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ON SOME PROBLEMS OF RODS WITH PERIODIC-VARIABLE CROSS-SECTIONS

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The aim of the paper is an application of the non-standard method of homogenization (a method of microlocal modelling), [5,6,7,8] to constrained torsion problems for straight linear–elastic rods with periodic–variable compact cross–sections. The paper is a continuation of the earlier contribution [3]. The method is based both on the concept of microlocal modelling and the notion of internal constraints [1,4].

1. Fundamentals

In sec.1 of [3] has been proposed a certain technical theory of constrained torsion for straight linear–elastic rods with the ε–periodic variable cross–sections. In the undeformed configuration the rod occupies a regular region Ω in the 3–space, parametrized by the orthogonal Carthesian coordinates X_1, X_2, X_3. We assume that X_3 coincides with the rod axis and X_1, X_2 are pararell to the principal central inertia axes of an arbitrary cross–section $F(X_3)$, $X_3 \in [0, l]$ and that $F(X_3) = F(X_3 + \varepsilon)$, $X_3 \in [0, l - \varepsilon]$. It means that the rod has the ε-periodic structure, with $\varepsilon << l$.

We shall confine ourselves to the rod deformations $\chi_k = \chi_k(\boldsymbol{X}, t)$, $\boldsymbol{X} = (X_1, X_2, X_3) \in \Omega$, $t \in [0, t_f]$, t being the time coordinate, admissible by the internal constraints of the form [1]

$$\chi^m{}_{,\alpha}\, \chi_{m,\beta} = \delta_{\alpha\beta}.$$

It means that projections of cross–sections of the deformed rod on the plane $0X_1X_2$ behave as rigid. Introducing the displacement vector field $\boldsymbol{u}(\boldsymbol{X}, t) = \chi(\boldsymbol{X}, t) - \boldsymbol{X}$, $\boldsymbol{X} \in \Omega$, $t \in [0, t_f]$, after the linearization of constraints with

[1]The Latin indices take the values 1,2,3; the Greek ones take the values 1,2. Summation convention holds for all kinds of indices.

respect to $u(X,t)$, we arrive at the following explicit form of the internal constraints [4]

$$\begin{aligned} u_1 &= -\Theta(X_3,t)X_2 + \psi(X_3,t), \\ u_2 &= \Theta(X_3,t)X_1 + \varphi(X_3,t), \\ u_3 &= u_3(X_1,X_2,X_3,t), \end{aligned} \tag{1.1}$$

where $\Theta(\cdot)$, $\psi(\cdot)$, $\varphi(\cdot)$ are arbitrary differentiable functions.
We introduce the extra constraints in the explicit form [2]

$$u_3(X_1,X_2,X_3,t) = \Phi(X_1,X_2)\zeta(X_3,t) + \eta(X_3,t), \tag{1.2}$$

where $\Phi(\cdot)$ is a certain a priori postulated function depending on the shape of rod cross–sections, and $\zeta(\cdot),\eta(\cdot)$ are arbitrary differentiable functions.

Functions $\Theta(\cdot)$, $\psi(\cdot)$, $\varphi(\cdot)$, $\zeta(\cdot)$, $\eta(\cdot)$, called the generalized coordinates [1], are assumed to be independent and are defined on $[0,l]\times[0,t_f]$.

The motion of the constrained body is governed by the equation of motion [1]

$$T^{ij}{}_{,j} + \rho b_i + \rho r_i = \rho\ddot{\chi}_i, \qquad X\in\Omega, \quad t\in[0,t_f], \tag{1.3}$$

where $T = T(X,t)$ is the stress tensor produced by the material reaction, $\rho = \rho(X)$ is the mass density in the reference configuration, $b = b(X,t)$ is the density of external loadings and $r = r(X,t)$ denotes the density of unknown reaction body forces due to the internal constraints.

At the boundary $\partial\Omega$ of the rod the following conditions hold [1]:

$$T^{ij}n_j = p_i + s_i, \quad \text{for almost every } X\in\partial\Omega, \ t\in[0,t_f], \tag{1.4}$$

where $n = n(X)$ is a unit outward normal to $\partial\Omega$, $p = p(X,t)$ are the known surface tractions and $s = s(X,t)$ stand for unknown surface reaction forces also due to the internal contraints.

We postulate that the contraints are ideal [1], i.e., that the condition

$$\int_\Omega \rho r\cdot\delta\chi d\Omega + \int_{\partial\Omega} s\cdot\delta\chi d(\partial\Omega) = 0, \tag{1.5}$$

holds for any virtual displacement $\delta\chi(X,t)$ admissible by the internal contraints.

Eliminating the reaction forces from eq.(1.5) by means of eqs.(1.3),(1.4) and substituting into the resulting relations the virtual displacements related to the internal contraints (1.1),(1.2), for homogenous isotropic materials

$$\begin{aligned} T^{11} &= T^{22} = \lambda(\Phi\xi_{,3} + \eta_{,3}), \\ T^{13} &= \mu(-X_2\Theta_{,3} + \psi_{,3} + \zeta\Phi_{,1}), \\ T^{23} &= \mu(X_1\Theta_{,3} + \varphi_{,3} + \zeta\Phi_{,2}), \\ T^{33} &= (\lambda+2\mu)(\Phi\zeta_{,3} + \eta_{,3}), \end{aligned} \tag{1.6}$$

where η and λ are Lame's modulae, we arrive at the system of the 5 variational equations (eqs.(1.11) in [3]) for the unknown generalized coordinates $\Theta(X_3,t)$, $\psi(X_3,t)$, $\varphi(X_3,t)$, $\zeta(X_3,t)$, $\eta(X_3,t)$, $X_3 \in [0,l]$, $t \in [0,t_f]$.
Because for the small values of ε as related to the rod lenght l, the obtaining of solutions of such differential equations system with variable ε-periodic coefficients is rather complicated. Hence we are going (in sec.2 of [3]) to approximate this system by a certain system of differential equations with the constant coefficients. We shall use the method of microlocal modelling [6], the general formulation of which was outlined in [7,8,5].

The microlocal approximation postulates that we look for the approximate solution in the class of functions given by

$$
\begin{aligned}
\Theta(X_3,t) &= \Theta_0(X_3,t) + \Theta_a(X_3,t)h^a(X_3),\\
\psi(X_3,t) &= \psi_0(X_3,t) + \psi_a(X_3,t)h^a(X_3),\\
\varphi(X_3,t) &= \varphi_0(X_3,t) + \varphi_a(X_3,t)h^a(X_3),\\
\zeta(X_3,t) &= \zeta_0(X_3,t) + \zeta_a(X_3,t)h^a(X_3),\\
\eta(X_3,t) &= \eta_0(X_3,t) + \eta_a(X_3,t)h^a(X_3),
\end{aligned}
\tag{1.7}
$$

where: $a = 1,2,...,n$, (summation convention holds), $h^a(\cdot)$ are postulated a priori ε-periodic regular functions such that

$$\int_0^{\varepsilon} h^a{}_{,3}(X_3)dX_3 = 0$$

and $\Theta_0(\cdot,t)$, $\Theta_a(\cdot,t)$, $\psi_0(\cdot,t)$, $\psi_a(\cdot,t)$, $\varphi_0(\cdot,t)$, $\varphi_a(\cdot,t)$, $\zeta_0(\cdot,t)$, $\zeta_a(\cdot,t)$, $\eta_0(\cdot,t)$, $\eta_a(\cdot,t)$ are sufficiently regular unknown functions. Functions $\Theta_0(\cdot)$, $\psi_0(\cdot)$, $\varphi_0(\cdot)$, $\zeta_0(\cdot)$, $\eta_0(\cdot)$ will be called generalized macro-deformations. Functions $\Theta_a(\cdot)$, $\psi_a(\cdot)$, $\varphi_a(\cdot)$, $\zeta_a(\cdot)$, $\eta_a(\cdot)$ describe the effects due to the microperiodic structure of the rod are called the microlocal parameters.
Defining

$$< f > = \frac{1}{\varepsilon}\int_0^{\varepsilon} f(X_3)dX_3,$$

for any integrable ε-periodic function $f(\cdot)$, denoting

$$
\begin{aligned}
S_1 &= S_1(X_3) = \int_{F(X_3)} X_2 dF \equiv 0,\\
S_2 &= S_2(X_3) = \int_{F(X_3)} X_1 dF \equiv 0,\\
J_0 &= J_0(X_3) = \int_{F(X_3)} (X_1^2 + X_2^2)dF,
\end{aligned}
$$

$$
\begin{aligned}
J_s &= J_s(X_3) = \int\limits_{F(X_3)} (X_1\Phi_{,2} - X_2\Phi_{,1})dF,\\
J &= J(X_3) = \int\limits_{F(X_3)} \Phi^2 dF,\\
J_k &= J_k(X_3) = \int\limits_{F(X_3)} (\Phi_{,1}^2 + \Phi_{,2}^2)dF,\\
K_1 &= K_1(X_3) = \int\limits_{F(X_3)} \Phi_{,2}\, dF,\\
K_2 &= K_2(X_3) = \int\limits_{F(X_3)} \Phi_{,1}\, dF,\\
S_\Phi &= S_\Phi(X_3) = \int\limits_{F(X_3)} \Phi dF,
\end{aligned}
\tag{1.8}
$$

and

$$
\begin{aligned}
P_k(0,t) &= \int\limits_{F(0)} p_k(0,t)dF,\\
P_k(l,t) &= \int\limits_{F(l)} p_k(l,t)dF,\\
M_s(0,t) &= \int\limits_{F(0)} [p_2(0,t)X_1 - p_1(0,t)X_2]dF,\\
M_s(l,t) &= \int\limits_{F(l)} [p_2(l,t)X_1 - p_1(l,t)X_2]dF,\\
M_\Phi(0,t) &= \int\limits_{F(0)} p_3(0,t)\Phi(X_1,X_2)dF,\\
M_\Phi(l,t) &= \int\limits_{F(l)} p_3(l,t)\Phi(X_1,X_2)dF,
\end{aligned}
\tag{1.9}
$$

for $X_3 \in (0,l)$:

$$
\begin{aligned}
\mathring{p}_k = \mathring{p}_k(X_3,t) &= \int\limits_{\partial F(X_3)} \sqrt{g(\gamma,X_3)}p_k(X_3,t)d(\partial F),\\
m_s = m_s(X_3,t) &= \int\limits_{\partial F(X_3)} \sqrt{g(\gamma,X_3)}[p_2(X_3,t)X_1 - p_1(X_3,t)X_2]d(\partial F),
\end{aligned}
$$

$$m_\Phi = m_\Phi(X_3,t) = \int_{\partial F(X_3)} \sqrt{g(\gamma,X_3)}p_3(X_3,t)\Phi(X_1,X_2)d(\partial F),$$

where $g(\gamma, X_3)$ is the discriminant of the first quadric form of the lateral surface of the rod, γ is the parameter of the curve $\partial F(X_3)$, we obtaining the following equations system

$$\begin{aligned} &\mu\Big(< J_0 > \Theta_{0,33} + < J_0 h^a{}_{,3} > \Theta_{a,3} + < J_s > \zeta_{0,3}\Big) = \rho < J_0 > \ddot{\Theta}_0 - m_s, \\ &< J_0 h^b{}_{,3} > \Theta_{0,3} + < J_0 h^a{}_{,3} h^b{}_{,3} > \Theta_a + < J_s h^b{}_{,3} > \zeta_0 = 0, \end{aligned} \qquad (i)$$

$$\begin{aligned} &\mu\Big(< F > \psi_{0,33} + < F h^a{}_{,3} > \psi_{a,3} + < K_2 > \zeta_{0,3}\Big) = \\ &= -\rho b_1 < F > + \rho < F > \ddot{\psi}_0 - \overset{\circ}{p}_1, \\ &< F h^b{}_{,3} > \psi_{0,3} + < F h^a{}_{,3} h^b{}_{,3} > \psi_a + < K_2 h^b{}_{,3} > \zeta_0 = 0, \end{aligned} \qquad (ii)$$

$$\begin{aligned} &\mu\Big(< F > \varphi_{0,33} + < F h^a{}_{,3} > \varphi_{a,3} + < K_1 > \zeta_{0,3}\Big) = \\ &= -\rho b_2 < F > + \rho < F > \ddot{\varphi}_0 - \overset{\circ}{p}_2, \\ &< F h^b{}_{,3} > \varphi_{0,3} + < F h^a{}_{,3} h^b{}_{,3} > \varphi_a + < K_1 h^b{}_{,3} > \zeta_0 = 0, \end{aligned} \qquad (iii)$$

$$(1.10)$$

$$\begin{aligned} &-\mu\Big(< J_s > \Theta_{0,3} + < J_s h^a{}_{,3} > \Theta_a + < J_k > \zeta_0\Big) + (\lambda + 2\mu)\cdot\Big(< J > \zeta_{0,33} + \\ &+ < J h^a{}_{,3} > \zeta_{a,3} + < S_\Phi > \eta_{0,33} + < S_\Phi h^a{}_{,3} > \eta_{a,3}\Big) = \\ &= -\rho b_3 < S_\Phi > + \rho < J > \ddot{\zeta}_0 + \rho < S_\Phi > \ddot{\eta}_0 - m_\Phi, \end{aligned} \qquad (iv)$$

$$\begin{aligned} &< J h^b{}_{,3} > \zeta_{0,3} + < J h^a{}_{,3} h^b{}_{,3} > \zeta_a + \\ &+ < S_\Phi h^b{}_{,3} > \eta_{0,3} + < S_\Phi h^a{}_{,3} h^b{}_{,3} > \eta_a = 0, \end{aligned}$$

$$\begin{aligned} &(\lambda + 2\mu)\Big(< S_\Phi > \zeta_{0,33} + < S_\Phi h^a{}_{,3} > \zeta_{a,3} + < F > \eta_{0,33} + \\ &< F h^a{}_{,3} > \eta_{a,3}\Big) = -\rho b_3 < F > + \rho < S_\Phi > \ddot{\zeta}_0 + \rho < F > \ddot{\eta}_0 - \overset{\circ}{p}_3, \end{aligned}$$

$$\begin{aligned} &< S_\Phi h^b{}_{,3} > \zeta_{0,3} + < S_\Phi h^a{}_{,3} h^b{}_{,3} > \zeta_a + < F h^b{}_{,3} > \eta_{0,3} + \\ &+ < F h^a{}_{,3} h^b{}_{,3} > \eta_a = 0, \end{aligned}$$

for $x_3 \in (0,l)$, $t \in [0,t_f]$, and boundary conditions

$$\mu\Big(< J_0 > \Theta_{0,3} + < J_0 h^a{}_{,3} > \Theta_a + < J_s > \zeta_0\Big) = M_s n_3,$$

$$\mu\Big(< F > \psi_{0,3} + < Fh^a{}_{,3} > \psi_a + < K_2 > \zeta_0\Big) = P_1 n_3,$$
$$\mu\Big(< F > \varphi_{0,3} + < Fh^a{}_{,3} > \varphi_a + < K_1 > \zeta_0\Big) = P_2 n_3, \tag{1.11}$$
$$(\lambda + 2\mu)\Big(< J > \zeta_{0,3} + < Jh^a{}_{,3} > \zeta_a + < S_\Phi > \eta_{0,3} +$$
$$+ < S_\Phi h^a{}_{,3} > \eta_a\Big) = M_\Phi n_3,$$
$$(\lambda + 2\mu)\Big(< S_\Phi > \zeta_{0,3} + < S_\Phi h^a{}_{,3} > \zeta_a + < F > \eta_{0,3} +$$
$$+ < Fh^a{}_{,3} > \eta_a\Big) = P_3 n_3,$$

for $X_3 = 0, \;\; X_3 = l, \;\; t \in [0, t_f]$.
If the exact analytical solution to the boundary–value problem given by eqs.(1.10) and (1.11) is known then the following approximation formulae may be used to evalute the solution to the primary (ε– periodic) problem:

$$\begin{aligned}
&\Theta(X_3,t) \sim \Theta_0(X_3,t), && \Theta_{,3}(X_3,t) \sim \Theta_{0,3}(X_3,t) + \Theta_a(X_3,t)h^a{}_{,3}(X_3),\\
&\psi(X_3,t) \sim \psi_0(X_3,t), && \psi_{,3}(X_3,t) \sim \psi_{0,3}(X_3,t) + \psi_a(X_3,t)h^a{}_{,3}(X_3),\\
&\varphi(X_3,t) \sim \varphi_0(X_3,t), && \varphi_{,3}(X_3,t) \sim \varphi_{0,3}(X_3,t) + \varphi_a(X_3,t)h^a{}_{,3}(X_3),\\
&\zeta(X_3,t) \sim \zeta_0(X_3,t), && \zeta_{,3}(X_3,t) \sim \zeta_{0,3}(X_3,t) + \zeta_a(X_3,t)h^a{}_{,3}(X_3),\\
&\eta(X_3,t) \sim \eta_0(X_3,t), && \eta_{,3}(X_3,t) \sim \eta_{0,3}(X_3,t) + \eta_a(X_3,t)h^a{}_{,3}(X_3).
\end{aligned} \tag{1.12}$$

We see that the microlocal parameters have the negleclible influence on the displacement field (1.1),(1.2), but they play an essential role if we calculate the stresses (1.6). We can also calculate the reaction forces produced by the internal contstraints (1.1),(1.2), using eqs.(1.3),(1.4).

2. Some special solutions

We consider the straight linear–elastic axial symmetric rod with lenght l. The radius of the cross–section is ε–periodic and given by the formula

$$R(X_3) = R_0\Big(1 + \delta\cos\frac{2\pi X_3}{\varepsilon}\Big), \tag{2.1}$$

where R_0 =const, δ =const, $\varepsilon << l$. R_0 stends for the adverage radius and $R_0\delta$ is its amplitude.

Taking into account the axial symmetry of the rod one should introduce also the axial symmetric function Φ which characterizes the out of plane displacement u_3. Let $\Phi(\cdot)$ has the form

$$\Phi(X_1, X_2) = X_1^2 + X_2^2. \tag{2.2}$$

In this case the characteristics (1.8) attain the following values

$$\begin{aligned}
J_0 &= J_0(X_3) = \frac{\pi R^4}{2}, \\
J_s &= J_s(X_3) \equiv 0 \text{ (for each axial symmetric function } \Phi), \\
J &= J(X_3) = \frac{\pi R^6}{3}, \\
J_k &= J_k(X_3) = 4J_0(X_3), \\
K_\alpha &= K_\alpha(X_3) \equiv 0 \text{ (for each axial symmetric function } \Phi), \\
S_\Phi &= S_\Phi(X_3) = J_0(X_3).
\end{aligned} \tag{2.3}$$

Using the microlocal approximation we are looking for the approximate solution given by (1.7). We assume the shape function $h^a(\cdot)$ in the form

$$h^a(X_3) = \frac{\varepsilon}{l} \sin \frac{a 2\pi X_3}{\varepsilon}. \tag{2.4}$$

The phase displacement between (2.1) and (2.4) follows from the simple reasoning that functions $\Theta(\cdot)$, $\psi(\cdot)$, $\zeta(\cdot)$, $\eta(\cdot)$ (which determine the displacement vector by means of eqs.(1.1),(1.2)) must increase if the rod radius decreases and inversely.

Introducing constant characteristics of the mean cross-section of the rod

$$\begin{aligned}
\bar{F} &= \pi R_0^2, \\
\bar{J}_0 &= \frac{\pi R_0^4}{2}, \\
\bar{J}_k &= 4\bar{J}_0, \\
\bar{S}_\Phi &= \bar{J}_0, \\
\bar{J} &= \frac{\pi R_0^6}{3},
\end{aligned} \tag{2.5}$$

after simple calculations we obtain the following averages (here for $n = 2$)

$$\begin{aligned}
&< J_0 >= \bar{J}_0(1 + 3\delta^2 + \frac{3}{8}\delta^4), \\
&< J_k >= 4 < J_0 >, \\
&< S_\Phi >=< J_0 >, \\
&< J_0 h^1{}_{,3} >= \bar{J}_0 \frac{\pi}{l}(4\delta + 3\delta^3), \\
&< J_0 h^2{}_{,3} >= \bar{J}_0 \frac{\pi}{l}(6\delta^2 + \delta^4), \\
&< J_0 h^1{}_{,3} h^1{}_{,3} >= \bar{J}_0 \frac{\pi^2}{l^2}(2 + 9\delta^2 + \frac{5}{4}\delta^4),
\end{aligned}$$

$$
\begin{aligned}
&< J_0 h^1{}_{,3} h^2{}_{,3} >= \bar{J}_0 \frac{\pi^2}{l^2}(8\delta + 8\delta^3),\\
&< J_0 h^2{}_{,3} h^2{}_{,3} >= \bar{J}_0 \frac{\pi^2}{l^2}(8 + 24\delta^2 + \frac{7}{2}\delta^4),\\
&< J >= \bar{J}(1 + \frac{15}{2}\delta^2 + \frac{45}{8}\delta^4 + \frac{5}{16}\delta^6),\\
&< J h^1{}_{,3} >= \bar{J}\frac{\pi}{l}(6\delta + 15\delta^3 + \frac{15}{4}\delta^5),\\
&< J h^2{}_{,3} >= \bar{J}\frac{\pi}{l}(15\delta^2 + 15\delta^4 + \frac{15}{16}\delta^6),\\
&< J h^1{}_{,3} h^1{}_{,3} >= \bar{J}\frac{\pi^2}{l^2}(2 + \frac{45}{2}\delta^2 + \frac{75}{4}\delta^4 + \frac{35}{32}\delta^6),\\
&< J h^1{}_{,3} h^2{}_{,3} >= \bar{J}\frac{\pi^2}{l^2}(12\delta + 40\delta^3 + \frac{45}{4}\delta^5),\\
&< J h^2{}_{,3} h^2{}_{,3} >= \bar{J}\frac{\pi^2}{l^2}(8 + 60\delta^2 + \frac{105}{2}\delta^4 + \frac{13}{4}\delta^6),\\
&< F >= \bar{F}(1 + \frac{\delta^2}{2}),\\
&< F h^1{}_{,3} >= \bar{F}\frac{\pi}{l}2\delta,\\
&< F h^2{}_{,3} >= \bar{F}\frac{\pi}{l}\delta^2,\\
&< F h^1{}_{,3} h^1{}_{,3} >= \bar{F}\frac{\pi^2}{l^2}(2 + \frac{3}{2}\delta^2),\\
&< F h^1{}_{,3} h^2{}_{,3} >= \bar{F}\frac{\pi^2}{l^2}4\delta,\\
&< F h^2{}_{,3} h^2{}_{,3} >= \bar{F}\frac{\pi^2}{l^2}(8 + 4\delta^2).
\end{aligned}
\tag{2.6}
$$

For simplification we assume that the rod is weightless and only static loading are applied. In this case

$$
\begin{aligned}
&\Theta_0 = \Theta_0(X_3), \quad \Theta_a = \Theta_a(X_3), \quad \varphi_0 = \varphi_0(X_3), \quad \varphi_a = \varphi_a(X),\\
&\psi_0 = \psi_0(X_3), \quad \psi_a = \psi_a(X_3), \quad \zeta_0 = \zeta_0(X_3), \quad \zeta_a = \zeta_a(X_3),\\
&\eta_0 = \eta_0(X_3), \quad \eta_a = \eta_a(X_3).
\end{aligned}
$$

On the lateral surface of the rod distributed torsional moments

$$
m_s(X_3) = \int_{\partial F(X_3)} \sqrt{1 + R_{,3}^2}[p_2(X_3)X_1 - p_1(X_3)X_2]d(\partial F)
$$

are given (for $X_3 \in (0, l)$), on the ends of the rod (for $X_3 = 0$ and $X_3 = l$)

concentrated torsional moments

$$M_s(0) = \int_{F(0)} [p_2(0)X_1 - p_1(0)X_2]dF,$$
$$M_s(l) = \int_{F(l)} [p_2(l)X_1 - p_1(l)X_2]dF,$$

and axial forces

$$P_3(0) = \int_{F(0)} p_3(0)dF, \qquad P_3(l) = \int_{F(l)} p_3(l)dF$$

are also known.

The transverse components of the resultant forces

$$\overset{\circ}{p}_\alpha = \int_{\partial F(X_3)} \sqrt{1 + R_{,3}^2}\, p_\alpha d(\partial F) \quad (\text{for } X_3 \in (0,l)),$$
$$P_\alpha(0) = \int_{F(0)} p_\alpha(0)dF, \qquad P_\alpha(l) = \int_{F(l)} p_\alpha(l)dF$$

and the axial external forces p_3 on the lateral surface of the rod (for $X_3 \in (0,l)$) vanish. Then eqs (1.9) attain the following form

$$\begin{aligned}
P_\alpha(0) &\equiv 0, \qquad P_\alpha(l) \equiv 0,\\
P_3(0) &= \int_{F(0)} p_3(0)dF, \qquad P_3(l) = \int_{F(l)} p_3(l)dF,\\
M_s(0) &= \int_{F(0)} [p_2(0)X_1 - p_1(0)X_2]dF,\\
M_s(l) &= \int_{F(l)} [p_2(l)X_1 - p_1(l)X_2]dF,\\
M_\Phi(0) &= \int_{F(0)} p_3(0)(X_1^2 + X_2^2)dF,\\
M_\Phi(l) &= \int_{F(l)} p_3(l)(X_1^2 + X_2^2)dF,
\end{aligned} \tag{2.7}$$

for $X_3 \in (0,l)$

$$\begin{aligned}
\overset{\circ}{p}_k &= \overset{\circ}{p}_k(X_3) \equiv 0\\
m_s &= m_s(X_3) = \int_{\partial F(X_3)} \sqrt{1 + R_{,3}^2}[p_2(X_3)X_1 - p_1(X_3)X_2]d(\partial F).
\end{aligned}$$

Substituting (2.6) and (2.7) into (1.10) we obtain the system of $5(n+1)$ linear differential equations (with constant coefficients) of the first order for $5n$ microlocal parameters $\Theta_a(\cdot)$, $\psi_a(\cdot)$, $\varphi_a(\cdot)$, $\zeta_a(\cdot)$, $\eta_a(\cdot)$ and the second order for 5 generalized macro-deformations $\Theta_0(\cdot)$, $\psi_0(\cdot)$, $\varphi_0(\cdot)$, $\zeta_0(\cdot)$, $\eta_0(\cdot)$.
In the some manner the boundary conditions (1.11) can be treated.

Because of $J_s \equiv 0$ and $K_\alpha \equiv 0$, eqs.(1.10) result in 4 independent system of equations. The microlocal parameters can be eliminated from this systems and thus we obtain 5 effective equations for 5 generalized coordinates (macro-deformations) – 3 equations are independent (obtained from systems (i) – (iii) in eqs.(1.10) – for $\Theta_0, \psi_0, \varphi_0$) and 2 equations are interrelated (for ζ_0 and η_0 – obtained from system (iv) in eqs.(1.10)).

Functions ψ_0, ψ_a, φ_0, φ_a disappear since equations (ii) and (iii) (1.10) together with the required boundary conditions for (2.7) are homogenous.

As an example assume $m_s(X_3) = m$, $p_3(0) = -q$, $p_3(l) = q$, where $m =$const, $q =$const (fig.1).

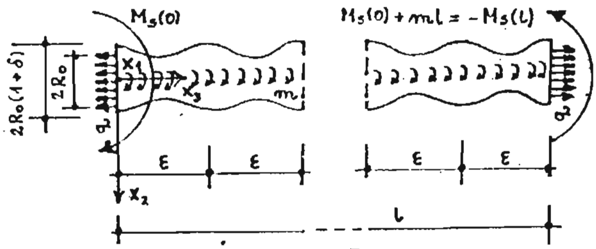

Fig. 1.

For $n = 1$ we obtain the following solution

$$\begin{aligned}
\Theta_0 &= -\frac{m}{\mu J_{01}^{\rm eff}}\frac{X_3^2}{2} - \frac{M_s(0)}{\mu J_{01}^{\rm eff}}X_3 + C,\\
\Theta_1 &= \Big[-\frac{m}{\mu J_{01}^{\rm eff}}X_3 - \frac{M_s(0)}{\mu J_{01}^{\rm eff}}\Big]\Big(-\frac{l(2\delta + 1.5\delta^3)}{\pi(1 + 4.5\delta^2 + 0.625\delta^4)}\Big),\\
\zeta_0 &= \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\rm eff}}\Big(\frac{5.5\delta^2 - 36\delta^4}{1 + 6.75\delta^2 - 98.625\delta^4}\Big)\frac{1}{\gamma_1}\frac{{\rm sh}\gamma_1(X_3 - \frac{l}{2})}{{\rm ch}\frac{\gamma_1 l}{2}},\\
\zeta_1 &= \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\rm eff}}\frac{l}{\pi}\Big[\frac{-16.5\delta^3 + 186.375\delta^5}{1 + 13.5\delta^2 - 151.6875\delta^4}\cdot\\
&\quad\cdot\frac{{\rm ch}\gamma_1(X_3 - \frac{l}{2})}{{\rm ch}\frac{\gamma_1 l}{2}} - \frac{\delta - 5.04545\delta^3 - 53.00552\delta^5}{1 + 24.454\delta^2 - 146.09659\delta^4}\Big], \qquad (2.8)\\
\eta_0 &= \frac{q\bar{F}}{(\lambda+2\mu)F_1^{\rm eff}}\Big[-\frac{16.5\delta^2 + 362.25\delta^4}{1 + 34.25\delta^2 + 63\delta^4}\cdot
\end{aligned}$$

$$\cdot\frac{1}{\gamma_1}\frac{\mathrm{sh}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}}+X_3\Big]+D,$$

$$\eta_1=\frac{q\bar{F}}{(\lambda+2\mu)F_1^{\mathrm{eff}}}\frac{l}{\pi}\Big[\frac{33\delta^3+642\delta^5}{1+40.5\delta^2+336.375\delta^4}\cdot$$

$$\cdot\frac{\mathrm{ch}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}}+\frac{2\delta-13.5\delta^3-13.5\delta^5}{1+21\delta^2+6.75\delta^4}\Big],$$

where

$$\begin{aligned}
J_{01}^{\mathrm{eff}} &= \Big(\frac{1-0.5\delta^2+2.5\delta^4-0.9375\delta^6+0.46875\delta^8}{1+4.5\delta^2+1.25\delta^4}\Big)\bar{J}_0,\\
J_1^{\mathrm{eff}} &= \Big(\frac{1-3.75\delta^2-6\delta^4}{1+6.75\delta^2-98.625\delta^4}\Big)\bar{J},\\
F_1^{\mathrm{eff}} &= \Big(\frac{1+20.5\delta^2-105.1875\delta^4}{1+24\delta^2+30.5625\delta^4}\Big)\bar{F},
\end{aligned}\tag{2.9}$$

and

$$\gamma_1=\sqrt{\frac{\mu(1+3\delta^2+0.375\delta^4)\bar{J}_k}{(\lambda+2\mu)0.25J_1^{\mathrm{eff}}}},$$

and C, D are arbitrary constants (that may be equal to zero).
Functions Θ_0 and Θ_1 have been exactly calculated (in eqs.(2.8)) while ζ_0, ζ_1, η_0, η_1 with the accuracy up to δ^4.
Becauce for $n>1$ to get the solution in the general form is rather complicated, to order to compare the results we are to use the solutions obtained for some fixed values of the parameter δ. Therefore we also calculate in the exact form functions ζ_0, ζ_1, η_0, η_1, and for $\delta=0.1$ we have

$$\begin{aligned}
\Theta_0 &= -\frac{m}{\mu J_{01}^{\mathrm{eff}}}\frac{X_3^2}{2}-\frac{M_s(0)}{\mu J_{01}^{\mathrm{eff}}}X_3+C,\\
\Theta_1 &= \Big(-\frac{m}{\mu J_{01}^{\mathrm{eff}}}X_3-\frac{M_s(0)}{\mu J_{01}^{\mathrm{eff}}}\Big)\Big(-0.19281\frac{l}{\pi}\Big),\\
\zeta_0 &= \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\mathrm{eff}}}0.04855\frac{1}{\gamma_1}\frac{\mathrm{sh}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}},\\
\zeta_1 &= \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\mathrm{eff}}}\frac{l}{\pi}\Big[-0.01309\frac{\mathrm{ch}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}}-0.07893\Big],\\
\eta_0 &= \frac{q\bar{F}}{(\lambda+2\mu)F_1^{\mathrm{eff}}}\frac{l}{\pi}\Big[-0.14285\frac{1}{\gamma_1}\frac{\mathrm{sh}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}}+X_3\Big]+D,\\
\eta_1 &= \frac{q\bar{F}}{(\lambda+2\mu)F_1^{\mathrm{eff}}}\frac{l}{\pi}\Big[0.0261359\frac{\mathrm{ch}\gamma_1(X_3-\frac{l}{2})}{\mathrm{ch}\frac{\gamma_1 l}{2}}+0.15393\Big],
\end{aligned}\tag{2.10}$$

where

$$\begin{aligned} J_{01}^{\text{eff}} &= 0.95228\bar{J}_0, \\ J_1^{\text{eff}} &= 0.90939\bar{J}, \\ F_1^{\text{eff}} &= 0.93742\bar{F}, \\ \gamma_1 &= \sqrt{\frac{4.12015\mu\bar{J}_k}{(\lambda+2\mu)J_1^{\text{eff}}}} = \frac{5.21384}{R_0}\sqrt{\frac{\mu}{\lambda+2\mu}}, \end{aligned} \tag{2.11}$$

we have

$$\begin{aligned} &\Theta \sim \Theta_0 = -\frac{m}{\mu J_{01}^{\text{eff}}}\frac{X_3^2}{2} - \frac{M_s(0)}{\mu J_{01}^{\text{eff}}}X_3 + C, \\ &\Theta_{,3} \sim \Theta_{0,3} + \Theta_1 h^1{}_{,3} = \Big[-\frac{mX_3}{\mu J_{01}^{\text{eff}}} - \frac{M_s(0)}{\mu J_{01}^{\text{eff}}}\Big]\Big(1 - 0.38562\cos\frac{2\pi X_3}{\varepsilon}\Big), \\ &\zeta \sim \zeta_0 = \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\text{eff}}}0.04855\frac{1}{\gamma_1}\frac{\text{sh}\gamma_1(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_1 l}{2}}, \\ &\zeta_{,3} \sim \zeta_{0,3} + \zeta_1 h^1{}_{,3} = \frac{q\bar{J}_k}{(\lambda+2\mu)J_1^{\text{eff}}}\Big[\frac{\text{ch}\gamma_1(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_1 l}{2}}\cdot \\ &\qquad \cdot\Big(0.04855 - 0.02618\cos\frac{2\pi X_3}{\varepsilon}\Big) - 0.15786\cos\frac{2\pi X_3}{\varepsilon}\Big], \\ &\eta \sim \eta_0 = \frac{q\bar{F}}{(\lambda+2\mu)F_1^{\text{eff}}}\Big[-0.14285\frac{1}{\gamma_1}\frac{\text{sh}\gamma_1(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_1 l}{2}} + X_3\Big] + D, \\ &\eta_{,3} \sim \eta_{0,3} + \eta_1 h^1{}_{,3} = \frac{q\bar{F}}{(\lambda+2\mu)F_1^{\text{eff}}}\Big[\frac{\text{ch}\gamma_1(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_1 l}{2}}\cdot \\ &\qquad \cdot\Big(-0.14285 + 0.05227\cos\frac{2\pi X_3}{\varepsilon}\Big) + 1 + 0.30786\cos\frac{2\pi X_3}{\varepsilon}\Big]. \end{aligned} \tag{2.12}$$

For $n = 2$, functions Θ_0 and Θ_a have the following general end exact form

$$\begin{aligned} \Theta_0 &= -\frac{m}{\mu J_{02}^{\text{eff}}}\frac{X_3^2}{2} - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}X_3 + C, \\ \Theta_1 &= \Big[-\frac{m}{\mu J_{02}^{\text{eff}}}X_3 - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}\Big]\frac{l}{\pi}\cdot \\ &\cdot\Big[-\frac{2\delta + 6.5\delta^3 + 6.375\delta^5 + 2.03125\delta^7 + 2.5\delta^9}{1 + 4.5\delta^2 + 10.0625\delta^4 + 6.40625\delta^6 + 0.1171875\delta^8 + 0.2734375\delta^{10}}\Big], \\ \Theta_2 &= \Big[-\frac{m}{\mu J_{02}^{\text{eff}}}X_3 - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}\Big]\frac{l}{\pi}\cdot \\ &\cdot\Big[\frac{1.25\delta^2 + 1.25\delta^4 + 0.46875\delta^6 + 0.390625\delta^8 - 0.078125\delta^{10}}{1 + 4.5\delta^2 + 10.0625\delta^4 + 6.40625\delta^6 + 0.1171875\delta^8 + 0.2734375\delta^{10}}\Big], \end{aligned} \tag{2.13}$$

where

$$J_{02}^{\text{eff}} = \left[\frac{A}{B}\right] \cdot \bar{J}_0, \tag{2.14}$$

$$\begin{aligned}
A &= (1 - 0.5\delta^2 - 0.5625\delta^4 + 2.03125\delta^6 + 6.171875\delta^8 + \\
&- 0.878906\delta^{10} + 0.3173828\delta^{12}), \\
B &= (1 + 4.5\delta^2 + 10.0625\delta^4 + 6.40625\delta^6 + \\
&+ 0.1171875\delta^8 + 0.2734375\delta^{10}).
\end{aligned}$$

Similarly, for $\delta = 0.1$ we have

$$\begin{aligned}
\Theta_0 &= -\frac{m}{\mu J_{02}^{\text{eff}}}\frac{X_3^2}{2} - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}X_3 + C, \\
\Theta_1 &= \left[-\frac{m}{\mu J_{02}^{\text{eff}}}X_3 - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}\right]\frac{l}{\pi}(-0.19748), \qquad (2.15)\\
\Theta_2 &= \left[-\frac{m}{\mu J_{02}^{\text{eff}}}X_3 - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}\right]\frac{l}{\pi}0.01207, \\
J_{02}^{\text{eff}} &= 0.95118\bar{J}_0. \qquad (2.16)
\end{aligned}$$

The remaining macro–deformations and microlocal parameters for $n = 2$ and $\delta = 0.1$ take the form

$$\begin{aligned}
\zeta_0 &= \frac{q\bar{J}_k}{(\lambda + 2\mu)J_2^{\text{eff}}}0.05748\frac{1}{\gamma_2}\frac{\text{sh}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}}, \\
\zeta_1 &= \frac{q\bar{J}_k}{(\lambda + 2\mu)J_2^{\text{eff}}}\frac{l}{\pi}\left[-0.01663\frac{\text{ch}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} - 0.08996\right], \\
\zeta_2 &= \frac{q\bar{J}_k}{(\lambda + 2\mu)J_2^{\text{eff}}}\frac{l}{\pi}\left[0.001353\frac{\text{ch}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} + 0.011918\right], \\
\eta_0 &= \frac{q\bar{F}}{(\lambda + 2\mu)F_2^{\text{eff}}}\left[\frac{-0.16820}{\gamma_2}\frac{\text{sh}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} + X_3\right] + D, \qquad (2.17)\\
\eta_1 &= \frac{q\bar{F}}{(\lambda + 2\mu)F_2^{\text{eff}}}\frac{l}{\pi}\left[0.03317\frac{\text{ch}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} + 0.18095\right], \\
\eta_2 &= \frac{q\bar{F}}{(\lambda + 2\mu)F_2^{\text{eff}}}\frac{l}{\pi}\left[-0.001888\frac{\text{ch}\gamma_2(X_3 - \frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} - 0.01928\right],
\end{aligned}$$

where

$$\begin{aligned}
J_2^{\text{eff}} &= 0.90120\bar{J}, \\
F_2^{\text{eff}} &= 0.92787\bar{F}, \\
\gamma_2 &= \frac{5.23747}{R_0}\sqrt{\frac{\mu}{\lambda + 2\mu}}.
\end{aligned} \tag{2.18}$$

Hence

$$
\begin{aligned}
&\Theta \sim \Theta_0 = -\frac{m}{\mu J_{02}^{\text{eff}}}\frac{X_3^2}{2} - \frac{M_s(0)X_3}{\mu J_{02}^{\text{eff}}} + C,\\
&\Theta_{,3} \sim \Theta_{0,3} + \Theta_1 h^1{}_{,3} + \Theta_2 h^2{}_{,3} = \Big[-\frac{mX_3}{\mu J_{02}^{\text{eff}}} - \frac{M_s(0)}{\mu J_{02}^{\text{eff}}}\Big]\cdot\\
&\quad\cdot\Big(1 - 0.39496\cos\frac{2\pi X_3}{\varepsilon} + 0.04828\cos\frac{4\pi X_3}{\varepsilon}\Big),\\
&\zeta \sim \zeta_0 = \frac{q\bar{J}_k}{(\lambda+2\mu)J_2^{\text{eff}}}0.05748\frac{1}{\gamma_2}\frac{\text{sh}\gamma_2(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}},\\
&\zeta_{,3} \sim \zeta_{0,3} + \zeta_1 h^1{}_{,3} + \zeta_2 h^2{}_{,3} = \frac{q\bar{J}_k}{(\lambda+2\mu)J_2^{\text{eff}}}\cdot\\
&\quad\cdot\Big[\frac{\text{ch}\gamma_2(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}}\Big(0.05748 - 0.03326\cos\frac{2\pi X_3}{\varepsilon} + 0.00541\cdot\\
&\quad\cdot\cos\frac{4\pi X_3}{\varepsilon}\Big) - 0.17992\cos\frac{2\pi X_3}{\varepsilon} + 0.04767\cos\frac{4\pi X_3}{\varepsilon}\Big],\\
&\eta \sim \eta_0 = \frac{q\bar{F}}{(\lambda+2\mu)F_2^{\text{eff}}}\Big[\frac{-0.1682}{\gamma_2}\frac{\text{sh}\gamma_2(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}} + X_3\Big] + D,\\
&\eta_{,3} \sim \eta_{0,3} + \eta_1 h^1{}_{,3} + \eta_2 h^2{}_{,3} = \frac{q\bar{F}}{(\lambda+2\mu)F_2^{\text{eff}}}\cdot\\
&\quad\cdot\Big[\frac{\text{ch}\gamma_2(X_3-\frac{l}{2})}{\text{ch}\frac{\gamma_2 l}{2}}\Big(-0.16820 + 0.06634\cos\frac{2\pi X_3}{\varepsilon} - 0.007552\cdot\\
&\quad\cdot\cos\frac{4\pi X_3}{\varepsilon}\Big) + 1 + 0.36190\cos\frac{2\pi X_3}{\varepsilon} - 0.07712\cos\frac{4\pi X_3}{\varepsilon}\Big].
\end{aligned}
\tag{2.19}
$$

In order to calculate the displacements it is enough to take $n = 1$, since almost identical values of J_{01}^{eff} and J_{02}^{eff}, J_1^{eff} and J_2^{eff}, F_1^{eff} and F_2^{eff} (table 1), here been obtained and the values of functions $\Theta(\cdot)$, $\zeta(\cdot)$, $\eta(\cdot)$ differ insignificantly.

Table 1

$\delta = 0.1$			
	$n = 1$	$n = 2$	
J_0^{eff}	$0.95228\bar{J}_0$	$0.95118\bar{J}_0$	$\frac{J_{01}^{\text{eff}} - J_{02}^{\text{eff}}}{J_{01}^{\text{eff}}}100\% = 0.11\%$
J^{eff}	$0.90939\bar{J}$	$0.90120\bar{J}$	$\frac{J_1^{\text{eff}} - J_2^{\text{eff}}}{J_1^{\text{eff}}}100\% = 0.90\%$
F^{eff}	$0.93742\bar{F}$	$0.92787\bar{F}$	$\frac{F_1^{\text{eff}} - F_2^{\text{eff}}}{F_1^{\text{eff}}}100\% = 1.02\%$

We consider the rod represented on fig.1 assuming the following data

$$\begin{aligned}
&M_s(0) = 0, \quad m = \text{const}, \quad q = \text{const},\\
&R_0 = 5\text{cm},\\
&\delta = 0.1,\\
&l = 100\text{cm},\\
&\varepsilon = \frac{l}{25} = 4\text{cm},\\
&\mu = 78.846\text{GPa},\\
&\lambda = 118.269\text{GPa}.
\end{aligned} \tag{2.20}$$

Hence, we obtain the diagrams of functions $\Theta(X_3)$, $\zeta(X_3)$, $\eta(X_3)$ for $n = 1$ and $n = 2$ (fig.2 – 7).

We can calculate that

$$\frac{\Theta(X_3)_{\text{for } n=1} - \Theta(X_3)_{\text{for } n=2}}{\Theta(X_3)_{\text{for } n=1}} \cdot 100\% = -0.11\%,$$

for $0 < X_3 \le l$, $(\Theta(0)_{\text{for } n=1} = \Theta(0)_{\text{for } n=2} = 0)$.
The maximum of function

$$\frac{\eta(X_3)_{\text{for } n=1} - \eta(X_3)_{\text{for } n=2}}{\eta(X_3)_{\text{for } n=1}} \cdot 100\%$$

for $X_3 = l$ equals to – 0.98 %, the maximum of function

$$\frac{\zeta(X_3)_{\text{for } n=1} - \zeta(X_3)_{\text{for } n=2}}{\zeta(X_3)_{\text{for } n=1}} \cdot 100\%$$

for $X_3 = 0$ and $X_3 = l$ is equal to – 15.31 %.

Basing on the analysis of the obtained functions $\Theta(\cdot)$, $\zeta(\cdot)$ and $\eta(\cdot)$ notice that the cross–sections of the twisted rod with ε–periodic variable radius loaded by m and $M_s(0)$, $M_s(l)$ remain plane. After the stretching of this rod the cross–sections are warping: this phenomenon occurs only on the end near segments of the rod, i.e. for $X_3 \in [0, 0.1l]$ and $[0.9l, l]$ cf. fig. 4,5).

To compare the results consider the rod loaded as shown on fig.1 (m = const, q = const) but with the constant radius R_0.
In this case (from (2.8) and assuming $\delta = 0$) we get

$$\begin{aligned}
&\Theta = -\frac{m}{\mu J_0}\frac{X_3^2}{2} - \frac{M_s(0)X_3}{\mu J_0} + C,\\
&\Theta_{,3} = -\frac{mX_3}{\mu J_0} - \frac{M_s(0)}{\mu J_0},\\
&\zeta \equiv 0,
\end{aligned} \tag{2.21}$$

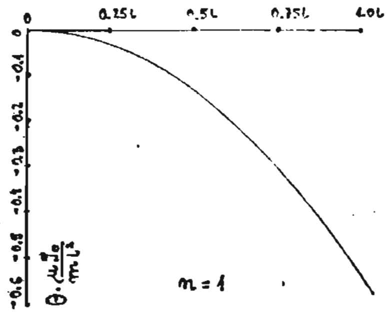

Fig. 2.

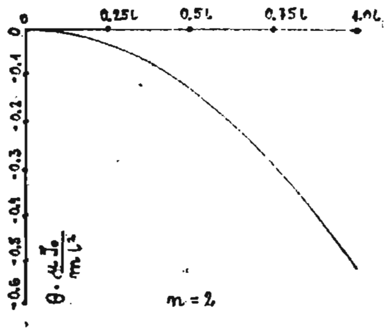

Fig. 3.

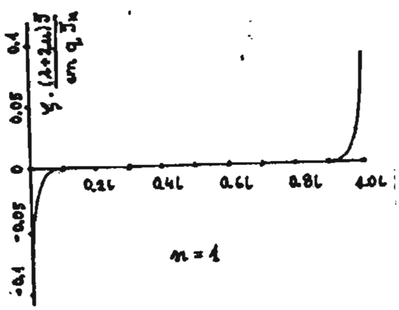

Fig. 4.

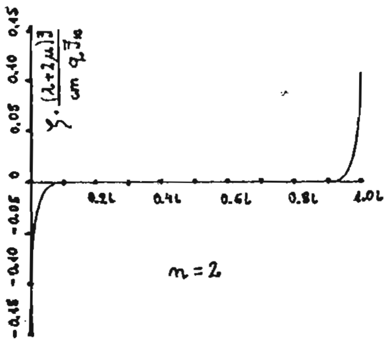

Fig. 5.

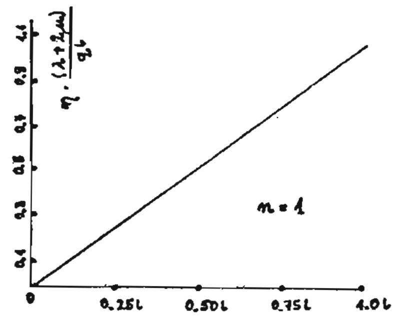

Fig. 6.

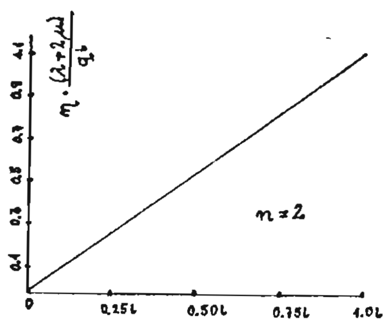

Fig. 7.

$$\eta = \frac{q\bar{F}}{(\lambda+2\mu)\bar{F}}X_3 + D,$$
$$\eta_{,3} = \frac{q\bar{F}}{(\lambda+2\mu)\bar{F}}.$$

For the some data (2.20) we obtain

$$\begin{aligned}
\Theta(l) &= -0.52506\frac{ml^2}{\mu\bar{J}_0} \quad \text{for } n=1,\ \delta=0.1,\\
\Theta(l) &= -0.52566\frac{ml^2}{\mu\bar{J}_0} \quad \text{for } n=2,\ \delta=0.1,\\
\Theta(l) &= -0.5\frac{ml^2}{\mu\bar{J}_0} \quad \text{for } \delta=0.
\end{aligned} \tag{2.22}$$

Analogously, we also have

$$\begin{aligned}
\eta(l) &= 1.06402\frac{ql}{\lambda+2\mu} \quad \text{for } n=1,\ \delta=0.1,\\
\eta(l) &= 1.07450\frac{ql}{\lambda+2\mu} \quad \text{for } n=2,\ \delta=0.1,\\
\eta(l) &= \frac{ql}{\lambda+2\mu} \quad \text{for } \delta=0.1.
\end{aligned} \tag{2.23}$$

Its evident that for $\delta = 0$ function $\zeta(X_3) \equiv 0$.

In the forthcoming paper [2], the stress analysis and the analysis of the reaction forces due to the internal constrains introduced in the problem considered will be carried on.

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Streszczenie

Tematem pracy jest zastosowanie niestandardowej metody homogenizacji (modelowania mikrolokalnego) [6,7,8,5], do rozwiązania problemu nieswobodnego skręcania prostego, liniowo-sprężystego pręta o okresowo zmieniającym się zwartym przekroju. Praca stanowi kontynuację artykułu [3]. Stosuje się modelowanie mikrolokalne, rozważając zagadnienie w ramach mechaniki analitycznej ośrodków ciągłych z wewnętrznymi więzami [1,4].

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