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OPTIMAL DESIGN OF HELICAL SPRINGS (A SURVEY)[1]

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1. Introductory remarks

Any helical spring problem has a multiplicity of practical solutions because of a larger number of the geometrical parameters than the governing equations. The conventional trial-and-error procedure for simple passive design of helical springs is to select the geometrical parameters so that the maximum stress does not exceed the allowable value. The functional relationship between the maximum stress, applied load and dimensions of the spring is usually known and permits evaluation of the maximum stress to a reasonably high degree of accuracy. The main disadvantage of such a method is that the resulting spring is not optimal and such solution is not unique.

Solution become unique only by introduction of certain space or operational restrictions. In the absence of any restrictive requirements the remaining "degrees of freedom" permit to apply an additional optimization procedure.

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2. Typical formulations of problems of optimal design of helical springs

Modern study and research concerning helical springs dates back at least a century (Thomson and Tait (1883)) but number of papers devoted to optimal design of such elements is rather scare.

Usually it is accepted that the first paper on optimal design of springs was published by Henderson (1894-1895). Nowadays, typical formulation of the optimization problem is almost unchanged. Only parametrical problems are considered and investigations are limited to cylindrical helical springs with a constant pitch; a spring is always replaced by an equivalent rod with the helix angle equals to zero. That simplifies the analysis of a spring.

In most cases, minimal weight or minimal volume of space that is taken up by the spring or minimal length of the spring is assumed to be a criterion of optimality. These two last criteria have two variants: volume and length in unloaded state and in closed up condition. In some cases also the maximal deflection of a spring is considered.

The geometrical parameters and maximal axial loadings are looked for under condition of a given rigidity of a spring (rate of a spring) and under stress condition. The calculation of the stresses including effects of curvature and direct shear is rather complicated. Hence, some correction factors, for example the Wahl factor, simplifying calculations are commonly used. Such a general factor including various effects is given by Życzkowski (1957).

Jennings (1938) designed helical springs with respect to discussed here all three basic criteria, namely he applied the version in closed up condition. Neither the effect of inactive coils nor correction factor was taken into account. Spotts (1953) applied only the criterion of minimal weight of a spring. Khare (1980) additionally investigated the effects of end coils (inactive coils), direct shear and wire curvature on optimal solutions. Similar problems were discussed by Tevelev (1960).

Graphical methods for obtaining optimum values of weight, volume and length were given by Henderson (1894-1895), Singh and Kulkarni (1976).

Hinkle and Morse (1959) gave a method to obtain minimal values of weight, volume of the space taken by a spring in closed up condition and free length of a spring. They introduced a special design constant combining several parameters. In their analysis, the effects of end coils and the Wahl factor were related and then minimized for optimum values. The results are presented in graphical form.

The springs of minimal free length were investigated by Tevelev (1960). Jogi and Singh (1973) designed helical springs for maximum deflection. Erisman (1961) applied a criterion of minimal weight and compared the results obtained for no stress correction factor with the solutions obtained for the Wahl factor. The procedure given by Agrawal (1975) can be used to design minimum weight springs with a spring index greater than 3. His equations account for end coil effect, direct shear, and wire curvature.

In the paper by Rao (1972) the size effect in springs was taken into account. The influence of limitations imposed on the outside or inside diameters was investigated.

Springs for special performance were studied by Jennings (1937), Jennings (1941), Shobert (1947), Tarinov (1949) and also by Whal (1942), Jain and Srivastava (1974), Bachtler and Rommel (1973).

In all papers mentioned above, the helix angle was assumed to be zero. Using the stress and deflection correction factors due to Ancker and Goodier, Kulkarni and Balasubrahmanyam (1979) showed that helix slope should be taken into account while optimizing weight, volume or length of a spring. Two cases were considered, namely solid and hollow circular wire springs.

Design of helical torsion springs for minimum weight with stress concentration factor and inactive coils taken into account was investigated by Agrawal (1976). His approach gives a straightforward formulae whereas most papers proposed graphical procedures for determination of optimal parameters.

Three different criteria of optimization which were investigated here lead to different proportions of optimal springs. The criterion of minimal volume gives a spring with a high slenderness ratio. Such a spring is less resistant to buckling. The criterion of minimal length leads to a thickset spring whereas the criterion of minimal weight gives additional profits, namely a lightweight spring has a high natural frequency, and is less likely to be affected by vibration and shock loads.

3. More general formulations of optimization problems

The problems of optimal design of springs can bealsoformulated under constraints in the form of weak inequalities. Then, the number of constraints can be larger than the number of design variables.

Such a formulation of parametrical optimization of helical springs (an equivalent rod) was proposed by Arora (1989). From condition of minimal weight, the diameter of the spring and of the wire and number of coils were looked for. The axial force was given and additional constraints connected with deflection, stress and surge wave frequency were applied.

In order to improve the performance of helical springs, such as increasing the fatigue life, suppressing resonance, buckling load, etc., variable pitch angle and variable helix radius may incorporate into the helical spring geometry.

Krużelecki and Życzkowski (1990) proposed the concept of an equivalent column to problems of stability of helical springs of arbitrary shape. That concept gives possibility to formulate optimization problem of springs with respect to their stability.

Making use of the equivalent column concept, Krużelecki (1990) looked for optimal varying helix angle which gives maximal buckling force for given constant weight of a spring. Additional inequality constraints were applied and nonlinear prebuckling state was considered.

4. Final remarks

In the present survey we discussed over 20 papers but most of them are connected with parametrical optimization under equality constraints. More difficult but ensuring higher profits are problems of optimal design of varying helix angle and radius with nonlinear behaviour of a spring taken into account.

Some problems, e.g., optimization of springs with respect to fatigue strength, optimization in combined state of loadings are completely not undertaken. Formulation and solving other ones, for example optimization with respect to stability, are found practically at the very beginning. Those and many other problems undoubtedly will be investigated in the nearest future.

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Summary

OPTYMALNE KSZTAŁTOWANIE SPRĘŻYN ŚRUBOWYCH - PRZEGLĄD

W pracy sklasyfikowano i omówiono ponad 20 prac z zakresu optymalnego kształtowania sprężyn zwitych. Ogromna większość tych prac dotyczy najprostszej optymalizacji parametrycznej sprężyn walcowych o stałym skoku. Wskazano również możliwości ujęcia zagadnienia w sposób bardziej ogólny.