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THEORETICAL MODEL OF EXTERNALLY PRESSURIZED CIRCULAR THRUST POROUS GAS BEARING WITH DEFORMABLE MATERIAL

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A method is proposed for determining dimensionless characteristics of an externally pressurized circular thrust bearing. An essential novelty in the present model as compared with many existing theoretical models consists in the deformability of the porous pad being taken into consideration. The unknowns of the present model are: the pressure distribution in the clearance and the thickness of the lubricating film (deflection of the porous pad). These quantities are determined by solving by method of successive approximations the set of governing equations. For the zero approximation it is assumed that the porous material is indeformable, which enables us to obtain a solution for the zero approximation to the pressure distribution. For the first and subsequent approximations the porous pad is treated as a thin elastic plate loaded in an axially symmetric manner by pressure which has been found in the preceding approximation. The equation of bending of the porous plate is integrated in an analytic manner in every approximation. The equation governing the pressure distribution is integrated numerically by using the method of orthogonal collocation. A detailed algorithm is given for the determination of the dimensionless load capacity and the dimensionless mass flow rate.

1. Introduction

Aerostatic thrust bearings are commonly used in industry, since they have exceedingly low frictional coefficients, even at slow speeds, and they are readily operated from the factory air-line. Conventional capillary or orifice-compensated bearings have, however, low load capacities for the high supply pressures and feed rates required, and their operating range is often limited by the pneumatic instability of the air film. These disadvantages may be overcome by using a porous pad in place of the combination of a solid pad and compensating elements. Thus, the aerostatic porous bearing, also has a stiffer film, ensuring greater positionala accuracy, and a smaller tendency to fail throughblockage.

Porous thrust bearings have been investigated by many authors. A review of the literature pertaining to the theory of such bearings was given in paper [1]. Almost all the research workers made the assumption that in bearing clearance exists a uniform gas film, as this drastically simplifies the solution of the Reynolds equation. This implies that the elastic strength of the porous material is such that deformation that does occur is negligible. In aerostatic thrust bearings, film thicknesses are small of the order of 12 μm and hence any apparently negligible deflection of the porous media may be of the same order of magnitude as the film thickness. The deflection of the porous pad depends upon its flexural rigidity. For certain materials the elastic strength of the pad will be insufficient to withstand the loading by pressure difference across it. Consequently, a diverging film will be produced. This effect was observed by Taylor & Lewis [2] in experiments with porous carbon as the media. The divergent film reduces the film pressure and hence the load-carrying capacity of the bearing.

The deformation of porous material, as yet, was taken into account only in Taylor & Lewis [2 - 3] and in the paper [4]. In papers [2 - 3] essential part of proposed model is determination the two-dimensional flow in porous material. However, in most applications the wall thickness of the pad is small compared to its radius. Thus, the gas flow in the bearing matrix is predominantly axial and it is immaterial whether the porous pad is sealed at the sides or open to the atmosphere. This assumption in essential way simplifies the mathematical model of the bearing. In paper [4] the method for determining characteristics of externally pressurized circular thrust bearings with deformable porous material with the mentioned above assumption on axial flow for incompressible lubricant was proposed.

The purpose of this paper is to present the mathematical model for the performance characteristics of the aerostatic porous thrust bearing with deformable porous material and compressible lubricant. We take into account also a slip flow at the boundary between the bearing clearance and the porous material. Opposite from papers [2 - 3] in this paper the radial flow in the porous material is neglected.

2. Assumptions

Figure 1 represents the flow model and coordinate system in the circular porous thrust bearing. We assume that known values are: p_s — supply pressure, p_p — ambient pressure, H — thickness of porous material, $2a$ — diameter of porous pad.

The assumptions made for this analysis are as follows:

a) The lubricant is a compressible viscous fluid with equation of state for perfect gas:

$$p = \varrho \mathcal{R} T, \tag{1}$$

where: p — pressure, ϱ —density of gas, $\mathcal{R}$ — gas constant, T — temperature.

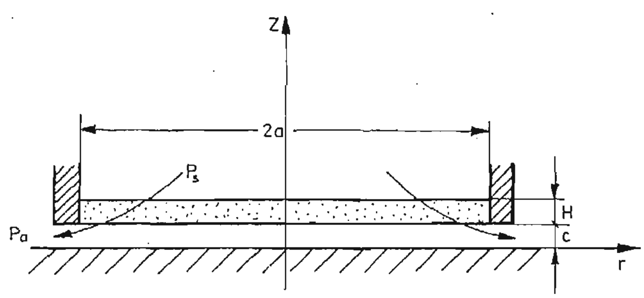

Fig. 1. Configuration of porous thrust bearing

b) The fluid flow through the bearing is isothermal and steady.

c) The flow in porous material is viscous and Darcy's law applies:

$$q = -\frac{k}{\mu}\operatorname{grad} p, \tag{2}$$

where: q — velocity in porous material, k — permeability coefficient, μ — viscosity of fluid.

d) The porous material is deformable. The thickness H of the pad of bearing is small as compared with the diameter $2a$; the deformability of the material may be described by the theory of thin plates, the deflection of which is discribed by the equation:

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{Q}{N}, \tag{3}$$

where: w — deflection of porous pad, Q — transverse loading, N — flexure rigidity of porous plate.

e) Since $H \ll a$, the radial flow in the porous material is neglected, The Darcy's equation (2) is in this way reduced to the form:

$$q_z = -\frac{k}{\mu}\frac{dp}{dz}. \tag{4}$$

f) The tangential stresses in lubricating layer penetrates on a distance δ in the bulk of porous material [5 - 6]. Therefore the condition that there is no sliding was proposed to apply but on surface inside porous material, not its nominal boundary.

g) The usual simplifications of the classical lubrication theory can be used for the bearing clearance, it being assumed that there is only radial flow governed by reduced equations of viscous compressible flow in the form:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\varrho v_r) + \frac{\partial(\varrho v_z)}{\partial z} = 0, \tag{5}$$

$$\frac{dp}{dr} = \mu\frac{\partial^2 v_r}{\partial z^2}, \tag{6}$$

where: v_r — radial velocity in bearing clearance, v_z — axial velocity in bearing clearance.

3. Governing equations and method of solution

Flow in the porous material is governed by the stedy-state mass continuity equation (axisymmetric case):

$$\frac{d}{dz}(\varrho q_z) = 0. \tag{7}$$

Substitution of Darcy's law (4) and equation of state (1) into equation (7), yields

$$\frac{d}{dz}\left(\frac{kp}{\mu \mathcal{R} T}\frac{dp}{dz}\right) = 0. \tag{8}$$

By noting that $2p\frac{dp}{dz} = \frac{dp^2}{dz}$, it can be shown, that

$$\left.\frac{dp^2}{dz}\right|_{z=c-w+\delta} = \frac{p_s^2-p^2}{H}. \tag{9}$$

Integrating equation (6) twice respect to z, applying the boudary conditions in the form:

$$v_r = 0 \quad \text{for} \quad z = 0, \tag{10}$$

$$v_r = 0 \quad \text{for} \quad z = c-w+\delta, \tag{11}$$

we have:

$$v_r = \frac{(c-w+\delta)^2}{2}\frac{dp}{dr}\left[\frac{z^2}{(c-w+\delta)^2} - \frac{z}{c-w+\delta}\right]. \tag{12}$$

Substitution of equation (12) to (5) and integration in the film region, yields:

$$\frac{(c-w+\delta)^3}{12}\cdot\frac{1}{r}\frac{d}{dr}\left(\varrho r\frac{dp}{dr}\right) = -(\varrho v_z)|_{z=c-w+\delta}. \tag{13}$$

Because $\varrho v_z = \varrho q_z$ for $z = c-w+\delta$, from (4) and (9) we have governing equation for pressure in bearing clearance:

$$\frac{d^2P^2}{dR^2} + \frac{1}{R}\frac{dP^2}{dR} - \frac{\Lambda_0 P^2}{(1-a_c W+\Delta)^3} = -\frac{\Lambda_0 P_s^2}{(1-a_c W+\Delta)^3}, \tag{14}$$

where:

$$R = \frac{r}{a}, \quad P = \frac{p}{p_a}, \quad P_s = \frac{p_s}{p_a}, \quad \Lambda_0 = \frac{12ka^2}{H}, \tag{15}$$

$$a_c = \frac{a}{c}, \quad \Delta = \frac{\delta}{c}.$$

After introducing dimensionless values (15) into (3) and puting $Q = p_s - p$, we have governing equation for deflection of porous plate:

$$\frac{1}{R}\frac{d}{dR}\left\{R\frac{d}{dR}\left[\frac{1}{R}\frac{d}{dR}\left(R\frac{dW}{dR}\right)\right]\right\} = S_b(P_s - P), \tag{16}$$

where:

$$S_b = \frac{a^2 p_a}{N} \tag{17}$$

is dimensionless parameter of stiffness.

In solution of equation (14), should satisfy the following boundary conditions:

$$\frac{dP}{dR} = 0 \quad \text{for} \quad R = 0, \tag{18}$$

$$P = 1 \quad \text{for} \quad R = 1, \tag{19}$$

While, in solution of equation (16), W — should satisfy the following boundary conditions:

$$\left.\begin{array}{l} W = 0 \\ \frac{dW}{dR} = 0 \end{array}\right\} \quad \text{for} \quad R = 1, \tag{20}$$

which are conditions for clamped edge of plate. It is also required that:

$$\frac{dW}{dR} = \frac{d^3W}{dR^3} = 0 \quad \text{for} \quad R = 0, \tag{21}$$

which results from the symmetry of the problem.

The unknowns of the present model are: the pressure distribution — P in the clearance, and deflection — W of the porous pad (the thickness of the lubricating film). These quantities are determined by solving by method of successive apprixomations the set of equations (14) and (16) with the boundary conditions (18), (19), (20) and (21). For the zero approximation it is assumed that the porous material is indeformable, $W^{(0)} = 0$, which enables to obtain the zero approximation to the pressure distribution $P^{(0)}$, by solution of equation (14) with $W = 0$. For the first and subsequent approximations the porous pad is treated as a thin elastic plate loaded in an axially symmetric manner by pressure which has been found in the preceeding approximations. The equation governing the pressure distribution (14) is integrated numerically in every approximation by means of orthogonal collocation [7]. In this way solution for P is given in polynomial form. The equation of bending of the plate (16) is integrated in analytic manner in every approximation, because it is a linear equation with load described by polynomials.

Solutions to the foregoing system of equations are in the form of pressure-squared distributions through the bearing clearance. The load capacity is simply found as the sum of forces created by the fouilm boundary pressure acting normally to the bearing area or

$$s = \int_0^{2\pi}\int_0^{a} (p-p_a)\, r\, dr\, d\Theta = 2\pi \int_0^{a} (p-p_a)\, r\, dr. \tag{22}$$

In dimensionless form this becomes:

$$S = \frac{s}{\pi a^2 (p_s - p_a)} = \frac{2}{P_s - 1}\int_0^1 (P-1)\, R\, dR. \tag{23}$$

The dimensionless load capacity is seen to be the ration of the actual load to the maximum load possible.

The mass flow rate required by the film may be calculated from the gas velocity crossing the film boundary:

$$m = -\int_0^{2\pi}\int_0^{a} (\varrho q_z)|_{z=c}\, r\, dr\, d\Theta. \tag{24}$$

Substitution of Darcy's law (4) and the equation of state (1) yields

$$m = -\frac{\pi k}{\mu \mathcal{R} T}\int_0^{a} \left(\frac{\partial p^2}{\partial z}\right)\bigg|_{z=c} r\, dr. \tag{25}$$

In dimensionless form the flow becomes:

$$M = \frac{2m\mu\mathcal{R}TH}{\pi a^2(p_s^2 - p_a^2)k} = \frac{-2}{\Lambda_0(P_s^2-1)} \left.\frac{dP^2}{dR}\right|_{R=1}. \tag{26}$$

4. Results

The convergence of the described above method of successive approximations is satisfactory. In almost all calculated cases with number of iterations less than 10, results are stable for the pressure distribution P and the deflection of porous plate W. This good convergence is illustrated in Tables 1 and 2.

Table 1. Load capacity S and mass flow rate M for succesive approximation i; $P_s = 9$, $\Lambda_0 = 10$, $a_c = 100$, $S_b = 0.05$, $\Delta = 0.01$

i	S	M
1	0.6237747	68.12282
2	0.6431721	69.77471
3	0.6407426	69.54882
4	0.6410518	69.57369
5	0.6410124	69.57052
6	0.6410168	69.57092
7	0.6410169	69.57087
8	0.6410168	69.57088

Table 2. Load capacity S and mass flow rate M for succecesive approximation i; $P_s = 9$, $\Lambda_0 = 90$, $a_c = 100$, $S_b = 0.05$, $\Delta = 0.1$

i	S	M
1	0.8452509	140.5341
2	0.8427989	140.1449
3	0.8428321	140.1500
4	0.8428317	140.1499
5	0.8428317	140.1499
6	0.8428317	140.1499
7	0.8428317	140.1499

In the proposed mathematical model the dimensionless characteristics such as the load capacity S and the mass flow rate M are functions of the following dimensionless parameters: Λ_0, P_s, S_b, a_c and Δ. The variation in the load capacity S and the mass flow rate M with the bearing number Λ_0 for various ration of radius pad to bearing clearance a_c are shown in Figs. 2 and 4. It is seen from these figures that deformation of

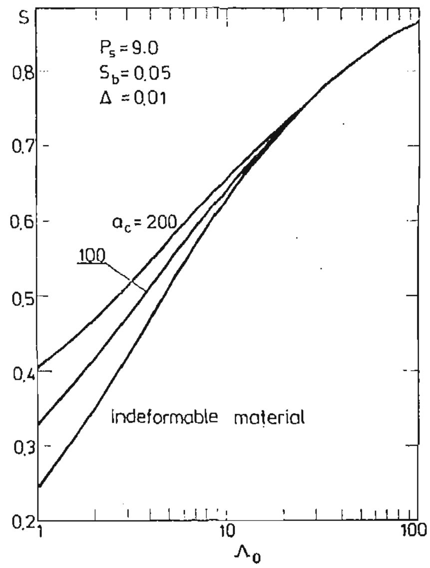

Fig. 2. Normalized load capacity S versus bearing number Λ_0 for a range of ratio of radius pad to bearing clearance a_c.

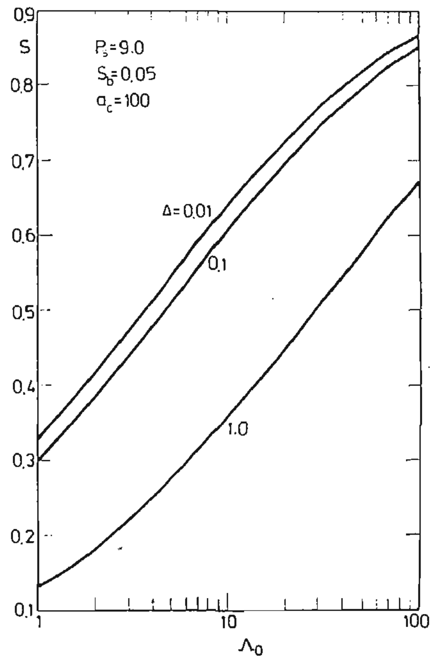

Fig. 3. Normalized load capacity S versus bearing number Λ_0, for a range of dimensionless depth of penetration of shear Δ

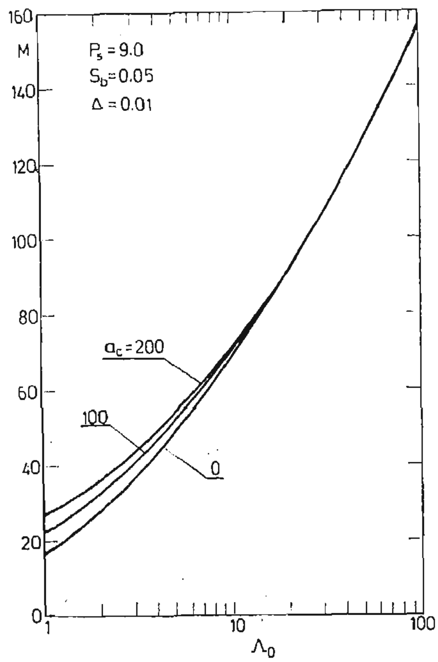

Fig. 4. Normalized mass flow rate M versus bearing number Λ_0 for a range of ration of radius pad to bearing clearance a_c

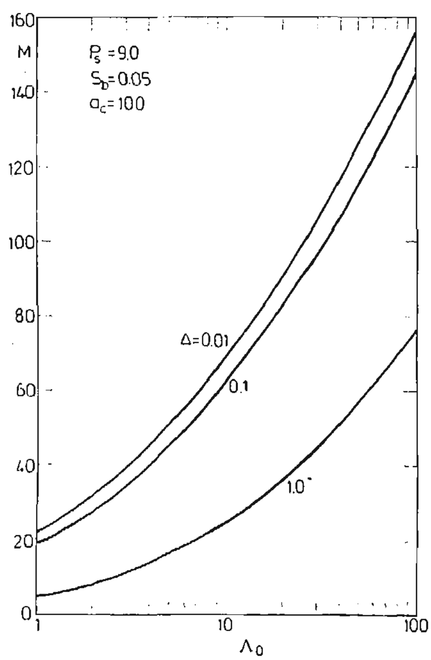

Fig. 5. Normalized mass flow rate M versus bearing number Λ_0 for a range of dimensionless depth of penetration of shear Δ

porous material can have significant influence on nondimensional load capacity and mass flow rate. This influence increases with decrease of parameter Λ_0, whereas this influence increases with increase of parameter a_c. While, Figs 3 and 5 show the variation in the load capacity S and the mass flow rate M with the bearing number Λ_0 for various dimensionless depths of penetration of shear Δ. It is seen from these figures that penetration of shear inside porous material (the slip flow on the boundary of porous region and of fluid region) can have significant influence on load capacity and mass flow rate. The Table 3 shows the variation in the load capacity and mass flow rate with dimensionless parameter of stiffness of porous pad S_b.

Table 3. Variation in the load capacity S and mass flow rate M with the dimensionless parameter of stiffness S ; $\Lambda_0 = 10$, $P = 9$, $\Delta = 0.01$, $a = 100$.

S_b	S	M
0.0	0.623774	68.123
0.002	0.624556	68.187
0.004	0.625329	68.251
0.006	0.626093	68.315
0.008	0.626849	68.378
0.01	0.627597	68.439
0.02	0.631218	68.742
0.03	0.634653	68.031
0.04	0.637915	69.307
0.05	0.641017	69.571

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Резюме

ТЕОРЕТИЧЕСКАЯ МОДЕЛЬ ЦИЛИНДРИЧЕСКОГО ПОДШИПНИКА-ПОДПЯТНИКА ВНЕШНЕ ПИТАЕМОГО ПРИ УЧЁТЕ ДЕФОРМАЦИИ ПОРИСТОГО МАТЕРИАЛА

В работе представлен способ определения безразмерных характеристик плоского цилиндрического подпятника, питаемого внешне. Существенной новостью в модели, по сравнению с многими уже существующими теоретическими моделями является учёт деформации пористой вкладки.

В предлагаемой модели неизвестными для определения являются: разложение давления в щели подшипника и функция толщины смазывающего слоя (прогиб пористой вкладки). Эти величины определяются методом очередных приближений. В нулевом приближении предлагается, что материал пористый недеформируемый, что позволяет получить решение распределения давления для этого приближения.

В первом и следующих приближениях пористая вкладка трактуется как тонкая эластичная плитка с осевой симметрической нагрузкой давления, определенного ранее в последним приближении. Уравнение прогиба плитки интегрируется аналитически в каждом приближении. Уравнение для распределения давления в каждом приближении интегруется численно методом ортогоналной коллокации.

Представлен подробный алгоритм определения безразмерной несущей силы и безразмерной скорости потока.

Streszczenie

TEORETYCZNY MODEL POROWATEGO ZEWNĘTRZNIE ZASILANEGO CYLINDRYCZNEGO GAZOWEGO ŁOŻYSKA WZDŁUŻNEGO Z ODKSZTAŁCALNYM MATERIAŁEM POROWATYM

W pracy przedstawia się sposób wyznaczania bezwymiarowych charakterystyk cylindrycznego łożyska wzdłużnego zasilanego zewnętrznie. Istotną nowością modelu, w porównaniu z wieloma istniejącymi modelami teoretycznymi jest uwzględnienie odkształcalności porowatej wkładki. W proponowanym modelu niewiadomymi są: rozkład ciśnienia w szczelinie łożyska oraz grubość filmu smarującego ugięcie wkładki porowatej. Wielkości te wyznacza się poprzez rozwiązanie układu równań rządzących metodą kolejnych przybliżeń. W przybliżeniu zerowym zakłada się, że materiał porowaty jest nieodkształcalny, co pozwala na uzyskanie zerowego przybliżenia dla rozkładu ciśnienia. W pierwszym i następnych przybliżeniach wkładkę porowatą traktuje się jako cienką sprężystą płytę obciążoną osiowosymetrycznie ciśnieniem wyznaczonym w poprzednim przybliżeniu. Równania zginania płyty całkuje się analitycznie w każdym przybliżeniu. Podaje się algorytm wyznaczania bezwymiarowej siły nośnej oraz bezwymiarowej prędkości przepływu gazi przez łożysko.

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