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VIBRATIONS OF A PRESTRESSED TWO-MEMBER COMPOUND COLUMN

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The dynamic bending response of a geometrically non-linear prestressed two-member compound beam with pinned ends is studied. The prestress was produced by initial lack of fit of the beam members. The problem is formulated as a variational one, the Hamilton principle being used. The solution was obtained by using the generalized Ritz method. The natural vibration frequency versus prestress and vibration amplitude are calculated. The orthogonality condition for the non-linear modes of vibration are also discussed.

1. Introduction

The linear elastic systems do not show the changes of natural vibration frequency regarding prestress and initial imperfections (cf [1]). Natural vibration frequencies of geometrically non-linear systems depend both on the value of prestress and the initial imperfections (cf [2 ÷ 6]).

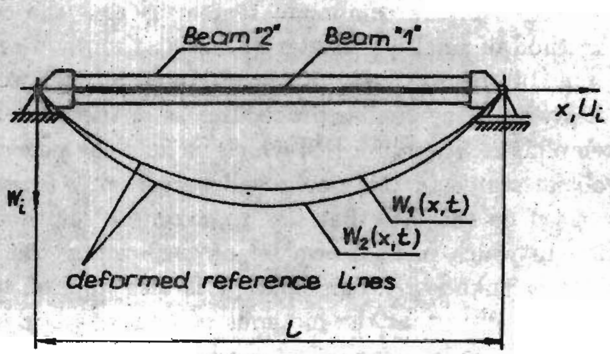

Fig. 1. Compound beam

This work concerns the dynamics of a geometrically non–linear prestressed perfect compound beam. Such a system may be presented in the form of two coaxial tubes or a tube and a rod (Fig.1) of different flexural and axial rigidities (E_iJ_i and E_iA_i, respectively) and different masses per unit length $(\rho_i A_i, i = 1,2)$ rigidly connected with each other (both in displacement and rotational sense) (cf [8,9]). The beam may also be in the form of a planar frame made of a strip located in the centre of the structure in which the second member is formed by two identical strips symmetrically located at both sides of the central strip (cf [10]). In refs. [8 ÷ 10] the divergence instability of such systems was detrermined, however their dynamics was not analysed.

For solving the problem of vibration of elastic systems characterized by geometrical nonlinearity the moderately large bending theory is usually applied. According to this theory one assumes a non–linear strain–displacement relationship of a bar in the following form

$$\varepsilon = \frac{1}{2}\Big(\frac{\partial W}{\partial x}\Big)^2 + \frac{\partial U}{\partial x}$$

Furthermore, one also assumes that the inertia in the in–plane direction is expected to be small as compared with the inertia in the transverse direction and hence it is neglected. The moderately large bending theory is in accord with the results of actually performed experiments [11,12].

In the works concerning the non–linear vibration of a beam with fixed or elastically supported ends in the longitudinal direction one applies continuum approach and finite elements method (cf [13 ÷ 16]). For solving the vibration of beams considered as a non–linear elastic continuous system one applies various methods. The space and time separation of variables is performed and then either one approximates the shape of the deflected beam or the time factor is assumed to be a trigonometrics function. The finite difference method, Ritz procedure, the perturbation method and multiple scales are used.

For solving vibration of the compound system as e.g. the compound beam considered, the method of small parameter or the Ritz method seems to be most suitable (cf [17] and [16], respectively). The considered problem is solved by using the Ritz method. By applying similar procedures as in the book by Leipholz [18] and work by Szemplińska–Stupnicka [19] which include the generalization of this method, the problem stated in this paper will be solved. Generalization of the Ritz method is based on the fact that the approximate solution in the form of a truncated series, in which instead of a set of coefficients, the set of functions of spatial variables is unknown. This concept is also applied by Lewandowski [16]. It is necessary to state that the methods of solutions of dynamic problems given above concern the time–independent loads. For the time–dependent loads the stochastic stability analysis of non–linear columns using the Liapunov direct method was presented by Tylikowski [20].

2. Mathematical formulation

In this work one analyses the dynamic bending response of the geometrically non–linear prestressed compound beam (Fig.1). The compound beam considered is prestressed due to the initial lack of fit $U_0 = l_{01} - l_{02}$ of members. If, before the members are connected, one of them is too long by an initial lack of fit, U_0, then, for any shortening U_{01}, U_{02} of members we have

$$U_0 = U_{01} - U_{02} \tag{2.1}$$

The Hook law states that

$$U_{0i} = \frac{S_{oi} L}{E_i A_i} \tag{2.2}$$

The equilibrium condition of internal forces in the bars yields

$$S_{0i} = (-1)^{i+1} S_0 \qquad S_0 = \frac{U_0}{\mu}$$
$$\mu = L\Big(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2}\Big) \tag{2.3}$$

After the value of prestress is determined, the dynamics of the system is formulated by applying the Hamilton principle (cf [17] chap.4) which in the case of this system takes the form

$$\delta J = \int\limits_{t_1}^{t_2} \int\limits_0^L \delta \mathcal{L} dx dt = 0 \tag{2.4}$$

where $\mathcal{L}$ is the Lagrangian density of the system. The Lagrangian density in our case may be presented as follows

$$\mathcal{L} = \sum_{i=1}^{2} \Big\{ T_i\Big(\frac{\partial W_i}{\partial t}\Big) - V_i^1\Big(\frac{\partial^2 W_i}{\partial x^2}\Big) - V_i^2\Big(\frac{\partial U_i}{\partial x}, \frac{\partial W_i}{\partial x}\Big) - V_i^3\Big(S_{i0}, \frac{\partial U_i}{\partial x}, \frac{\partial W_i}{\partial x}\Big)\Big\}$$

where T_i, V_i^m, $i = 1,2$, $m = 1,2,3$, denote the densities of the kinetic and potential energy, respectively. For the system considered these densities are expressed by formulae

$$\begin{aligned} T_i &= \frac{1}{2}\rho_i A_i \Big(\frac{\partial W_i}{\partial t}\Big)^2 \\ V_i^1 &= \frac{1}{2} E_i J_i \Big(\frac{\partial^2 W_i}{\partial x^2}\Big)^2 \\ V_i^2 &= \frac{1}{2} E_i A_i \Big[\frac{\partial U_i}{\partial x} + \frac{1}{2}\Big(\frac{\partial W_i}{\partial x}\Big)^2\Big]^2 \\ V_i^3 &= -S_{0i}\Big[\frac{\partial U_i}{\partial x} + \frac{1}{2}\Big(\frac{\partial W_i}{\partial x}\Big)^2\Big]^2 \end{aligned} \tag{2.5}$$

The Hamilton's principle (2.4), after considering (2.5) and integrating by parts, can be transformed into the form

$$
\begin{aligned}
&\sum_{i=1}^{2}\int_{t_1}^{t_2}\int_{0}^{L}\Big\{\Big[-\frac{\partial}{\partial t}\Big(\frac{\partial T_i}{\partial W_{i,t}}\Big)-\frac{\partial^2}{\partial x^2}\Big(\frac{\partial V_i^1}{\partial W_{i,xx}}\Big)+\frac{\partial}{\partial x}\Big(\frac{\partial V_i^2}{\partial W_{i,x}}\Big)+\\
&+\frac{\partial}{\partial x}\Big(\frac{\partial V_i^3}{\partial W_{i,x}}\Big)\Big]\delta W_i+\Big[\frac{\partial}{\partial x}\Big(\frac{\partial V_i^2}{\partial U_{i,x}}\Big)+\frac{\partial}{\partial x}\Big(\frac{\partial V_i^3}{\partial U_{i,x}}\Big)\Big]\delta U_i\Big\}dxdt+\\
&+\int_0^L\frac{\partial T_i}{\partial W_{i,t}}\delta W_i\Big|_{t_1}^{t_2}dx+\int_{t_1}^{t_2}\Big\{-\frac{\partial V_i^1}{\partial W_{i,xx}}\delta W_{i,x}+ \qquad\qquad (2.6)\\
&+\Big[\frac{\partial}{\partial x}\Big(\frac{\partial V_i^1}{\partial W_{i,xx}}\Big)-\frac{\partial V_i^2}{\partial W_{i,x}}-\frac{\partial V_i^3}{\partial W_{i,x}}\Big]\delta W_i-\Big[\frac{\partial V_i^2}{\partial U_{i,x}}+\frac{\partial V_i^3}{\partial U_{i,x}}\Big]\delta U_i\Big\}\Big|_0^L dt=0
\end{aligned}
$$

where variations δW_i, δU_i and their derivatives with respect to the coordinate x are independent of each other. The conditions of vanishing the variations at $t = t_1$ and $t = t_2$ must be utilized

$$\delta W_i(x,t_1)=\delta W_i(x,t_2)=0 \qquad i=1,2 \tag{2.7}$$

3. Solution of the problem

The displacements $W_i(x,t)$ and $U_i(x,t)$ being searched for and the force $S_i(t)$ resulting from bending the bar i, as being the approximate solution of the problem are assumed in the form

$$W_i(x,t)=aw_i(x)\cos\omega t \qquad U_i(x,t)=a^2u_i(x)\cos^2\omega t \tag{3.1}$$

$$S_i(t)=\bar{S}_i\cos^2\omega t \tag{3.2}$$

where $w_i(x)$ is a unknown eigenfunction of lateral beam vibration and a is the amplitude of constant value depending on the initial conditions. The geometrical and continuity conditions of our problem for the functions $w_i(x)$ and $u_i(x)$ are the following

$$
\begin{aligned}
&w_i\Big|_{x=0}=w_i\Big|_{x=L}=0 \qquad &&\frac{\partial w_1}{\partial x}\Big|_{\substack{x=0\\x=L}}=\frac{\partial w_2}{\partial x}\Big|_{\substack{x=0\\x=L}}\\
&u_1\Big|_{x=0}=u_2\Big|_{x=0}=0 \qquad &&u_1\Big|_{x=L}=u_2\Big|_{x=L}
\end{aligned} \tag{3.3}
$$

By using the concept of the generalized Ritz method [18] we have

$$\begin{aligned} \delta W_i(x,t) &= a\delta w_i(x)\cos\omega t \\ \delta U_i(x,t) &= a^2\delta u_i(x)\cos^2\omega t \end{aligned} \tag{3.4}$$

After substituting Eq (2.5) and Eqs (3.1), (3.2) and (3.4) into Eq (2.6) and then integrating this equation within the limits $t_1 = 0$, $t_2 = 2\Pi$ and making use of boundary conditions (3.3) one obtains the ordinary differential equations for the unknown $w_i(x)$, $u_i(x)$ and the associated (natural) boundary conditions

$$\begin{aligned} &E_iJ_iw_i^{\rm IV} + S_i^*w_i'' - \rho_iA_i\omega^2w_i = 0 \qquad && i = 1,2 \\ &E_iA_i\Big[u_i' + \frac{1}{2}(w_i')^2\Big]' = 0 && i = 1,2 \end{aligned} \tag{3.5}$$

$$\begin{aligned} &E_1J_1w_1'' + E_2J_2w_2'' = 0 \qquad \text{for} \quad x = 0 \quad \text{and} \quad x = L \\ &E_1A_1\Big[u_1' + \frac{1}{2}(w_1')^2\Big] + E_2A_2\Big[u_2' + \frac{1}{2}(w_2')^2\Big] = 0 \\ &\qquad\qquad \text{for} \quad x = 0, L \end{aligned} \tag{3.6}$$

where

$$\begin{aligned} S_i^* &= S_{i0} + \frac{3}{4}\bar{S}_i \\ E_iA_i\Big[u_i' + \frac{1}{2}(w_i')^2\Big] &\equiv -\bar{S}_i \end{aligned} \tag{3.7}$$

By integrating both sides of Eq (3.7) within the limits $0, L$ and considering conditions $(3.3)_2$ the following relationship

$$a^2u_i(L) = -\frac{\bar{S}_iL}{E_iA_i} - \frac{1}{2}a^2\int_0^L (w_i')^2\,dx \tag{3.8}$$

is obtained. On the basis of conditions $(3.3)_2$ and (3.7) we have

$$S = \frac{a^2}{2\mu}\int_0^L \Big[(w_2')^2 - (w_1')^2\Big]\,dx \tag{3.9}$$

where: $\bar{S}_1 = -\bar{S}_2 = S$.

The solution of Eq $(3.5)_1$ satisfying conditions (3.6) is in the form

$$\begin{aligned} w_i(x) &= (w_0' + w_1')\gamma_{i1}\Big[\sinh\frac{\alpha_i L}{2}\sin\beta_i(x-\frac{L}{2}) - \sin\frac{\beta_i L}{2}\sinh\alpha_i(x-\frac{L}{2})\Big] + \\ &+ (w_0' - w_1')\gamma_{i2}\Big[\cosh\frac{\alpha_i L}{2}\cos\beta_i(x-\frac{L}{2}) - \cos\frac{\beta_i L}{2}\cosh\alpha_i(x-\frac{L}{2})\Big] \end{aligned} \tag{3.10}$$

where one assumes

$$\begin{aligned} w_0' &= w_1'(0) = w_2'(0) \\ w_1' &= w_1'(L) = w_2'(L) \end{aligned} \tag{3.11}$$

$$\gamma_{i1} = \Big[\beta_i L\sinh\frac{\alpha_i L}{2}\cos\frac{\beta_i L}{2} - \alpha_i L\cosh\frac{\alpha_i L}{2}\sin\frac{\beta_i L}{2}\Big]^{-1}$$
$$\gamma_{i2} = \Big[\beta_i L\cosh\frac{\alpha_i L}{2}\sin\frac{\beta_i L}{2} + \alpha_i L\sinh\frac{\alpha_i L}{2}\cos\frac{\beta_i L}{2}\Big]^{-1}$$

$$\alpha_i^2 = \frac{L^2}{2E_iJ_i}\Big[-\bar{S}_i + \sqrt{\bar{S}_i^2 + 4\omega^2 E_iJ_i\rho_iA_i}\,\Big]$$
$$\beta_i^2 = \frac{L^2}{2E_iJ_i}\Big[\bar{S}_i + \sqrt{\bar{S}_i^2 + 4\omega^2 E_iJ_i\rho_iA_i}\,\Big]$$

Substituting solution (3.10) into conditions $(3.6)_1$ leads to a characteristic equation for vibration frequency ω.

Then, one considers two cases

a) Symmetric modes of vibration, where $w_1' = -w_0'$.
The characteristic equation is then in the form

$$\gamma_{12}b_{12} + e\gamma_{22}b_{22} = 0 \tag{3.12}$$

and the solution w_i is expressed by the formula

$$w_i(x) = w_0'\gamma_{i2}\Big[\cosh\frac{\alpha_i L}{2}\cos\beta_i(x-\frac{L}{2}) - \cos\frac{\beta_i L}{2}\cosh\alpha_i(x-\frac{L}{2})\Big] \tag{3.13}$$

b) Antisymmetric modes of vibration, where $w_1' = w_0'$.
The characteristic equation and solution w_i are the following

$$\gamma_{11}b_{11} + e\gamma_{21}b_{21} = 0 \tag{3.14}$$

$$w_i(x) = w_0'\gamma_{i2}\Big[\sinh\frac{\alpha_i L}{2}\sin\beta_i(x-\frac{L}{2}) - \sin\frac{\beta_i L}{2}\sinh\alpha_i(x-\frac{L}{2})\Big] \tag{3.15}$$

where

$$e = \frac{E_2 J_2}{E_1 J_1}$$

$$b_{i1} = (\alpha_i^2+\beta_i^2)\sinh\frac{\alpha_i L}{2}\sin\frac{\beta_i L}{2}$$

$$b_{i2} = (\alpha_i^2+\beta_i^2)\cosh\frac{\alpha_i L}{2}\cos\frac{\beta_i L}{2}$$

The coefficient w_0' in solutions (3.13) and (3.15) is determined from the normalizing condition

$$\frac{1}{2}\Big[w_1\Big(\frac{L}{4}\Big) + w_2\Big(\frac{L}{4}\Big)\Big] = 1 \tag{3.16}$$

One takes the normalization for $x = L/4$ due to the possibility of determining an antisymmetric shape of vibration. The average displacement value guarantees the symmetry of solutions for both beams of a compound column.

4. Orthogonality condition

Let us assume that $W_{in}(x,t)$ and $W_{ik}(x,t)$ $(i = 1,2)$ are two different known solutions with corresponding axial forces S_n and S_k, amplitudes a_n and a_k, frequencies ω_n and ω_k, and modes of vibration w_{in} and w_{ik}. The orthogonality condition will have the following form

$$(\omega_n^2-\omega_k^2)\sum_{i=1}^{2}\Big[\rho_i A_i\int_0^L w_{in}w_{ik}\,dx + \frac{3}{4}(S_k - S_n)\int_0^L w_{in}\frac{d^2 w_{ik}}{dx^2}\,dx\Big] = 0 \tag{4.1}$$

This condition is satisfied for symmetric modes of vibrations w_{in}^s and w_{ik}^s, and also for antisymmetric modes w_{in}^a, w_{ik}^a, if the forces corresponding to them fulfill the relationship

$$S_n^s(a_n) = S_k^s(a_k) \qquad \text{for symmetric modes}$$

$$S_n^a(a_n) = S_k^a(a_k) \qquad \text{for antisymmetric modes.}$$

However, fulfilling condition (4.1) for symmetric modes and antisymmetric modes w_{in}^s and w_{ik}^a, respectively, results directly from the properties of even and odd respectively parity of the functions $w_{in}(x + L/2)$ and $w_{ik}(x + L/2)$ determined by formulae (3.13) and (3.15). This leads to the following equality

$$\int_0^L w^s_{in} w^a_{ik} dx = \int_0^L w^s_{in} \frac{d^2 w^a_{ik}}{dx^2} dx = 0$$

5. Numerical results

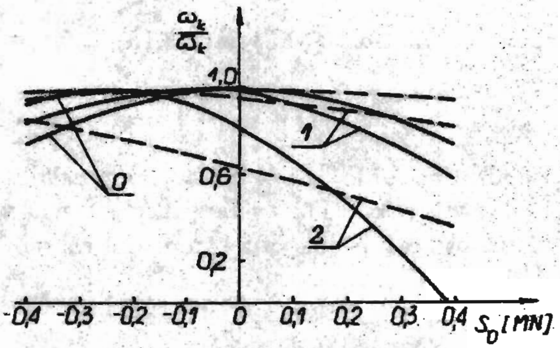

Fig. 2. Frequency ratio (first frequency — ; second frequency - - -) versus prestressing force S_0 for different physical data of the compound column as listed in Table 1

Fig.2 presents the variation of the two first non–linear frequency ratios $(\omega_k/\bar{\omega}_k)$ versus prestressing force S_0 ($S_0 > 0$ and $S_0 < 0$ denote compressive and streching forces, respectively), $\bar{\omega}_k$ is the frequency of natural vibration of a beam for $S_0 = 0$ and other data are the same as those in exammple "0" (Table 1). The largest variations of natural vibration frequencies occur for asymmetrical distribution of flexural rigidities of both beams of a compound column.

Table 1. Physical and geometrical data of the compound beam

Beam No.	$\frac{E_1 J_1}{E_2 J_2}$	$\frac{E_1 A_1}{E_2 A_2}$	$\frac{\rho_1 A_1}{\rho_2 A_2}$
0	1	1	1
1	3/5	1	1
2	1/5	1	1
$L = 4$ m, $E_1 J_1 + E_2 J_2 = 1.2$ MNm^2			
$E_1 A_1 + E_2 A_2 = 10.0$ GN			
$\rho_1 A_1 + \rho_2 A_2 = 35.0$ kgm^{-1} $a = 0.002$			

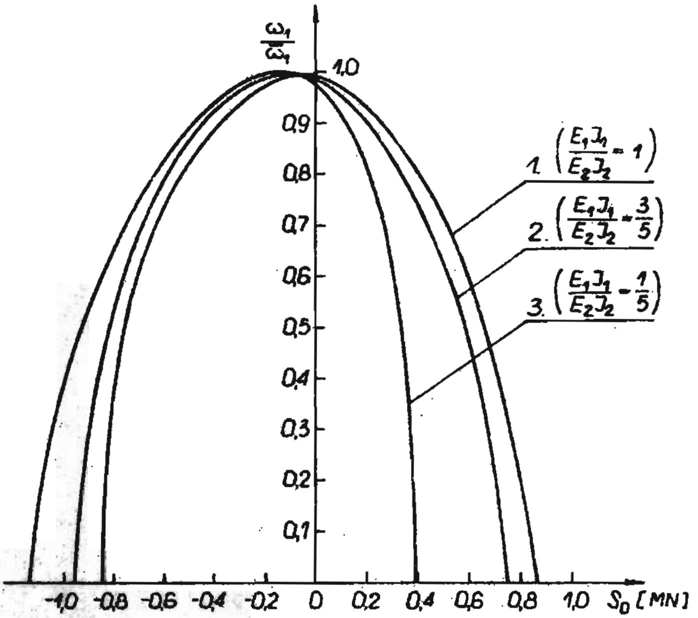

Fig. 3. First frequency ratio versus prestressing force S_0 for different flexural rigidity ratios (E_1J_1/E_2J_2), where $E_1J_1 + E_2J_2 = 1.2$ MNm^2, $a = 0.002$ m

Fig.3 illustrates the change of the first natural frequency versus the internal force S_0 for different flexural rigidities. One assumes here that $E_1 = 1 \cdot 10^{11}$ N/m^2, $E_2 = 2 \cdot 10^{11}$ N/m^2, $\rho_1 = 3000$ kg/m^3, $\rho_2 = 8000$ kg/m^3, and the cross section value A_i is estabilished treating beam "1" as a rod and beam "2" as a tube. The tube is fixed on the rod with radial clearance. The value of the sum of flexural rigidities is constant: $E_1J_1 + E_2J_2 = 1.2$ MNm^2. Depending on the asymmetry of distributions of the flexural rigidities, there are certain values of the internal forces S_0 when the natural frequency of the system lowers to zero. Fig.4 illustrates the effect of the amplitude a on the value of the ratio ω_1/ω_{01}, where ω_{01} is the frequency of natural vibrations of the column for the amplitude $a \to 0$. The curves $1(a-c)$ were calculated for $S_0 = 3.0 \cdot 10^5$ N and the curves $2(a-c)$ for $S_0 = 3.5 \cdot 10^5$ N.

The results of the calculation concerning variations of frequency ratio versus the value of prestressing force S_0 for various values of amplitude a are gathered in Fig.5 (the ratio of flexural rigidities $E_1J_1/E_2J_2 = 1/5$). The boundary curve $F(S_{01}, S_{02}, \omega = 0) = 0$, for which the natural frequencies are equal to zero is presented in Fig.6. The shape of the column loaded according to this curve can be either recitilinear or curvilinear (compare [8]).

Let us consider the behaviour of the column with flexural rigidity ratio

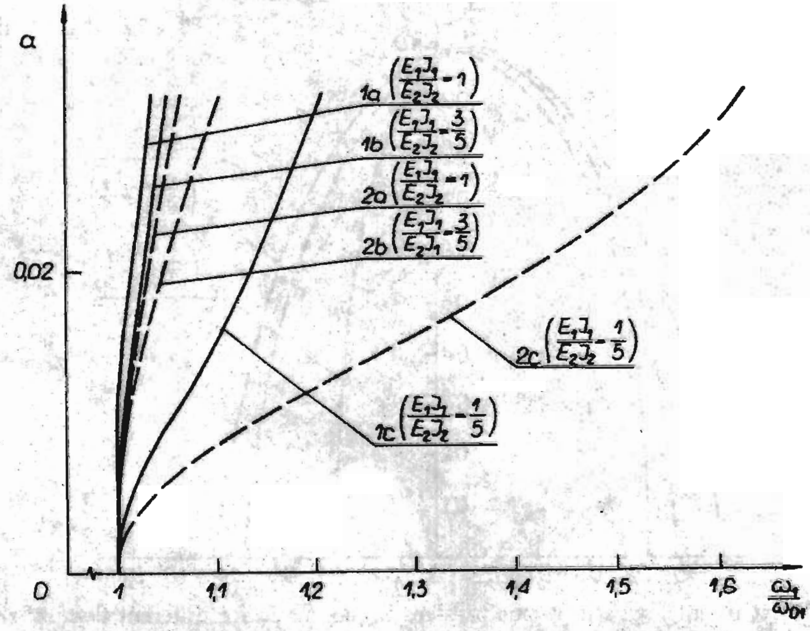

Fig. 4. Changes of first frequency ratio vs. vibration amplitude ($S_0 = 3 \cdot 10$ N —— and $S_0 = 3.5 \cdot 10$ N - - -) for different flexural rigidity ratio, where $E_1J_1 + E_2J_2 = 1.2$ MNm^2

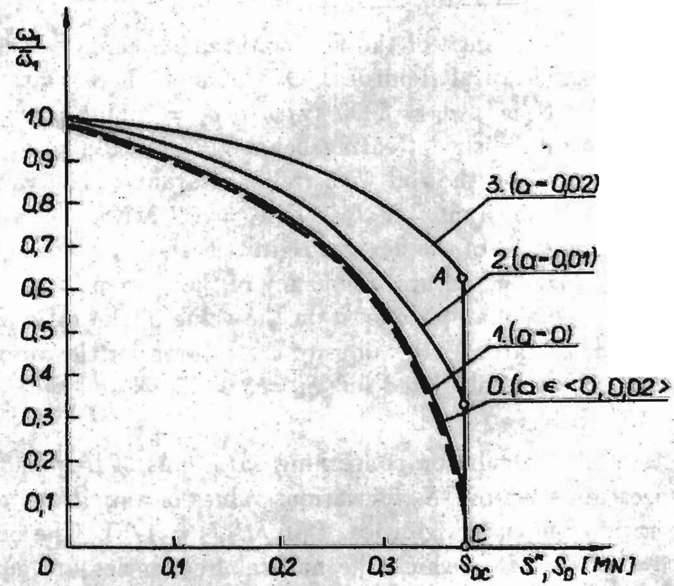

Fig. 5. Changes of first frequency ratios versus internal force S and prestressing force S_0 for $E_1J_1/E_2J_2 = 1/5$ and $E_1J_1 + E_2J_2 = 1.2$ MNm^2 and arbitrary amplitude a

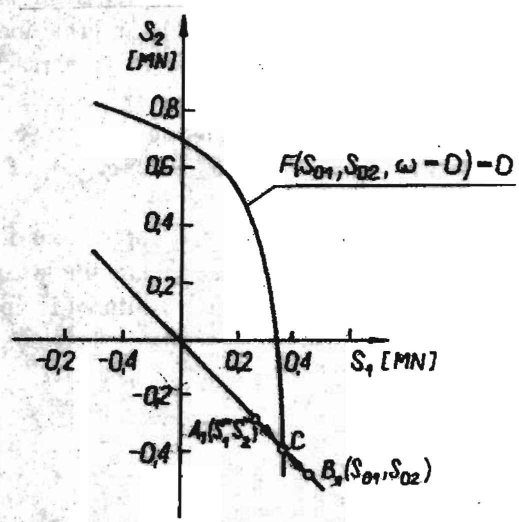

Fig. 6. Relationship among internal forces S_{01} and S_{02} for which natural frequency is equal to zero

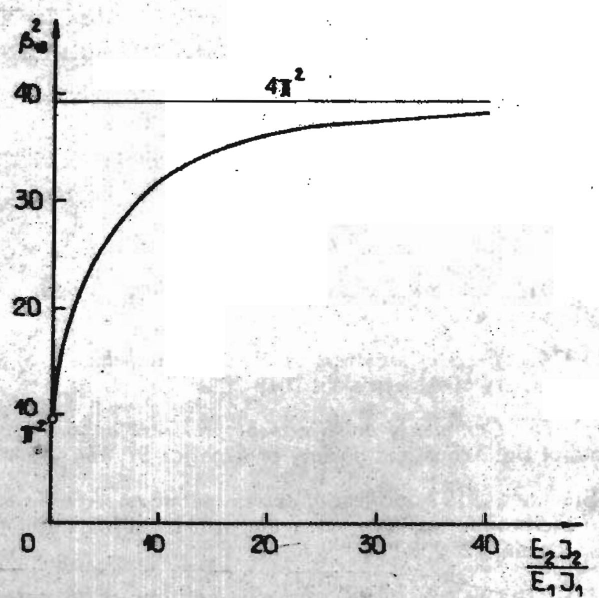

Fig. 7. Changes of $\beta_{10} = S_0 L^2 / E_1 J_1$ versus flexural rigidity ratio $E_2 J_2 / E_1 J_1$

$E_1J_1/E_2J_2 = 1/5$ for the amplitude $a = 0.02$ m (curve 3, Fig.5). The system is prestressed with the force $S_0 = S_{0C}$. This column, in the initial instant $t = 0$, has got the natural frequency marked by the point A. The internal longitudinal forces are marked by the point A in the Fig.6. When the column reaches the internal forces defined by the point C on the boundary curve (Fig.6), its natural frequencies vanish ($\omega = 0$) and the column takes the rectilinear shape. The loading of the system with the prestressing force $S_0 > S_{0C}$ (point B_1, Fig.6) causes the curvilinear state of equilibrium at the point C (Fig.6).

Fig.7 shows the variation of $\beta_{10}^2 = S_0L^2/E_1J_1$ for various flexural rigidities E_2J_2. For $E_2J_2 \to 0$ the first column behaves as the single column and loses stability for $\beta_{10}^2 = \Pi^2$. When $E_2J_2 \to \infty$ the column "1" loses stability for $\beta_{10}^2 = 4\Pi^2$ as the cantilever column does.

6. Conclusion

The study indicates that prestressing of the geometrically non–linear structure has an effect on its vibrations. The prestressing force changes the natural frequency. The asymmetry of distributions of the flexural rigidities (as constant sum of this quantities) also affects the values of the natural vibration frequencies. For the discussed compound column there occurs the divergence instability ($\omega = 0$) as a result of prestressing without application of an additional external load.

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Appendix: List of general symbols

A_i – cross-section area
a – amplitude of vibration
E_i – modulus of elasticity
$i = 1,2$ – number of beam
J_i – moment of inertia
l_{0i} – initial length
k, n – number of vibration mode

L	–	length of the structure
S_{0i}	–	prestressing axial force of i–th beam ($S_{0i} = (-1)^{i+1} S_0$)
$S_i(t)$	–	internal axial force due to flexural deflection of i–th beam
$\bar{S}_i$	–	value of the force $S_i(t)$ for $t = 0$
S	–	force defined in equation (3.9)
S^s, S^a	–	force S for symmetric and antisymmetric mode of vibrations
T_i	–	density of kinetic energy
t	–	time
$U_i(x,t)$	–	axial displacement
$u_i(x)$	–	amplitude of axial displacement
U_0	–	initial lack of fit of beams
U_{0i}	–	initial axial displacement (at $x = L$)
V_i^m	–	denisity of potential energy of the system, m–th component
$W_i(x,t)$	–	lateral deflection
$w_i(x)$	–	mode of vibrations
w_{ik}^a, w_{in}^s	–	symmetrical and antisymmetrical modes of vibrations
x	–	axial coordinate
ρ_i	–	mass denisity
ω_k	–	natural frequency of k–th mode of vibrations
ω	–	natural frequency
ω_{0k}	–	natural frequency of vibration for amplitude $a \to 0$
$\bar{\omega}_k$	–	natural frequency of vibration of system "0" (Table 1) for $S_0 = 0$

Drgania wstępnie sprężonej columny dwuprętowej

Streszczenie

Przedmiotem pracy jest dynamiczna odpowiedź układu geometrycznie nieliniowego, który stanowi dwuczłonowa belka swobodnie podparta, wstępnie sprężona. Sprężenie wywołane jest przez początkową różnicę długości poszczególnych prętów. Podano wariacyjne sformułowanie problemu i zastosowano zasadę Hamiltona. Rozwiązanie otrzymano przez zastosowanie uogólnionej metody Ritza. Wyznaczono częstości drgań i wartości sił wewnętrznych w zależności od sprężenia i amplitudy drgań. Przeprowadzono dyskusję warunku ortogonalności dla nieliniowych postaci drgań.

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