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UBIQUITIFORM IN APPLIED MECHANICS

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We demonstrate that a physical object in nature should not be described as a fractal, despite an ideal mathematical object, rather a ubiquitiform (a terminology coined here for a finite order self-similar or self-affine structure). It is shown mathematically that a ubiquitiform must be of integral dimension, and that the Hausdorff dimension of the initial element of a fractal changes abruptly at the point at infinity, which results in divergence of the integral dimensional measure of the fractal and makes the fractal approximation to a ubiquitiform unreasonable. Therefore, instead of the existing fractal theory in applied mechanics, a new type of ubiquitiformal one is needed.

Key words: ubiquitiform, fractal, Hausdorff dimension

1. Introduction

A fractal (Mandelbrot, 1967, 1977, 1982) or, generally, a self-similar (or self-affine) structure of infinite order, has been extensively used as a nonlinear mathematical tool to describe various irregular and complex phenomena in many fields of science and technology such as fracture mechanics (Mandelbrot *et al.*, 1984; Saouma and Barton, 1994; Carpinteri, 1994; Borodich, 1997, 1999; Carpinteri and Paggi, 2010), quantum mechanics (Argyris *et al.*, 2000), material science (Fleury, 1997; Ma *et al.*, 2009), nonlinear dynamics and chaos (Aguirre *et al.*, 2009), biology (Gözen *et al.*, 2010) and even city planning (Batty, 2008). In fracture mechanics, for example, since Mandelbrot's pioneering work (1984) in which the fractality of an impacting fracture surface in metal was found, fractal fracture mechanics has been developed quickly in several aspects. For example, on the one hand, the fractality of fracture surfaces in various kinds of materials such as steel (Mandelbrot *et al.*, 1984), concrete (Saouma *et al.*, 1990; Saouma and Barton, 1994), ceramic (Mecholsky *et al.*, 1989) and rock (Krohn and Thompson, 1986) was confirmed by experimental methods. On the other hand, it was found that the distribution of micro-cracks and micro-damage could be described by fractals (Barenblatt and Botvina, 1986; Chudnovsky and Wu, 1992; Barenblatt, 1993). Moreover, the fractal geometry was also used to describe the mechanical behavior of materials such as the size effect on material strength (Carpinteri, 1994; Carpinteri and Puzzi, 2008; Carpinteri and Paggi, 2010) and crack propagation (Xie and Sanderson, 1995), and, at the same time, the fracture problem in regions with fractal boundaries also became a research focus (Panagiotopoilos *et al.*, 1995; Wnuk and Yavari, 2005).

With the development of fractal application, some inherent difficulties appear gradually because that a fractal is just an ideal mathematical object with a self-similar (or self-affine) structure of infinite hierarchy. For example, in fractal fracture mechanics, some authors have endeavored to link the fractal dimension of the fracture surface to fracture toughness, but a general conclusion cannot be drawn easily. Actually, all three kinds of the correlation, namely, the negative correlations (Mandelbrot *et al.*, 1984; Saouma and Barton, 1994), the positive correlations (Mecholsky *et al.*, 1989; Ray and Mandal, 1992) as well as no correlation (Pande *et al.*, 1987) can be found. It is not hard to imagine that such difficulties should be resulted from the uncertainty in describing a fractal. That is, a fractal could not be uniquely determined just by its fractal dimension. In fact, as is well known, besides the dimension, in general, a finite measure is another significant parameter in describing a geometric object, such as that all the line segments are of one-dimension, but they can have different length or measure and hence two line segments with different lengths are always not equivalent in practice. However, it is difficult to calculate the measure of a fractal, let alone that some fractals even have no finite measure at all. Moreover, the integral dimensional measure of a fractal is usually divergent or singular, namely, the integral dimensional measure of a fractal is either infinite or zero. In addition, traditional physical parameters are always defined in integral dimensional measure, but the direct extension of these parameters in fractal application seems not available and may cause unreasonable results. For example, the fracture energy is defined as the work done by the external force to generate a unit fracture surface (smooth or in integral dimension of $D = 2$), but for a fractal fracture surface with the fractal dimension $D > 2$, the corresponding area is infinite because of the divergence of the integral dimensional measure of the fractal surface. This implies that the infinite amount of fracture energy will be needed for the fracture of a material, which is certainly not the case. To accommodate the integral dimensional divergence of a fractal, new density kinds of fractal parameters defined on the unit fractal measure had to be introduced, such as the specific energy-absorbing capacity of unit fractal measure (Borodich, 1997, 1999), the fractal tensile strength and the critical fractal strain (Carpinteri, 1994; Carpinteri *et al.*, 2002), but these fractal parameters are both difficult to be determined in practice and seem lacking of unambiguous physical meanings (Bažant and Yavari, 2005). Moreover, it should also be mentioned that, in general, the boundary-value problem in a region with fractal boundaries is difficult to be constructed because the normal direction of the fractal boundary cannot be defined due to that a mathematical fractal curve can be continuous everywhere but differentiable nowhere. Although some homogenization methods were used by previous researchers (Davey and Alonso Rasgado, 2011; Davey and Prosser, 2013), but these methods are usually approximate and lacking of theoretical foundation.

In fact, from a mathematical point of view, a fractal is an infinite set with a self-similar (or self-affine) structure of infinite hierarchy, which can usually be constructed by an infinite iterative procedure. On the contrary, however, a physical object in nature can never experience such an infinite iterative process but only a finite one, and thus has a lower bound of the scale (such as the scale of the elementary particle or the time limit of a physical process). Two questions arise then: Is a physical object in nature really a fractal? If not, is it reasonable or available to describe a natural object approximately by a fractal? It was believed that there is not any real fractal in nature (Mandelbrot, 1977; Falconer, 2003), but a physical object in nature could be described as or approximated by a fractal, just as presented by Mandelbrot (1977), "Intuitively as well as pragmatically (from the point of view of the simplicity and naturalness of the corrective terms required), it is reasonable to consider a very close approximation to the Koch curve as closer to a curve of dimension $\log 4/\log 3$ than to a curve of dimension 1", However, as will be shown later, it is not the case but antipodal. That is to say, such a close approximation to the Koch curve with a finite iterative procedure must be a curve of integral dimension 1 rather than a curve of fractional dimension $\log 4/\log 3$. Actually, it has been noticed that a physical object in nature could be described as or approximated by a fractal only in a finite range of scaling invariance in all fractal applications (Mandelbrot, 1982; Cherepanov *et al.*, 1995; Balankin, 1997; Borodich, 1999; Addison, 2000; Martinez-Lopez *et al.*, 2001). Moreover, we will show that the fractal approximation of a physical object is unreasonable especially in the sense of measure for the divergence of the integral dimensional measure or the discontinuity of the Hausdorff dimension of a fractal.

In order to avoid the weakness of fractals used in applied mechanics as mentioned above, a new concept of ubiquitiform, a new word (with its adjective form "ubiquitiformal") coined here from the abbreviated word combination of "ubiquitous" and "form" to imply the ubiquitous physical forms, is introduced in this paper. A ubiquitiform is defined as a finite order self-similar (or self-affine) physical configuration constructed usually by a finite iterative procedure, and thus it has the same Hausdorff dimension of the initial element which is always integral in practice. Moreover, a ubiquitiform is also different with a "prefractal" which was used by some previous authors for "a fractal in a finite range of scaling invariance" (Borodich, 1997; Addison, 2000). Although a prefractal is also produced by a finite iterative procedure, but by definition, it is also regarded as a fractal such as it has the same fractal dimension as that of the corresponding fractal, etc. However, a ubiquitiformal curve is just one-dimensional. Additionally, for the sake of simplicity, just the self-similar case is considered in the following.

2. Ubiquitiform or fractal?

Consider in the two-dimensional Euclidean space a fractal curve with the length of the initial element l and fractal dimension D $(1 < D < 2)$. Noticing that the length of the corresponding polygonal line is

$$L(\delta) = l^D \delta^{1-D} \tag{2.1}$$

where δ is the length of the "yardstick" used for the measurement, the length or the one-dimensional measure of the fractal curve can be obtained by an infinite limiting process of $\delta \to 0$ from Eq. (2.1) as

$$L = \lim_{\delta \to 0} L(\delta) = \infty \tag{2.2}$$

This just implies the divergence of one-dimensional measure for a fractal curve, which is one of the basic characteristics of a mathematical fractal. However, differently from a mathematical fractal, any real physical object cannot have detail on arbitrarily small scales, for some physical conditions such as the scale of the elementary particle or the time limit of a physical process must be taken into account and thus the corresponding physical process cannot be infinite either. In other words, the infinite limiting process of $\delta \to 0$ is impossible, instead of which there must be a finite process of $\delta \to \delta_{min}$, where δ_{min} is the lower bound to scale invariance. We called such a configuration constructed by a finite self-similar iterative procedure as a ubiquitiform. The length or one-dimensional measure of the ubiquitiformal curve L_{ubif} is thus determined directly from Eq. (2.1), as

$$L_{ubif} = l^D \delta_{min}^{1-D} < \infty \tag{2.3}$$

which shows that a ubiquitiformal curve has a finite one-dimensional measure, and is essentially different from that of the corresponding fractal. This indicates that a ubiquitiformal curve is one-dimensional while the Hausdorff dimension of a fractal curve is always not equal to one. On the other hand, it can be seen that for a certain ubiquitiform, there is always a fractal generated by the corresponding infinite limiting process. Hereinafter, this fractal is termed as "the associated fractal" of the ubiquitiform, and its fractal dimension will be called "complexity" of the ubiquitiform to indicate its non-fractal characteristic.

Moreover, from the viewpoint of the Hausdorff measure and the Hausdorff dimension, it can also be shown by induction that a ubiquitiform which is constructed from an initial element whose Hausdorff dimension is integer by a finite self-similar iterative procedure must be of integral dimension. In fact, the Hausdorff measure $H^{(d)}$ of a certain set F is defined as (Falconer, 2003)

$$H^{(d)}(F) = \lim_{\delta \to 0} H_\delta^{(d)}(F) = \lim_{\delta \to 0} \inf \sum_{i=1}^{\infty} |U_i|^d = \begin{cases} \infty & 0 \leqslant d < D \\ 0 & d > D \end{cases} \tag{2.4}$$

where $\{U_i\}$ and D are the δ-cover and the Hausdorff dimension of F, respectively, and $|\bullet|$ denotes a certain Euclidean metric. As is well known, a self-similar structure can usually be expressed as a union of finite or infinite (corresponding to a ubiquitiform and fractal, respectively) number of subsets resulted from a series of repetitious contractions starting from an initial element, say F_0 of integral dimension d, with the finite d-dimensional Hausdorff measure, namely

$$H^{(d)}(F_0) = \lim_{\delta\to 0} \inf \sum_{i=1}^{\infty} |U_i|^d < \infty \tag{2.5}$$

where $\{U_i\}$ is the δ-cover of F_0. Without loss of generality, assume that the ubiquitiform F_n is generated by n steps of contractions and can be mapped into the corresponding subsets F_{n+1}^k $(k = 1, \ldots, m)$ of the ubiquitiform F_{n+1} generated by $n + 1$ steps of contractions with the scaling $\lambda_k < 1$ $(k = 1, \ldots, m)$, where m is the number of contractions in the iterated function system (IFS). According to induction, supposing that F_n is of dimension d, i.e.

$$H^{(d)}(F_n) = \lim_{\delta\to 0} \inf \sum_{i=1}^{\infty} |V_i|^d < \infty \tag{2.6}$$

where $\{V_i\}$ is the δ-cover of F_n, and if $\delta_k = \lambda_k\delta$, then $\{\lambda_k V_i\}$ will be the δ-cover of F_{n+1}^k $(k = 1, \ldots, m)$. According to the scaling properties of the Hausdorff measure (Falconer, 2003), one can obtain

$$H^{(d)}(F_{n+1}^k) = \lambda_k^d H^{(d)}(F_n) \leqslant \lambda_{k\,max}^d H^{(d)}(F_n) \tag{2.7}$$

where $\lambda_{k\,max}$ is the maximum of λ_k $(k = 1, \ldots, m)$. For

$$F_{n+1} = \bigcup_{k=1}^{m} F_{n+1}^k \tag{2.8}$$

there is

$$H^{(d)}(F_{n+1}) = H^{(d)}\Big(\bigcup_{k=1}^{m} F_{n+1}^k\Big) \leqslant \sum_{k=1}^{m} H^{(d)}(F_{n+1}^k) \leqslant m\lambda_{k\,max}^d H^{(d)}(F_n) \tag{2.9}$$

Considering that $m\lambda_{k\,max}^d$ is a finite value, form Eqs. (2.6) and (2.9), one obtains

$$H^{(d)}(F_{n+1}) < \infty \tag{2.10}$$

which means that F_{n+1} has the finite d-dimensional Hausdorff measure. This implies that the Hausdorff dimension of F_{n+1} is d by considering the uniqueness of the Hausdorff dimension. Therefore, a given ubiquitiform must be of integral dimension.

The above proof has also revealed an important fact that the Hausdorff dimension is discontinuous at the point at infinity of $\delta \to 0$ in the iterative procedure of a fractal, or, in other words, the Hausdorff dimension of a fractal changes abruptly in the point of $\delta = 0$, and the corresponding configuration changes from a ubiquitiform with an integral dimension to a fractal with a fractal dimension. The Hausdorff dimension remains all through an integer for $\delta > 0$ and hence the configuration is a ubiquitiform, but it jumps to a fractional value just when $\delta \to 0$, and the configuration becomes a fractal. Thus, it can be easily understood that a fractal with an infinite iteration process may have a fractional Hausdorff dimension, but "a fractal in a finite range of scaling invariance" must be of an integral Hausdorff dimension, which can then be described only by a ubiquitiform. Taking the Koch curve as an example (see Fig. 1), its Hausdorff dimension remains to be 1 for arbitrary finite times of iteration k, which can be called as a

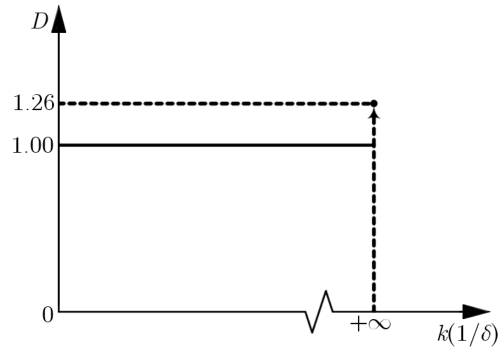

Fig. 1. Hausdorff dimension of the ubiquitiformal and the fractal Koch curve

ubiquitiformal Koch curve, and jumps from 1 to 1.26 at the infinite point of $k \to \infty$, which is rigorously the fractal Koch curve.

As mentioned above, a physical object in nature is a ubiquitiform. In such a case, is it reasonable to describe a ubiquitiform as a fractal? We will show that such an approximation is unavailable in the sense of the Hausdorff measure (in most cases of fractal application). Comparison Eq. (2.2) with Eq. (2.3) shows that the difference in the one-dimensional measure between a ubiquitiformal and its associated fractal curve is infinitely large, which makes the fractal approximation to a ubiquitiform unreasonable. The basic reason is that the Hausdorff dimension of a ubiquitiform is different from that of its associated fractal. Moreover, it is worthy of special mention that Panagiotopoulos *et al.* (1993) and Falconer (2003) have argued that one can obtain very good approximations of a fractal by an iterated function system constructed from a given set in the sense of the Hausdorff distance. However, on the one hand, such an iterative process is after all finite and then could not realize the approximation of the Hausdorff dimension because of the discontinuity of the Hausdorff dimension at the infinite point with respect to the Hausdorff distance, or, in other words, the approximation in the sense of the Hausdorff distance does not imply that in the Hausdorff dimension. On the other hand, it must be noticed that a physical object in nature is the objective reality, and, from the viewpoint of scientific research, what we need is to use the fractal as an available mathematical tool to describe such objectivity, but not vice versa. Unfortunately, although there may exist some set sequences approximating to a certain fractal within a satisfactory precision, the inverse is not true. That is, a given physical object in nature can never be well approximated by any fractal especially in the sense of measure. It is also worthy of mention that some similar approximations seem existing in practice, such as the mass density in continuum mechanics, which is defined mathematically in the limit case of the mass-to-volume ratio $\Delta M/\Delta V$ when the small volume ΔV shrinks to zero, but physically, ΔV is still kept above some finite value $\varepsilon > 0$ larger compared with the mean free path of the molecules. Such an ideal of "macro-infinitesimal and micro-infinite" is the foundation of continuum mechanics. It must be noticed that the mass density can always tend to a stable value when the small volume shrinks, say, to a finite value ε, and hence the density can be defined well. However, for a fractal, its integral dimensional measure is always divergent with the decreasing measure scale and cannot reach a stable value. In addition, it has also been noticed that the stress $\lim_{\Delta S\to 0} \Delta F/\Delta S$ (ΔS and ΔF is a small area and the resultant surface force acting on ΔS respectively) has played an important role in macro continuum mechanics, but in micromechanics such as nanomechanics, researchers have paid attention to some new integrated parameter such as energy-momentum tensor and configurational force, because the divergence of such a limit has caused more and more difficulties. In summary, it is unreasonable to describe a physical object in nature as a fractal because of the divergence of the integral dimensional measure of the fractal. From this perspective, the real physical object in nature which had ever been approximated by a fractal should be described as a ubiquitiform, or further, the real physical object in nature is a ubiquitiform. In fact, the splendid "fractal pictures" drawn on a piece of paper can only be two-dimensional, and the Piano curve of a finite order is also not able to cover any finite area (this is imaginable considering that a curve is of zero-width), which is just a one-dimensional ubiquitiformal curve.

3. Discussion

As described above, we emphasize again that the geometry of nature is ubiquitiformal rather than fractal, and any physical object in nature should not be described as a fractal, rather a ubiquitiform, which is of integral dimension in the Euclidean space. There is inherent difference between a fractal and a ubiquitiform, especially in the case of the Hausdorff dimension, which has been ignored for a long time. Comparing with a fractal, one of the most important characterizations of a ubiquitiform is that it has a finite integral dimensional measure. Equation (2.3) shows that the one-dimensional measure of a ubiquitiformal curve depends not only on the measure of the initial element but also on the complexity D and the lower bound to the scale invariance δ_{min}. As has been shown above, the complexity of a ubiquitiform is equal to the fractal dimension of the corresponding fractal, and this indicates that achievements obtained in fractal application will play an important role in the ubiquitiformal theory.

It can be seen from Eq. (2.3) that the lower bound to the scale invariance δ_{min} is a crucial parameter for a ubiquitiform, which is usually assumed to be a material constant. However, until now, the understanding of this parameter is far from complete. The key points are which factor will affect the lower bound to the scale invariance δ_{min} and how to determine its value accurately for a certain material. For a certain ubiquitiform with unknown δ_{min}, it seems that δ_{min} can be obtained from Eq. (2.3) under a given iterative procedure, which can be termed as the computational lower bound δ_{minc}. However, it should be pointed out that δ_{minc} depends on the measure of the initial element l as well as the complexity D and then may not be equal to δ_{min}. For clarity, as an example, numerical results are presented by taking the ubiquitiformal Koch curve. Assume that $l = 1\,\mathrm{mm}$ and $\delta_{min} = 1\,\mu\mathrm{m}$, the yardstick length of the nth iteration can be obtained as $\delta_n = 1/3^n\,\mathrm{mm}$, and then the maximum number of the iteration n_{max} can be determined by taking δ_n the minimum value greater than δ_{min}, i.e., $n_{max} = [3/\log 3] = 6$, where the square brackets represent the maximum integer less than the argument. The computational lower bound is then $\delta_{minc} = 1/3^6\,\mathrm{mm} \cong 1.37\,\mu\mathrm{m}$. Moreover, taking $l = 10\,\mathrm{mm}$, it can be verified that $n_{max} = 8$ and $\delta_{minc} \cong 1.52\,\mu\mathrm{m}$, which is about 10.9% greater than that for $l = 1\,\mathrm{mm}$ and implies the dependence of the computational lower bound δ_{minc} on the length of the initial element l. On the other hand, the influence of the relative error of δ_{min} on the measure of the ubiquitiform should also be noticed. For example, in the case of $l = 1\,\mathrm{mm}$, although the relative error of δ_{min} is 37%, it can be obtained from Eq. (2.3) that $L_{ubif}(\delta_{min}) \cong 6.10\,\mathrm{mm}$ and $L_{ubif}(\delta_{minc}) \cong 5.62\,\mathrm{mm}$ with the relative error of 7.9%, much less than that of δ_{min}, and such a relative error is acceptable in engineering application.

Moreover, in general, it was believed that the dimension d of both the integral dimensional objects and fractals can be defined simply by (Turcotte, 1997)

$$N(\delta)\delta^d = C \tag{3.1}$$

where $N(\delta)$ is the number of objects (i.e. fragments) with a characteristic linear dimension δ, C is a constant. However, as described above, all ubiquitiforms are of integral dimension such as d, but satisfy

$$N(\delta)\delta^D = C \qquad \delta \geqslant \delta_{min} \tag{3.2}$$

where D is generally a fractional value which is obviously not the same as d, which is also the real justification that we call D as the "complexity" of the ubiquitiform. Therefore, it appears that Eq. (3.2) should be a sufficient rather than a necessary condition for the definition of the dimension of a object. Moreover, it is seen that the complexity D may be an important parameter to classify objects in integral dimensions. That is to say, the objects in the integral dimension d could be classified into two categories: one is that they satisfy Eq. (3.1), or $D = d$, including for example a rectilinear segment or a smooth circular arc; the other one is that they satisfy Eq. (3.2) with $D \neq d$, as ubiquitiforms. Such a difference between the objects in a common dimension but different complexities seems to have been ignored for a long time.

On the other hand, it is worth to mention that a fractal can be constructed from initial elements with different dimensions because the Hausdorff dimension of the fractal can change in an infinite iterative procedure. This is very much different from that for a ubiquitiform. The Hausdorff dimension of the ubiquitiform is equal to the Hausdorff dimension of its initial element, but it is usually not the case for the fractal. For example, a Cantor set can be constructed from either an initial unit line segment or an initial point (Edger, 2008). The Hausdorff dimension of the initial line element is 1 and that of the initial point element is 0, but the fractal Cantor sets constructed from both initial elements have a common Hausdorff dimension of $\ln 2/\ln 3$. However, the ubiquitiformal Cantor sets come from such different initial elements, i.e. the initial point element and the initial line element, have different Hausdorff dimensions of 0 and 1, respectively, and can never be equivalent to each other. Obviously, the inherent difference between a ubiquitiform and its associated fractal comes from the jump characteristic of the Hausdorff dimension in the infinite iteration process. Moreover, in this way, it can be easily understood that the concepts of the invasive and the lacunar fractals (Bažant and Yavari, 1997) will be meaningless in the sense of ubiquitiform, because the Hausdorff dimension of the ubiquitiform is always equivalent to that of its initial element.

Moreover, it should be pointed out that, based on the concept of ubiquitiform, theoretically, some intrinsic difficulties in applied fractal science may be avoided. For example, on the one hand, a ubiquitiform is measurable in integral dimension and hence no any ambiguous physical variable (such as the specific energy-absorbing capacity (Borodich, 1997, 1999), the fractal tensile strength and the critical fractal strain (Carpinteri, 1994; Carpinteri *et al.*, 2002) proposed in fractal fracture mechanics) defined on the unit fractal measure seems necessary. All the corresponding physical variables can now be defined normally on the unit measure of integral dimension and thus will have unambiguous physical meanings. On the other hand, the finite integral measures of any two ubiquitiforms in a common dimension but different complexities are now comparable, while on the contrary, it is meaningless to compare the measures of two fractal curves with different fractal dimensions. It should be emphasized that waking up to this point is of significance in practical applications because most of scientific criteria can be established based on such a comparison. For example, the fractal fracture energy can not be considered as one of important parameters to characterize the material toughness. The two fracture energies of two specimens with fracture surfaces in different fractal dimensions will be expressed in different mechanical units. Under the concept of ubiquitiform, such an embarrassment will disappear naturally. Moreover, it can be expected that the incoming universal ubiquitiformal theory will be more easy-to-use in practice than the existing fractal one, since it can be constructed on the base of "classical" mathematics in the Euclidean space. For example the arduous fractal boundary value problems are difficult to be dealt with because the normal direction of a fractal boundary can not be defined, but using the ubiquitiformal theory, the problems turn into common boundary value problems defined on piecewise smooth boundaries and can be treated by traditional mathematical methods.

Additionally, it can be expected that the concept of ubiquitiform will play an important role in the development of the critical distance theory (Taylor, 2004, 2007, 2008) and the quantized (finite or discrete) fracture mechanics (Pugno and Ruoff, 2004) proposed recently by some authors, because of the characteristic finite number iterative procedure. Moreover, combining the ubiquitiformal theory with the concept of incubation time in dynamic failure mechanics will also be propitious for the investigation of strain rate effects on material strength, in which the discrete structural response characteristic has been noticed by some authors recently (Cotsovos and Pavlović, 2008a,b,c; Ou *et al.*, 2010).

4. Conclusion

To overcome the intrinsic weakness of fractals used in applied mechanics, a new concept of ubiquitiform is introduced and discussed in this paper, and some conclusions can be drawn out as follows.

- A physical object or a geometric configuration in nature can never be constructed by an infinite iterative procedure. In applied mechanics, therefore, instead of a fractal, despite an ideal mathematical object, a new concept of ubiquitiform should be introduced for a finite order self-similar (or self-affine) structure in nature in an attempt to overcome the weakness of the fractal such as, especially, its divergence or singularity of the integral dimensional measure.
- It is shown mathematically that a ubiquitiform constructed from an initial element whose Hausdorff dimension is integer by a finite self-similar iterative procedure must be of integral dimension, which implies theoretically that all real physical objects in nature could not be of fractional dimension and then are not fractals. It is also found that the Hausdorff dimension is discontinuous at the point at infinity in the infinite iterative procedure of a fractal, which results in the radical difference between the ubiquitiform and the fractal.
- It is also demonstrated that a given physical object in nature can never be well approximated by any fractal, especially in the sense of the Hausdorff measure, because the ubiquitiform always has an finite Hausdorff measure, whereas the corresponding fractal has not.
- Based on the concept of ubiquitiform, some intrinsic difficulties of fractals in applied mechanics can be avoided, e.g. it will not be necessary to introduce any kinds of density of fractal parameters defined on the unit fractal measure, and that the so-called fractal boundary-value problem could be constructed expediently in the sense of ubiquitiform.

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