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STRESSES IN A NON-HOMOGENEOUS AELOTROPIC SOLID WITH SPHERICAL INCLUSION HAVING A RIGID SPHERICAL CORE AT THE CENTRE

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The object of this paper is to consider the problem of the stress distribution in an infinite, non-homogeneous spherically isotropic solid due to a nucleus in the form of a centre of rotation. It also aims at the consideration of influence of a spherical inclusion having a rigid spherical core of non-homogeneous material of different kinds at the centre on the stress distributions, while the elastic constants are considered to be functions of position.

1. Introduction

The introduction of non-homogeneity in aelotropic materials has recently been one of the main pursuits in mechanics of solids. During the process of manufacturing and due to various technological processes, the elastic solid bodies sometimes not only occur to be anisotropic but also display a non-homogeneity of various types. It was pointed out by Lekhnitskii [5] that anisotropies appear not only in the manufacturing and technological processes but also in the natural course of growth of bodies, like natural wood which is transversely isotropic in nature. Moreover, non-homogeneity is also exhibited in the problems of stress concentration around holes, cavitles, fillets, grooves and inclusions. These problems were throughly discussed by Savin [8] in his monograph.

The stresses in an infinite isotropic solid due to nucleus in the form of a centre of rotation have been discussed by Love [6]. The influence of an isotropic spherical inclusion of a different material in the foregoing case have been considered by Chatterjee and Dutta [2], Das [3] has also determined the stresses due to a nucleus in the form of a centre of rotation in an elastic sphere embeded in an infinite elastic solid of another material. Chatterjee and Bose [1] have investigated the deformations and stresses in an earth model with a rigid core. Das [4] has obtained the stresses due to a nucleus in the form of a centre of rotation in an elastic sphere embedded in an infinite elastic solid of another material. Subramanium and Das [9] have obtained the stresses due to nucleus in the form of a centre of rotation in an aelotropic solid with spherical inclusion, the nucleus being taken outside of the inclusion. Maity [7] has obtained the stresses due to nucleus in the form of a centre of rotation in the interior of a spherical shell of aeolotropic material.

The author has discussed the problem of the stress distribution in an infinite, non-homogeneous spherically isotropic solid due to a nucleus in the form of a centre of rotation. The problem of influence of a spherical inclusion having a rigid spherical core of a different non-homogeneous material at the centre on the stress distributions, when the elastic constants are considered to be functions of position has also been discussed by the author. The displacements and stresses for some interesting particular cases have been discussed and are compared with the results obtained by previous researches for homogeneous cases. Finally, the author shows graphically the variations of stresses and compares them with those for the homogeneous cases.

2. Formulation and solution of the problem

We take the axis of rotation for the z-axis with the origin at the centre of the spherical inclusion of radius a.

The centre of rotation is located at a distance c from the pole, where $c < a$.

The stress-strain relations for a spherically isotropic material in spherical polar coordinates (r, θ, ϕ) are given by Love [6]

$$\begin{aligned}
\sigma_{rr} &= c_{33}e_{rr} + c_{13}e_{\theta\theta} + c_{13}e_{\phi\phi} \\
\sigma_{\theta\theta} &= c_{13}e_{rr} + c_{11}e_{\theta\theta} + c_{12}e_{\phi\phi} \\
\sigma_{\phi\phi} &= c_{13}e_{rr} + c_{12}e_{\theta\theta} + c_{11}e_{\phi\phi} \\
\sigma_{r\phi} &= c_{44}e_{r\phi} \\
\sigma_{\theta\phi} &= c_{66}e_{\theta\phi} \\
\sigma_{r\theta} &= c_{44}e_{r\theta}
\end{aligned} \tag{2.1}$$

where

$$c_{11} = 2c_{66} + c_{13} \tag{2.2}$$

and c_{ij} $(i, j = 1, 2, 3, ..., 6)$ are the elastic constants which are in general functions of the location of the point.

We assume the displacement components as

$$u_r = u_\theta = 0 \qquad u_\phi = \omega(r, \theta)$$

So the components of strain are given by Love [6]

$$
\begin{aligned}
e_{rr} &= e_{\theta\theta} = e_{\phi\phi} = e_{r\theta} = 0 \\
e_{\theta\phi} &= \frac{1}{r}\Big(\frac{\partial \omega}{\partial \theta} - \omega \cot\theta\Big) \\
e_{r\phi} &= \frac{\partial \omega}{\partial r} - \frac{\omega}{r}
\end{aligned}
\tag{2.3}
$$

For non-homogeneity of the material we assume

$$
c_{44} = \lambda_{44} r^m \qquad c_{66} = \lambda_{66} r^m \qquad (m < 0) \tag{2.4}
$$

where λ_{44}, λ_{66} are constants being the values of c_{44} and c_{66}, respectively in the homogeneous case $m = 0$.

Using Eqs (2.3) and (2.4) we get the following form of Eq (2.1)

$$
\begin{aligned}
\sigma_{rr} &= \sigma_{\theta\theta} = \sigma_{\phi\phi} = \sigma_{r\theta} = 0 \\
\sigma_{\theta\phi} &= \lambda_{66} r^{m-1}\Big(\frac{\partial \omega}{\partial \theta} - \omega \cot\theta\Big) \\
\sigma_{r\phi} &= \lambda_{44} r^m \Big(\frac{\partial \omega}{\partial r} - \frac{\omega}{r}\Big)
\end{aligned}
\tag{2.5}
$$

Two equations of equilibrium are identically satisfied and the third takes the form

$$
\frac{\partial}{\partial r}(\sigma_{r\phi}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\sigma_{\theta\phi}) + \frac{1}{r}[3\sigma_{r\phi} + 2\sigma_{\theta\phi}\cot\theta] = 0 \tag{2.6}
$$

Substituting Eq (2.5) into (2.6) one obtaines

$$
\begin{aligned}
&\frac{\lambda_{44}}{\lambda_{66}}\Big[r^2\frac{\partial^2 \omega}{\partial r^2} + (m+2) r \frac{\partial \omega}{\partial r} - (m+2)\omega\Big] + \\
&+ \Big[\frac{\partial^2 \omega}{\partial \theta^2} + \cot\theta \frac{\partial \omega}{\partial \theta} + (1 - \cot^2\theta)\omega\Big] = 0
\end{aligned}
\tag{2.7}
$$

Let us seek a solution to this equation in the form

$$
\omega = R(r)\Theta(\theta) \tag{2.8}
$$

where R is a function of r alone and Θ is a function of θ alone. Substituting Eq (2.8) into (2.7), we have

$$
\frac{\lambda_{44}}{\lambda_{66}}\frac{r^2}{R}\Big[\frac{d^2R}{dr^2} + \frac{m+2}{r}\frac{dR}{dr} - \frac{m+2}{r^2}R\Big] = -\frac{1}{\Theta}\Big[\frac{d^2\Theta}{\partial \theta^2} + \cot\theta\frac{d\Theta}{d\theta} + (1 - \cot^2\theta)\Theta\Big]
$$

Assuming that each side of the above equation is a constant and is equal to $n(n+1) - 2$, we get

$$
\frac{d^2R}{dr^2} + \frac{m+2}{r}\frac{dR}{dr} - \frac{1}{r^2}\Big[(m+2) + \frac{\lambda_{66}}{\lambda_{44}}(n-1)(n+2)\Big]R = 0 \tag{2.9}
$$

and

$$\frac{d^2\Theta}{\partial\theta^2} + \cot\theta\frac{d\Theta}{d\theta} + [n(n+1) - (1+\cot^2\theta)]\Theta = 0 \tag{2.10}$$

As a solution to Eq (2.9), we take

$$R_n = A_n\left(\frac{r}{a}\right)^{\alpha_n} + B_n\left(\frac{a}{r}\right)^{\alpha_n+1+m} \tag{2.11}$$

where

$$\alpha_n = -\frac{m+1}{2} + \frac{1}{2}\sqrt{(m+1)^2 + 4\Big[(m+2) + \frac{\lambda_{66}}{\lambda_{44}}(n-1)(n+2)\Big]} \tag{2.12}$$

and $-1 - 2\alpha_n < m < 0$. Solution Eq (2.10) suitable for our problem is

$$\Theta = \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \tag{2.13}$$

where $P_n(\cos\theta)$ is the Legendre function of the first kind and of nth degree.

Thus

$$\omega = \sum_{n=1}^{\infty}\left\{A_n\left(\frac{r}{a}\right)^{\alpha_n} + B_n\left(\frac{a}{r}\right)^{\alpha_n+1+m}\right\}\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \tag{2.14}$$

where A_n and B_n are constants.

2.1. Solution in the case of nucleus in the form of a centre of rotation

When $n = 1$, we have $\alpha_n = 1$ and we may take

$$\omega = A\frac{\sin\theta}{r^{m+1}} \tag{2.15}$$

Which obviously satisfies Eq (2.7).

This gives rise to the stress distribution given by

$$\begin{aligned}\sigma_{r\phi} &= -A\lambda_{44}(m+3)\frac{\sin\theta}{r^3} \\ \sigma_{\theta\phi} &= 0\end{aligned} \tag{2.16}$$

The foregoing stress distribution has a singularity at $r = 0$ that is, at the centre of the sphere, but the traction on a sphere of small radius ϵ gives rise to a couple of moment

$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}[\sigma_{r\phi}]_{r=\epsilon}\,\epsilon^3\sin^2\theta\,d\theta\,d\phi \tag{2.17}$$

about z-axis.

The value of the integral (2.17) is $-\frac{8}{3}\pi A\lambda_{44}(m+3)$ and remains unaffected as $\epsilon \to \infty$. Hence this can be taken as the moment of the nucleus in the form of a centre of rotation about the z-axis situated at the pole.

Thus Eqs (2.15) and (2.16) give respectively the displacement and stresses in an infinite spherically isotropic solid due to a nucleus situated at the centre, in the form of a centre of rotation, respectively.

If $2P$ represents the moment of the stress couple about z-axis, then

$$A = -\frac{3P}{8\pi(m+3)\lambda_{44}} \tag{2.18}$$

If the nucleus is situated at the point $(0, 0, c)$ then the only non-vanishing component is given by

$$\omega = -\frac{3P}{8\pi(m+3)\lambda_{44}} \frac{r^{1-m}\sin\theta}{(r^2 - 2rc\cos\theta + c^2)^{3/2}} \tag{2.19}$$

2.2. Solution in the case of inclusion

Let the spherical inclusion have at the centre a small spherical rigid core having radius $r = \epsilon$ $(\epsilon \to 0)$.

We assume

$$c_{ij} = \lambda_{ij} r^m \qquad d_{ij} = \mu_{ij} r^{m'} \qquad (m' < 0) \tag{2.20}$$

as the elastic constants inside and outside the spherical inclusion of the infinite solid respectively; here μ_{ij} are constants and represent the values of d_{ij} in the homogeneous case $m' = 0$.

Thus, as a solution suitable to this problem we may take

$$\begin{aligned} \omega_1 \; &= -\frac{3P}{4\pi(m+3)\lambda_{44}} \frac{r^{1-m}\sin\theta}{(r^2 - 2rc\cos\theta + c^2)^{3/2}} + \\ &+ \sum_{n=1}^{\infty} A_n \left(\frac{r}{a}\right)^{\alpha_n} \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \end{aligned} \tag{2.21}$$

inside the sphere $r = a$, and

$$\omega_2 = \sum_{n=1}^{\infty} B_n \left(\frac{a}{r}\right)^{\beta_n + 1 + m'} \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \qquad (-1 - 2\beta_n < m' < 0) \tag{2.22}$$

Outside the sphere $r = a$ where β_n is given by Eq (2.12) with λ_{ij}, m being replaced by μ_{ij}, m', respectively.

Thus

$$\sigma_{r\phi_1} = \frac{3P}{4\pi(m+3)}\Big[\frac{m\sin\theta}{(r^2-2rc\cos\theta+c^2)^{3/2}} + \frac{3r\sin\theta(r-c\cos\theta)}{(r^2-2rc\cos\theta+c^2)^{5/2}}\Big] + \\ +\lambda_{44}\sum_{n=1}^{\infty}\frac{A_n}{a}(\alpha_n-1)r^m\Big(\frac{r}{a}\Big)^{\alpha_n-1}\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \tag{2.23}$$

$$\sigma_{\theta\phi_1} = \frac{9P}{4\pi(m+3)}\frac{\lambda_{66}}{\lambda_{44}}\frac{cr\sin^2\theta}{(r^2-2rc\cos\theta+c^2)^{5/2}} + \\ +\lambda_{66}\sum_{n=1}^{\infty}A_n\Big(\frac{r}{a}\Big)^{\alpha_n}r^{m-1}\Big\{\frac{d^2}{d\theta^2}\Big[P_n(\cos\theta)\Big] - \cos\theta\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]\Big\} \tag{2.24}$$

$$\sigma_{r\phi_2} = -\mu_{44}\sum_{n=1}^{\infty}\frac{B_n}{a}r^{m'}\Big(\frac{a}{r}\Big)^{\beta_n+2+m'}\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \tag{2.25}$$

$$\sigma_{\theta\phi_2} = \mu_{66}\sum_{n=1}^{\infty}\frac{B_n}{a}r^{m'}\Big(\frac{a}{r}\Big)^{\beta_n+2+m'}\Big\{\frac{d^2}{d\theta^2}\Big[P_n(\cos\theta)\Big] - \\ -\cot\theta\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]\Big\} \tag{2.26}$$

where the lower indices 1 and 2 denote the displacements and stresses inside and outside of the inclusion, respectively.

The boundary conditions to be fulfilled in this case are

$$\omega_1 = \omega_2 \qquad \sigma_{r\phi_1} = \sigma_{r\phi_2} \qquad \text{on} \quad r = a \tag{2.27}$$

Now the formulae for displacements and stresses in Eqs (2.21) ÷ (2.26) on the surface on inclusion $r = a$ can be written as

$$\Big[\omega_1\Big]_{r=a} = \frac{3P}{4\pi(m+3)\lambda_{44}}\frac{1}{a^m}\sum_{n=1}^{\infty}\frac{c^{n-1}}{a^{n+1}}\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] + \\ +\sum_{n=1}^{\infty}A_n\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]$$

$$\Big[\sigma_{r\phi_1}\Big]_{r=a} = -\frac{3P}{4\pi(m+3)}\sum_{n=1}^{\infty}(n+2-m)\frac{c^{n-1}}{a^{n+2}}\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] + \\ +\lambda_{44}a^m\sum_{n=1}^{\infty}\frac{A_n}{a}(\alpha_n-1)\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \tag{2.28}$$

$$\Big[\sigma_{\theta\phi_1}\Big]_{r=a} = \frac{9P}{4\pi(m+3)}\frac{\lambda_{66}}{\lambda_{44}}\frac{ca\sin^2\theta}{(a^2-2ac\cos\theta+c^2)^{5/2}} + \\ +\lambda_{66}\sum_{n=1}^{\infty}A_n a^{m-1}\Big\{\frac{d^2}{d\theta^2}\Big[P_n(\cos\theta)\Big] - \cot\theta\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]\Big\}$$

$$
\begin{aligned}
\left[\omega_2\right]_{r=a} &= \sum_{n=1}^{\infty} B_n \frac{d}{d\theta}\left[P_n(\cos\theta)\right] \\
\left[\sigma_{r\phi_2}\right]_{r=a} &= -\mu_{44} a^{m'} \sum_{n=1}^{\infty} \frac{B_n}{a}(\beta_n + 2 + m')\frac{d}{d\theta}\left[P_n(\cos\theta)\right] \\
\left[\sigma_{\theta\phi_2}\right]_{r=a} &= \mu_{66} \sum_{n=1}^{\infty} B_n a^{m'-1}\left\{\frac{d^2}{d\theta^2}\left[P_n(\cos\theta)\right] - \cot\theta \frac{d}{d\theta}\left[P_n(\cos\theta)\right]\right\}
\end{aligned}
\tag{2.29}
$$

Using Eq (2.27), we have

$$
\begin{aligned}
A_n &= \frac{3P}{4\pi(m+3)} \frac{c^{n-1}}{a^{m+m'+2+n}} \cdot \\
&\cdot \frac{\lambda_{44}(n+2-m)a^{m-m'} - \mu_{44}(\beta_n + 2 + m')}{\lambda_{44}\left[a^{m-1}\lambda_{44}(\alpha_n - 1) + \mu_{44}a^{m'-1}(\beta_n + 2 + m')\right]}
\end{aligned}
\tag{2.30}
$$

$$
B_n = \frac{3P}{4\pi(m+3)} \frac{c^{n-1}}{a^{m+n+1}} \frac{a^{m-m'}(\alpha_n + n + 1 - m)}{\lambda_{44}(\alpha_n - 1)a^{m-m'} + \mu_{44}(\beta_n + 2 + m')}
\tag{2.31}
$$

The displacements and stresses on the boundary $r = a$ one can obtain from Eqs (2.28) or after substituting for the values of the constants A_n and B_n from Eqs (2.30) and (2.31) into Eqs (2.29).

3. Some particular cases

Case I. For the homogeneous material spherically isotropic, we have $m = m' = 0$.

Thus

$$
\begin{aligned}
A_n &= \frac{P}{4\pi} \frac{c^{n-1}}{a^{n+1}} \frac{(n+2)c_{44} - (\beta_n + 2)d_{44}}{c_{44}\left[(\alpha_n - 1)c_{44} + d_{44}(\beta_n + 2)\right]} \\
B_n &= \frac{P}{4\pi} \frac{c^{n-1}}{a^{n+1}} \frac{\alpha_n + n + 1}{(\alpha_n - 1)c_{44} + d_{44}(\beta_n + 2)} \\
\alpha_n &= -\frac{1}{2} + \frac{1}{2}\sqrt{9 + 4\frac{c_{66}}{c_{44}}(n-1)(n+2)} \\
\beta_n &= -\frac{1}{2} + \frac{1}{2}\sqrt{9 + 4\frac{d_{66}}{d_{44}}(n-1)(n+2)}
\end{aligned}
\tag{3.1}
$$

Then the displacement and stresses on the boundary of the inclusion $r = a$ are obtained from (2.29) as

$$
\begin{aligned}
\omega &= \sum_{n=1}^{\infty} B_n \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \\
\sigma_{r\phi} &= -d_{44} \sum_{n=1}^{\infty} \frac{B_n}{a}(\beta_n + 2)\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \\
\sigma_{\theta\phi} &= d_{66} \sum_{n=1}^{\infty} \frac{B_n}{a}\Big\{\frac{d^2}{d\theta^2}\Big[P_n(\cos\theta)\Big] - \cot\theta\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]\Big\}
\end{aligned}
\tag{3.2}
$$

where B_n, α_n, β_n are given by Eqs (3.1).

Case II. We consider the case of homogeneous isotropic inclusion within a homogeneous isotropic solid. In this case

$$
m = m' = 0 \qquad c_{44} = c_{66} = G_1 \qquad d_{44} = d_{66} = G_2
$$

where G_1, G_2 are the proper material shear moduli.

Then

$$
\begin{aligned}
\alpha_n &= \beta_n = n \\
A_n &= \frac{P}{4\pi}\frac{c^{n-1}}{a^{n+1}}\frac{(n+2)(G_1 - G_2)}{G_1\Big[G_1(n-1) + G_2(n+2)\Big]} \\
B_n &= \frac{P}{4\pi}\frac{c^{n-1}}{a^{n+1}}\frac{2n+1}{G_1(n-1) + G_2(n+2)}
\end{aligned}
$$

Then the displacement and stresses on the surface of the inclusion $r = a$ are obtained by Eqs (2.29) as

$$
\begin{aligned}
\omega &= \frac{P}{4\pi G_2}\sum_{n=1}^{\infty} L_n M_n \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \\
\sigma_{r\phi} &= -\frac{P}{4\pi}\sum_{n=1}^{\infty}(n+2)L_n M_n \frac{d}{d\theta}\Big[P_n(\cos\theta)\Big] \\
\sigma_{\theta\phi} &= -\frac{P}{4\pi}\sum_{n=1}^{\infty}\frac{L_n M_n}{a}\Big\{\frac{d^2}{d\theta^2}\Big[P_n(\cos\theta)\Big] - \cot\theta\frac{d}{d\theta}\Big[P_n(\cos\theta)\Big]\Big\}
\end{aligned}
\tag{3.3}
$$

where

$$
L_n = \frac{c^{n-1}}{a^{n+1}} \qquad M_n = \frac{G_2(2n+1)}{G_1(n-1) + G_2(n+2)}
$$

These results agree withthose of Das [4].

Case III. In the case when the material outside the inclusion is rigid, $\mu_{44} = \mu_{66} \to \infty$, then

$$A_n = -\frac{3P}{4\pi(m+3)}\frac{c^{n-1}}{a^{m+n+1}}\frac{1}{\lambda_{44}}$$
$$B_n = 0$$

The displacement and stresses on the surface of inclusion $r = a$ are obtained using Eqs (2.28) as

$$\begin{aligned}
\omega &= 0 \\
\sigma_{r\phi} &= -\frac{3P}{4\pi(m+3)}\sum_{n=1}^{\infty}(\alpha_n + n + 1 - m)\frac{c^{n-1}}{a^{n+2}}\frac{d}{d\theta}\left[P_n(\cos\theta)\right] \\
\sigma_{\theta\phi} &= \frac{3P}{4\pi(m+3)}\frac{\lambda_{66}}{\lambda_{44}}\Big[\frac{3ac\sin^2\theta}{(a^2 - 2ac\cos\theta + c^2)^{5/2}} - \\
&- \sum_{n=1}^{\infty}\frac{c^{n-1}}{a^{n+2}}\Big\{\frac{d^2}{d\theta^2}\left[P_n(\cos\theta)\right] - \cot\theta\frac{d}{d\theta}\left[P_n(\cos\theta)\right]\Big\}\Big]
\end{aligned} \tag{3.4}$$

Case IV. In case when the material outside the inclusion is absent, then

$$\mu_{44} = \mu_{66} = 0$$
$$A_n = DE_n\frac{n+2-m}{\alpha_n - 1}$$
$$B_n = DE_n\frac{\alpha_n + n + 1 - m}{\alpha_n - 1}$$

where

$$D = \frac{3P}{4\pi(m+3)\lambda_{44}} \qquad E_n = \frac{c^{n-1}}{a^{n+m+1}}$$

The displacement and stresses on the boundary $r = a$ are given by

$$\begin{aligned}
\omega &= D\sum_{n=2}^{\infty}E_n\frac{\alpha_n + n + 1 - m}{\alpha_n - 1}\frac{d}{d\theta}\left[P_n(\cos\theta)\right] \\
\sigma_{r\phi} &= 0
\end{aligned} \tag{3.5}$$

$$\sigma_{\theta\phi} = \lambda_{66} D \Big[\frac{3ac\sin^2\theta}{(a^2 - 2ac\cos\theta + c^2)^{5/2}} + \\ + \sum_{n=2}^{\infty} \frac{c^{n-1}}{a^{n+2}} \frac{n+2-m}{\alpha_n - 1} \Big\{ \frac{d^2}{d\theta^2}\big[P_n(\cos\theta)\big] - \cot\theta \frac{d}{d\theta}\big[P_n(\cos\theta)\big]\Big\}\Big]$$

All the infinite series occuring in this problem are obviously convergent.

For all the other possible cases namely

(i) if the material of inclusion is homogeneous isotropic or spherically isotropic and the material outside the inclusion is non–homogeneous isotropic or spherically isotropic,

(ii) if the material of the inclusion is non–homogeneous isotropic or spherically anisotropic and that of outside the inclusion is homogeneous isotropic or spherically isotropic,

the stresses and displacement on the surface of the inclusion can be easily found out by simple substituting the suitable values for m and m'.

4. Numerical results and discussion

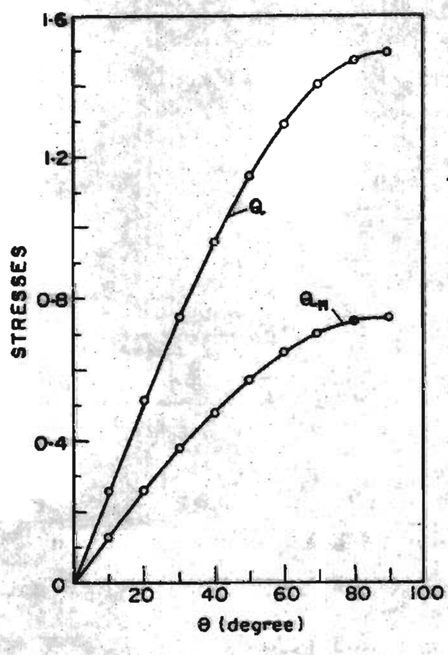

Fig. 1.

In the adjoining Fig.1 we exhibit the variations of Q $(Q = \frac{\pi a^3}{P}\sigma_{r\phi})$ on the surface of the non–homogeneous anisotropicspherical inclusion within non–homogeneous anisotropic solid assuming $m = m' = -1$ and $n = 1$ (in particular) for different values of θ $(0 \leq \theta \leq \pi/2)$. Q_H shows the variations of Q in the associated homogeneous case where $c_{44} = \lambda_{44}$, $c_{66} = \lambda_{66}$, $d_{44} = \mu_{44}$ and $d_{66} = \mu_{66}$. It is observed in Fig.1 that for all values of θ $(0 \leq \theta \leq \pi/2)$, Q are greater than those in homogeneous cases.

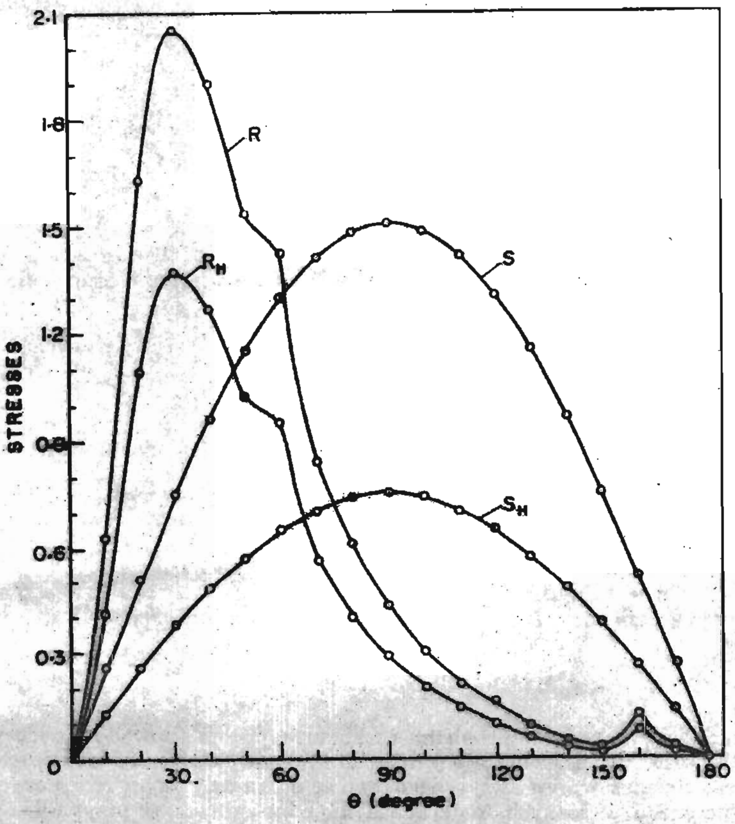

Fig. 2.

Further in Fig.2 for an interesting case, we exhibit the variations of S and R $(S = \frac{\pi a^3}{P}\sigma_{r\phi}$, $R = \frac{\lambda_{44}}{\lambda_{66}}\frac{\pi a^3}{3P}\sigma_{\theta\phi}$) on the surface of the non–homogeneous spherical inclusion when the material outside the inclusion is rigid assuming $m = -1, n = 1$ (in particular) and $c/a = 0.5$ for different values of θ $(0 \leq \theta \leq \pi)$. S_H and R_H show the variations of S and R in the associated homogeneous cases, respectively.

It is observed at a glance that there are some peculiarities in Fig.2, R, R_H both in homogeneous and non–homogeneous cases within $(50^o \leq \theta \leq 60^o)$ and

$(150^\circ \leq \theta \leq 160^\circ)$. In the first interval $(50^\circ \leq \theta \leq 60^\circ)$ we observed a rapid change of R & R_H but not in a systematic patern. While in $(150^\circ \leq \theta \leq 160^\circ)$ unlikely of the expectation both of them have some peculiar elevation.

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References

1. CHATTERJEE P.P., BOSE S.C., 1963, Ind.J.Theor.Phys., 11
2. CHATTERJEE P.P., DUTTA S.B., 1959, Bull.Cal.Math.Soc., 51
3. DAS B.R., 1965, J.Sc.Eng.Res., VI, 2
4. DAS S.N., 1965, Ind.J.Theor.Phys., 13
5. LEKHNITSKII S.G., 1963, *Theory of elasticity of anisotropic elastic bodies*, Oxford University Press
6. LOVE A.E.H., 1944, *A Treatise on the mathematical theory of elasticity*, 4th Edit. Cambridge University Press
7. MAITY S.R., 1968, Bull.Cal.Math.Soc., 60, 179-184
8. SAVIN G.N., 1968, *Stress distribution around Holes*, Naukova Dumka, Kiev
9. SUBRAMANIUM R., DAS S.C., 1964, Ind.J.Mech.Maths., IV, 1

Naprężenia w niejednorodnym ciele alotropowym ze sferycznymi wtrąceniami z kulistym sztywnym rdzeniem w środku

Streszczenie

Celem pracy jest rozważenie problemu rozkładu naprężeń spowodowanych jądrem w postaci środka obrotu w nieskończonym, niejednorodnym, sferycznie izotropowym ciele. Rozważany jest również wpływ sferycznego wtrącenia ze sztywnym rdzeniem w środku na rozkłady naprężeń w różnych materiałach niejednorodnych w przypadku gdy stałe sprężyste przyjmowane są w postaci funkcji położenia.

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