MECHANIKA
TEORETYCZNA
I STOSOWANA
1-2, 28 (1990)

PARAMETRIC OPTIMIZATION OF VISCOPLASTIC BARS UNDER DYNAMIC TWISTING LOADING

EDWARD CEGIELSKI (KRAKÓW)

1. Introduction

The dynamic short overloading are often the reason of failure of various machine elements. This is due to residual large viscoplastic deformations. The optimization of machine elements with respect to the minimization of these deformations is hardly examined in the literature. The main reasons result from the difficulties in formulating the constitutive equations and from the complicated form or, more often, even unknown impulsive loading.

The problem of optimization of rigid-plastic elements under dynamic impulsive loadings was considered by Lepik (1970) and by Lepik and Mróz (1977). The viscoplastic properties of material were applied by Życzkowski and Cegielski (1981),(1983),(1986),(1988) and Cegielski (1981) when dealing with the optimization problems of bars, beams and momentless shells under various types of the loading impulses.

Despite of the fact that the twisted shafts are often subjected to dynamic loadings there are no papers devoted to the optimization of such elements.

In the present paper some results of the optimal parametrical design of viscoplastic bar under the dynamic twisting loading are given. The impact of twisting moment is assumed as a certain function of time $M=M(t)$ for $0<t<t_1$, $M=0$ for $t<0$ and $t>t_1$. As in many other similar problems the minimal volume of the bar can be regarded as the design objective under the constraint of a given residual deflection (twisting angle). However, it is more convenient to use the dual formulation and hence, to look for the minimal residual deflection under the constraint of a given volume of the bar. The shape of the bar is described by one or few parameters which are the only design variables. Due to the inertia forces, a prismatic bar is not optimal in the case under consideration.

The material of the bar is assumed to be rigid - viscoplastic and described by a nonlinear (power) constitutive equation of viscoplasticity. However, the effective solutions are given for the linear viscoplastic material only.

2. Nonprismatic bar under twisting dynamic impact

Let us consider a nonprismatic circular bar clamped at one end, subject to dynamic twisting loading (twisting moment $\bar{M}=\bar{M}(\bar{t})$) at the other end. Physical (dimensional) quantities are marked here by a bar over the corresponding symbol. The elastic properties of the material are neglected and hence, the constitutive equation of the rigid - viscoplastic material is written in the form

$$\dot{\gamma} = \left< f \left[\frac{\bar{\tau}}{\bar{\tau}_o} - 1 \right] \right> , \tag{2.1}$$

where the dot denotes the derivative with respect to time $\bar{t}$, f is an arbitrary function of its argument, γ denotes shear strain, $\bar{\tau}_o$ denotes the yield point for pure shear and the symbol <f> denotes either f if the argument is positive or 0, if it is negative. Due to the assumed physical law the nonprismatic bar under applied load is divided into two zones at least: the rigid one, undeformed and the viscoplastic one, deformed (fig.1). The effect of time delay is neglected and then at the rigid -viscoplastic interface ($\bar{x} = \bar{x}_{bi}$) it must be $\bar{\tau} = \bar{\tau}(\bar{r}) = \bar{\tau}_o$. Assuming that after deformation a cross section remains plane and a radius remains straight (which is strong simplyfying assumption usually met for circular cross - sections) it is required that the stress $\bar{\tau}$ and the velocity $\bar{v} = d\bar{\phi}/d\bar{t}$ are continuous at the interface (governing equations will be given for $\bar{v}$ but not for $\bar{\phi}$), where $\bar{\phi} = \bar{\phi}(\bar{x})$ denotes the angle of twist. The acceleration $\dot{\bar{v}}$ may be discontinues.

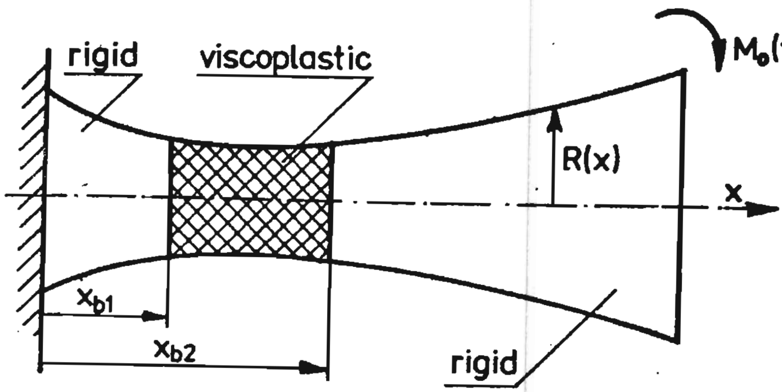

Fig. 1. Viscoplastic zone of twisted bar.

Taking into account the above assumptions one can describe the stress $\bar{\tau}$ as a function of the radius $\bar{r}$. At first let us consider an active (deformable) viscoplastic zone. The function f is assumed to be described by the following power law of pure shear

$$\dot{\bar{\gamma}} = \bar{D}\left[\frac{\bar{\tau}}{\bar{\tau}} - 1\right]^n \text{ for } \bar{\tau} \geq \bar{\tau}_0 \ , \tag{2.2}$$

where the viscosity coefficient $\bar{D}$ and the exponent n are constants. Describing strain $\bar{\gamma}$ by the unit angle $\bar{\vartheta}$

$$\dot{\bar{\gamma}} = \dot{\bar{\vartheta}}\ \bar{r} = \dot{\bar{\phi}}'\ \bar{r} \ , \tag{2.3}$$

and substituting this to (2.2) we obtain

$$\dot{\bar{\phi}}' = \bar{v}' = \frac{\bar{D}}{\bar{r}}\left[\frac{\bar{\tau}}{\bar{\tau}_0} - 1\right]^n \qquad \text{for } \bar{\tau} \geq \bar{\tau}_0, \tag{2.4}$$

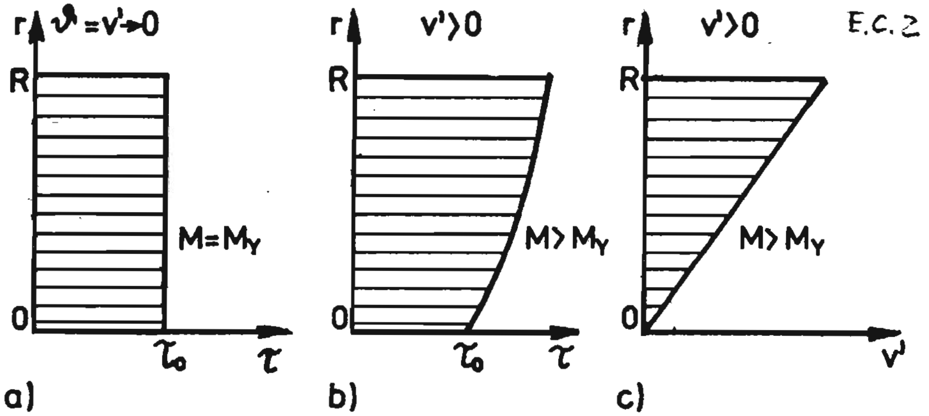

Fig. 2. Stresses and strains in the cross-section.

where prime denotes differentiation with respect to the space variable $\bar{x}$. The equation (2.4) results in the fact that the case $\bar{v}' \to 0^+$, which corresponds to the initial viscoplastic flow at the time t_Y, gives $\bar{\tau}(\bar{r})$ $\bar{\tau}_0$. It means that the stress at the moment t_Y is constant with respect to the radius $\bar{r}$ (fig.2.). At this instant the torque takes the form

$$\bar{M}(\bar{x}, \bar{t}_Y) = \bar{M}_Y(\bar{x}) = \pi\bar{\tau}_0\bar{R}^4/2, \tag{2.5}$$

where $\bar{M}_Y$ denotes the yield carrying capacity of the cross section and $\bar{R} = \bar{R}(\bar{x})$ is the radius of the bar. Similar assumptions on rectangular stress distributions are often met in the papers devoted to rigid - viscoplastic flow of beams or plates e.g. Bodner and Symonds (1960), Wierzbicki (1980). Inverting (2.4) to find the stress τ

$$\bar{\tau} = \bar{\tau}_0\left[1 + \left(\frac{\bar{r}\,\bar{v}'}{\bar{D}}\right)^{1/n}\right], \tag{2.6}$$

and integrating $\bar{r}\bar{\tau}(\bar{r})$ over the cross section area we finally come to

$$\bar{M}(\bar{x},\bar{t}) = \bar{M}(\bar{x}) \left[1 + \frac{3n}{3n+1} \left(\frac{\bar{R}\,\bar{v}'}{\bar{D}} \right)^{1/n} \right], \tag{2.7}$$

In what follows, we introduce the dimensionless quantities

$$x = \frac{\bar{x}}{\bar{l}}, \qquad R = \frac{\bar{R}}{\bar{l}}, \qquad M = \frac{\bar{M}}{\bar{M}_{Yp}}, \qquad A = \frac{\bar{A}}{\bar{A}_p},$$
$$t = \frac{4\,\bar{\tau}_0}{\bar{l}^2\,\bar{D}\,\bar{\rho}}\,\bar{t}, \qquad v = \frac{1}{\bar{D}}\frac{\partial\bar{\phi}}{\partial\bar{t}}, \tag{2.8}$$

where $\bar{l}$ denotes the length of the bar, ρ is the unit density of material, $\bar{M}_{Yp}$ stands for the yielding torque (2.5) of the prismatic bar, $\bar{A}_p = \pi\bar{R}_p^2$ is cross section area of the prismatic bar.

Substituting (2.7) into the equation of motion of a nonprismatic bar under twisting loading we obtain

$$\left.\begin{aligned} \dot{v} &= \frac{3}{R}M' + \frac{R'}{R^2}M, \\ v' &= \frac{1}{R}\left[(M-1)\,\frac{3n+1}{3n} \right]^n \end{aligned}\right\} \quad \text{for } M \geq 1, \tag{2.9}$$

$$v' = 0 \qquad\qquad \text{for } M < 1,$$

where the dot and the prime denotes differentiation with respect to the dimensionless time and spatial variables, respectively. In what follows we confine the effective calculations to the physically linear case: n = 1. In such the case, after elimination of M, we can write (2.9) as the one parabolic - type equation

$$v'' + \frac{4}{3}\frac{R'}{R}v' + \frac{1}{4}\frac{R'}{R^2} - \dot{v} = 0 \qquad \text{for } M \geq 1,$$
$$v' = 0 \qquad \text{for } M < 1. \tag{2.10}$$

In the case of the prismatic bar ($R' = 0$) we obtain the equation analogous to that describing the heat conduction and, hence, the velocity of propagation is infinite and propagation starts when $M(x) = M_{Yp} = 1$. On the contrary, in a nonprismatic bar velocity of propagation is finite and is the same as the the speed of the rigid - viscoplastic interface coordinate x_b. Within the rigid zones (plastically passive) we obviously have:

$$v = v(t) = \text{const}(x), \qquad v'(x,t) = 0. \tag{2.11}$$

The mixed boundary conditions are assumed as follows

$$v(0,t) = 0, \qquad v(x,0) = 0, \qquad M(1,t) = M_0(t). \tag{2.12}$$

3. Numerical integration and optimization

One may face many serious difficulties when integrating analytically the parabolic equations of motion (2.9) or (2.10) for arbitrarily varying radius of the circular cross section. Having in mind that the rigid - viscoplastic interfaces $\dot{x}_{bi}$ as well as the loading torque M_0 change in time one finds that the analytical solution of these equations is practically impossible. Hence, the numerical methods have to be applied. The finite differences method based on simple implicit difference scheme, in conjunction with the Euler algorithm was chosen here.

The rigid-viscoplastic zone interfaces x_{bi} were determined for each time step using the conditions $M(x_{bi})=M_Y$. The number of nodes in x-direction, within the dimensionless integration interval [0,1], was taken between 30 and 50. Discretization in t-direction was introduced to divide the duration of impulse t_1 to 15-30 intervals. As it has been mentioned above, the minimization problem was confined to a parametrical one. Some independent design parameters a_i of the dimensionless function of cross-section area $A = A(x;a_i)$ were sought for. The form of the function A was chosen so as to make its integral, the volume of the bar V, insensitive to design parameters a_i

$$A(x,a_i) = 1 + \sum_i a_i \left[x^i - \frac{1}{i+1} 1^i \right] . \tag{3.1}$$

The additional, natural constraint imposed on the parameters a_i implied from the condition of nonnegative value of area A.

The computer time is a strongly increasing function of the number of design parameters what results in high costs of the optimization. Hence, the iterative and the more effective method, based on the variational Euler-Lagrange approach was proposed by Życzkowski and Cegielski (1988)

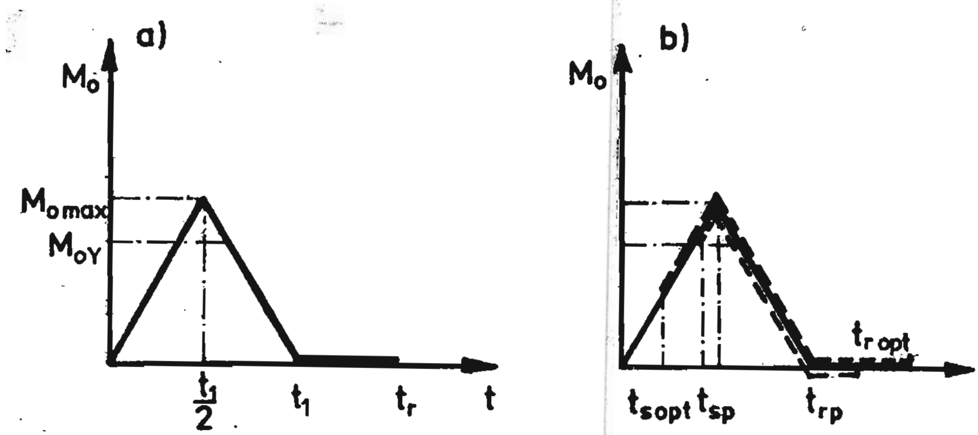

Fig.3. Triangular loading impulse.

4. Discussion of the results

The optimal shape of the bar depends, in the case under consideration, on the shape of the impulse function $M_0=M_0(t)$. In many papers, which deal with dynamic loadings, the rectangular impulse functions are assumed although the real excitation impulses are closer to a triangular-like type, with less rapid time derivative at the beginning and/or at the end of the impulse. For this reason in, the present paper only the triangle impulse is discussed (fig.3a). Two parameters describe the shape of the impulse: the maximal value of the twisting moment M_0 and the duration of the impulse t_1. It could be easily proved that in optimal bars the viscoplastic flow starts always at a lower level of applied torque than in the prismatic one (fig.3b). In all cases under consideration the process was examined until the full rigidification was reached ($t = t_r$) and residual displacement $\phi_r = \phi_r(1)$ was calculated and minimized. As a rule t_r was longer than t_1 whereas, time of viscoplastic flow for optimal bar was than for the prismatic one $t_{ropt} > t_{rp}$ (although $\phi_{ropt} < \phi_{rp}$).

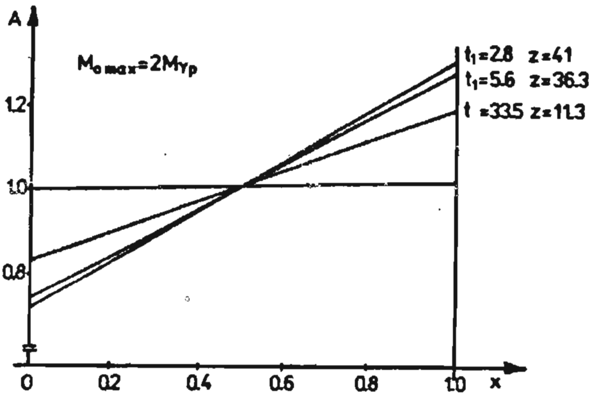

Fig.4. Optimal cross-section function one-parametrical optimization.

To compare the presented results, the profit parameter z is defined

$$z \overset{def}{=} \frac{\phi_{rp} - \phi_{r\ opt}}{\phi_{rp}} 100\ [\%]\ , \tag{4.1}$$

where ϕ_{rp} denotes the residual angular displacement of prismatic bar at the free end.

The optimal shape function $A(x;a_{1opt})$ for the case of one parametrical optimization is shown in the fig. 4. Parameter a_{1opt} was sought for the three various impulse time t_1; the maximal value of external torque was assumed $M_{Omax} = 2M_{Yp}$.

The optimal shapes for

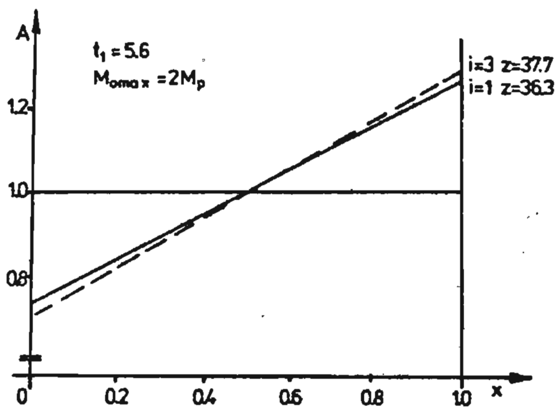

Fig. 5. Optimal cross-section function three-parametrical optimization.

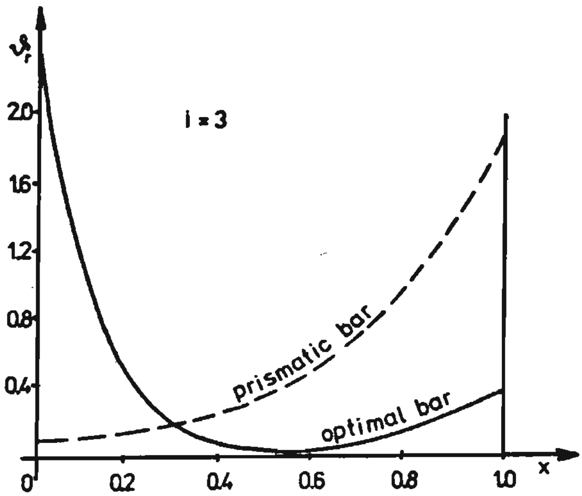

Fig. 6. Unit angle distribution in optimal bar; three-parametrical optimization.

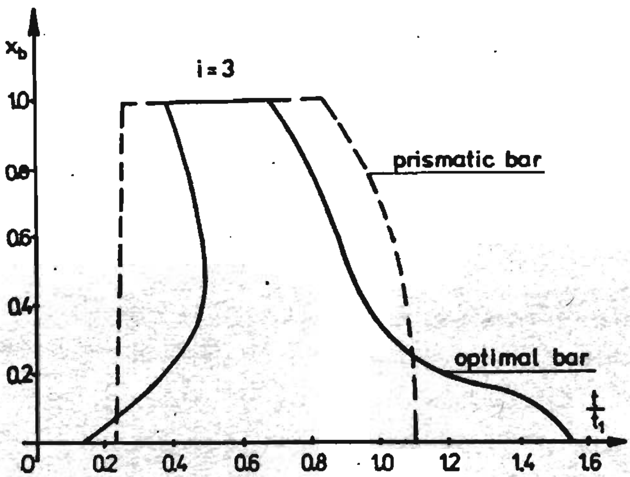

Fig. 7. Rigit-viscoplastic zone inter-face function; three-parametrical optimization.

one- and three- parametrical optimizations are compared in the fig. 5. As it may be se-en, there are small differen-ce in both the compared sha-pes and the respectively cal-culated profits z. Fig.6 points out the differences in residual unit angle $\vartheta_r(x) = \phi'_r(x)$ for the prismatic and the optimal bars in the case of three parametrical op-timization. One can see main-ly qualitative differences. This could be easily explai-ned if we bear in mind that the the viscoplastic flow in the optimal bar always starts at the clamped end (x=0) where optimal cross - section function $A_{opt}(x)$ ta-kes the minimal value. At the end of the viscoplastic flow this part of the bar is intensive-ly deformed. The process of arising and disappearing of rigid viscoplastic interfaces is shown in the fig.6, where the inter-face coordinate x_b is plotted versus time t.

References

Bodner, D., Symonds, P.S.(1960): Plastic deformations in impact and impulsive loading, Plasticity, Pergamon Press, London, 1960.
Cegielski, E.(1981): Opimization of rigid viscoplastic bar and beems under dynamic loadings, Proc.IV Bulg. Congr. Mech., Vol. 1,Varna 1981.
Cegielski, E., Życzkowski, M.(1981): Parameric optimization of viscoplastic bars under dynamic loading, *Rozpr. Inz.*, 1,29.
Cegielski, E., Życzkowski, M.(1983): Optimization of some viscoplastic stuctures under variable loads,Proc EUROMECH Coll. 174, Palermo 1983, COGRAS Palermo 1984.
Cegielski, E., Życzkowski, M.(1988): Optimal design of viscoplastic structures under dynamić loadings, Proc. GAMM Semin., Oct.1988, Siegen FRG, Springer-Verlag 1988,102-109.
Lepik, Ju.(1970): Optimizaton of nonelastic beams with additional support in the case of dynamic loading (in Russian) *UZTGU* 430,132-143.
Mróz, E., Lepik, Ju.(1977): Optimal design of plastic structures under impulsive end dynamic pressure loading, *Int. J. Solid Str.*,13,657-674.
Wierzbicki, T.(1980): Design of structures under dynamic loadings (in Polish), Arkady, Warszawa 1980.
Życzkowski, M., Cegielski, E.(1986): Parametrical optimization of rigid viscoplastic beams under dynamic loadings (in Polish), *Arch. Inz. Ladow.*,32, 95-104.

Summary

PARAMETRYCZNA OPTYMALIZACJA SKRĘCANYCH PRETÓW LEPKOPLASTYCZNYCH PRZY OBCIĄŻENIACH DYNAMICZNYH

W pracy przedstawiono problem parametrycznej optymalizacji kształtu pręta kołowego obciążonego dynamicznym impulsem momentu skręcającego. Poszukiwano minimalnego resztkowego kąta skręcenia pręta przy ograniczeniu stałej objętości pręta. Rozważono pręt wykanany z materiału sztywno - lepkoplastycznego oraz dwuliniowy symetryczny (trójkątny) impuls momentu skręcającego.