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BALANCE EQUATIONS FOR MIXTURE AND POROUS MEDIA IN THE LIGHT OF NONSTANDARD ANALYSIS

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Introduction

The list of papers and monographs concerning mixtures and porous media is very extensive (for example, cf. [1 - 20]). Most of them is based on the rather unphysical postulate that in every point of the region occupied by the mixture there are all its components. At the same time, also porosity is assigned to every such point. Moreover, in [1 - 19], the balance equations for mixture and porous media are introduced in a form of certain a priori assumptions, which take into account the forementioned postulates being not related to the balance equations of the classical continuum mechanics.

The aim of the paper is to derive the balance equations for the mixture and porous media directly from the well known balance equations of continuum mechanics. We are to show that it can be done using the methods of the nonstandard analysis. Such approach makes it possible to assign the clear physical interpretation to all terms in the resulting balance equations for mixture and porous media. At the same time, the extra unphysical postulates, such as mentioned in the begining of the Introduction, are avoided. The present paper is self contained but the partial results and ideas concerning the proposed approach were published in [24 - 26]. The approach proposed here has certain features common with that of [14], [21 - 22].

Denotations. Throughout the paper n is the fixed positive integer. Subscripts α, β run over the sequence $1, 2, \ldots, n$, subscript γ runs over $0, 1, 2, \ldots, n$. Moreover, t_0, t_1 are the known real numbers, $t_0 < t_1$, which determine certain time interval.

1. Physical basis

Let ε be Euclidean real 3-space of places, O be a set of all regular regions in ε, V be a translation space for ε, V_f be a Euclidean real 3-space of forces, U be the material universe of all 3-dimensional elementary differentiable manifolds (deformable bodies).

Define $\mathcal{M}$ as a set of all n-tuples $(\chi_1, \ldots, \chi_n)$ where $\chi_\alpha\colon \Omega_\alpha \times [t_0, t_1] \to \varepsilon$, $\alpha = 1, \ldots, n$ are deformation functions of n disjointed deformable bodies $\mathcal{B}_\alpha$, $\mathcal{B}_\alpha \in U$ such that

$$(\forall \alpha, \beta)(\forall t)\big((\alpha \neq \beta) \Rightarrow (\chi_\alpha(\Omega_\alpha, t) \cap \chi_\beta(\Omega_\beta, t) \neq \emptyset)\big).$$

Let Ω_α, $\alpha = 1, \ldots, n$, be the disjointed regions in ε occupied by bodies $\mathscr{B}_\alpha$ in the known reference configurations $\varkappa_\alpha$: $\mathscr{B}_\alpha \to \varepsilon$. Let $v_\alpha(\cdot, t) \in V$, where $v_\alpha(x, t) \equiv \dot{\chi}(X, t)$, $X = \chi^{-1}(x, t)$, $X \in \Omega_\alpha$, $t \in [t_0, t_1]$, be the velocity field defined, for every t, on the region $\chi_\alpha(\Omega_\alpha, t)$ occupied by the body $\mathscr{B}_\alpha$ at the time instant t. Define $\Omega_\alpha^t \equiv \chi_\alpha(\Omega_\alpha, t)$ and $\Omega_0^t \equiv \varepsilon \setminus \cup \overline{\Omega}_\alpha^t$. By virtue of the well known assumptions of solid mechanics, to every $(\chi_1, \ldots, \chi_n) \in \mathscr{M}$ and to every $\Delta \in O$ the following functions are uniquely assigned:

$$
\begin{gathered}
[t_0, t_1] \ni t \to \int\limits_{\Delta \cap \Omega_\alpha^t} \varrho_\alpha(x, t)\, dv \in R_+, \\
[t_0, t_1] \ni t \to \int\limits_{\Delta \cap \Omega_\alpha^t} \varrho_\alpha(x, t) b_\alpha(x, t)\, dv \in V_f, \\
[t_0, t_1] \ni t \to \int\limits_{\partial(\Delta \cap \Omega_\alpha^t) \cap \partial \Omega_\gamma^t} t_{\alpha\gamma}(x, t)\, da \in V_f, \qquad t_{\alpha\alpha}(\cdot) \equiv 0, \\
[t_0, t_1] \ni t \to \int\limits_{\partial \Delta \cap \Omega_\alpha^t} T_\alpha(x, t) n_{\partial \Delta}(x)\, da \in V_f, \\
[t_0, t_1] \ni t \to \int\limits_{(\Delta \cap \Omega_\alpha^t) \cap \partial \Omega_\beta^t} f_{\alpha\beta}(x, t)\, da \in V_f.
\end{gathered} \tag{1.1}
$$

Functions (1.1) are assumed to be continuous for almost every $t \in [t_0, t_1]$. The values of (1.1) represent for any fixed t, the following objects: mass of $\mathscr{B}_\alpha$ in Δ, resultant body forces acting at $\mathscr{B}_\alpha$ in Δ, resultant of the boundary tractions executed by $\mathscr{B}_\gamma$ (for $\gamma \neq 0$) on $\mathscr{B}_\alpha$ in Δ, resultant of the surface tractions on $\partial\Delta$ within body $\mathscr{B}_\alpha$($T_\alpha(x, t)$ is Cauchy stress tensor), respectively. Moreover $f_{\alpha\beta}(x, t)$ is the density of the forces due to the friction between $\mathscr{B}_\alpha$ and $\mathscr{B}_\beta$, which act on $\mathscr{B}_\alpha$. Here $f_{\alpha\beta}(x, t) + f_{\beta\alpha}(x, t) = 0$. Term $t_{\alpha 0}$ in $(1.1)_3$ represents boundary tractions due to the external forces acting at $\mathscr{B}_\alpha$.

For an arbitrary Δ, $\Delta \in O$, define

$$
\begin{gathered}
\Delta^e \equiv \varepsilon \setminus \Delta, \\
\Gamma_{\alpha\gamma}^t(\Delta, \Delta^e) \equiv \partial(\Omega_\alpha^t \cap \Delta) \cap \partial(\Omega_\gamma^t \cap \Delta^e) \subset \partial\Delta, \\
\Gamma_{\alpha\gamma}^t(\Delta, \Delta) \equiv \partial(\Omega_\alpha^t \cap \Delta) \cap \partial(\Omega_\gamma^t \cap \Delta).
\end{gathered}
$$

As it is known, for each $(\mathscr{B}_1, \ldots, \mathscr{B}_n)$, $\mathscr{B}_\alpha \in U$, and every $(\chi_1, \ldots, \chi_n) \in \mathscr{M}$, the integrands in formulas (1.1) satisfy the well known system of conditions

$$
\frac{D}{Dt} \int\limits_{\Delta \cap \Omega_\alpha^t} \varrho_\alpha(x, t)\, dv = 0^{1)}, \tag{1.2}
$$

[1]) Here

$$
\frac{D}{Dt} \int\limits_{\Delta \cap \Omega_\alpha^t} f(x, t)\, dv \equiv \int\limits_{\Delta \cap \Omega_\alpha^t} \left[\frac{\partial f(x, t)}{\partial t} + \operatorname{div}(f(x, t) v_\alpha(x, t)) \right] dv,
$$

for an arbitrary function $f(x, t)$ on the LHS of Eqs. (I.2)

$$\frac{D}{Dt}\int\limits_{\Delta\cap\Omega^t_\alpha} v_\alpha(x,t)\varrho_\alpha(x,t)\,dv = \sum_{\gamma=0}^{n}\int\limits_{\Gamma^t_{\alpha\gamma}(\Delta,\Delta^e)} t_{\alpha\beta}(x,t)\,da + \sum_{\gamma=0}^{n}\int\limits_{\Gamma^t_{\alpha\gamma}(\Delta,\Delta)} t_{\alpha\gamma}(x,t)\,da +$$

$$+\int\limits_{\partial\Delta\cap\Omega^t_\alpha} T_\alpha(x,t)n_{\partial\Delta}(x)\,da + \int\limits_{\Delta\cap\Omega^t_\alpha} b_\alpha(x,t)\varrho_\alpha(x,t)\,dv, \tag{1.2}$$

[cont.]

$$\frac{D}{Dt}\int\limits_{\Delta\cap\Omega^t_\alpha}(x-x_0)\times v_\alpha(x,t)\varrho_\alpha(x,t)\,dv = \sum_{\gamma=0}^{n}\int\limits_{\Gamma^t_{\alpha\gamma}(\Delta,\Delta^e)}(x-x_0)\times t_{\alpha\gamma}(x,t)\,da +$$

$$+\sum_{\gamma=0}^{n}\int\limits_{\Gamma^t_{\alpha\gamma}(\Delta,\Delta)}(x-x_0)\times t_{\alpha\gamma}(x,t)\,da + \int\limits_{\partial\Delta\cap\Omega^t_\alpha}(x-x_0)\times T_\alpha(x,t)n_{\partial\Delta}(x)\,da +$$

$$+\int\limits_{\Delta\cap\Omega^t_\alpha}(x-x_0)\times b_\alpha(x,t)\varrho_\alpha(x,t)\,dv,$$

which has to hold for every $\Delta\in O$ and a.e. $t\in[t_0,t_1]$. Here x_0 is an arbitrary point in ε and $t_{\alpha\beta}(\cdot)\equiv 0$, for $\alpha=\beta$, while

$$t_{\alpha\beta}(x,t) = T_\alpha(x,t)n_{\partial\Omega^t_\alpha}(x) = -T_\beta(x,t)n_{\partial\Omega^t_\beta}(x)+f_{\alpha\beta}(x,t), \quad \alpha\neq\beta, \tag{1.3}$$

holds for a.e. $x\in\partial\Omega^t_\alpha\cap\partial\Omega^t_\beta$. Eqs. (1.2) represent the well known balance equations of mass, momentum and moment of momentum, respectively, for the system of n non intersecting deformable bodies.

2. Nonstandard definition of mixtures and porous media

Let $\mathfrak{M}$ be the full structure in which ε, V, V_f, R are disjointed relations of the type (0) (cf. [23], p. 19). The balance equations (1.2), which hold in $\mathfrak{M}$, hold also in $^*\mathfrak{M}$ as certain internal relations. It means that for every (internal) $\mathscr{B}_\alpha\in^* U$, $\alpha=1,\dots,n$, for every internal $(\chi_1,\dots,\chi_n)\in^*\mathscr{M}$, for every internal $\Delta\in^* O$ and $x_0\in\varepsilon$, as well as for every internal function of the form (1.1) (with domain $^*[t_0,t_1]$ in $^*R$ and with the values in $^*R$ and $^*V_f$, respectively), relations (1.2) and (1.3) hold in $^*\mathfrak{M}$.

Define in $^*\mathfrak{M}$

$$m_\alpha(x,r)\equiv\int\limits_{B(x,r)\cap\Omega^t_\alpha}\varrho_\alpha(x,t)\,dv,$$

where $\mathscr{B}(x,r)$ stands for a ball with a center x and the radius r, $r\in^* R_+$.

Let $\mathscr{M}^0$ be a subset of $^*\mathscr{M}$ which satisfies the following conditions:

1° For every $(\chi_1,\dots,\chi_n)\in\mathscr{M}^0$ there is $^\circ(\chi_\alpha)=\chi$ for $\alpha=1,\dots,n$ where χ is deformation function in $\mathfrak{M}$, i.e., $\chi:\Omega\times[t_0,t_1]\to\varepsilon$[2]. Hence we see that there are regions Ω^t in ε, such that

$$^\circ(\Omega^t_\alpha)=\overline{\Omega}^t \quad \text{for } \alpha=1,\dots,n \quad \text{and} \quad t\in[t_0,t_1]. \tag{2.1}$$

[2] Symbols $^\circ(\cdot)$ stands for a standard part of function or set, [23], p. 115.

We also assume that all points belonging to each Ω_α^t are near standard and Ω^t are regular regions in ε.

2° For every $x \in S-\text{int}^*\Omega^t$ [3], the internal sequences

$$\frac{\text{vol}\,[B(x, r_m)\cap\Omega_\alpha^t]}{\text{vol}\,B(x, r_m)}, \qquad \frac{m_\alpha(x, r_m)}{\text{vol}\,[B(x, r_m)\cap\Omega_\alpha^t]}, \tag{2.2}$$

$$r_m = \frac{r_1}{m}, \qquad {}^\circ(r_1) > 0, \qquad m = 1, 2, \ldots,$$

have F-limits (cf. [23], p. 109). By virtue of the theorem which can be found in [23], p. 110, there exists $\lambda_0 \in {}^*N\setminus N$ such that for every $\nu \in {}^*N\setminus N$ and $\nu < \lambda_0$ each value

$$\frac{\text{vol}[B(x, r_\nu)\cap\Omega_\alpha^t]}{\text{vol}\,B(x, r_\nu)}, \qquad \frac{m_\alpha(x, r_\nu)}{\text{vol}[B(x, r_\nu)\cap\Omega_\alpha^t]}, \tag{2.3}$$

is the F-limit of any sequence (2.2).

3° There exist functions $\Omega^t \ni x \to \nu_\alpha(x, t) \in [0, 1]$, $\Omega^t \ni x \to \tilde{\varrho}_\alpha(x, t) \in R_+$, continuous a.e. on Ω^t for $t \in [t_0, t_1]$ (where $\tilde{\varrho}_\alpha(\cdot, t)$ is also differentable) which are the standard parts of functions

$$S-\text{int}^*\Omega^t \ni x \to \frac{\text{vol}[B(x, r_\nu)\cap\Omega_\alpha^t]}{\text{vol}B(x, r_\nu)} \in^* [0, 1],$$

$$S-\text{int}^*\Omega^t \ni x \to \frac{m_\alpha(x, r_\nu)}{\text{vol}[B(x, r_\nu)\cap\Omega_\alpha^t]} \in^* R_+.$$

Now we formulate the following.

Definition. The system of bodies $\mathscr{B}_\alpha \in {}^*U$, $\alpha = 1, \ldots, n$, for which $(\chi_1, \ldots, \chi_n) \in \mathscr{M}^0$, will be called the mixture. Every $\mathscr{B}_\alpha$ is said to be the component of the mixture. The value $\nu_\alpha(x, t)$ will be called the saturation of the mixture by the α-th component.

Define function

$$\Omega^t \ni x \to \nu(x, t) \in [0, 1],$$

putting

$$\nu(x, t) \equiv 1 - \sum_{\alpha=1}^{n} \nu_\alpha(x, t).$$

If $\nu(x, t)$ is not identically equal to zero and $n = 1$, then the mixture is called the porous medium. Function $\nu(\cdot, t)$ is known as a porosity, and has been derived here by the non-standard approach from the real porous structure of the body. In the traditional approaches, porosity $\nu(\cdot, t)$ and saturation $\nu_\alpha(\cdot, t)$ are postulated a priori.

3. General form of the balance equation in $\mathfrak{M}$ and $^*\mathfrak{M}$

All balance equations (1.2) can be written down in what is called the general form of the balance equation

[3] Symbol S-int, where $A \subset {}^*R^n$ stands for S-interior of A, cf. [23], p. 107. Moreover, for every A in $\mathfrak{M}$, by $^*A$ we define the corresponding standard entity in $\mathfrak{M}^*$.

$$\frac{D}{Dt}\int_{\Delta\cap\Omega_\alpha^t}\Psi_\alpha(x,t)\varrho_\alpha(x,t)dv=\sum_{\gamma=0}^{n}\int_{\Gamma_{\alpha\gamma}^t(\Delta,\Delta^e)}G_{\alpha\gamma}(x,t)da+$$
$$+\sum_{\gamma=0}^{n}\int_{\Gamma_{\alpha\gamma}^t(\Delta,\Delta)}G_{\alpha\gamma}(x,t)da+\int_{\partial\Delta\cap\Omega_\alpha^t}\Phi_\alpha(x,t)n_{\partial\Delta}(x)da+ \tag{3.1}$$
$$+\int_{\Delta\cap\Omega_\alpha^t}B_\alpha(x,t)\varrho_\alpha(x,t)dv,\qquad \alpha=1,\dots,n,$$

where $G_{\alpha\beta}(\cdot)\equiv 0$ for $\alpha=\beta$ and where

$$G_{\alpha\beta}(x,t)=\Phi_\alpha(x,t)n_{\partial\Omega_\alpha^t}(x)=-\Phi_\beta(x,t)n_{\partial\Omega_\beta^t}(x)+F_{\alpha\beta}(x,t);\qquad \alpha\neq\beta, \tag{3.2}$$

here $F_{\alpha\beta}(x,t)=-F_{\beta\alpha}(x,t)$ hold for a.e. $x\in\Gamma_{\alpha\beta}^t(\Delta,\Delta^e)\subset\partial\Delta$ and $t\in[t_0,t_1]$. Eqs. (3.1) have to be satisfied by an arbitrary $\mathscr{B}_\alpha\in U$, $\alpha=1,\dots,n$, by every $(\chi_1,\dots,\chi_n)\in\mathscr{M}$ and by every $\Delta\in O$. In Eqs. (3.1), scalar field $\varrho_\alpha^t(\cdot,t)$ has the same meaning as before, $\Psi_\alpha(\cdot,t)$, $B_\alpha(\cdot,t)$ are tensor fields of the k-th order, defined a.e. on Ω_α^t. Moreover, $G_{\alpha\gamma}(\cdot,t)$ are tensor fields of the k-th order defined a.e. on $\partial\Omega_\alpha^t\cap\partial\Omega_\beta^t$ and $\Phi_\alpha(\cdot,t)$ are tensor fields of the $k+1$-th order, defined a.e. on Ω_α^t. Here $\Phi_\alpha(\cdot,t)$ is the flux field, $B_\alpha(\cdot,t)$ is the internal and $G_{\alpha\gamma}(\cdot,t)$ is the external supply in the α-th component. In what follows instead of (1.2), (1.3) we shall deal with the general form of the balance equation (3.1) and with the continuity conditions (3.2).

On passing to enlargement $^*\mathfrak{M}$ of $\mathfrak{M}$ we shall take the internal conditions (3.1) and (3.2) as the basis of the analysis.

4. From micro- to macro- general balance equations

The general balance equation in $^*\mathfrak{M}$ given by (3.1), (3.2), under assumptions that $(\chi_1,\dots,\chi_n)\in\mathscr{M}^0\subset{}^*\mathscr{M}$, will be called the general micro-balance equation for mixtures and porous media. The forementioned micro-balance equation constitutes only the starting point for further considerations, being the basis for obtaining in $\mathfrak{M}$ what will be called the the general macro-balance equation. In order to pass from the micro-balance equation in $^*\mathfrak{M}$ to the macro-balance equation in $\mathfrak{M}$, a number of the extra assumptions has to be introduced. It must be emphasized that the macro-balance equations exist only for rather special kinds of structure of mixtures and porous media (which in the nonstandard sense were defined in Sec. 2).

Let us substitute to (3.1): $\Delta=B(z,r_\nu)$, $z\in S-\text{int}^*\Omega^t$, $\nu<\lambda_0$, $\nu\in{}^*N\setminus N$. Then for a.e. $z\in S-\text{int}^*\Omega^t$, $t\in{}^*[t_0,t_1]$, from (3.1) we obtain

$$\frac{D}{Dt}\frac{1}{\text{vol}B(z,r_\nu)}\int_{B(z,r_\nu)\cap\Omega_\alpha^t}\Psi_\alpha(x,t)\varrho_\alpha(x,t)dv=$$
$$=\sum_{\gamma=0}^{n}\frac{1}{\text{vol}B(z,r_\nu)}\int_{\Gamma_{\alpha\gamma}^t(B(z,r_\nu),B^e(z,r_\nu))}G_{\alpha\gamma}(x,t)da+ \tag{4.1}$$

$$+\sum_{\gamma=0}^{n}\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{\Gamma^t_{\alpha\gamma}(B(z,r_\nu),\,B(z,r_\nu))} G_{\alpha\gamma}(x,t)\,da+ \tag{4.1}$$

[cont.]

$$+\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{\partial B(z,r_\nu)\cap\Omega^t_\alpha}\Phi_\alpha(x,t)n_{\partial B(z,r_\nu)}(x)\,da+$$

$$+\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}B_\alpha(x,t)\varrho_\alpha(x,t)\,dv.$$

We shall assume that all terms in (4.1) are near standard and that in $\mathfrak{M}$ there is field $\tilde{\Psi}_\alpha(\cdot,t)$,

$$\tilde{\Psi}(z,t)={}^\circ\!\left(\frac{1}{\operatorname{vol}[B(z,r_\nu)\cap\Omega^t_\alpha]}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\Psi_\alpha(x,t)\,dv\right),$$

such that the following formulas hold[4)]

$$\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\Psi_\alpha(x,t)\varrho_\alpha(x,t)\,dv\simeq\frac{{}^*\tilde{\varrho}(z,t)}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\Psi_\alpha(x,t)\,dv\simeq$$

$$\simeq{}^*\tilde{\varrho}_\alpha(z,t){}^*\nu_\alpha(z,t)\frac{1}{\operatorname{vol}[B(z,r_\nu)\cap\Omega^t_\alpha]}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\Psi_\alpha(x,t)\,dv\simeq{}^*\tilde{\varrho}(z,t){}^*\nu_\alpha(z,t){}^*\tilde{\Psi}(z,t),$$

$$\frac{D}{Dt}\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\Psi_\alpha(x,t)\varrho_\alpha(x,t)\,dv\simeq\frac{D}{Dt}\left[{}^*\tilde{\varrho}_\alpha(z,t){}^*\nu_\alpha(z,t){}^*\tilde{\Psi}_\alpha(z,t)\right].$$

Analogously, we assume that in $\mathfrak{M}$ there are fields $\tilde{B}_\alpha(\cdot,t)$ and $S_{\alpha\gamma}(\cdot,t)$ such that

$${}^*\tilde{B}_\alpha(z,t)\simeq\frac{1}{\operatorname{vol}[B(z,r_\nu)\cap\Omega^t_\alpha]}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}B_\alpha(x,t)\,dv,$$

$${}^*S_{\alpha\gamma}(z,t)\simeq\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{\Gamma^t_{\alpha\gamma}(B(z,r_\nu),\,B(z,r_\nu))}G_{\alpha\gamma}(x,t)\,da,\qquad\gamma=1,\dots,n,$$

$${}^*\tilde{\varrho}(z,t){}^*\nu_\alpha(z,t){}^*\tilde{B}_\alpha(z,t)\simeq\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}B_\alpha(x,t)\varrho_\alpha(x,t)\,dv,$$

holds in S-int$^*\Omega^t$, $t\in[t_0,t_1]$.

Let Γ be an arbitrary smooth surface in S-int$^*\Omega^t$ oriented by the unit normal $n_\Gamma(z)$, $z\in\Gamma$. Let the minimum radius r_{min} of the curvature of Γ is greater than r_ν. Define by $B^+(z,r)$, $B^-(z,r)$, $z\in\Gamma$ the semisphers of the ball $B(z,r)$, which are situated on the positively and negatively oriented side of Γ, respectively.

Define

$$\Gamma^t_{\alpha\gamma}[B^+(z,r),B^-(z,r)]\equiv\partial[B^+(z,r)\cap\Omega^t_\alpha]\cap\partial[B^-(z,r)\cap\Omega^t_\gamma],$$

---

4) We write $a\simeq b$, where a, b are finite numbers in $^*R$, if $|a-b|$ is infinitesimal

for $\alpha = 1, \ldots, n$; $\gamma = 0, \ldots, n$. We shall assume that for every $z \in \Gamma \cap (S\text{-int}^*\Omega^t)$, the internal sequences

$$\frac{\operatorname{area}\Gamma^t_{\alpha\gamma}[B^+(z, r_m), B^-(z, r_m)]}{\operatorname{area}[\Gamma \cap B(z, r_m)]},$$

$$r_m = \frac{r_1}{m}, \quad m = 1, 2, \ldots, {}^\circ(r_1) > 0$$

have F-limits. Then by virtue of [23], p. 110 there exist $\lambda_1 \in {}^*N \setminus N$ such that for every $\delta \in {}^*N \setminus N$ and $\delta < \lambda_1$, the following values

$$\frac{\operatorname{area}\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]}{\operatorname{area}[\Gamma \cap B(z, r_\delta)]}, \tag{4.2}$$

are the F-limits.

We shall assume that for every pair of surfaces $'\Gamma$, $''\Gamma$ in S-int$^*\Omega^t$, which satisfy conditions analogous to those imposed on Γ, and for every $z \in {}'\Gamma \cap {}''\Gamma$ there is

$$\frac{\operatorname{area}{}'\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]}{\operatorname{area}[{}'\Gamma \cap B(z, r_\delta)]} \simeq \frac{\operatorname{area}{}''\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]}{\operatorname{area}[{}''\Gamma \cap B(z, r_\delta)]},$$

where $'\Gamma^t_{\alpha\gamma}(\cdot)$, $''\Gamma^t_{\alpha\gamma}(\cdot)$ have the same meaning as $\Gamma^t_{\alpha\gamma}(\cdot)$.

Define the system of functions

$$\Gamma \cap (S-\operatorname{int}^*\Omega^t) \ni z \to \frac{\operatorname{area}\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]}{\operatorname{area}[\Gamma \cap B(z, r_\delta)]} \in^* [0, 1],$$

and assume that there exist functions

$$\Omega^t \in x \to \mu_{\alpha\gamma}(x, t) \in [0, 1],$$

which are standard parts of functions defined above. We also postulate that there exist functions

$$\Omega^t \ni x \to \Phi_{\alpha\beta}(x, t),$$

constituting tensor fields of the $k+1$-th order, such that

$$^*\Phi_{\alpha\gamma}(z, t) n_\Gamma(z) \simeq \frac{1}{\operatorname{area}\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]} \int\limits_{\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]} \Phi_\alpha(x, t) n_\Gamma(x) da,$$

Hence

$$^*\mu_{\alpha\gamma}(z, t)^*\Phi_{\alpha\gamma}(z, t) n_\Gamma(z) \simeq \frac{1}{\operatorname{area}[\Gamma \cap B(z, r_\delta)]} \int\limits_{\Gamma^t_{\alpha\gamma}[B^+(z, r_\delta), B^-(z, r_\delta)]} \Phi_\alpha(x, t) n_\Gamma(x) da.$$

Now assume that

$$\operatorname{div}[^*\mu_{\alpha\beta}(z, t)^*\Phi_{\alpha\beta}(z, t)] \simeq \frac{1}{\operatorname{vol} B(z, r_\nu)} \int\limits_{\partial B(z, r_\nu)} {}^*\mu_{\alpha\beta}(x, t)^*\Phi_{\alpha\beta}(x, t) n_{\partial B(z, r_\nu)}(x) da \simeq$$

$$\simeq \frac{1}{\operatorname{vol} B(z, r_\nu)} \int\limits_{\Gamma^t_{\alpha\beta}[B(z, r_\nu), B^e(z, r_\nu)]} \Phi_\alpha(x, t) n_{\partial B(z, r_\nu)}(x) da.$$

Taking into account all obtained results, we conclude that (4.1) implies that

$$\frac{D}{Dt}[\tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{\Psi}_\alpha(z,t)] = \sum_{\gamma=0}^{n}\operatorname{div}[\mu_{\alpha\gamma}(z,t)\Phi_{\alpha\gamma}(z,t)]+$$

$$+\sum_{\gamma=0}^{n}S_{\alpha\gamma}(z,t)+\tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{B}_\alpha(z,t), \tag{4.3}$$

hold for a.e. $z \in \Omega^t$ and for a.e. $t \in R$.

Let us observe, that from

$$\int\limits_{\Gamma^t_{\alpha\beta}[B^+(z,r),(B^-(z,r)]} \Phi_\alpha(x,t)n_\Gamma(x)\,da = \int\limits_{\Gamma^t_{\alpha\beta}[B^-(z,r),B^+(z,r)]} [-\Phi_\beta(x,t)n_\Gamma(x)+F_{\alpha\beta}(x,t)]\,da,$$

and under assumption that $\mu_{\alpha\beta}(x,t) = \mu_{\beta\alpha}(x,t)$ and $F_{\alpha\beta}(\cdot) \equiv 0$ we obtain $\Phi_{\alpha\beta}(x,t) = \Phi_{\beta\alpha}(x,t)$.

Thus, for every $\mathscr{P} \subset \Omega$ and every $t \in [t_0, t_1]$, there is

$$\frac{D}{Dt}\int\limits_{\chi(\mathscr{P},t)}\tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{\Psi}_\alpha(z,t)\,dv = \sum_{\gamma=0}^{n}\int\limits_{\partial\chi(\mathscr{P},t)}\mu_{\alpha\gamma}(z,t)\Phi_{\alpha\gamma}(z,t)n_{\partial\chi(\mathscr{P},t)}(z)\,da+$$

$$+\sum_{\gamma=0}^{n}\int\limits_{\chi(\mathscr{P},t)}S_{\alpha\gamma}(z,t)\,dv+\int\limits_{\chi(\mathscr{P},t)}\tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{B}_\alpha(z,t)\,dv. \tag{4.4}$$

At the same time, assuming that $\mu_{\alpha\beta}(\cdot,t) = \mu_{\beta\alpha}(\cdot,t)$ we define

$$\tilde{F}_{\alpha\beta}(z,t) \equiv \Phi_{\alpha\beta}(z,t)-\Phi_{\beta\alpha}(z,t),$$

as a field determining the effect of friction. Condition (4.4) will be called the general integral macro-balance equation for mixtures and porous media, while (4.3) be the local form of this equation.

From the foregoing consideration it follows that the general macro-balance equation in the form (4.3) or (4.4) holds under rather strong regularity conditions, which have been succesively introduced in this Section. The mixtures and porous media for which Eqs. (4.3) and Eqs. (4.4) take place, will be called the ideal mixtures and ideal porous media, respectively.

5. Macro-balance equations for ideal mixtures and porous media

From the general balance equation (4.4) we shall obtain now the macro-balance equations of mass, momentum and moment of momentum. It will be done by the specification of fields $\Psi_\alpha(\cdot,t)$, $G_{\alpha\gamma}(\cdot,t)$, $\Phi_\alpha(\cdot,t)$ and $B_\alpha(\cdot,t)$ in (3.1) and (3.2).

5.1. Mass conservation. In order to obtain the principle of mass conservation from the general balance equations (for α-th component of the mixture), we have to substitute

$$\Psi_\alpha(\cdot) \equiv 1, \quad G_{\alpha\gamma}(\cdot) \equiv 0, \quad \Phi_\alpha(\cdot) \equiv 0, \quad B_\alpha(\cdot) \equiv 0,$$

into Eq. (3.1), where $\varrho_\alpha(\cdot, t)$ stands for a mass density of the α-th component in configuration Ω^t_α. Taking into account the forementioned substitutions, we obtain fields $\Psi_\alpha(\cdot, t)$, $\Phi_{\alpha\gamma}(\cdot, t)$, $S_{\alpha\beta}(\cdot, t)$, $\tilde{B}_\alpha(\cdot, t)$, $\tilde{\varrho}_\alpha(\cdot, t)$ and $\nu_\alpha(\cdot, t)$ in $\mathfrak{M}$ where in $^*\mathfrak{M}$

$$
\begin{aligned}
{}^*\tilde{\Psi}_\alpha(z, t) &= \frac{1}{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]} \int\limits_{B(z, r_\nu)\cap\Omega^t_\alpha} \Psi_\alpha(x, t)\,dv = 1,\\
{}^*\Phi_{\alpha\gamma}(z, t)n_\Gamma(z) &= \frac{1}{\operatorname{area}\Gamma^t_{\alpha\gamma}[B^+(z, r_\nu), B^-(z, r_\nu)]} \int\limits_{\Gamma^t_{\alpha\beta}[B^+(z, r_\nu), B^-(z, r_\nu)]} \Phi_\alpha(x, t)n_\Gamma(x)\,da = 0,\\
{}^*S_{\alpha\gamma}(z, t) &= \frac{1}{\operatorname{vol}B(z, r_\nu)} \int\limits_{\Gamma^t_{\alpha\gamma}[B(z, r_\nu), B(z, r_\nu)]} G_{\alpha\gamma}(x, t)\,da = 0,\\
{}^*\tilde{B}_\alpha(z, t) &= \frac{1}{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]} \int\limits_{B(z, r_\nu)\cap\Omega^t_\alpha} B_\alpha(x, t)\,dv = 0,\\
{}^*\tilde{\varrho}_\alpha(z, t) &\simeq \frac{1}{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]} \int\limits_{B(z, r_\nu)\cap\Omega^t_\alpha} \varrho_\alpha(x, t)\,dv,\\
{}^*\nu_\alpha(z, t) &\simeq \frac{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]}{\operatorname{vol}B(z, r_\nu)},
\end{aligned}
\tag{5.1}
$$

and where $z \in S\text{-int}^*\Omega^t$, $\nu, \lambda_0 \in {}^*N\setminus N$, $\nu < \lambda_0$. Now, from (5.1) and from Eq. (4.3) we obtain

$$
\frac{D}{Dt}\left(\tilde{\varrho}_\alpha(z, t)\nu_\alpha(z, t)\right) = 0,
\tag{5.2}
$$

where $\tilde{\varrho}_\alpha(z, t)$ and $\nu_\alpha(z, t)$ satisfy the postulated regularity condition and

$$
\begin{aligned}
\tilde{\varrho}_\alpha(z, t) &= {}^\circ\!\left(\frac{1}{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]} \int\limits_{B(z, r_\nu\cap\Omega^t_\alpha} \varrho_\alpha(x, t)\,dv\right),\\
\nu_\alpha(z, t) &= {}^\circ\!\left(\frac{\operatorname{vol}[B(z, r_\nu)\cap\Omega^t_\alpha]}{\operatorname{vol}B(z, r_\nu)}\right),
\end{aligned}
\tag{5.3}
$$

hold for every $z \in S-\operatorname{int}^*\Omega^t$. Moreover if $\mathscr{P}$ is an arbitrary regular subregion of Ω, then

$$
\frac{D}{Dt}\int\limits_{\chi(\mathscr{P}, t)} \tilde{\varrho}_\alpha(z, t)\nu_\alpha(z, t)\,dv = 0,
\tag{5.4}
$$

holds. The resulting equations (5.2), (5.4) represent macro-mass balance equations in the local and the integral form, respectively. Fields $\tilde{\varrho}^t_\alpha(\cdot, t)$, $\nu_\alpha(\cdot, t)$ are not postulated a priori (which takes place in the known approaches to mechanics of porous media) but are given by formulas (5.3).

5.2. Conservation of momentum. In this case we have to assume that in the general balance equation (3.1)

$$
\begin{aligned}
\Psi_\alpha(\cdot, t) &\equiv v_\alpha(\cdot, t),\\
G_{\alpha\gamma}(\cdot, t) &\equiv t_{\alpha\gamma}(\cdot, t),
\end{aligned}
$$

$$\Phi_\alpha(\cdot, t) \equiv T_\alpha(\cdot, t),$$
$$B_\alpha(\cdot, t) \equiv b_\alpha(\cdot, t).$$

By virtue of the assumptions formulated in Sec. 4 we have

$$*\tilde{v}_\alpha(z, t) \simeq \frac{1}{\mathrm{vol}[B(z, r_\nu)\cap\Omega_\alpha^t]} \int\limits_{B(z, r_\nu)\cap\Omega_\alpha^t} v_\alpha(x, t)\,dv,$$

$$*s_{\alpha\gamma}(z, t) \simeq \frac{1}{\mathrm{vol}B(z, r_\nu)} \int\limits_{\Gamma_{\alpha\gamma}^t[B(z, r_\nu), B(z, r_\nu)]} t_{\alpha\gamma}(x, t)\,da,$$

$$*T_{\alpha\gamma}(z, t) \simeq \frac{1}{\mathrm{area}\Gamma_{\alpha\gamma}^t[B^+(z, r_\nu), B^-(z, r_\nu)]} \int\limits_{\Gamma_{\alpha\gamma}^t[B^+(z, r_\nu), B^-(z, r_\nu)]} T_\alpha(x, t)n_\Gamma(x)\,da, \tag{5.5}$$

$$*\tilde{b}_\alpha(z, t) \simeq \frac{1}{\mathrm{vol}[B(z, r_\nu)\cap\Omega_\alpha^t]} \int\limits_{B(z, r_\nu)\cap\Omega_\alpha^t} b_\alpha(x, t)\,dv,$$

and

$$\frac{D}{Dt}\frac{1}{\mathrm{vol}B(z, r_\nu)} \int\limits_{B(z, r_\nu)\cap\Omega_\alpha^t} v_\alpha(x, t)\varrho_\alpha(x, t)\,dv \simeq$$

$$\simeq \frac{D}{Dt}[*\tilde{\varrho}_\alpha(z, t)*\nu_\alpha(z, t)*\tilde{v}_\alpha(z, t)],$$

for $z \in S-\mathrm{int}{*\Omega^t}$, $\lambda_0, \nu \in {*N}\setminus N$, $\nu < \lambda_0$. Using the procedure analogous to that applied in Sec. 4, instead of (4.3) we obtain

$$\frac{D}{Dt}[\tilde{\varrho}_\alpha(z, t)\nu_\alpha(z, t)\tilde{v}_\alpha(z, t)] = \sum_{\gamma=0}^{n} \mathrm{div}[\mu_{\alpha\gamma}(z, t)T_{\alpha\gamma}(z, t)]+$$

$$+\sum_{\gamma=0}^{n} s_{\alpha\gamma}(z, t)+\tilde{\varrho}_\alpha(z, t)\nu_\alpha(z, t)\tilde{b}_\alpha(z, t). \tag{5.6}$$

The meaning of $\mu_{\alpha\gamma}(\cdot, t)$ has been explained in Sec. 4. Here

$$\mathrm{div}[*\mu_{\alpha\gamma}(z, t)*T_{\alpha\gamma}(z, t)] \simeq \frac{1}{\mathrm{vol}B(z, r_\nu)} \int\limits_{\Gamma_{\alpha\gamma}^t[B^+(z, r_\nu), (B^-(z, r_\nu)]} t_{\alpha\gamma}(x, t)n_{\partial B}(x)\,da.$$

If $\mu_{\alpha 1}(z, t) = \ldots = \mu_{\alpha n}(z, t)$ then the following equalities hold

$$\sum_{\gamma=0}^{n} \mathrm{div}[\mu_{\alpha\gamma}(z, t)T_{\alpha\gamma}(z, t)] = \mathrm{div}\Big[\mu_{\alpha\gamma}(z, t)\sum_{\gamma=0}^{n} T_{\alpha\gamma}(z, t)\Big].$$

Putting

$$\bar{T}_\alpha(z, t) \equiv \sum_{\gamma=0}^{n} T_{\alpha\gamma}(z, t),$$

we shall refer $\bar{T}_\alpha(\cdot, t)$ to as a partial stress tensor related to the α-th component of the mixture. At the same time, for an arbitrary regular subregion by virtue of (4.4) we obtain

$$\frac{D}{Dt}\int\limits_{\chi(\mathscr{P},t)} \tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{v}_\alpha(z,t)\,dv =$$

$$= \sum_{\gamma=0}^{n}\int\limits_{\chi(\mathscr{P},t)} \mu_{\alpha\gamma}(z,t)T_{\alpha\gamma}(z,t)\,da + \sum_{\gamma=0}^{n}\int\limits_{\chi(\mathscr{P},t)} s_{\alpha\gamma}(z,t)\,dv + \tag{5.7}$$

$$+ \int\limits_{\chi(\mathscr{P},t)} \tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\tilde{b}_\alpha(z,t)\,dv.$$

Eqs. (5.6), (5.7) represent the local and global form, respectively, of the macro-balance equation of momentum. The macro-fields occuring in the forementioned equations have been not postulated a priori but are related to the micro-structure of the body by means of Eqs. (5.5).

5.3. Conservation of moment of momentum. Applying the procedure analogous to that of Sec. 5.1, 5.2, we assume now that

$$\begin{aligned}
\Psi_\alpha(x,t) &\equiv (x-x_0)\times v_\alpha(x,t),\\
G_{\alpha\gamma}(x,t) &\equiv (x-x_0)\times t_{\alpha\gamma}(x,t),\\
\Phi_\alpha(x,t)n_{\partial\Delta}(x) &\equiv (x-x_0)\times T_\alpha(x,t)n_{\partial\Delta}(x),\\
B_\alpha(x,t) &\equiv (x-x_0)\times b_\alpha(x,t).
\end{aligned}$$

Hence in $\mathfrak{M}$ there are fields $\tilde{v}_\alpha(\cdot,t)$, $s_{\alpha\gamma}(\cdot,t)$, $T_{\alpha\gamma}(\cdot,t)$ such that in $^*\mathfrak{M}$ the following relations hold

$$\begin{gathered}
(z-z_0)\times{}^*\tilde{v}(z,t) \cong \frac{1}{\operatorname{vol}[B(z,r_\nu)\cap\Omega^t_\alpha]}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}(x-x_0)\times v_\alpha(x,t)\,dv,\\
(z-x_0)\times{}^*s_{\alpha\gamma}(z,t) \simeq \frac{1}{\operatorname{vol}B(z,r_\gamma)}\int\limits_{\Gamma^t_{\alpha\gamma}[B(z,r_\nu),B(z,r_\nu)]}(x-x_0)\times t_{\alpha\gamma}(x,t)\,da,\\
(z-x_0)\times{}^*T_{\alpha\gamma}(z,t)n_\Gamma(z) \simeq \frac{1}{\operatorname{area}[B(z,r_\nu)\cap\Gamma]}\int\limits_{\Gamma^t_{\alpha\gamma}[B^+(z,r_\nu),B^-(z,r_\nu)]}(x-x_0)\times T_\alpha(x,t)n_\Gamma(x)\,da,\\
(z-x_0)\times{}^*\tilde{b}_\alpha(z,t) \approx \frac{1}{\operatorname{vol}[B(z,r_\nu)\cap\Omega^t_\alpha]}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}(x-x_0)\times b_\alpha(x,t)\,dv,
\end{gathered}\tag{5.8}$$

and where

$$\frac{D}{Dt}\frac{1}{\operatorname{vol}B(z,r_\nu)}\int\limits_{B(z,r_\nu)\cap\Omega^t_\alpha}\varrho_\alpha(x,t)(x-x_0)\times v_\alpha(x,t)\,dv \simeq$$

$$\simeq \frac{D}{Dt}[{}^*\tilde{\varrho}_\alpha(z,t){}^*\nu_\alpha(z,t)(z-x_0)\times{}^*\tilde{v}_\alpha(z,t)].$$

Applying the forementioned relations we arrived

$$\frac{D}{Dt}[\tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)(z-x_0)\times\tilde{v}_\alpha(z,t)] =$$
$$= \sum_{\gamma=0}^{n}\operatorname{div}[\mu_{\alpha\gamma}(z,t)(z-x_0)\times T_{\alpha\gamma}(z,t)]+ \tag{5.9}$$
$$+\sum_{=0}^{n}(z-x_0)\times s_{\alpha\gamma}(z,t)+\nu_\alpha(z,t)(z-x_0)\times\tilde{b}_\alpha(z,t).$$

Taking the time derivation of Eq. (5.9) and bearing in mind (5.6), we obtain

$$\mu_{\alpha\gamma}(z,t)\operatorname{div}z\times T_{\alpha\gamma}(z,t) = \tilde{\varrho}_\alpha(z,t)\nu_\alpha(z,t)\frac{Dz}{Dt}\times\tilde{v}_\alpha(z,t) = 0,$$

under the extra assumption $\tilde{v}_1(\cdot,t) = \ldots = \tilde{v}_n(\cdot,t) = \tilde{v}(\cdot,t)$ where

$$\frac{Dz}{Dt}(X,t) = \tilde{v}(\chi(X,t),t), \qquad X\in\Omega.$$

Hence, after simple calculation, we arrive at

$$\sum_{\gamma=0}^{n}\mu_{\alpha\gamma}(z,t)T_{\alpha\gamma}(z,t) = \Big[\sum_{\gamma=0}^{n}\mu_{\alpha\gamma}(z,t)T_{\alpha\gamma}(z,t)\Big]^T. \tag{5.10}$$

Defining

$$\tilde{T}_\alpha(z,t) \equiv \sum_{\gamma=0}^{n}\mu_{\alpha\gamma}(z,t)T_{\alpha\gamma}(z,t),$$

as a total stress tensor of the α-th component of the mixture, we obtain here the symmetry condition of this tensor.

The resulting equation (5.10) represents the macro-balance law of the moment of momentum in its local form.

Final remarks

The main feature of the resulting macro-balance equations is that they are not postulated a priori but are derivied from the balance equations for system of unintersecting and coacting deformable bodies, i.e., from Eqs. (1.2). Such procedure has been realized here by applying the methods of the nonstandard analysis. On this way we are able to give the exact fenomenological definitions of mixtures, ideal mixtures, porous media, and ideal porous media. The approach used in the paper assignes to every term in the resulting macro-balance equations its physical interpretation in terms occuring in Eqs. (1.2), which have the clear physical meaning. For the particulars and the further analysis of the obtained results, the reader is referred to [27].

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Резюме

УРАВНЕНИЯ БАЛАНСА ДЛЯ СМЕСЕЙ И ПОРИСТИХ ТЕЛ
В ВИДУ НЕСТАНДАРТНОГО АНАЛИЗА

Цел настоящего сообщения — это получить, методами нестандартного анализа, уравнения баланса для смесей и пористих тел из известных уравнений баланса механики сплошных сред. Такий подход дает возможность ясной интерпретации всех полей в полученых уравнениях баланса.

Streszczenie

RÓWNANIA BILANSU DLA MIESZANIN I CIAŁ POROWATYCH
W ŚWIETLE ANALIZY NIESTANDARDOWEJ

Celem pracy jest otrzymanie, za pomocą metod analizy niestandardowej, równań bilansu dla mieszanin i ciał porowatych wprost ze znanych równań bilansu mechaniki kontinuum. Podejście takie umożliwia jasną interpretację wszystkich pól w otrzymanych równaniach bilansu.

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