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THE INFLUENCE OF MOISTURE AND TEMPERATURE ON THE BEHAVIOUR OF ORTHOTROPIC, VISCOELASTIC PLATES

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1. Introduction

The influence of humidity and temperature variation on the properties of different materials is commonly known. In the case of wood and plywood this influence is of great moment for the durability and the behaviour in constructions of wood as well. Numerous investigators have shown that the mechanical properties of such materials strongly depend on moisture, also. The influence of temperature on the elastic properties of wood is also perceptible.

There are many papers in which mathematical models, involving the variations of moisture content and temperature are considered (e.g. [1 - 3]) basing on experimental investigations. Recently, Bažant [4] has formulated the constitutive relation for steady states conditions basing on the Maxwell chain model whose viscosity coefficients depend on moisture content and temperature. However, in the cases mentioned above one dimensional problems were studied only and an anisotropy of the material was omitted.

The object of this paper is an attempt to formulate the equations describing the behaviour of orthotropic, viscoelastic plates, subjected to the influence of temperature and moisture variation.

The paper consist of two general parts. The first one is concerned with deriving the fundamental equations for the body considered basing on the principles of mechanics and thermodynamics. We restrict our considerations to the linear case, only. It indicates that the displacements, temperature changes and moisture concentration are assumed to be small. The second part is devoted to formulate the basic differential equations for thin plate, where the equations that were just derived in the first part will be applied.

2. Basic equations

The point of departure of our considerations are the balance equations that result from the fundamental laws of mechanics and thermodynamics of continuous media. The local balance laws which must be satisfied are:

1. conservation of mass

$$\dot{c} = -\eta_{i,i}+r_m, \quad c = \varrho_m/\varrho \tag{2.1}$$

2. balance of linear momentum and angular momentum

$$\sigma_{ji,j}+X_i = \varrho\ddot{u}_i, \quad \sigma_{ji} = \sigma_{ij} \tag{2.2}$$

3. balance of energy

$$\dot{e} = \sigma_{ji}\dot{\varepsilon}_{ij}-q_{i,i}+(\mu\eta_i)_{,i}+r_h-\mu r_m \tag{2.3}$$

4. entropy inequality

$$\dot{s} \geqslant \frac{1}{T}r_h-\left(\frac{q_i}{T}\right)_{,i} \tag{2.4}$$

where c denotes the concentration of moisture, ϱ_m and ϱ are the densities of moisture and the material, respectively, σ_{ij} is the stress tensor, ε_{ij} is the deformation tensor, X_i is the body force vector, u_i is the displacement vector, q_i is the heat flux vector, η_i is the vector of moisture mass flux, μ is the potential of moisture transmision, e, s, r_h, r_m and T are respectively the internal energy, the entropy, the internal heat source, internal source of diffusing matter and the absolute temperature. A superposed dot denotes differentiation with respect to the time variable t and $(\,)_{,i}$ denotes partial differentiation with respect to the coordinate x_i, referred to a system of rectangular cartesian axes fixed in space.

A different form of the entropy inequality (2.4) will be more convenient in the further considerations. We will obtain it by eliminating from (2.3) and (2.4) the heat source r_h and introducing the function of free energy:

$$\psi = e-sT. \tag{2.5}$$

We now get:

$$-\frac{1}{T}(\dot{\psi}+s\dot{T})+\frac{1}{T}(\sigma_{ij}\dot{\varepsilon}_{ij}+\eta_i\mu_{,i}-\mu\dot{c})-\frac{q_i}{T^2}T_{,i}, \tag{2.6}$$

where the equation (2.1) has been used.

As can be seen, the field equations (2.1) - (2.3) and entropy inequality (2.6) do not constitue a closed system. Therefore, it must be supplemented with suitable constitutive equations defining the class of considered material [5, 6]. In our case we will assume the following constitutive equations:

$$\sigma_{ij} = \sigma_{ij}(\{\mathscr{F}\}), \quad q_i = q_i(\{\mathscr{F}\}), \quad \eta_i = \eta_i(\{\mathscr{F}\}), \quad \psi = \psi(\{\mathscr{F}\}), \tag{2.7}$$

where:

$$\{\mathscr{F}\} = \{\varepsilon_{ij}, \dot{\varepsilon}_{ij}, T, T_{,k}, c, c_{,k}\} \tag{2.8}$$

is a set of independent constitutive variables.

Substituting for ψ from (2.5) into (2.6), and carrying out the indicated differentiations of ψ, we obtain:

$$\begin{aligned} &-\frac{1}{T}\left(\frac{\partial\psi}{\partial T}+s\right)\dot{T}+\frac{1}{T}\left(\sigma^E_{ij}-\frac{\partial\psi}{\partial\varepsilon_{ij}}\right)\dot{\varepsilon}_{ij}+\frac{1}{T}\sigma^D_{ij}\dot{\varepsilon}_{ij}-\frac{1}{T}\left(\mu+\frac{\partial\psi}{\partial c}\right)\dot{c} \\ &-\frac{1}{T}\left(\frac{\partial\psi}{\partial\dot{\varepsilon}_{ij}}\ddot{\varepsilon}_{ij}+\frac{\partial\psi}{\partial\dot{c}}\ddot{c}+\frac{\partial\psi}{\partial T_{,i}}\dot{T}_{,i}+\frac{\partial\psi}{\partial c_{,i}}\dot{c}_{,i}\right)+\frac{1}{T}\eta_i\mu_{,i}-\frac{q_i}{T^2}T_{,i} \geqslant 0. \end{aligned} \tag{2.9}$$

Here decomposition of the stress tensor into elastic σ_{ij}^E and dissipative part σ_{ij}^D have been introduced.

The inequality (2.9) must hold for all independent variation of $\dot{\varepsilon}_{ij}$, $\ddot{\varepsilon}_{ij}$, $\dot{T}$, $\dot{T}_{,i}$, $\dot{c}$ and $\dot{c}_{,i}$. These variables appear linearly in (2.9) and thus their coefficients must vanish. It then follows that:

$$\sigma_{ij}^E = \frac{\partial\psi}{\partial\varepsilon_{ij}}, \quad \mu = -\frac{\partial\psi}{\partial c}, \quad s = -\frac{\partial\psi}{\partial T}, \tag{2.10}$$

$$\frac{\partial\psi}{\partial\dot{\varepsilon}_{ij}} = 0, \quad \frac{\partial\psi}{\partial\dot{c}} = 0, \quad \frac{\partial\psi}{\partial T_{,i}} = 0, \quad \frac{\partial\psi}{\partial c_{,i}} = 0, \tag{2.11}$$

and inequality (2.9) reduces to:

$$\frac{1}{T}\sigma_{ij}^D\dot{\varepsilon}_{ij} + \frac{1}{T}\eta_i\mu_{,i} - \frac{q_i}{T^2}T_{,i} \geqslant 0. \tag{2.12}$$

From conditions (2.10) and (2.11) it results that:

$$\psi = \psi(\varepsilon_{ij}, T, c), \quad \sigma_{ij}^E = \sigma_{ij}^E(\varepsilon_{ij}, T, c), \quad \mu = \mu(\varepsilon_{ij}, T, c), \quad s = s(\varepsilon_{ij}, T, c). \tag{2.13}$$

The inequality (2.12) is a constraint of functions σ_{ij}^D, η_i and q_i but does not lead to a more general conclusion before the choice of these functions.

Let us now proceed to determining the final form of constitutive relations. We begin by specifying σ_{ij}^E, s and μ. To this end we develop the free energy function ψ into Taylor series about the reference state ($\varepsilon_{ij} = 0, T = T_0, c = c_0$) with accuracy to quadratic terms. We have:

$$\psi(\varepsilon_{ij}, T, c) = \frac{1}{2}C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} + \frac{1}{2}m\Theta^2 + \frac{1}{2}nC^2 - \beta_{ij}\varepsilon_{ij}\Theta \tag{2.14}$$
$$-\gamma_{ij}\varepsilon_{ij}C + \xi\Theta C, \quad \Theta = T - T_0, \quad C = c - c_0.$$

On the basis of (2.10) we obtain:

$$\begin{aligned} \sigma_{ij}^E &= C_{ijkl}\varepsilon_{kl} - \beta_{ij}\Theta - \gamma_{ij}C, \\ \mu &= \gamma_{ij}\varepsilon_{ij} - \xi\Theta - nC, \quad n = \frac{c_m}{T_0}, \\ s &= \beta_{ij}\varepsilon_{ij} - m\Theta - \xi C, \quad m = -\frac{c_V}{T_0}, \end{aligned} \tag{2.15}$$

where C_{ijkl}, β_{ij}, γ_{ij}, ... etc. are constants characterizing the mechanical and thermal properties of the medium.

We pass now to determine σ_{ij}^D, q_i and η_i. We will determine these functions from the condition of satisfaction of inequality (2.12), limiting ourselves to linear relations. This makes it possible to use phenomena of the cross effect and Onsager's symmetry relations. Thus, making use of the well known procedure we finally obtain:

$$\begin{aligned} \sigma_{ij}^D &= G_{ijkl}\dot{\varepsilon}_{kl} + \gamma_{ijk}\mu_{,k} - \beta_{ijk}\Theta_{,k}, \\ \eta_i &= \gamma_{ijk}\dot{\varepsilon}_{jk} + \alpha_{ik}\mu_{,k} - \xi_{ik}\Theta_{,k}, \\ q_i &= T_0\beta_{ikl}\dot{\varepsilon}_{kl} + T_0\xi_{ik}\mu_{,k} - k_{ik}\Theta_{,k}, \end{aligned} \tag{2.16}$$

where G_{ijkl}, γ_{ikl}, ξ_{ik}, ... are material constants.

Summing the result obtained up to now we can write the final form of the constitutive equations. Hence, making use of (2.15) and (2.16) we obtain:

$$\begin{aligned} \sigma_{ij} &= \sigma^E_{ij}+\sigma^D_{ij} = C^*_{ijkl}\varepsilon_{kl}+E_{ijkrs}\varepsilon_{r,k}-\beta_{ij}\Theta-\beta^*_{ijk}\Theta_{,k}-\gamma_{ij}C-\gamma^*_{ijk}C_{,k},\\ \eta_i &= \gamma_{ikl}\dot{\varepsilon}_{kl}+\alpha_{ikrs}\varepsilon_{rs,k}-\xi^*_{ik}\Theta_{,k}-\alpha^*_{ij}C_{,k},\\ q_i &= T_0\beta_{ikl}\dot{\varepsilon}_{kl}+T_0\xi_{ikrs}\varepsilon_{rs,k}-k^*_{ik}\Theta_{,k}-T_0d_{ik}C_{,k}, \end{aligned} \tag{2.17}$$

where the following abbreviations have been introduced

$$C^*_{ijkl} = C_{ijkl}+G_{ijkl}\partial_t,\qquad E_{ijkrs}=\gamma_{ijk}\gamma_{rs},\qquad \beta^*_{ijk}=\beta_{ijk}+\xi\gamma_{ijk},\qquad \partial_t=\frac{\partial}{\partial t},$$

$$\alpha_{ikrs}=\alpha_{ik}\gamma_{rs},\qquad \xi^*_{ik}=\xi_{ik}+\xi\alpha_{ik},\qquad \alpha^*_{ik}=n\alpha_{ik}=\frac{c_m}{T_0}\alpha_{ik},$$

$$\xi_{ikrs}=\xi_{ik}\gamma_{rs},\qquad k^*_{ik}=k_{ik}+T_0\xi\xi_{ik},\qquad \gamma^*_{ikl}=n\gamma_{ikl},\qquad d_{ik}=n\xi_{ik}.$$

Proceeding now to writing the equation of heat conductivity we will use equation (2.3) in which we will take into consideration the substitution of (2.5), the derivative with respect to time of function $(2.13)_1$, and relations (2.10) and $(2.15)_3$. We get:

$$T(\beta_{ij}\dot{\varepsilon}_{ij}-m\dot{\Theta}-\xi\dot{C})=\sigma^D_{ij}\dot{\varepsilon}_{ij}-q_{i,i}+\eta_i\mu_{,i}+r_h, \tag{2.18}$$

where q_i is defined by relation $(2.16)_3$.

Assuming further that $|\theta/T|\ll 1$, i.e. restricting our considerations to small temperature changes and omitting the non-linear terms $\sigma^D_{ij}\dot{\varepsilon}_{ij}$ and $\eta_i\mu_{,i}$ as higher-order smalls, we finally obtain after taking into account (2.17):

$$k^*_{ij}\Theta_{,ij}+T_0m\dot{\Theta}+d^*_{ij}C_{,ji}+d\dot{C}=\xi^*_{ijrs}\varepsilon_{rs,ji}+T_0\beta_{ikl}\dot{\varepsilon}_{kl,i}+T_0\beta_{ij}\dot{\varepsilon}_{ij}-r_h, \tag{2.19}$$

where:

$$\xi^*_{ijrs}=T_0\xi_{ijrs},\qquad d^*_{ij}=T_0d_{ij}=c_m\xi_{ij},\qquad d=T_0\xi.$$

The equation of concentration of moisture can be obtained from mass continuity equation (2.1). After taking into consideration $(2.17)_2$ we have:

$$\alpha^*_{ik}C_{,ki}-\dot{C}+\xi^*_{ik}\Theta_{,ki}=\alpha_{ikrs}\varepsilon_{rs,ki}+\gamma_{ikl}\dot{\varepsilon}_{kl,i}-r_m, \tag{2.20}$$

where:

$$\alpha^*_{ik}=n\alpha_{ik}=\frac{1}{T_0}c_m\alpha_{ik}.$$

Equations (2.2), (2.17), (2.19) and (2.20) represent the full set of equations of considered medium.

3. Basic plate equations

In this section an attempt will be made to derive the basic differential equations for thin orthotropic, viscoelastic plate of thickness h, whose median plan lies in the x_1x_2 plane with x_3 denoting the distance from this plane. The deflection of the middle surface is assumed small in relation to the thickness of the plate. Moreover, it is assumed that all simplifying assumptions that are usually used in the classical thin plates theory [7, 8] are also valid in the present case. In accordance with these assumptions we can write $u_3(x_1, x_2, x_3, t) \approx w(x_1, x_2, t)$ where $w(x_1, x_2, t)$ is the deflection of the middle surface of the plate. Moreover, the strain tensor ε_{ij} can be divided into two parts [7]:

$$\varepsilon_{\alpha\beta} = \varepsilon^0_{\alpha\beta} + \varepsilon'_{\alpha\beta} = \frac{1}{2}(u^0_{\alpha,\beta} + u^0_{\beta,\alpha}) - x_3 w_{,\alpha\beta}, \quad (\alpha, \beta = 1, 2), \tag{3.1}$$

where u^0_α denotes the displacement due to uniform tension of the middle surface, $u'_\alpha = -x_3 w_{,\alpha}$ stands for the displacement due to the deflection of the plate.

As it was already mentioned above, we are dealing with the thin plates. In this case the temperature distribution along the thickness can be assumed to be linear. We shall introduce this simplification also for the concentration field, i.e.:

$$\begin{aligned} \Theta(x_1, x_2, x_3, t) &\approx \tau_0(x_1, x_2, t) + x_3 \tau(x_1, x_2, t), \\ C(x_1, x_2, x_3, t) &\approx \varkappa_0(x_1, x_2, t) + x_3 \varkappa(x_1, x_2, t), \end{aligned} \tag{3.2}$$

where the following notations have been introduced:

$$\tau_0(x_1, x_2, t) = \frac{1}{h}\int_{-h/2}^{h/2} \Theta(x_1, x_2, x_3, t)dx_3 \approx \frac{\Theta_U + \Theta_L}{2},$$

$$\varkappa_0(x_1, x_2, t) = \frac{1}{h}\int_{-h/2}^{h/2} C(x_1, x_2, x_3, t)dx_3 \approx \frac{C_U + C_L}{2},$$

$$\tau(x_1, x_2, t) = \frac{12}{h^3}\int_{-h/2}^{h/2} x_3\Theta(x_1, x_2, x_3, t)dx_3 \approx \frac{\Theta_U - \Theta_L}{h},$$

$$\varkappa(x_1, x_2, t) = \frac{12}{h^3}\int_{-h/2}^{h/2} x_3 C(x_1, x_2, x_3, t)dx_3 \approx \frac{C_U - C_L}{h}.$$

Here τ_0 and $\varkappa_0$ are the mean temperature and concentration of moisture which do not vary in x_3 direction, respectively, Θ_j and C_j ($j = U, L$) are temperatures and concentrations on upper (U) and lower (L) sides of the plate.

Further, let us define forces and moments per unit width of the plate cross-section, as it is usually done in the plate theory:

$$\begin{aligned} N_{\alpha\beta}(x_1, x_2, t) &= \int_{-h/2}^{h/2} \sigma_{\alpha\beta} dx_3, \quad M_{\alpha\beta}(x_1, x_2, t) = \int_{-h/2}^{h/2} x_3 \sigma_{\alpha\beta} dx_3, \\ Q_{\alpha 3}(x_1, x_2, t) &= Q_{3\alpha}(x_1, x_2, t) = \int_{-h/2}^{h/2} \sigma_{\alpha 3} dx_3. \end{aligned} \tag{3.3}$$

Making now use of (2.17), (3.1) and (3.3) we obtain:

$$\begin{aligned} N_{\alpha\beta} &= h(C^*_{\alpha\beta\gamma\delta}\varepsilon^0_{\gamma\delta} + E_{\alpha\beta\gamma\delta\omega}\varepsilon^0_{\gamma\delta,\omega} - \beta^*_{\alpha\beta\delta}\tau_{0,\delta} - \beta_{\alpha\beta}\tau_0 - \gamma^*_{\alpha\beta\delta}\varkappa_{0,\delta} - \gamma_{\alpha\beta}\varkappa_0), \\ M_{\alpha\beta} &= -\frac{h^3}{12}(C^*_{\alpha\beta\gamma\delta} w_{,\gamma\delta} + E_{\alpha\beta\gamma\delta\omega} w_{,\gamma\delta\omega} - \beta^*_{\alpha\beta\gamma}\tau_{,\gamma} - \beta_{\alpha\beta}\tau - \gamma^*_{\alpha\beta\delta}\varkappa_{,\delta} - \gamma_{\alpha\beta}\varkappa). \end{aligned} \tag{3.4}$$

Returning to the equation of motion (2.2) we express it in a different form:

$$\begin{aligned}\sigma_{\alpha\beta,\beta}+\sigma_{\alpha 3,3}+X_\alpha &= \varrho_0 \ddot{u}_\alpha,\\ \sigma_{3\beta,\beta}+\sigma_{33,3}+X_3 &= \varrho_0 \ddot{w}.\end{aligned} \tag{3.5}$$

If we integrate now these equations along the thickness of the plate, and later on doing the same with equation $(3.5)_1$, after having first multiplied it by x_3, and taking into account the expressions (3.3) we arrive at the following equations

$$N_{\alpha\beta,\beta}+p_\alpha = \varrho_0 h\ddot{u}_\alpha, \tag{3.6}$$

$$Q_{3\alpha,\alpha}+p_3 = \varrho_0 h\ddot{w}, \tag{3.7}$$

$$M_{\alpha\beta,\beta}+m_{3\alpha}-Q_{3\alpha} = 0, \tag{3.8}$$

where ϱ_0 is the plate density per unit area of the middle surface and:

$$p_\alpha = \sigma_{\alpha 3}\Big|_{-h/2}^{h/2}+\int_{-h/2}^{h/2} X_\alpha dx_3, \quad p_3 = \sigma_{33}\Big|_{-h/2}^{h/2}+\int_{-h/2}^{h/2} X_3 dx_3,$$

$$m_{3\alpha} = (\sigma_{3\alpha}x_3)\Big|_{-h/2}^{h/2}+\int_{-h/2}^{h/2} x_3 X_3 dx_3.$$

Equation (3.6) concerns the state of displacement in the plane of the plate.

Let us return now to the equations (3.7) and (3.8). Eliminating $Q_{3\alpha}$ from it we arrive at the equation of motion:

$$M_{\alpha\beta,\beta\alpha}+q_w = \varrho_0 h\ddot{w}, \tag{3.9}$$

where:

$$q_w = p_3+m_{3\alpha}.$$

If we now introduce $M_{\alpha\beta}$ from $(3.4)_2$, into the equation of motion (3.9), we obtain in the general case of anisotropy the differential dynamic equation of the bent plate in the form:

$$\begin{aligned}C^*_{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta}+E_{\alpha\beta\gamma\delta\omega} w_{,\alpha\beta\gamma\delta\omega}+\beta_0\varrho_0\ddot{w} &= \beta_{\alpha\gamma}\tau_{,\alpha\gamma}\\ +\beta^*_{\alpha\gamma\omega}\tau_{,\alpha\gamma\omega}+\gamma_{\alpha\beta}\varkappa^{\prime}_{,\alpha\beta}+\alpha^*_{\beta\gamma\delta}\varkappa_{,\beta\gamma\delta}+\frac{12}{h^3}q_w, &\quad \beta_0 = \frac{12}{h^2}.\end{aligned} \tag{3.10}$$

For the orthotropic plate the equation (3.10) simplified and take the following form:

$$C^*_{\alpha\beta\gamma\delta} w_{,\alpha\beta\gamma\delta}+\beta_0\varrho_0\ddot{w} = \frac{12}{h^3}q_w+\beta_{\alpha\gamma}\tau_{,\alpha\gamma}+\gamma_{\alpha\beta}\varkappa_{,\alpha\beta}, \tag{3.11}$$

where:

$$C^*_{1111} = A(E_1+\lambda_1\partial_t), \quad C^*_{2222} = A(E_2+\lambda_2\partial_t), \quad C^*_{1122} = A(E_1+\lambda_1\partial_t),$$

$$C^*_{1212} = (G_{12}+\eta_{12}\partial_t)\frac{h^3}{12}, \quad A = \frac{h^3}{12(1-\nu^2)}, \quad \nu^2 = \nu_1\nu_2,$$

$$\beta_{11} = \frac{h^3}{12}\left(\frac{E_1}{1-\nu^2}\alpha_1+G_{12}\alpha_2\right), \quad \beta_{22} = \frac{h^3}{12}\left(G_{12}\alpha_1+\frac{E_2}{1-\nu^2}\alpha_2\right).$$

In these formulas $E_\alpha(\alpha = 1,2)$ is the Young's modulus in the x_α direction, ν_α is the Poisson's ratio, G_{12} is the shear modulus in the x_1x_2 plane, α_β is the thermal expansion coefficient in the x_β direction, λ_α and η_{12} are the viscosity coefficients.

Equation (3.11) must be suplemented by the equations describing heat conduction τ and concentration of the diffusing matter $\varkappa$. In order to derive these equations we turn to equations (2.19) and (2.20) and integrate over the plate thickness, before this multiplying them by x_3.

If, in addition, the boundary conditions of the form:

$$\lambda_0 \frac{\partial \Theta}{\partial x_3}\bigg|_{x_3=\frac{h}{2}} = p_U(x_1, x_2, t), \quad \lambda_0 \frac{\partial \Theta}{\partial x_3}\bigg|_{x_3=-\frac{h}{2}} = p_L(x_1, x_2, t),$$
$$D_0 \frac{\partial C}{\partial x_3}\bigg|_{x_3=\frac{h}{2}} = f_U(x_1, x_2, t), \quad D_0 \frac{\partial C}{\partial x_3}\bigg|_{x_3=-\frac{h}{2}} = f_L(x_1, x_2, t), \tag{3.12}$$

are assumed, where λ_0 and D_0 are the coefficients of heat conduction and diffusion, respectively, then equations (2.19) and (2.20), in the absence of heat and diffusion sources, for the considered orthotropic plate reduce to:

$$k^*_{\alpha\beta}\tau_{,\alpha\beta} - c_v\dot{\tau} - a_1\tau + d^*_{\alpha\beta}\varkappa_{,\alpha\beta} + d\dot{\varkappa} - a_2\varkappa = \frac{a_1}{2\lambda_0}(p_L - p_U) + \\ + \frac{a_2}{2D_0}(f_L - f_U) - \xi^*_{\alpha\beta\gamma\delta}w_{,\alpha\beta\gamma\delta} - T_0\beta_{\alpha\beta}\dot{w}_{,\alpha\beta}, \tag{3.13}$$

$$\alpha^*_{\beta\gamma}\varkappa_{,\beta\gamma} - \dot{\varkappa} - a_3\varkappa + \xi^*_{\alpha\beta}\tau_{,\alpha\beta} - a_4\tau = \frac{a_4}{2\lambda_0}(p_L - p_U) + \frac{a_3}{2D_0}(f_L - f_U) - \alpha_{\beta\gamma\delta\omega}w_{,\beta\gamma\delta\omega}, \tag{3.14}$$

where:

$$a_1 = k^*_{33}\beta_0, \quad a_2 = d^*_{33}\beta_0, \quad a_3 = \alpha_{33}\beta_0, \quad a_4 = \xi_{33}\beta_0.$$

The system of equations (3.11), (3.13) and (3.14) formed a mutually coupled system of differential equations for the case, when the boundary conditions are given by (3.12). The solution of the set of equations mentioned above must satisfy boundary and initial conditions appropriate to the given problem.

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Резюме

ВЛИЯНИЕ ИЗМЕНЕНИЯ ВЛАЖНОСТИ И ТЕМПЕРАТУРЫ НА ПОВЕДЕНИЕ ОРТОТРОПНЫХ ВЯЗКОУПРУГИХ ПЛАСТИНОК

В работе выведено основные уравнения для ортотропных вязкоупругих, тонких пластинок подверженых действию влажности и температуры переменных во времени. Предположено, что распределение так влажости как и температуры линейное по толщине пластинки. Принято, что упрощения классической теории тонких пластинок здесь также справедливы.

Формулировка перечисленых уравнений основана на уравнениях движения, конститутивных, а также уравнениях теплопроводности и концентрации влажности для анизотропной вязкоупругой среды. Уравнения построены в первой части работы, при использовании основных принципов механики и термодинамики сплошных сред и ограничении до линейных соотношений.

Streszczenie

WPŁYW ZMIAN WILGOTNOŚCI I TEMPERATURY NA ZACHOWANIE SIĘ ORTOTROPOWYCH PŁYT LEPKOSPRĘŻYSTYCH

Zasadniczym celem pracy było wyprowadzenie podstawowych równań dla ortotropowych, lepkosprężystych płyt cienkich poddanych równoczesnemu działaniu wilgotności i temperatury zmiennymi w czasie. Założono, że rozkład zarówno temperatury jak i wilgotności na grubości płyty jest liniowy. Przyjęto również, iż obowiązują założenia upraszczające stosowane w klasycznej teorii płyt cienkich.

Podstawę do sformułowania wyżej wymienionych równań stanowiły równania ruchu, związki konstytutywne oraz równania przewodnictwa ciepła i koncentracji wilgotności dla anizotropowego ośrodka lepkosprężystego. Równania te, co stanowi przedmiot pierwszej części pracy, zbudowano wykorzystując podstawowe prawa mechaniki i termodynamiki ośrodków ciągłych, ograniczając się do relacji liniowych.

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