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DYNAMIC STABILITY OF ANTISYMMETRICALLY LAMINATED CROSS-PLY CYLINDRICAL SHELLS

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1. Introduction

Dynamic behaviour of thin laminated cylindrical shells is of great importance to engineers. The coupling between bending and tension in laminates results in the necessity to modify the classic equilibrium equations and the boundary conditions for thin uniform cylindrical shells in order to apply them to laminated shells. The reformulation of boundary conditions and the solution of the static buckling problems for the cylindrical shells was done by Almroth [1]. Numerous papers are available on free vibrations of laminated shells (see for example papers by Bert, Baker and Egle [2], Dong [3], Alam and Asnani [4]). While parametric vibrations and dynamic stability problems for uniform isotropic cylindrical shells under time-dependent membrane forces have drawn much attention, the dynamic stability of cylindrical shells has not been investigated yet.

The purpose of the paper is to analyse the dynamic asymptotic stability of thin elastic cylindrical shells for cross-ply antisymmetric configuration. Membrane forces acting in the shell midsurface are assumed to be deterministic functions of time or stochastic processes with differentiable realizations. The shell consists of an even number of equal thickness orthotropic laminae laid on each other with principal material directions alternating at 0 and $\pi/2$ to the shell axial and circumferential directions. Using the direct Liapunov method we have derived the sufficient conditions for the asymptotic stability and the almost sure asymptotic stability. The influence of geometric and material properties of the shell as well as characteristics of loading on stability regions have been examined numerically.

2. Problem formulation

Let us consider a closed elastic simply supported cylindrical shell of radius a, length l and total thickness h, $a \gg h$, $l \gg h$. The shell consists of an even number of equal thickness orthotropic layers antisymmetrically laminated with respect to its midsurface from both the geometric and the material property standpoint. The Kirchhoff-Love hypothesis on nondeformable normal element is taken into account. Tangential, rotary and coupling inertias are neglected. For the shell subjected to a concentrated load P and a uniformly distributed radial loading q, the initial membrane loads can be determined by assuming that the shell remains circular and undergoes a uniform compression circumferentially. Consequently:

$$N_x = P/2\pi a,$$
$$N_\Theta = aq.$$

Taking into account a linear damping in the radial direction we obtain the equations of the technical theory of thin laminated shells in terms of displacements u, v, w in tangential, circumferential and radial direction, respectively [3]:

$$\begin{aligned}
&A_{11}u_{,xx}+A_{66}u_{,\Theta\Theta}/a^2+(A_{12}+A_{66})v_{x,\Theta}/a-B_{11}w_{,xxx}+A_{12}w_{,x}/a=0,\\
&(A_{12}+A_{66})u_{,x\Theta}/a+A_{66}v_{,xx}+A_{11}v_{,\Theta\Theta}/a^2+B_{11}w_{,\Theta\Theta\Theta}/a^3+A_{11}w_{,\Theta}/a^2=0,\\
&-B_{11}u_{,xxx}+A_{12}u_{,x}/a+B_{11}v_{,\Theta\Theta\Theta}/a^3+A_{11}v_{,\Theta}/a^2+D_{11}w_{,xxxx}+\\
&+2(D_{12}+2D_{66})w_{,xx\Theta\Theta}/a^2+D_{22}w_{,\Theta\Theta\Theta\Theta}/a^4+2B_{11}w_{,\Theta\Theta}/a^3+A_{11}w/a^2+\\
&+\varrho hw_{,tt}+2\varrho\beta hw_{,t}-\overline{N}_x w_{,xx}-\overline{N}_\Theta w_{,\Theta\Theta}/a^2=0,\\
&\qquad (x,\Theta)\in\Omega\equiv(0,1)\times(0,2\pi).
\end{aligned} \tag{1}$$

Internal forces and moments are expressed by the displacements as follows:

$$\begin{aligned}
N_x &= A_{11}u_{,x}+A_{12}v_{,\Theta}a+A_{12}w/a-B_{11}w_{,xx},\\
N_\Theta &= A_{12}u_{,x}+A_{11}v_{,\Theta}/a+A_{11}w/a+B_{11}w_{,\Theta\Theta}/a^2,\\
N_{x\Theta} &= A_{66}(v_{,x}+u_{,\Theta}/a),\\
M_x &= B_{11}u_{,x}-D_{11}w_{,xx}-D_{12}w_{,\Theta\Theta}/a^2,\\
M_\Theta &= -B_{11}v_{,\Theta}/a-B_{11}w/a-D_{12}w_{,xx}-D_{22}w_{,\Theta\Theta}/a^2,\\
M_{x\Theta} &= -2D_{66}w_{,x\Theta}/a.
\end{aligned} \tag{2}$$

The closed shell is assumed to be simply supported without displacement in circumferential direction at $x = 0,1$. The conditions imposed on displacements, internal forces and moments, called according to Almroth's classification $S2$, can be written down as:

$$w=0,\quad v=0,\quad N_x=0,\quad M_x=0 \quad \text{at} \quad x=0,1. \tag{3}$$

Our purpose is to investigate the stability of undisturbed shell surface $u = v = w = 0$ (the trivial solution). A disturbed state is estimated by means of a distance of the solution of system (1) with nontrivial initial conditions from the trivial solution. Under assumption that the membrane forces are the deterministic functions of time we will study the asymptotic stability of trivial solution, i.e. we will derive conditions that imply:

$$\lim_{t\to\infty}||w||=0. \tag{4}$$

If the forces are stochastic "nonwhite" processes with sufficiently smooth realizations we shall consider the almost sure asymptotic stability which holds if a probability of event defined by (4) is equal to one:

$$P\{\lim_{t\to\infty}||w||=0\}=1. \tag{5}$$

We shall examine the foregoing kinds of stability using the direct Liapunov method, which provides a significant advantage in that the conditions for stability can be obtained without the explicite solving the equations of motion.

3. Derivation of the sufficient stability conditions

We construct a functional as a sum of a modified kinetic energy and the potential energy of the shell in order to apply it as a Liapunov functional:

$$V = \frac{1}{2}\int_{\Omega} [z^2+2\beta zw+2\beta^2w^2-M_x w_{,xx}-M_\Theta w_{,\Theta\Theta}/a^2-M_{x\Theta}2w_{,x\Theta}/a+ \\ +N_x u_{,x}+N_\Theta(v_{,\Theta}+w)/a+N_{x\Theta}(v_{,x}+u_{,\Theta}/a)]\,ad\Omega, \tag{6}$$

where $z = w_{,t}$.

The functional is positive-definite since the first three terms of integrand can be rearranged as a sum of squares. Therefore, we can choose the square root of functional (6) as the distance used in the stability definitions. Under the previous assumptions imposed on the membrane forces the classic differentiation rule can be applied to calculate the time-derivative of functional (6). Dividing equations of motion (1) by ϱh and retaining for convenience the same symbols for coefficients we obtain the time-derivative of functional (6) in the following form:

$$\frac{dV}{dt} = \frac{1}{2}\int_{\Omega} [2(z+\beta w)(-2\beta z+\overline{N}_x w_{,xx}+\overline{N}_\Theta w_{,\Theta\Theta}/a^2+B_{11}u_{,xxx}+ \\ -A_{12}u_{,x}/a-B_{11}v_{,\Theta\Theta\Theta}/a^3-A_{11}v_{,\Theta}/a^2-D_{11}w_{,xxxx}+ \\ -2(D_{12}+2D_{66})w_{,xx\Theta\Theta}/a^2-D_{22}w_{,\Theta\Theta\Theta\Theta}/a^4-2B_{11}w_{,\Theta\Theta}/a^3-A_{11}w/a^2)+ \\ -M_{x,t}w_{,xx}-M_x z_{,xx}-M_{\Theta,t}w_{,\Theta\Theta}/a^2-M_\Theta z_{,\Theta\Theta}/a^2-2M_{x\Theta,t}w_{,x\Theta}/a+ \\ -2M_{x\Theta}z_{,x\Theta}/a]\,ad\Omega+(I_1)_{,t}, \tag{7}$$

where I_1 denotes an additional functional:

$$I_1 = \frac{1}{2}\int_{\Omega} [N_x u_{,x}+N_\Theta(v_{,\Theta}+w)/a+N_{x\Theta}(v_{,x}+u_{,\Theta}/a)]\,ad\Omega.$$

Integrating by parts, using boundary conditions (3) and periodicity conditions with respect to variable Θ we prove the following formulae:

$$\int_{\Omega} M_{x,xx}z\,ad\Omega = \int_0^{2\pi} M_{x,x}z\Big|_0^l ad\Theta-\int_{\Omega} M_{x,x}z_{,x}ad\Omega = -\int_0^{2\pi} M_x z_{,x}\Big|_0^l ad\Theta+ \\ +\int_{\Omega} M_x z_{,xx}ad\Omega = \int_{\Omega} M_x z_{,xx}ad\Omega,$$

$$\int_{\Omega} M_{\Theta,\Theta\Theta}z\,ad\Omega = \int_{\Omega} M_\Theta z_{,\Theta\Theta}\,ad\Omega,$$

$$\int_{\Omega} M_{x\Theta,x\Theta}z\,ad\Omega = \int_{\Omega} M_{x\Theta}z_{,x\Theta}\,ad\Omega,$$

$$\int_\Omega M_{x,t} w_{,xx} a d\Omega = \int_\Omega M_x z_{,xx} a d\Omega,$$

$$\int_\Omega M_{\Theta,t} w_{,\Theta\Theta} a d\Omega = \int_\Omega M_\Theta z_{,\Theta\Theta} a d\Omega,$$

$$\int_\Omega M_{x\Theta,t} w_{,x\Theta} a d\Omega = \int_\Omega M_{x\Theta} z_{,x\Theta} a d\Omega.$$

In a similar way integrating by parts we convert the functional I_1 to the following form:

$$I_1 = \frac{1}{2}\int_\Omega [-(N_{x,x}+N_{x\Theta,\Theta}/a)u-(N_{x\Theta,x}+N_{\Theta,\Theta}/a)v+N_\Theta w/a] a d\Omega.$$

Recognizing the expressions in the parentheses as left hand side expressions of the first two equations of motion (1) we omit them so we can write:

$$I_1 = \frac{1}{2}\int_\Omega N_\Theta w d\Omega.$$

Using the above relations we rewrite the time-derivative of functional (7) as:

$$\frac{dV}{dt} = -2\beta V + 2U, \tag{8}$$

where:

$$U = \frac{1}{2}\int_\Omega [(z+\beta w)(\bar{N}_x w_{,xx}+\bar{N}_\Theta w_{,\Theta\Theta}/a^2)+2\beta^2 wz+2\beta^3 w^2] a d\Omega. \tag{9}$$

Now we attempt to construct a bound:

$$U \leqslant \lambda V, \tag{10}$$

where the function λ is to be determined.

Proceeding similarly as Kozin [5] we solve an additional variational problem $\delta(U-\lambda V) = 0$ and we obtain:

$$\lambda = \max_{m,n=1,2,\ldots} |\beta^2+(N_x k_m^2+N_\Theta k_n^2)/2| [\beta^2+D_{11}k_m^4+2(D_{12}+2D_{66})k_m^2k_n^2+ \\ +D_{22}k_n^4-2B_{11}k_n^2/a+A_{11}/a^2+(2T_{12}T_{13}T_{23}-T_{11}T_{23}^2-T_{22}T_{13}^2)/ \\ /(T_{11}T_{22}-T_{12}^2)]^{-1/2}, \tag{11}$$

where:

$$k_m = m\pi/l, \quad k_n = n/a,$$
$$T_{11} = A_{11}k_m^2+A_{12}k_n^2, \quad T_{12} = -(A_{12}+A_{66})k_m k_n,$$
$$T_{22} = A_{11}k_n^2+A_{66}k_m^2, \quad T_{13} = -k_m(A_{12}/a+B_{11}k_m^2),$$
$$T_{23} = k_n(-B_{11}k_n^2+A_{11}/a).$$

Substituting inequality (10) into equation (8) we obtain the differential inequality, from which we have the following estimation of functional (6):

$$V(t) \leqslant V(0)\exp\left\{-2t\left[\beta-\frac{1}{t}\int_0^t \lambda(s)ds\right]\right\}.$$

Thus, it immediately follows that the sufficient stability condition for the asymptotic stability with respect to the distance $\|\cdot\| = V^{1/2}$ is:

$$\beta \geqslant \lim_{t\to\infty} \frac{1}{t} \int_0^t \lambda(s)ds, \tag{12}$$

or for the almost sure asymptotic stability, provided processes N_x and N_Θ are ergodic and stationary is:

$$\beta \geqslant E\lambda. \tag{13}$$

where E denotes the operator of the mathematical expectation.

4. Results

Expression (11) and inequality (13) give us possibility to obtain the critical damping coefficient guaranteeing the almost sure asymptotic stability as a function of laminate parameters and statistic characteristics of membrane forces. In order to obtain stability regions, we choose discrete values of force (N_x or N_Θ) and compute λ_{mn}. Then we choose the largest value corresponding to the given value of the force and take the expectation numerically integrating the product of λ by the probability density function. This is accomplished for various values of parameters by choosing the variance and varying the damping coefficient until inequality (13) will be satisfied.

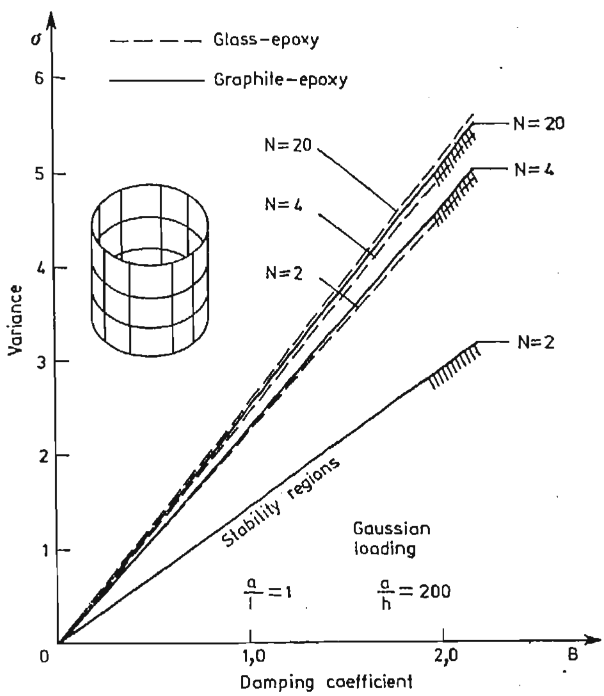

Fig. 1.

Numerical calculations are performed for the gaussian process with zero mean and variance σ^2 and the harmonic process with variance $\sigma^2 = A^2/2$, where A denotes its amplitude, for different number of layers and the shell aspect ratio a/l.

The almost sure asymptotic stability regions as functions of β, σ and number of layers N in the case, when the shell with $a/l = 1$ is loaded by the gaussian process, are shown in Fig. 1. The stability regions are not changed in going from the axial loading to the circumferential one. As the number of layers increases the orthotropic solution is rapidly approached. The coupling between bending and extension depends on the orthotropic moduli ratio E_1/E_2. It is seen from the figure that for greater ratios E_1 to E_2 the effect of coupling increases.

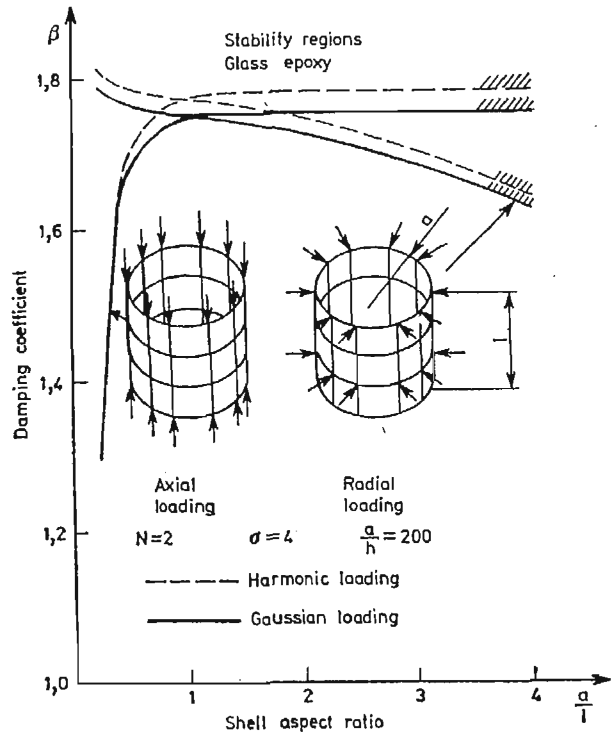

Fig. 2.

The dependence of stability regions as functions of β and the shell aspect ratio a/l for twolayered shell made of glass-epoxy is shown in Fig. 2. It is found that the stability regions are not changed substantially in going from the gaussian process to the harmonic one. The dependence of stability regions on the direction of loading is quite essential.

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Резюме

ДИНАМИЧЕСКАЯ УСТОЙЧИВОСТЬ АНТИСИММЕТРИЧНО СЛОИСТЫХ ЦИЛИНДРИЧЕСКИХ ОБОЛОЧЕК

Принимая во внимание, что замкнутая круговая оболочка составлена из четного числа ортотропных слоев антисимметрично расположеных относительно срединной поверхности, исследована асимптотическая и почти наверно асимптотическая устойчивость невыпученной формы оболочки. Слоя оболочки изготовлены из однородного материала, которого главные направления ортотропии переменно совпадают с аксиальным и меридиональным направлением. В срединной поверхности оболочки действуют усилия зависящие от времени отвечающие исходному безмоментному состоянию. Вводя соответствующий функционал Ляпунова и исследуя его приращение по траектории решения уравнений движения получены достаточные условия устойчивости. Исследовано влияние числа слоев, геометрии оболочки и направления нагрузки на области устойчивости оболочек изготовленых из стекла и эпоксидной смолы или графита и эпоксидной смолы.

Streszczenie

DYNAMICZNA STATECZNOŚĆ ANTYSYMETRYCZNIE POPRZECZNIE LAMINOWANYCH POWŁOK WALCOWYCH

Zakładając, że zamknięta powłoka walcowa zbudowana jest z parzystej liczby ortotropowych warstw antysymetrycznie rozmieszczonych względem powierzchni środkowej zbadana jest asymptotyczna i prawie pewnie asymptotyczna stateczność nieodkształconej powierzchni. Warstwy powłoki wykonane są z ortotropowego materiału którego kierunki główne mają przemiennie kierunek osiowy lub obwodowy. W powierzchni środkowej powłoki działają siły membranowe jawnie zależne od czasu. Konstruując odpowiedni funkcjonał Lapunowa i badając jego wzrost wzdłuż rozwiązań równań ruchu wyznaczono dostateczne warunki stateczności. Przedyskutowano wpływ liczby warstw i współczynników geometrycznych na obszar stateczności powłoki wykonanej z włókna szklanego na bazie żywicy epoksydowej i włókna grafitowego na bazie żywicy epoksydowej.

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