point measurements $(l=0)$. The functions F_{11} and F_{11}^m may be expressed by

$$F_{11}(k_1) = \int\limits_{k_1}^{\infty} \frac{E(k)}{k^3}(k^2-k_1^2)\,dk \tag{2.2}$$

$$F_{11}^m(k_1,l) = \int\limits_{k_1}^{\infty}\Big[\frac{2}{\pi}\int\limits_0^{\kappa} T_2(k_2,l)\frac{dk_2}{\sqrt{\kappa^2-k_2^2}}\Big]\frac{E(k)}{k^3}(k^2-k_1^2)\,dk \tag{2.3}$$

where $\kappa^2 = k_2^2+k_3^2$, $k^2=k_1^2+\kappa^2$ and k_i is a component of a wavenumber vector.

The expression $T_2(k_2, l)$ in Eq (2.3)

$$T_2(k_2, l) = \Big| \int_{-\infty}^{\infty} S_1(\xi_2) \exp(ik_2\xi_2)\, d\xi_2 \Big|^2 \tag{2.4}$$

is the square of the Fourier transform of the function $s_1(\xi_2, l)$ describing the distribution of a hot-wire sensitivity to velocity fluctuations along the wire length.

The effect of the adopted form of sensitivity distribution has already been thoroughly analysed by Domagała et al. (1992). For a given $S - 1(\xi_2)$ the only term in Eq (2.3) which would affect the correction function $\varphi(k_1, l)$ is $E(k)$ which describes the 3D turbulence energy spectrum. Via Eqs (2.2) and (2.3) this term appears in both the numerator and denominator of the expression (2.1) describing the correction function $\varphi(k_1, l)$.

In the literature dealing with the problem there are at least several different suggestions for $E(k)$ compiled among others by Greichen (1979). At a sufficiently high Reynolds number a specially good representation of a 3D-spectrum function is ensured by the Pao's formula, Pao (1965)

$$E(k) = \alpha\varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}} \exp\Big[-\frac{2}{3}\alpha(k\eta)^{\frac{4}{3}}\Big] \tag{2.5}$$

where η is a Kolmogorov's micro-length scale and α is an universal constant originally assumed to be $\alpha = 1.7$. It should be noted here that e.q. the Heisenberg's formula, Heisenberg (1948)

$$E(k) = \alpha(\varepsilon\nu^5)^{\frac{1}{4}}(k\eta)^{-\frac{5}{3}}\Big[1 + \frac{27}{8}\alpha^3(k\eta)^4\Big]^{-\frac{4}{3}} \tag{2.6}$$

or Lin's proposal (1972)

$$E(k) = \alpha\varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}[1 + (k\eta)^{\frac{2}{3}}] \exp\alpha\Big[\frac{3}{2}(k\eta)^{\frac{4}{3}} + (k\eta)^2\Big] \tag{2.7}$$

are also frequently recommended. For comparison, all these functions in the form of energy dissipation spectra $D(k) = k^2E(k)$ are plotted in Fig.1. As can be seen, the highest values of $D(k)$ correspond to the Heisenberg's formula, while the lowest ones to the Pao's spectrum. Thus, for the further comparative analysis, only these two functions have been finally accepted.

The turbulence energy dissipation $\varepsilon^m(l)$ may be calculated from the dissipation spectra $\varepsilon_{11}^m(k_1, l) = k_1 F_{11}^m(k_1, l)$ measured with the hot sensor of an arbitrary wire length l. For the assumed isotropic turbulence structure we have

$$\varepsilon^m(l) = 15\nu \int_0^{\infty} D_{11}(k_1, l)\, dk_1 = 15\nu \int_0^{\infty} k_1^2 F_{11}^m(k_1, l)\, dk_1 \tag{2.8}$$

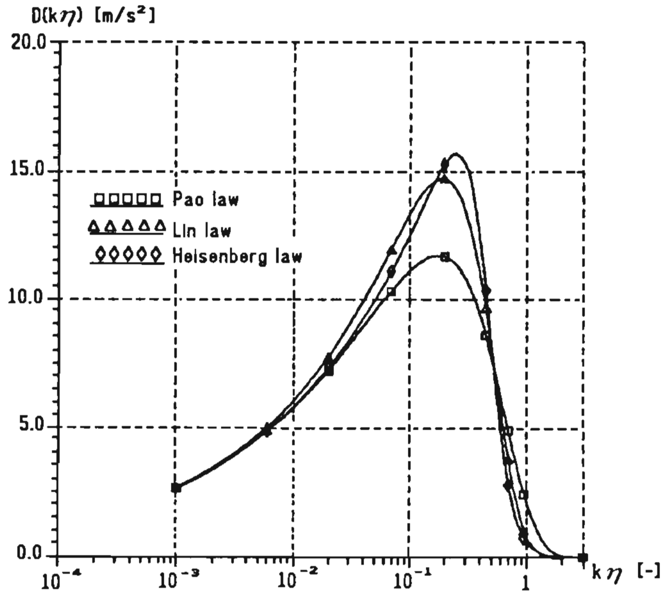

Fig. 1. Energy dissipation spectra according to the Pao, Lin and Heisenberg laws, respectively

and respectively

$$\varepsilon = \varepsilon^m(l=0) = 15\nu \int\limits_0^\infty D_{11}(k_1)\, dk_1 = 15\nu \int\limits_0^\infty k_1^2 F_{11}(k_1)\, dk_1 \tag{2.9}$$

The ratio of Eqs (2.8) and (2.9) gives us the correction function for turbulence energy dissipation

$$\Gamma_\varepsilon(l) = \frac{\int\limits_0^\infty k_1^2 F_{11}^m(k_1,l)\, dk_1}{\int\limits_0^\infty k_1^2 F_{11}(k_1)\, dk_1} = \frac{\int\limits_0^\infty k_1^2 \varphi(k_1,l) F_{11}(k_1)\, dk_1}{\int\limits_0^\infty k_1^2 F_{11}(k_1)\, dk_1} \tag{2.10}$$

It is worth noting that with the help of the Kolmogorov's micro-length scale η both the φ and Γ_ε correction functions may also be expressed in the more universal form as

$$\varphi = \varphi\Big(k_1\eta, \frac{l}{\eta}\Big) \qquad\qquad \Gamma_\varepsilon = \Gamma_\varepsilon\Big(\frac{l}{\eta}\Big)$$

3. Computational results and their analysis

The comparison of the functions $\varphi(k_1\eta, l/\eta)$ based on the hot-wire sensitivity proportional to $\Theta_w(\xi_2)$ and corresponding to Pao's and Heisenberg's formula is depicted in Fig.2.

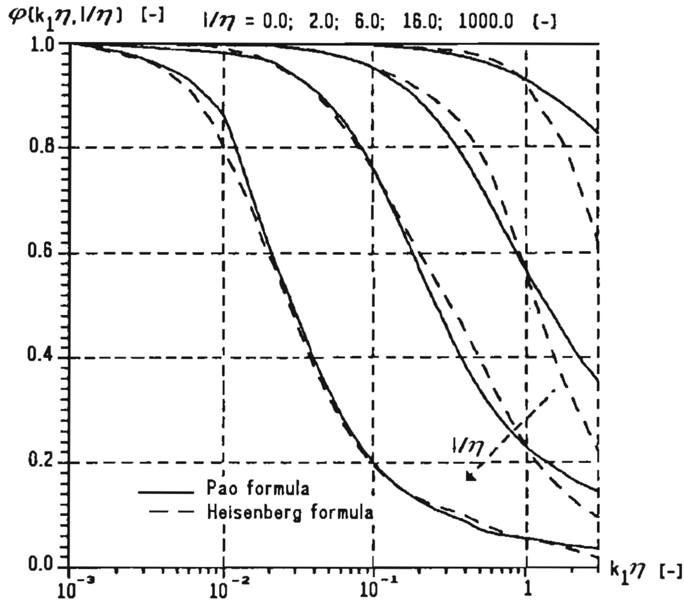

Fig. 2. Comparison of $\varphi\left(k_1\eta, \frac{l}{\eta}\right)$ functions given by Pao's and Heisenberg's formulae

The differences between both the families of curves are rather small and more visible divergences may only be observed in a certain range of variables $k_1\eta$ and l/η. This statement is confirmed by the relative deviation of the correction function

$$\frac{\Delta\varphi}{\varphi} = \frac{\varphi_H - \varphi_P}{\varphi_H} \tag{3.1}$$

plotted in terms of l/η in Fig.3. As can be seen, the ratio $\Delta\varphi/\varphi$ takes its highest values (up to 12% for $k_1\eta \simeq 0.633$) within the limit of $l/\eta < 10 - 26$ which represents typical laboratory flow conditions.

The similar influence is exerted by the adopted form of $E(k)$-spectrum on the correction function $\Gamma_\varepsilon(l/\eta)$ presented in Fig.4. The curve corresponding to the Heisenberg's formula gives evidently higher values what is in agreement with the tendency previously observed in Fig.1. It should be emphasized that the curves plotted here are of an universal character and have been obtained from numerical calculation without the help of any experimental data. The relative

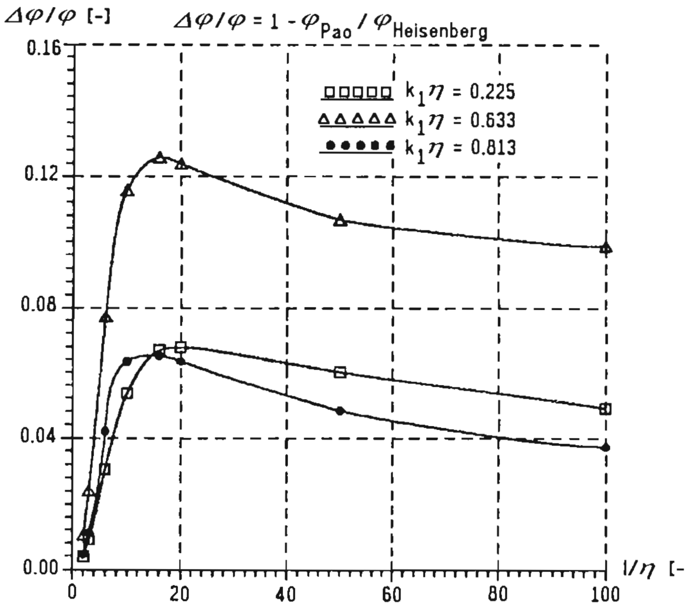

Fig. 3. Relative deviation of the correction functions $\frac{\Delta\varphi}{\varphi}$

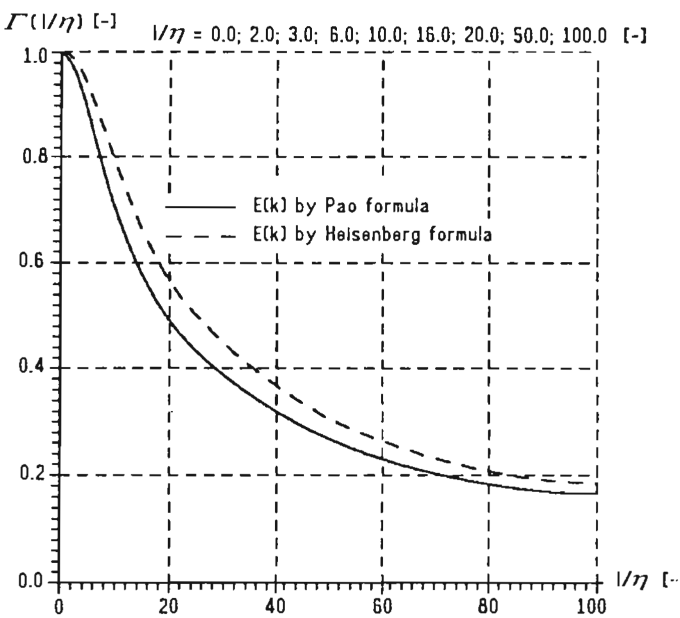

Fig. 4. Correction functions $\Gamma_\varepsilon\left(\frac{l}{\eta}\right)$ of energy dissipation

deviation (Fig.5)

$$\frac{\Delta\Gamma_\varepsilon}{\Gamma_\varepsilon} = \frac{\Gamma_H - \Gamma_P}{\Gamma_H} \tag{3.2}$$

has the maximum of about 13% at $l/\eta \simeq 25$. As the true form of $E(k)$-spectrum in a high wavenumber range is not precisely known, thus the curve shown in Fig.5 gives only a general idea on the possible scatter of results corresponding to the adopted $E(k)$-function.

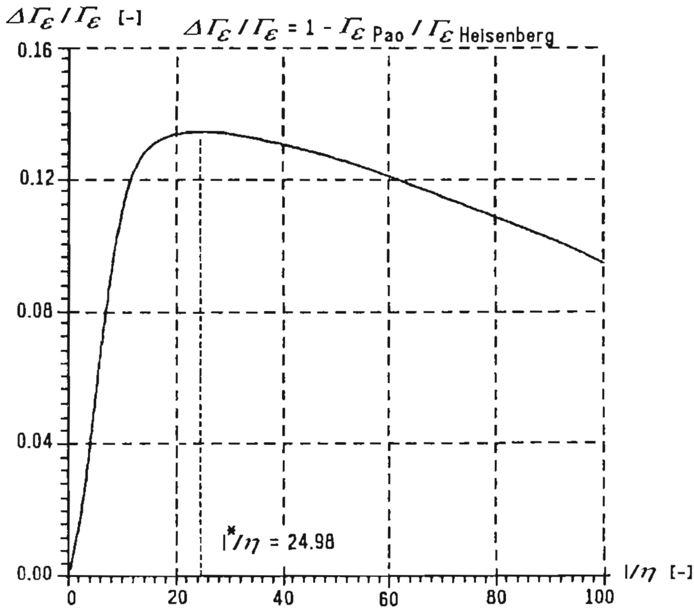

Fig. 5. Relative deviation $\frac{\Delta\Gamma_\varepsilon}{\Gamma_\varepsilon}$ of the correction functions

4. Comparison with the experimental data

In order to verify the validity of the numerical calculations presented here, their results have been compared with the available experimental data.

Fig.6 shows the normalized dissipation spectra presented in commonly used nondimensional coordinates. The experimental data plotted after Stewart and Townsend (1989) as well as Domagała (1992) show a much better agreement with the numerical results corresponding to the Pao's formula. This tendency is also confirmed by the correction function $(\Gamma_\varepsilon)_P$ plotted in Fig.7, which coincides quite well with the values obtained by Elsner et al. (1993). On the other hand, the

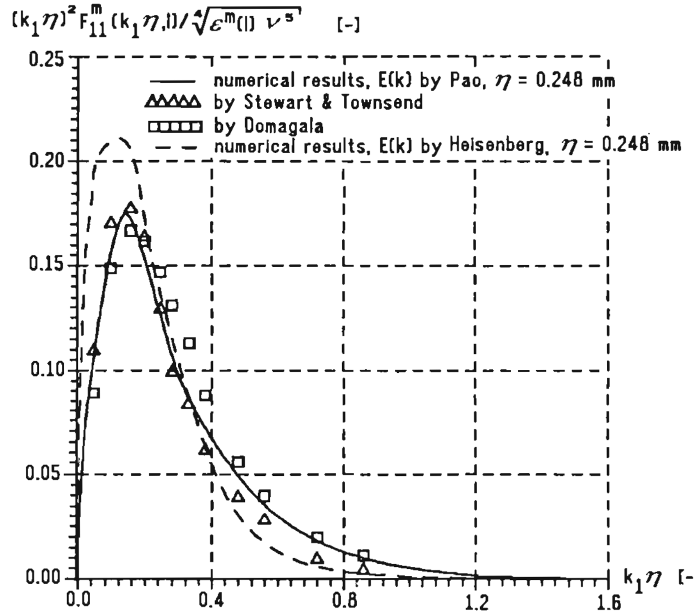

Fig. 6. Comparison of the calculated and experimental dissipation spectra plotted in nondimensional coordinates

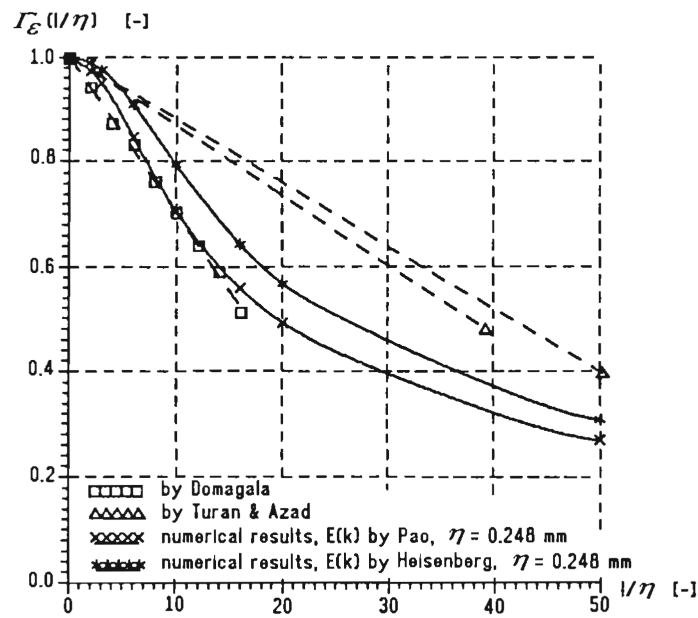

Fig. 7. Comparison of the calculated and experimental Γ_ε functions

linear regression lines found by Turan and Azad (1989) exhibit neither quantitive nor qualitative resemblance with the numerical results. This disparity may be, in a certain measure, explained by the fact that the experiment of Turan and Azad was performed in channel and diffuser flows which did not exhibit the isotropy and homogeneity of a turbulence structure.

5. Concluding remarks

The results of computational analysis presented here have, shaken the existing views that the form of 3D turbulence energy spectrum has no influence on the correction functions taking into account the imperfect spatial resolution of a hot-wire anemometry. The comparative calculations based on the Pao's and Heisenberg's formulae for $E(k)$ spectra have shown that the maximum divergence in φ and Γ_ε values is noted within the limits of wire length to the Kolmogorov's scale ratio $l/\eta \simeq 12 \div 26$ and does not exceed 13%. Numerical results have been compared with the available experimental data obtained in a grid generated turbulence, because this kind of flow ensures the isotropy and homogeneity of a turbulence structure being the basis of the predicted theoretical relations. The analysis of such a comparison allows to draw the conclusion that the Pao's formula for $E(k)$ spectrum may be regarded as a good basis for the construction of the required correction functions.

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Uwagi o przestrzennej rozdzielczości techniki termoanemometrycznej

Streszczenie

Ze względu na ograniczoną rozdzielczość przestrzenną czujnika termoanemometrycznego w procesie opracowywania uzyskanych przy jego pomocy wyników konieczne jest wprowadzenie odpowiedniej procedury korekcyjnej. Przy formułowaniu odpowiednich funkcji korekcyjnych opartych na zachowaniu homogenicznej i izotropowej struktury turbulencji, niezbędne jest określenie funkcji $E(k)$ opisującej 3-wymiarowe spektrum energii turbulencji w obszarze wysokich liczb falowych. W pracy rozważono jaki wpływ wywiera postać funkcji $E(k)$ na numeryczne wyznaczanie funkcji korekcyjnych. Uzyskane wyniki porównane zostały z dostępnymi danymi literaturowymi.

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