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FLEXURAL WAVES IN A FLUID-SATURATED POROUS STRATUM EMBEDDED IN AN ELASTIC INFINITE MEDIUM

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The flexural waves in a fluid-saturated porous stratum embedded in an homogeneous, isotropic infinite medium, impermeable to a fluid flow have been studied in the paper. The outer half-space has been assumed to reveal the same mechanical properties. Basing on the Biot's equations for wave propagation in the porous media and the theory of elasticity for the outer half-spaces, the dispersion equation relating the phase velocity to the wave number has been derived in a complex form. The simplified version of the dispersion equation for which the dissipation caused by the relative fluid flow is neglected has been also presented. The roots of this equation have been found numerically. The way of passing to the well-known classical results has also been shown.

1. Introduction

The propagation of waves in layered media due to its important applications in geophysics and seismology, has been the subject of several investigations. Perhaps the best source of inforamtion on this subject is the monograph of Ewing, Jardetzky and Press [1]. However, most of the work done in this field does not concern a multilayer medium including a fluid-saturated porous layer. To the best of Author's knowledge, there are few papers only in which the propagation of waves in such media has been investigated. It has also to be noted that majority of the work on multilayered media considers mainly the propagation of shears waves because of the relative mathematical simplicity.

Many authors have considered Love wave propagation in a fluid-saturated porous layer resting on an elastic half-space. The work of Deresiewicz [2], Chattopadhyay and De [3], Chakraborty and Dey [4], Chattopadhyay et al. [5] and Kończak [6] can be mentioned for instance. The propagation of SH-type waves in porous layer of nonuniform thickness has been examined by Chattopadhyay et.al. [7] and the effect of a double surface layer on the propagation of Love waves has been investigated by Deresiewicz [8].

Most recently the propagation of shear waves in a multilayer medium including a fluid–saturated porous stratum has been studies by Kończak [9]. The Rayleigh–Lamb type waves in a fluid–filled porous layer embedded in an infinite impermeable elastic medium has been investigated receutly [10].

In all papers mentioned above the equations formulated by Biot [11], [12] have been applied to describe the behaviour of a fluid–saturated porous medium. Although another theories has been recently proposed be several investigators e.g. [13 ÷ 16] the Biot's theory remines widely accepted for the study of wave motions in fluid–saturated porous media.

The purpose of the present paper is to examine a more complex problem i.e. the propagation of flexural waves in a fluid–saturated porous stratum embedded in an infinite impermeable elastic medium. Basing on the Biot's theory for wave propagation in the porous media [11] and the theory of elasticity for the outher half–spaces, the dispersion equations has been derived. It relates the phase velocity of propagation to the wave number and shear and longitudinal wave speeds of the porous layer and the half–spaces. The simplified version of the dispersion equation in which the dissipation caused by the relative fluid flow has been omitted, has also been obtained.

2. Formulation of the problem and governing equations

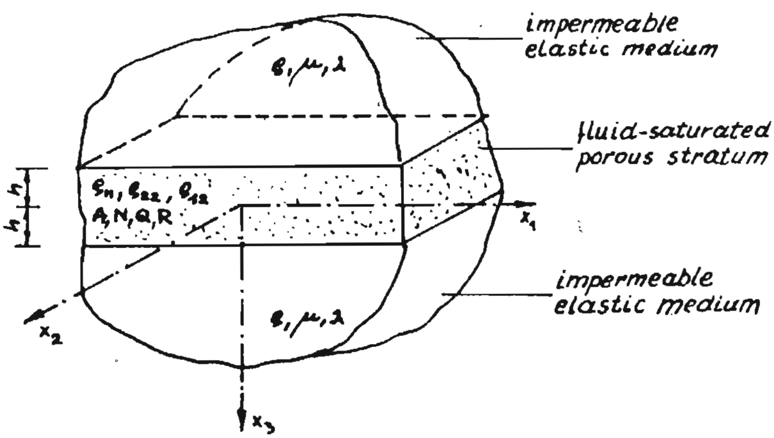

Fig. 1. Schematic outline of the problem considered

Let us consider a fluid–saturated porous stratum of finite thickness $2h$ embedded in an infinite isotropic elastic medium impermeable for the fluid flow, as shown in Fig.1. The outer semi-infinite spaces and the porous layer are assumed to be perfectly bonded at the interfaces. We refer the considered medium to a (x_1, x_2, x_3) coordinate system in which the $(x_1 - x_2)$ – plane coincides with mid-plane of the porous layer. The x_3 –axis is taken vertically downward and the x_1 –axis is chosen in the direction of wave propagation.

As our point of departure we take the basic equations that describe the behaviour of the considered medium. We have
– for the outer semi-infinite layers [17]

$$\mu\nabla^2\boldsymbol{v} + (\lambda+\mu)\mathrm{graddiv}\boldsymbol{v} = \rho\frac{\partial^2\boldsymbol{v}}{\partial t^2} \tag{2.1}$$

$$\boldsymbol{\tau} = \mu\Big(\mathrm{grad}\boldsymbol{v} + (\mathrm{grad}\boldsymbol{v})^{\top}\Big) + \lambda\mathrm{div}\boldsymbol{v}\mathbf{1} \tag{2.2}$$

– for the fluid-saturated porous layer [11]

$$\begin{aligned} N\nabla^2\boldsymbol{u} + (A+N)\mathrm{graddiv}\boldsymbol{u} + Q\mathrm{graddiv}\boldsymbol{U} &= \\ = \rho_{11}\frac{\partial^2\boldsymbol{u}}{\partial t^2} + \rho_{12}\frac{\partial^2\boldsymbol{U}}{\partial t^2} - b\frac{\partial}{\partial t}(\boldsymbol{U}-\boldsymbol{u}) \\ Q\mathrm{graddiv}\boldsymbol{u} + R\mathrm{graddiv}\boldsymbol{U} = \rho_{12}\frac{\partial^2\boldsymbol{u}}{\partial t^2} + \rho_{22}\frac{\partial^2\boldsymbol{U}}{\partial t^2} + b\frac{\partial}{\partial t}(\boldsymbol{U}-\boldsymbol{u}) \end{aligned} \tag{2.3}$$

$$\begin{aligned} \boldsymbol{T}_s &= N\Big(\mathrm{grad}\boldsymbol{u} + (\mathrm{grad}\boldsymbol{u})^{\top}\Big) + (A\mathrm{div}\boldsymbol{u} + Q\mathrm{div}\boldsymbol{U})\mathbf{1} \\ \boldsymbol{T}_f &= (Q\mathrm{div}\boldsymbol{u} + R\mathrm{div}\boldsymbol{U})\mathbf{1} \end{aligned} \tag{2.4}$$

where $\boldsymbol{v}$, $\boldsymbol{u}$ and $\boldsymbol{U}$ are the displacement vectors of the outher layers, the solid skeleton and the fluid of the porous layer, respectively, $\boldsymbol{\tau}$, $\boldsymbol{T}_s$, $\boldsymbol{T}_f$ denote the stress tensors of the outer layers, the porous skeleton and the fluid, $\mathbf{1}$ is a unit tenosr, b determines the resistance of fluid flow through the porous skeleton and λ, μ, A, N, Q, R are the material constants. ρ is the density of the outer layers and ρ_{11}, ρ_{22}, ρ_{12} are the dynamical coefficients that are related to mass densities of the porous solid ρ_s and fluid ρ_f by [11]

$$\rho_{11} + \rho_{12} = (1-f)\rho_s, \qquad \rho_{12} + \rho_{22} = f\rho_f$$
$$\rho_{11} > 0, \qquad \rho_{22} > 0, \qquad \rho_{12} \le 0$$

where ρ_{12} is the coupling parameter.

If we now adopt the Helmholtz resolution for the displacement vectors $\boldsymbol{v}$, $\boldsymbol{u}$, and $\boldsymbol{U}$ as

$$\begin{aligned} \boldsymbol{v} &= \mathrm{grad}p + \mathrm{curl}\boldsymbol{q} & \mathrm{div}\boldsymbol{q} &= 0 \\ \boldsymbol{u} &= \mathrm{grad}\varphi + \mathrm{curl}\boldsymbol{\psi} & \mathrm{div}\boldsymbol{\psi} &= 0 \\ \boldsymbol{U} &= \mathrm{grad}\chi + \mathrm{curl}\boldsymbol{\eta} & \mathrm{div}\boldsymbol{\eta} &= 0 \end{aligned} \tag{2.5}$$

then equations (2.1) and (2.3) are satisfied, provided that

$$\left(\nabla^2 - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2}\right)p = 0, \qquad \left(\nabla^2 - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2}\right)q = 0 \tag{2.6}$$

$$\begin{aligned} \left(P\nabla^2 - \rho_{11}\frac{\partial^2}{\partial t^2} - b\frac{\partial}{\partial t}\right)\varphi + \left(Q\nabla^2 - \rho_{12}\frac{\partial^2}{\partial t^2} + b\frac{\partial}{\partial t}\right)\chi = 0 \\ \left(Q\nabla^2 - \rho_{12}\frac{\partial^2}{\partial t^2} + b\frac{\partial}{\partial t}\right)\varphi + \left(R\nabla^2 - \rho_{22}\frac{\partial^2}{\partial t^2} - b\frac{\partial}{\partial t}\right)\chi = 0 \end{aligned} \tag{2.7}$$

$$\begin{aligned} \left(N\nabla^2 - \rho_{11}\frac{\partial^2}{\partial t^2} - b\frac{\partial}{\partial t}\right)\psi - \left(\rho_{12}\frac{\partial^2}{\partial t^2} - b\frac{\partial}{\partial t}\right)\eta = 0 \\ \left(\rho_{12}\frac{\partial^2}{\partial t^2} - b\frac{\partial}{\partial t}\right)\psi + \left(\rho_{22}\frac{\partial^2}{\partial t^2} + b\frac{\partial}{\partial t}\right)\eta = 0 \end{aligned} \tag{2.8}$$

where $P = A + 2N$, $c_1^2 = (\lambda + 2\mu)/\rho$, $c_2^2 = \mu/\rho$.

Eliminating from the system of equations (2.7) first the quantity φ and then χ we arrive at the following decoupled equations for φ and χ

$$\left\{a_0\nabla^2\nabla^2 - \left(a_1\frac{\partial^2}{\partial t^2} + bH\frac{\partial}{\partial t}\right)\nabla^2 + \left(a_2\frac{\partial^4}{\partial t^4} + b\rho^*\frac{\partial^3}{\partial t^3}\right)\right\}(\varphi, \chi) = 0 \tag{2.9}$$

where

$$a_0 = PR - Q^2, \qquad a_1 = P\rho_{22} + R\rho_{11} - 2Q\rho_{12}, \qquad a_2 = \rho_{11}\rho_{22} - \rho_{12}^2$$

$$\rho^* = \rho_{11} + \rho_{22} + 2\rho_{12}, \qquad H = P + 2Q + R.$$

If a similar operation is carried out for the system of equations (2.8) then we obtain

$$\left\{N\left(\rho_{22}\frac{\partial^2}{\partial t^2} + b\frac{\partial}{\partial t}\right)\nabla^2 - \left(a_2\frac{\partial^4}{\partial t^4} + \rho^* b\frac{\partial^3}{\partial t^3}\right)\right\}(\psi, \eta) = 0. \tag{2.10}$$

By substituting (2.5) for the parts of equations (2.2) and (2.4) the stresses may be written in a cartesian coordinate system as

$$\tau_{ij} = \mu\{2p_{,ij} + e_{ikl}q_{l,kj} + e_{jkl}q_{l,ki}\} + \lambda\nabla^2 p\delta_{ij} \tag{2.11}$$

$$\sigma_{ij} = N\{2\varphi_{,ij} + e_{ikl}\psi_{l,kj} + e_{jkl}\psi_{l,ki}\} + (A\nabla^2\varphi + Q\nabla^2\chi)\delta_{ij} \tag{2.12}$$

$$\sigma = Q\nabla^2\varphi + R\nabla^2\chi \tag{2.13}$$

where δ_{ij} is the Kronecker delta, e_{rsq} is the permutation tensor.

Since we are dealing with the waves propagating in the (x_1, x_2) -plane all the field variables depend only on x_1, x_3 and t. In this case the displacement vectors become $\boldsymbol{v} = (v_1, 0, v_3)$, $\boldsymbol{u} = (u_1, 0, u_3)$, $\boldsymbol{U} = (U_1, 0, U_3)$ and for the vector potentials $\boldsymbol{q}$, $\boldsymbol{\psi}$, and $\boldsymbol{\eta}$ only the x_2 component is relevant. Thus we have

$$\begin{aligned} &q_1 = q_3 = 0, \qquad q_2 \equiv q, \qquad \psi_1 = \psi_3 = 0 \\ &\psi_2 \equiv \psi, \qquad \eta_1 = \eta_3 = 0, \qquad \eta_2 \equiv \eta. \end{aligned} \tag{2.14}$$

The equations of motion (2.1) and (2.3) have to satisfy the continuity conditions on displacements and stresses at the interfaces between the fluid–saturated porous layer and the outer semi–infinite spaces, and the boundary conditions for the fluid filling the pores as well. At $x_3 = \pm h$ these conditions are given by

$$v_1 = u_1, \qquad v_3 = u_3, \qquad \tau_{33} = \sigma_{33} + \sigma, \qquad \tau_{31} = \sigma_{31}, \qquad \sigma_{,3} = 0 \tag{2.15}$$

which in terms of the displacement potentials (2.5) become

$$\begin{aligned} &p_{,1} - q_{,3} = \varphi_{,1} - \psi_{,3} \\ &p_{,3} + q_{,1} = \varphi_{,3} + \psi_{,1} \\ &2\mu(p_{,33} + q_{,13}) + \lambda\nabla^2 p = 2N(\varphi_{,33} + \psi_{,13}) + (A+Q)\nabla^2\varphi + (Q+R)\nabla^2\chi \\ &\mu(2p_{,31} + q_{,11} - q_{,33}) = N(2\varphi_{,31} + \psi_{,11} - \psi_{,33}) \\ &Q\nabla^2\varphi + R\nabla^2\chi)_{,3} = 0 \end{aligned} \tag{2.16}$$

where the last condition of (2.15) or (2.16) denotes that the boundary $x_3 = \pm h$ is impermeable to the fluid flow.

Moreover, the solutions to equations (2.6) have to be augemented by regularity conditions at infinity.

Equations (2.6), (2.9) and (2.10) with the conditions (2.15) serve as governing equations of the problem censidered.

3. Solution to the problem, dispersion equation

We are concerned with the wave changing harmonically with time and propagating along the x_1 –direction. Thus we seek, as usual, solutions to the displacement potentials (2.5) in the form

$$\begin{aligned} &p(x_1, x_3, t) = p^*(x_3)e^{i\varphi(x_1,t)}, \qquad q(x_1, x_3, t) = q^*(x_3)e^{i\varphi(x_1,t)} \\ &\varphi(x_1, x_3, t) = \varphi^*(x_3)e^{i\varphi(x_1,t)}, \qquad \psi(x_1, x_3, t) = \psi^*(x_3)e^{i\varphi(x_1,t)} \\ &\chi(x_1, x_3, t) = \chi^*(x_3)e^{i\varphi(x_1,t)}, \qquad \eta(x_1, x_3, t) = \eta^*(x_3)e^{i\varphi(x_1,t)} \end{aligned} \tag{3.1}$$

where k is a wave number, ω is the angular frequency, $\varphi(x_1, t) = kx_1 - \omega t$ and $i^2 = -1$.

Putting expressions (3.1) into equations (2.6), (2.9) and (2.10) with the aid of (2.14) we obtain

$$\Big(\frac{\partial^2}{\partial x_3^2}-\kappa_1^2\Big)p^*=0,\qquad \Big(\frac{\partial^2}{\partial x_3^2}-\kappa_2^2\Big)q^*=0 \tag{3.2}$$

$$\Big(\frac{\partial^2}{\partial x_3^2}-m_1^2\Big)\Big(\frac{\partial^2}{\partial x_3^2}-m_2^2\Big)(\varphi^*,\chi^*)=0 \tag{3.3}$$

$$\Big(\frac{\partial^2}{\partial x_3^2}-m_3^2\Big)(\psi^*,\eta^*)=0 \tag{3.4}$$

where

$$\kappa_j^2=k^2-\frac{\omega^2}{c_j^2},\qquad j=1,2,\quad c_1^2=\frac{\lambda+2\mu}{\rho},\quad c_2^2=\frac{\mu}{\rho} \tag{3.5}$$

$$m_{1,2}^2=k^2-\frac{\omega^2}{c_L^2}\zeta_{1,2},\qquad c_L^2=\frac{H}{\rho^*} \tag{3.6}$$

$$m_3^2=k^2-\frac{\omega^2}{c_T^2}\zeta_3,\qquad c_T^2=\frac{N}{\rho^*}. \tag{3.7}$$

In (3.6) and (3.7) the following abbreviations are introduced

$$\zeta_{1,2}=\frac{H(a_1\omega+\mathrm{i}bH)}{2a_0\rho^*\omega}\Big(1\pm\sqrt{1-4\frac{a_0(a_2\omega^2-\mathrm{i}b\rho^*\omega)}{(a_1\omega+\mathrm{i}bH)^2}}\Big) \tag{3.8}$$

$$\zeta_3=\alpha_1+\mathrm{i}\alpha_2,\qquad \alpha_1=\frac{a_2\rho_{22}\omega^2+b\rho^*}{\rho^*(\rho_{22}^2\omega^2+b^2)},\qquad \alpha_2=\frac{(a_2-\rho^*\rho_{22})b\omega}{\rho^*(\rho_{22}^2\omega^2+b^2)} \tag{3.9}$$

where $c_T/\mathrm{Re}(\sqrt{\zeta_3})$ is the velocity of shear wave and $c_L/\mathrm{Re}(\sqrt{\zeta_{1,2}})$, when the Biot's notation is applied, are the velocities of longitudinal waves of the first and second kind in the porous stratum. c_L and c_T are reference velocities of dilatational and rotational waves, respectively, for the case of no relative motion between fluid and solid [11].

The solutions to equations (3.2), for the outer layers with the aid of (3.1) are for $x_3\geq h$

$$\begin{aligned} p(x_1,x_3,t)&=A_1\mathrm{e}^{-\kappa_1(x_3-h)}\mathrm{e}^{\mathrm{i}(kx_1-\omega t)}\\ q(x_1,x_3,t)&=A_2\mathrm{e}^{-\kappa_2(x_3-h)}\mathrm{e}^{\mathrm{i}(kx_1-\omega t)}\end{aligned} \tag{3.10}$$

and for $x_3\leq -h$

$$\begin{aligned} p(x_1,x_3,t)&=-A_1\mathrm{e}^{\kappa_1(x_3+h)}\mathrm{e}^{\mathrm{i}(kx_1-\omega t)}\\ q(x_1,x_3,t)&=A_2\mathrm{e}^{\kappa_2(x_3+h)}\mathrm{e}^{\mathrm{i}(kx_1-\omega t)}\end{aligned} \tag{3.11}$$

where the requirement of vanishing the disturbance as $x_3 \to \pm\infty$ have been taken into account. In the solutions given above A_1 and A_2 are the arbitrary constants.

Let us pass now to examine a disturbance in the fluid–saturated porous layer occupying the region $-h \le x \le h$.

The solutions to equations (3.3) and (3.4) with the aid of (3.1) are

$$\begin{aligned} \varphi(x_1, x_3, t) &= (A_3 \sinh\ m_1 x_3 + A_4 \sinh\ m_2 x_3) e^{i(kx_1 - \omega t)} \\ \chi(x_1, x_3, t) &= (\bar{A}_3 \sinh\ m_1 x_3 + \bar{A}_4 \sinh\ m_2 x_3) e^{i(kx_1 - \omega t)} \\ \psi(x_1, x_3, t) &= A_5 \cosh\ m_3 x_3 e^{i(kx_1 - \omega t)} \\ \eta(x_1, x_3, t) &= \bar{A}_5 \cosh\ m_3 x_3 e^{i(kx_1 - \omega t)} \end{aligned} \tag{3.12}$$

where $A_1,...,\bar{A}_5$ are arbitrary constants. Of course, these constans are not entirely independent of each other, because of equations (2.7) and (2.8). Using equations $(2.7)_2$ and $(2.8)_2$, the relations between those constants can be written in the following form

$$\begin{aligned} \bar{A}_{3,4} &= -r_{1,2} A_{3,4}, \qquad r_{1,2} = \frac{Q(k^2 - m_{1,2}^2) - \rho_{12}\omega^2 + ib\omega}{R(k^2 - m_{1,2}^2) - \rho_{22}\omega^2 - ib\omega} \\ \bar{A}_5 &= -r_3 A_5, \qquad r_3 = \frac{\rho_{12}\rho_{22}\omega^2 - b^2}{(\rho_{22}\omega)^2 + b^2} - i\frac{(\rho_{12} + \rho_{22})b\omega}{(\rho_{22}\omega)^2 + b^2}. \end{aligned} \tag{3.13}$$

Putting the solutions (3.10) or (3.11) and (3.12) together with the relations (3.13) into the conditions (2.16), we obtain a system of five homogeneous linear equations in $A_1,...,A_5$.

$$\begin{aligned} & ikA_1 + \kappa_2 A_2 - ik(S_1 A_3 + S_2 A_4) + m_3 S_3 A_5 = 0 \\ & -\kappa_1 A_1 + ikA_2 - m_1 C_1 A_3 - m_2 C_2 A_4 - ikC_3 A_5 = 0 \\ & \alpha^* A_1 - 2ik\mu\kappa_2 A_2 - \beta_1 S_1 A_3 - \beta_2 S_2 A_4 - 2ikN m_3 S_3 A_5 = 0 \\ & 2ik\mu\kappa_1 A_1 - \mu(k^2 + \kappa_2^2) A_2 + 2ikN(m_1 C_1 A_3 + m_2 C_2 A_4) - N(k^2 + m_3^2) C_3 A_5 = 0 \\ & \gamma_1 C_1 A_3 + \gamma_2 C_2 A_4 = 0 \end{aligned} \tag{3.14}$$

where we introduced the following notation

$$\begin{aligned} & S_j = \sinh\ m_j h, \qquad C_j = \cosh\ m_j h, \qquad (j = 1, 2, 3) \\ & \alpha^* = (\lambda + 2\mu)\kappa_1^2 - \lambda k^2 \\ & \beta_{1,2} = [P + Q - r_{1,2}(Q + R)] m_{1,2}^2 - [A + Q - r_{1,2}(Q + R)] k^2 \\ & \gamma_{1,2} = m_{1,2}(k^2 - m_{1,2}^2)(r_{1,2} R - Q). \end{aligned} \tag{3.15}$$

In order to have a nontrivial solution to equations (3.14), the determinant of the coefficient matrix must be equal to zero. This condition gives us

$$\begin{vmatrix} ik & \kappa_2 & -ik(S_1-\gamma_{12}C_{12}S_2) & m_3S_3 \\ \kappa_1 & -ik & (m_1-m_2\gamma_{12})C_1 & ikC_3 \\ -\alpha^* & 2ik\mu\kappa_2 & \beta_1S_1-\beta_2\gamma_{12}C_{12}S_2 & 2ikNm_3S_3 \\ 2ik\mu\kappa_1 & \mu(k^2+\kappa_2^2) & 2ikN(m_1-m_2\gamma_{12})C_1 & -N(k^2+m_3^2)C_3 \end{vmatrix} = 0 \tag{3.16}$$

which is the dispersion equation for the considered problem.

In (3.16) the relation

$$A_4 = -\gamma_{12}A_3, \qquad \gamma_{12} = \frac{\gamma_1}{\gamma_2}, \qquad C_{12} = \frac{C_1}{C_2} \tag{3.17}$$

that results from the last equation of the system (3.14) has been applied.

It is easy to see that the elements of (3.16) are complex due to the quantities $m_{1,2}$ and m_3 expressed by the formulae (3.6) and (3.7) with the aid of (3.8) and (3.9). Thus the wave number k is also complex and hence κ_q $(q = 1,2)$. Therefore it is difficult to obtain any analytical informaton from the dispersion equation (3.16). It seems that the only way to get the roots of (3.16) is to apply an approximation method.

The dispersion equation (3.16) simplifies somehow if the dissipation caused by the relative fluid flow is neglected. This can be acchieved if we assume $b = 0$ (see equation (2.3)). Such an assumption corresponds to the case of elastic waves propagation only. Now, according to the above assumption it results from (3.8) and (3.9) that $\mathrm{Im}(\zeta_j) = 0$ $(j = 1,2,3)$ and thus all the ζ_j are real. Moreover, also the wave number k as well as κ_q $(q = 1,2)$ are real, and the simplified dispersion equation becomes

$$\Big(\frac{\rho^*}{\rho}\Big)^2 a_{11}D_1 + \Big(\frac{\rho^*}{\rho}\Big)d_1D_2 - d_1d_2a_{41}D_3 = 0 \tag{3.18}$$

where we introduced the following notation

$$\begin{aligned}
D_1 &= (1+\nu_3^2)(\beta_1^*S_1^* - \beta_2^*\gamma_{12}C_{12}S_2^*)C_3^* - 4d_4\nu_3(\bar{\nu}_1 - \nu_2\gamma_{12})C_1^*S_3^*,\\
D_2 &= d_4(a_{13}a_{34} - a_{23}a_{44}) - (a_{14}a_{33} - a_{24}a_{43}),\\
D_3 &= \bar{\nu}_1\nu_3C_1^*S_3^* - S_1^*C_3^* + (S_2^*C_3^* - \bar{\nu}_2\nu_3S_3^*C_2^*)\gamma_{12}C_{12},\\
d_1 &= \frac{c_2^2}{c_T^2}, \qquad d_2 = \frac{c_2^2}{c_L^2}, \qquad d_4 = \frac{N}{H}\\
a_{11} &= \nu_1\nu_2 - 1, \qquad a_{13} = S_1^* - \bar{\nu}_1\nu_2C_1^* - \bar{\gamma}_{12}(\bar{C}_{12}S_2^* - \nu_2\bar{\nu}_2C_1^*),\\
a_{14} &= \nu_3S_3^* + \nu_2C_3^*, \qquad a_{41} = 4\nu_1\nu_2 - (1+\nu_2^2)^2\\
a_{23} &= \nu_1(S_1^* - \bar{\gamma}_{12}\bar{C}_{12}S_2^*) + (\bar{\nu}_1 - \bar{\nu}_2\bar{\gamma}_{12})C_1^*, \qquad a_{24} = \nu_1\nu_3S_3^* + C_3^*\\
a_{33} &= 2\{\nu_1(\beta_1^*S_1^* - \beta_2^*\bar{\gamma}_{12}\bar{C}_{12}S_2^*) + d_4(1+\nu_2^2)(\bar{\nu}_1 - \bar{\nu}_2\bar{\gamma}_{12})C_1^*\}
\end{aligned}$$

$$a_{34} = 4\nu_1\nu_3 S_3^* + (1+\nu_2^2)(1+\nu_3^2)C_3^*$$
$$a_{43} = (1+\nu_2^2)(\beta_1^* S_1^* - \beta_2^*\bar{\gamma}_{12}\bar{C}_{12}S_2^*) + 4d_4\nu_2(\bar{\nu}_1 - \bar{\nu}_2\bar{\gamma}_{12})C_1^*$$
$$a_{44} = 2\{\nu_3(1+\nu_2^2)S_3^* + \nu_2(1+\nu_3^2)C_3^*\}.$$

In these expressions the following abbreviations are introduced

$$\begin{aligned}
&S_j^* = \sinh \bar{\nu}_j kh, \qquad C_j^* = \cosh \bar{\nu}_j kh, \qquad (j=1,2)\\
&S_3^* = \sinh \nu_3 kh, \qquad C_3^* = \cosh \nu_3 kh\\
&\beta_j^* = \bar{\nu}_j^2 - \frac{A+Q}{H} - \frac{Q+R}{H}\Big[\bar{\nu}_j^2 - \bar{r}_j(1-\nu_j^2)\Big], \qquad \bar{r}_j = \frac{Q\xi_j - \rho_{12}c_L^2}{R\xi_j - \rho_{22}c_L^2}\\
&\bar{\gamma}_j = \bar{\nu}_j(1-\bar{\nu}_j^2)(Q - \bar{r}_j R), \qquad \bar{\gamma}_{12} = \frac{\bar{\gamma}_1}{\bar{\gamma}_2}, \qquad \bar{C}_{12} = \frac{C_1^*}{C_2^*} \qquad\qquad (3.19)\\
&\xi_{1,2} = \frac{Ha_1}{2a_0\rho^*}\Big(1 \pm \sqrt{1-4\delta}\Big), \qquad \delta = \frac{a_0 a_2}{a_1^2}, \qquad \xi_3 = \frac{a_2}{\rho^*\rho_{22}}\\
&\nu_j^2 = 1 - \frac{c^2}{c_j^2}, \qquad \bar{\nu}_j^2 = 1 - \frac{c^2}{c_L^2}\xi_j, \qquad \nu_3^2 = 1 - \frac{c^2}{c_T^2}\xi_3.
\end{aligned}$$

Here $c = \omega/k$ is the phase velocity.

A condition for the existence of exponential decay is that both ν_1 and ν_2 (3.19) are real-valued. This implies that $c < c_2$. The condition $c_T/\xi_3 < c_2$ is also assumed to be hold.

Let us now consider a particular cases.

a) If we assume $\rho/\rho^* = 0$ (or $\mu = \lambda = 0$) what corresponds to an absence of outer semi-infinite spaces, equation (3.18) reduces to

$$\begin{aligned}(\nu_1\nu_2 - 1)\{(1+\nu_3^2)(\beta_1^* S_1^* - \beta_2^*\bar{\gamma}_{12}C_{12}S_2^*)C_3^* +\\ -4d_4(\bar{\nu}_1 - \bar{\nu}_2\bar{\gamma}_{12})\nu_3 C_1^* S_3^*\} = 0 \qquad (3.20)\end{aligned}$$

which will be satisfied provided that

$$\nu_1\nu_2 - 1 = 0 \qquad (3.21)$$

$$(1+\nu_3^2)(\beta_1^* S_1^* - \beta_2^*\bar{\gamma}_{12}\bar{C}_{12}S_2^*)C_3^* - 4d_4(\bar{\nu}_1 - \bar{\nu}_2\bar{\gamma}_{12})\nu_3 C_1^* S_3^* = 0. \qquad (3.22)$$

The equations (3.21) can be satisfied if the phase velocity is equal to zero, and thus the solutons are trivial.

The solution to (3.22) is the dispersion equation for flexural wave propagation in a fluid-saturated porous stratum with the edges $x_3 = \pm h$ free of stresses and impermeable to the fluid flow.

Furthermore, if we assume that $Q = R = 0$ what implies that the layer is not filled with fluid, then $\bar{\gamma}_{12} = 0$, $\beta_1^* = d_4(1+\nu_3^2)$, $\beta_2^* = 0$ and the equation (3.22) now yields

$$\frac{\tanh\left(kh\sqrt{1-\frac{c^2}{c_L^2}}\right)}{\tanh\left(kh\sqrt{1-\frac{c^2}{c_T^2}}\right)}=\frac{4c_T^4\sqrt{1-\frac{c^2}{c_L^2}}\sqrt{1-\frac{c^2}{c_T^2}}}{\left(2c_T^2-c^2\right)^2} \tag{3.23}$$

which, apart from the notation, is the classical dispersion equation for flexural waves in a plate of elastic isotropic material ([1], p.283).

b) When $\rho^*/\rho = 0$ what, for ρ being infinite, corresponds to a rigidly clamped porous plate filled with fluid, then equation (3.18) reduces to

$$[4\nu_1\nu_2-(1+\nu_2^2)^2]\{S_1^*C_3^*-\bar{\nu}_1\nu_3C_1^*S_3^*+\bar{\gamma}_{12}(\bar{\nu}_2\nu_3C_2^*S_3^*-S_2^*C_3^*)\bar{C}_{12}\}=0 \tag{3.24}$$

which will be satidfied provided that

$$4\nu_1\nu_2-(1-\nu_2^2)^2=0 \tag{3.25}$$

$$S_1^*C_3^*-\bar{\nu}_1\nu_3C_1^*S_3^*+\bar{\gamma}_{12}(\bar{\nu}_2\nu_3C_2^*S_3^*-S_2^*C_3^*)\bar{C}_{12}=0. \tag{3.26}$$

Equation (3.25) is the well known dispersion equation of Rayleigh waves ([17], p.606) for the outer layers, when the inner fluid–saturated porous stratum is absent.

Equation (3.26), however, is the dispersion equation for flexural waves in a rigidly clamped porous plate filled with fluid and edges of which are impermeable to the fluid flow.

Moreover, if we assume $Q = R = 0$, $\rho_{12} = \rho_{22} = 0$ what corresponds to the elastic non porous plate, then $\bar{\gamma}_{12} = 0$, $\xi_2 = 0$, $\xi_1 = 1$ and the equation (3.26) reduces to

$$\tanh\left(kh\sqrt{1-\frac{c^2}{c_L^2}}\right)=\sqrt{1-\frac{c^2}{c_L^2}}\sqrt{1-\frac{c^2}{c_T^2}}\tanh\left(kh\sqrt{1-\frac{c^2}{c_T^2}}\right) \tag{3.27}$$

which, apart from the notation, is identical with equation (3.65) of [18].

The simplified dispersion equation will be the subject of our interest in the following part.

4. Numerical results

Because of the difficulty of solving the dispersion equation (3.16) analytically, even in the simplified case (3.18), the roots of dispersion equation have been found numerically. The calculations, however, have been restricted to the simplified equation (3.18). The roots have been calculated on IBM-PC/XT computer for different values of c_2^2/c_T^2 and ρ/ρ^* using the following parameters

$$\frac{c_2^2}{c_1^2} = 0.25, \qquad \frac{c_2^2}{c_L^2} = 0.5, \qquad \frac{N}{H} = 0.234.$$

The results of these computations are given as plots of dimensionless phase velocity c/c_2 versus dimensionless wave number kh.

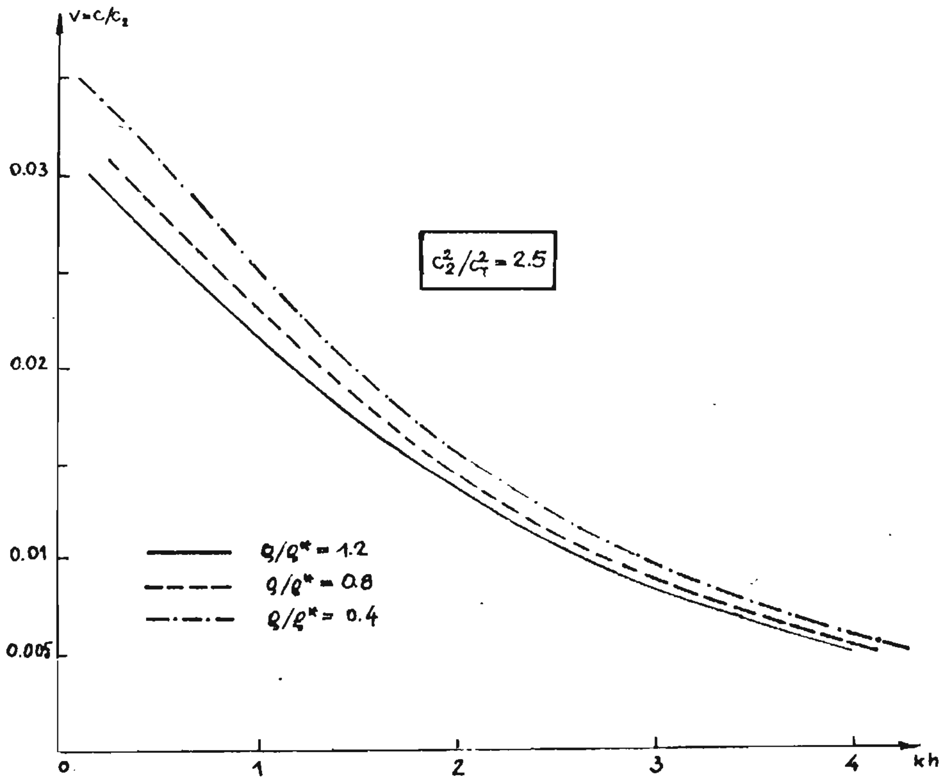

Fig. 2. Variation of phase velocity with wave number

Fig.2 shows the variation of c/c_2 versus kh for various values of the ratio ρ/ρ^*. The plots indicate that the phase velocity c/c_2 decreases as the wave number kh increases. We also note that for a fixed value of c_2^2/c_T^2 and different ρ/ρ^* the dispersion curves are qualitatively similar. The differences concern the values of phase velocity only. They are greater when ratios ρ/ρ^* become smaller.

5. Conclusions

The analysis of preceding analytical and numerical work, dealing with the propagation of flexural waves in a fluid-saturated porous stratum embedded in an infinite elastic medium allows us to draw the following conclusions:

1. The flexural waves are, in general (when the disspiation caused by the relative fluid flow is not omitted), attenuated and dispersive.

2. For a fixed wave number the magnitude of phase velocity is greater when the ratio ρ/ρ^* grows small.

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Streszczenie

W pracy rozpatrzono propagację fali giętnej w warstwie porowatej nasyconej cieczą, zanurzonej w jednorodnej, izotropowej nieograniczonej przestrzeni sprężystej nieprzepuszczalnej dla cieczy. Przyjęto, że ośrodek zewnętrzny względem porowatej warstwy charakteryzuje się tymi samymi własnościami mechanicznymi. Przyjmujac za podstawę rozważań równania sformułowane przez Biota dla warstwy porowatej oraz równania teorii sprężystości dla warstw zewnętrznych, wyprowadzono równanie dyspersji wiążące prędkość fazową z liczbą falową. Postać tego równania jest zespolona. Podano również uproszczoną formę równania dyspersji wynikającą z pominięcia wpływu tłumienia spowodowanego przepływem cieczy. Pierwiastki tego równania określono na drodze numerycznej. Pokazano także przejścia do znanych rozwiązań klasycznych.

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