MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
1, 30, 1992

EFFECT OF SOME PHYSICAL PROPERTIES OF HYDRAULIC TUBES UPON SHOCK WAVE PRESSURE PROPAGATION VELOCITY

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The purpose of this paper is to examine the effect of some physical properties of hydraulic pipelines, in particular the relative thickness and elasticity, on the accuracy of determining the propagation velocity of pressure shock wave. Considering the two-direction state of stress, which occurs in pipeline walls, the improved formula for the propagation velocity of pressure wave was derived. In the final part of the paper suitableness of the derived formula for hydrodynamic and strength calculations was verified. The calculations concerned water hammer phenomena in pipelines manufactured of different constructional materials. It was assumed that strains did not exceed the elastic range.

List of symbols

a – internal radius of the thick-walled tube conduit,

A – cross-section area of the tube,

b – external radius of the thick-walled tube,

B – modulus of the fluid compressibility,

c – mean fluid velocity,

d – internal diameter of the tube,

e – deformation of the tube wall in the radius direction,

E – elasticity modulus (Young) of the tube wall material,

m – relative thickness of the tube wall,

p – pressure,

r – radius,

t – time,

u	–	pressure propagation velocity of shock wave within thin-walled tube,
u_1	–	presssure propagation velocity of shock wave in a thick-walled tube,
δ	–	thickness of tube walls,
ν	–	Poisson number of the tube wall material,
ρ	–	fluid density/thickness,
σ	–	stress,
Δp	–	pressure increment due to the water hammer phenomenon.

1. Introduction

Pressure increment Δp, for a simple water hammer, caused by inertia of the moving fluid molecules is expressed by the following formula [2,12]

$$\Delta p = \rho u(c_0 - c) \tag{1.1}$$

where

u	–	propagation velocity of the pressure wave,
c	–	mean flow velocity of fluid in steady state motion,
c_0	–	mean flow velocity of fluid at instant t.

From the presented expression it follows that the pressure increment Δp at the moment of shock depends to a great extent on the propagation velocity of the pressure shock wave. To determine the pressure shock wave propagation velocity advantage is taken of the following dependence [3,7,10]

$$u = \sqrt{\frac{\frac{1}{\rho}}{B + \frac{d}{\delta E}}}. \tag{1.2}$$

In this equation account has been taken of the conduit deformability and the fluid instantaneous compressibility and the following simplifying assumptions have been made [2,12]

- energy dissipation due to the viscosity of the flowing fluid, as well as interaction of the forces of gravity is neglected,
- thickness of the tube walls during the water hammer is not subject to changes,

- within the tube walls there occurs one-directional state of stress (only circumferential stresses), and the value of it throughout the conduit cross-section is constant.

The assumptions confine the applicability of eq.(1.2) to thin-walled tubes, in which the ratio of the wall thickness to the radius of curvature $\delta/r < 40$, hence the state of stresses due to the fluid pressure is satisfied by the Laplace equation [4], utilized in the linear membrane theory of shells [1].

2. Pressure wave propagation velocity in thick-walled tubes

The stress effect in radial direction which results in variation of the pipeline wall thickness should be also taken into consideration in the relationship describing the pressure wave propagation velocity in thick-walled tubes. This leads in consequence to an increase of the hydraulic pipe flow area. For this reason, during the water hammer phenomenon, the fluid flowing through the tube will be stopped at instant dt, along a shorter interval dl.

In order to derive a more precise dependence, enabling to determine the pressure wave propagation velocity in thick-walled pipelines the following assumptions have been made

- the effect of the gravity forces, and energy dissipation due to the fluid viscosity is neglected,
- the instantaneous fluid compressibility and the deformability of the pipeline are of primary significance as far as the pressure wave velocity is concerned,
- within the walls of the conduit a two-dimensional state of stress occurs (circumferential and radial stress) of nonuniform distribution across the wall.

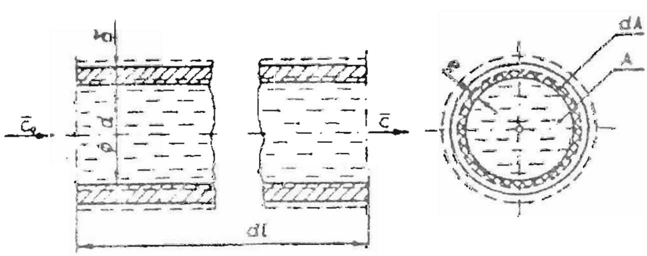

Fig. 1.

Taking into account the deformability of the pipeline, and the instantaneous compressibility of the fluid, the propagation velocity of insignificant disturbances in a horizontal conduit (Fig.1.) can be expressed by the following equation [6]

$$u_1^2 = A\frac{\partial p}{\partial(\rho A)}. \tag{2.1}$$

Denoting by

Δp – pressure increment, which accompanies the water hammer phenomenon,

dA – density variation of fluid due to compressibility,

expression (2.1) can be presented in the following form

$$u_1^2 = \frac{A\delta p}{(\rho + d\rho)(A + dA) - \rho A}. \tag{2.2}$$

Having neglected term $d\rho\, dA$ as a very small one and after dividing the numerator and the denominator of right hand side of the equation (2.2) by $\rho A\Delta p$, we obtain

$$u_1^2 = \frac{\frac{1}{\rho}}{\frac{dA}{A}\frac{1}{\Delta p} + \frac{d\rho}{\rho}\frac{1}{\Delta p}}. \tag{2.3}$$

From compressibility definition [2]

$$\frac{d\rho}{\rho} = B\Delta p$$

therefore

$$u_1^2 = \frac{1}{\rho\left(B + \frac{dA}{A}\frac{1}{\Delta p}\right)}. \tag{2.4}$$

For the increment of the flow cross–section radius equal to e, the expression

$$\frac{dA}{A} = \frac{\pi de}{\frac{\pi d^2}{4}} = \frac{4e}{d}$$

is valid. Hence

$$u_1^2 = \frac{1}{\rho\left(B + \frac{4e}{d}\frac{1}{\Delta p}\right)}. \tag{2.5}$$

In the walls of a thick–walled tube – in compliance with the assumptions – there occurs a two–directional state of stress, namely, the radial stress δ_r, and the circumferential one σ_t.

In view of the above, displacement e_r of an arbitrary point of the wall, situated at distance r from the tube axis can be determined by means of the Lame's formula, which is one of the particular function solutions of the Airy stress [1,5].

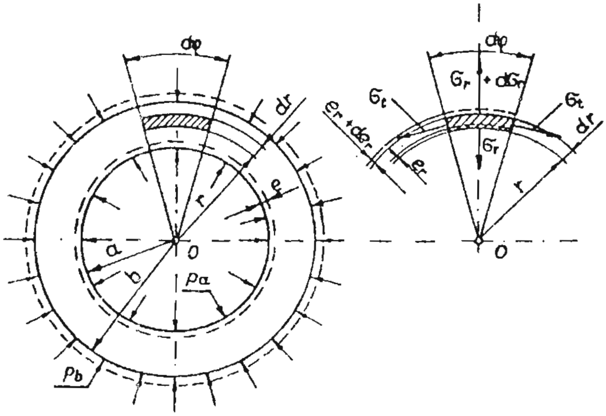

Fig. 2.

Therefore we shall have

$$e_r = \frac{1-\nu}{E(b^2-a^2)}\Big[\frac{p_a a^2 - p_b b^2}{1+\nu} r + \frac{(p_a - p_b)a^2 b^2}{(1-\nu)r}\Big]. \tag{2.6}$$

If

$$p_b = 0, \qquad p_a = \Delta p, \qquad r = a$$

then the increment of the flow cross-section radius

$$e = \frac{\Delta p a}{E(b^2-a^2)}\Big[a^2(1-\nu) + b^2(1+\nu)\Big]. \tag{2.7}$$

After substibuting for

$$a = \frac{d}{2} \;\text{ i }\; b = \frac{d}{2} + \delta$$

into the last equation and some simplifications

$$e = \frac{\Delta p d}{2E}\Big[\frac{d^2}{2(d\delta + \delta^2)} + 1 + \nu\Big] \tag{2.8}$$

or

$$e = \frac{\Delta p d}{4E}\Big[\frac{1}{\delta/d + \delta^2/d^2} + 2(1+\nu)\Big]. \tag{2.9}$$

Taking into consideration expression (2.9) in equation (2.5) after appropriate transformations, we arrive at the pressure propagation velocity of the shock wave in thick–walled tubes.

Thus

$$u_1 = \sqrt{\frac{\frac{1}{\rho}}{B + \frac{1}{E}\Big[\frac{1}{\delta/d+\delta^2/d^2} + 2(1+\nu)\Big]}}. \tag{2.10}$$

3. Analysis of relationship describing the pressure propagation velocity of shock wave

In order to determine the effect of some physical properties of hydraulic conduits, and in particular the relative thickness

$$m = \frac{\delta}{d} \tag{3.1}$$

upon the calculation accuracy of pressure shock wave propagation velocity, the velocity ratio was determined on the basis of formulae (1.2) and (2.10)

$$\frac{u_1}{u} = \sqrt{\frac{EB + \frac{1}{m}}{EB + \frac{1}{m+m^2} + 2(1+\nu)}}. \tag{3.2}$$

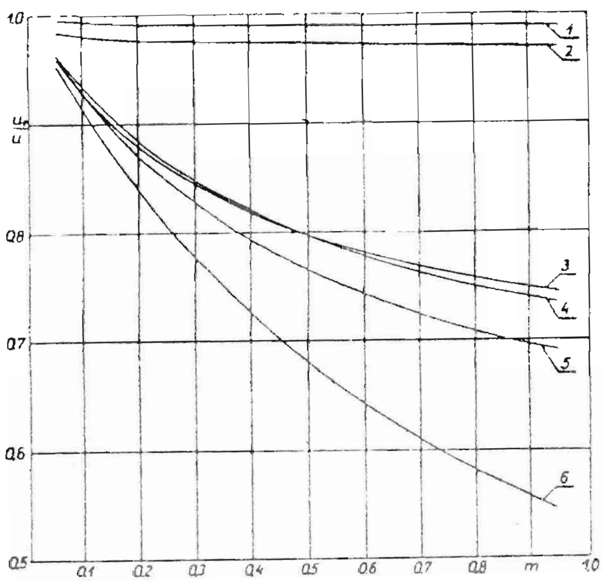

Fig. 3.

Subsequently the $u_1/u = f(m)$ diagrams were determined (Fig.3.) for conduits filled up with water and constructed of the following stuctural materials

1 – steel,
2 – aluminium,
3 – polymethacrylate,
4 – polyester resin,

5 – polyamide,

6 – hard rubber (78°Sh) applied in manufacture of high-pressure hydraulic tubes.

Material constants of respective structural materials were determined during tests carried out at the Laboratory of Strength of Materials at the Chair of Mechanics and Machine Construction Fundamentals of the Agricultural Academy in Olsztyn. The results of the material constant being obtained coincide with the data published in domestic and foreign references [8,9].

From curves 1 and 2 (Fig.3.) for metal tubes it follows that together with the change of the relative thickness m of $0.05 \div 0.95$, the velocity ratio u_1/u declines insignificantly (from 0.997 to 0.990 for steel and from 0.983 to 0.970 for aluminium).

However, in the case of conduits made out of non-metallic materials, an evident effect of the relative thickness upon the quotient value u_1/u can be noticed.

For instance, with a conduit made of hard rubber (curve 6), u_1/u varies between 0.95 for $m = 0.05 \div 0.55$, and for $m = 0.95$.

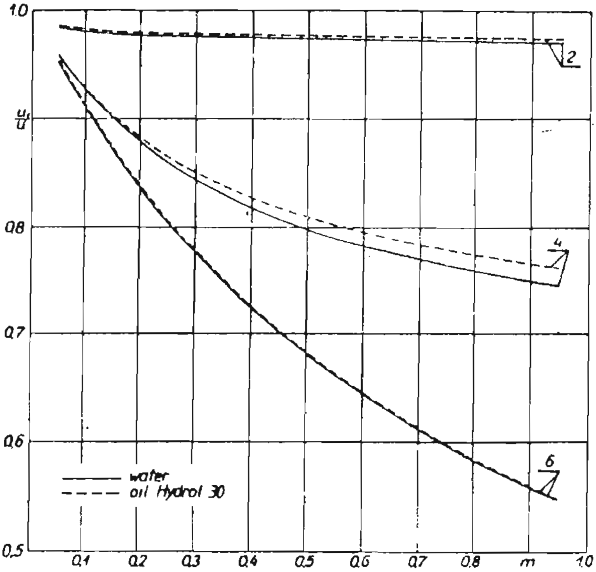

Fig. 4.

Fig.4. presents some characteristics illustrating the effect of the fluid compressibility upon variation of the $u_1/u = f(m)$ function. They were determined for hydraulic tubes made of the following materials

2 – aluminium,

4 – polyester resin,

6 – rubber.

A similar shape/outline of curves, drawn for the same structural material confirms the insignificant effect of the fluid compressibility upon function $u_1/u = f(m)$.

4. Summary and conclusions

1. The relationship derived in this paper makes it possible to determine more accurately the pressure shock wave propagation velocity, in particular in the case of thick–wall tubes.
2. The characteristics illustrated in Figs.3. and 4. prove a considerable influence of compressibility, and the relative thickness of tube upon the propagation velocity of the pressure shock wave. The presented curves prove the necessity to make use of the obtained formula in hydrodynamic and strength calculations concerning the water hammer phenomenon in conconduits made of non–metallic structural materials.
3. Determination of the pressure wave velocity within thick–walled non–metallic tubes, basing on the currently applied relationship (eq.(1.2)) can be burdened with a substantial error.

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Streszczenie

Celem niniejszej pracy jest określenie wpływu niektórych właściwości fizycznych przewodów hydraulicznych, a w szczególności grubości względnej oraz sprężystości, na dokładość wyznaczania prędkości propagacji ciśnieniowej fali uderzeniowej.

Uwzględniając dwukierunkowy stan naprężeń, jaki występuje w ściankach rurociągu, wyprowadzono uściśloną zależność na prędkość rozprzestrzeniania się fali uderzeniowej. W końcowej części pracy sprawdzono celowość stosowania otrzymanego wzoru do obliczeń hydrodynamicznych i wytrzymałościowych, dotyczących uderzenia hydraulicznego, występującego w przewodach wykonanych z różnych tworzyw konstrukcyjnych. Założono przy tym, że odkształcalność ścianek rurociągu mieści się w zakresie sprężystym.

Praca wpłynęła do Redakcji dnia 11 grudnia 1990 roku