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IDENTIFICATION OF THE MATHEMATICAL MODEL OF AN INSPECTION MOBILE ROBOT WITH FUZZY LOGIC SYSTEMS AND NEURAL NETWORKS

Józef Giergiel
Krzysztof Kurc

Rzeszow University of Technology, Department of Applied Mechanics and Robotics, Rzeszów, Poland
e-mail: bartek@prz.edu.pl; kkurc@prz.edu.pl

The paper focuses on the comparison of identification of the mathematical model of an inspection mobile robot by making use of fuzzy logic systems and neural networks. The solution to the problem was carried out through simulations and experimentally.

Key words: modeling, identification, neural network, fuzzy logic, robot

1. Introduction

It is hard to take all phenomena into consideration when modelling manipulators or robots, therefore the corresponding mathematical models are not known exactly. Correct analysis of dynamics of such complex systems requires identification of dynamical equations of motion (Giergiel *et al.*, 2000, 2002).

The identification of mathematical models with the use of neural networks and fuzzy logic systems enables one to recognize unknown parameters and adjust the mathematical model to the real object. The proposed in the paper methods of identification were verified on a prototype object.

2. Identification using fuzzy logic

In the design of fuzzy sets, a the most important is specification of the consideration set. In the case of an ambiguous term "high temperature", another value will be considered too high, if we accept the temperature interval 0-10°C, and other, if we accept the temperature interval 0-1000°C. The consideration domain or the set of action will be marked with the X letter. We must remember that X is a fuzzy set. The definition of the fuzzy set (Rutkowska *et al.*, 1997) was formulated as follows:

A fuzzy set in a non-empty space X, written down as $A \subseteq X$, is called a set of pairs

$$A = \{(x, \mu_A(x));\ x \in X\} \tag{2.1}$$

where

$$\mu_A : X \to [0, 1] \tag{2.2}$$

is a membership function of the A fuzzy set. This function assigns for every element $x \in X$ some affiliation degree to the fuzzy set A. Three cases are distinuished: full affiliation of the element x to the fuzzy set A, when $\mu_A(x) = 1$, no affiliation of the element x to the fuzzy set A, when $\mu_A(x) = 0$, and partial affiliation to the fuzzy set A of the element x, when $0 < \mu_A(x) < 1$. There are many standard forms of the membership function, which have been described in literature, see e.g. Rutkowska *et al.* (1997), however the most common are: gauss functions, triangular functions and trapezoidal functions (Driankow *et al.*, 1996; Osowski, 1996).

In systems with fuzzy logic the rules are symbolic "**IF-THEN**", quality variables are described with linguistic variables and there are fuzzy operators like "**AND**", so the sample rule can be written as follows

$$\textbf{IF}\ x_1 \text{ is small } \textbf{AND}\ x_2 \text{ is large } \textbf{THEN}\ y \text{ is average} \tag{2.3}$$

A mathematical model was adopted (Fig. 1b) for description of motion of an inspection robot (Fig. 1a).

The dynamic equation of motion (Giergiel and Kurc, 2006a,b,c, 2007) is

$$\begin{aligned}
&\Big[\frac{3600(l_4+r)^2 z_s^2 \tan^2\varphi}{z_k^2\pi^2}\Big(3m_1+m_2+m_3+\frac{3I_{Gx}}{r^2}+\frac{1}{\cos^2\delta}\Big)+\frac{I_{Fy}z_s^2}{z_k^2}+\\
&\quad+\frac{z_s^2}{z_k^2\cos^2\delta}\Big(I_{By}+3m_5 l_4^2\cos^2\varphi+\frac{3I_{Cy}l_4^2\cos^2\varphi}{r^2}\Big)\Big]\ddot{\alpha}+\\
&\quad+\Big(\frac{60m_1 l_4(l_4+r)z_s\tan\varphi(1-2\sin\psi)}{z_k\pi}\Big)\ddot{\beta}+\\
&\quad+\frac{60z_s\tan\varphi(l_4+r)}{z_k\pi}\Big(\frac{3N_1 f_1}{r}+(3G_1+G_2+G_3)\sin\beta+\frac{G_4\sin\gamma}{\cos\delta}\Big)+\\
&\quad+\frac{3\pi z_s l_4\cos\varphi(G_5 r\sin\gamma+N_2 f_2)}{r\cos\delta}=\frac{\cos^2\varphi}{\cos\delta}M
\end{aligned} \tag{2.4}$$

where: m_i, $i = 1,\ldots,5$ are masses. I_{By}, I_{Cy}, I_{Fy}, I_{Gx} – mass moments of inertia, N_1, N_2 – forces of pressure of wheels, f_1, f_2 – arms of resistance of

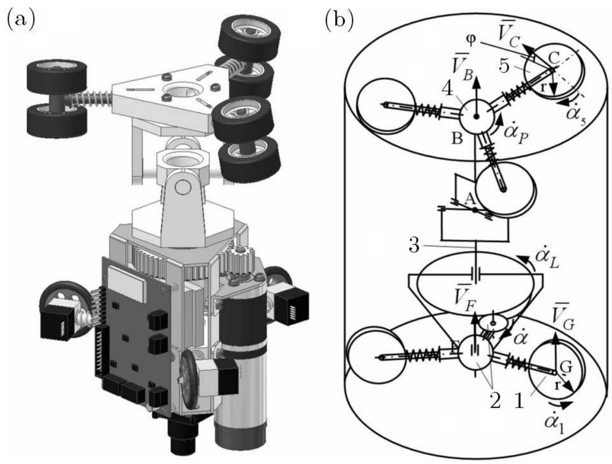

Fig. 1. (a) Inspection robot, (b) model of the robot

the rolling wheels, M – torque of the motor, l_4 – distance between FG and BC points, r – radius of wheels, z_s, z_k – number of teeth of cogwheels.

The assumed data is

$$
\begin{array}{lll}
m_1 = 0.004\,\mathrm{kg} & m_2 = 0.554\,\mathrm{kg} & \\
m_3 = 0.075\,\mathrm{kg} & m_4 = 0.05\,\mathrm{kg} & \\
m_5 = 0.015\,\mathrm{kg} & I_{By} = 0.000034692\,\mathrm{kgm}^2 & \\
I_{Cy} = 0.000001971\,\mathrm{kgm}^2 & I_{Fy} = 0.000018807\,\mathrm{kgm}^2 & \\
I_{Gx} = 0.000000312\,\mathrm{kgm}^2 & f_1 = 0.0015\,\mathrm{m} & \\
f_2 = 0.003\,\mathrm{m} & N_1 = 4.4\,\mathrm{N} & \\
N_2 = 6.1\,\mathrm{N} & z_s = 12 & z_k = 48
\end{array}
$$

Equation (2.4) is written down in the state space

$$
\dot{\boldsymbol{\alpha}} = \mathbf{A}\boldsymbol{\alpha} + \boldsymbol{B}[f(\alpha,\beta,\gamma) + G(\alpha,\beta,\gamma)u(t)] \tag{2.5}
$$

or in the form of a vector matrix

$$
\begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Big[-\frac{1}{s_1}(s_2\ddot{\beta} + s_3) + \frac{s_4}{s_1}u(t)\Big] \tag{2.6}
$$

where

$$
\begin{aligned}
s_1 = \Big[&\frac{3600(l_4+r)^2 z_s^2 \tan^2\varphi}{z_k^2\pi^2}\Big(3m_1 + m_2 + m_3 + \frac{3I_{Gx}}{r^2} + \frac{1}{\cos^2\delta}\Big) + \\
&+ \frac{I_{Fy}z_s^2}{z_k^2} + \frac{z_s^2}{z_k^2\cos^2\delta}\Big(I_{By} + 3m_5 l_4^2\cos^2\varphi + \frac{3I_{Cy}l_4^2\cos^2\varphi}{r^2}\Big)\Big]
\end{aligned}
$$

$$
\begin{aligned}
s_2 &= \frac{60m_1l_4(l_4+r)z_s\tan\varphi(1-2\sin\psi)}{z_k\pi} \\
s_3 &= \frac{60z_s\tan\varphi(l_4+r)}{z_k\pi}\Big(\frac{3N_1f_1}{r}+(3G_1+G_2+G_3)\sin\beta+\frac{G_4\sin\gamma}{\cos\delta}\Big)+ \\
&\quad +\frac{3\pi z_sl_4\cos\varphi(G_5r\sin\gamma+N_2f_2)}{r\cos\delta} \\
s_4 &= \frac{\cos^2\varphi}{\cos\delta}
\end{aligned}
\tag{2.7}
$$

Variables β, γ, δ which occur in equations (2.7) are dependent on the pipe profile. Elements $f(\alpha,\beta,\gamma)$ and $G(\alpha,\beta,\gamma)$ in formula (2.5) are non-linear functions approximated by systems with fuzzy logic

$$
f(\alpha,\beta,\gamma)=-\frac{1}{s_1}(s_2\ddot{\beta}+s_3) \qquad\qquad G(\alpha,\beta,\gamma)=\frac{s_4}{s_1} \tag{2.8}
$$

Because functions $f(\alpha,\beta,\gamma)$ and $G(\alpha,\beta,\gamma)$ do not have the linear form with regard to parameters (2.7), there are some inaccuracies in the modelling. The identification system takes the form

$$
\dot{\widehat{\boldsymbol{\alpha}}}=\mathbf{A}\widehat{\boldsymbol{\alpha}}+\boldsymbol{B}[\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})+\widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})u]+\mathbf{K}\widetilde{\boldsymbol{\alpha}} \tag{2.9}
$$

where vector $\widehat{\boldsymbol{\alpha}}$ is an estimation of the state vector $\boldsymbol{\alpha}$, $\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})$, $\widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})$ are estimations of the non-linear functions in equation (2.5). Accepting the error of the estimation of the state vector in the form

$$
\widetilde{\boldsymbol{\alpha}}=\boldsymbol{\alpha}-\widehat{\boldsymbol{\alpha}} \tag{2.10}
$$

and subtracting equation (2.9) from equation (2.5), a description of the identification system in the error space is acquired

$$
\dot{\widetilde{\boldsymbol{\alpha}}}=\mathbf{A}_H\widetilde{\boldsymbol{\alpha}}+\boldsymbol{B}[\widetilde{f}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})+\widetilde{G}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})u] \tag{2.11}
$$

where: $\mathbf{A}_H=\mathbf{A}-\mathbf{K}$ and the matrix $\mathbf{K}$ is such that the characteristic equation of the matrix $\mathbf{A}_H$ is strictly stable.

3. Identification using neural networks

Another kind of the solution to the task of identification is an application of artificial neural networks.

Adding and subtracting the expression $\mathbf{A}_m\boldsymbol{\alpha}$ from equation (2.5) where $\mathbf{A}_m$ is a stable design matrix (Giergiel *et al.*, 2002), we receive

$$\dot{\boldsymbol{\alpha}} = \mathbf{A}_m\boldsymbol{\alpha} + (\mathbf{A} - \mathbf{A}_m)\boldsymbol{\alpha} + \boldsymbol{B}[f(\alpha,\beta,\gamma) + G(\alpha,\beta,\gamma)u] \tag{3.1}$$

Equation (3.1) defines the series-parallel structure of the identification system which is in the form

$$\dot{\widehat{\boldsymbol{\alpha}}} = \mathbf{A}_m\widehat{\boldsymbol{\alpha}} + (\mathbf{A} - \mathbf{A}_m)\boldsymbol{\alpha} + \boldsymbol{B}[\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma}) + \widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})u] \tag{3.2}$$

where $\widehat{\boldsymbol{\alpha}}$ is the estimator of the vector of the state $\boldsymbol{\alpha}$, $\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})$ and $\widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})$ are estimators of the non-linear functions from equation (3.1).

The error of the estimation of the state is given in the form (2.10).

Substracting equation (3.2) from (3.1), a description of the task of identification is received in the error space

$$\dot{\widetilde{\boldsymbol{\alpha}}} = \mathbf{A}_m\widetilde{\boldsymbol{\alpha}} + \boldsymbol{B}[\widetilde{f}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \widetilde{G}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})u] \tag{3.3}$$

where

$$\mathbf{A}_m\widetilde{\boldsymbol{\alpha}} = \mathbf{A}_m\boldsymbol{\alpha} - \mathbf{A}_m\widehat{\boldsymbol{\alpha}} \tag{3.4}$$

and

$$\begin{aligned}
\widetilde{f}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) &= f(\alpha,\beta,\gamma) - \widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})\\
\widetilde{G}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) &= G(\alpha,\beta,\gamma) - \widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})
\end{aligned} \tag{3.5}$$

To determine the function $\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})$ and $\widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})$, neural networks have been applied.

Since the functions $f(\alpha,\beta,\gamma)$ and $G(\alpha,\beta,\gamma)$ are supposed to be approximated by neural networks, then

$$\begin{aligned}
f(\alpha,\beta,\gamma) &= \mathbf{W}_f^\top\boldsymbol{S}_f(\alpha,\beta,\gamma) + \varepsilon_f(\alpha,\beta,\gamma)\\
G(\alpha,\beta,\gamma) &= \mathbf{W}_G^\top\boldsymbol{S}_G(\alpha,\beta,\gamma) + \varepsilon_G(\alpha,\beta,\gamma)
\end{aligned} \tag{3.6}$$

where $\varepsilon_f(\alpha,\beta,\gamma)$ and $\varepsilon_G(\alpha,\beta,\gamma)$ are the inaccuracies of approximation of the function $f(\alpha,\beta,\gamma)$ and $G(\alpha,\beta,\gamma)$ through neural networks, $\mathbf{W}_f$ and $\mathbf{W}_G$ – matrices of weights of neural connections, $\boldsymbol{S}_f(\alpha,\beta,\gamma)$ and $\boldsymbol{S}_G(\alpha,\beta,\gamma)$ – vectors of base functions.

These networks have the structure of a network with the radial functional extension in form of Gauss' function

$$S_j(x) = \exp(-\beta\|x - c_j\|^2) \tag{3.7}$$

where c_j is the j-th centre.

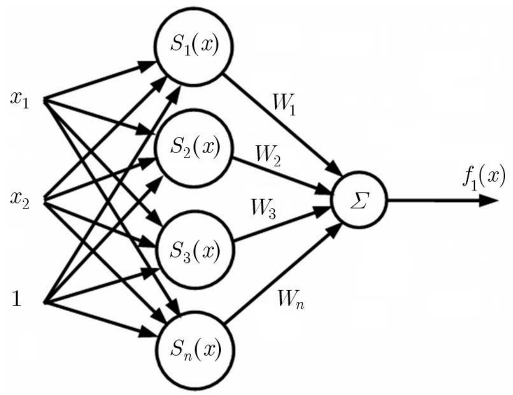

Fig. 2. Structure of radial networks approximating functions $\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma})$ and $\widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma})$

A general structure of the system is shown in Fig. 2.

Setting the estimations of functions in equation (3.5) in the form

$$\begin{aligned}\widehat{f}(\alpha,\widehat{\beta},\widehat{\gamma}) &= \widehat{\mathbf{W}}_f^\top \boldsymbol{S}_f(\alpha,\widehat{\beta},\widehat{\gamma})\\ \widehat{G}(\alpha,\widehat{\beta},\widehat{\gamma}) &= \widehat{\mathbf{W}}_G^\top \boldsymbol{S}_G(\alpha,\widehat{\beta},\widehat{\gamma})\end{aligned} \tag{3.8}$$

formulas (3.5) are written in the form

$$\begin{aligned}\widetilde{f}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) &= \widetilde{\mathbf{W}}_f^\top \boldsymbol{S}_f(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \varepsilon_f(\alpha,\beta,\gamma)\\ \widetilde{G}(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) &= \widetilde{\mathbf{W}}_G^\top \boldsymbol{S}_G(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \varepsilon_G(\alpha,\beta,\gamma)\end{aligned} \tag{3.9}$$

where $\varepsilon_f(\alpha,\beta,\gamma)$ and $\varepsilon_G(\alpha,\beta,\gamma)$ are the errors of approximation of the network, $\widetilde{\mathbf{W}}_f$ and $\widetilde{\mathbf{W}}_G$ – errors of the estimation of weights of the network.

Then equation (3.3) will be in the form

$$\dot{\widetilde{\boldsymbol{\alpha}}} = \mathbf{A}_m\widetilde{\boldsymbol{\alpha}} + \boldsymbol{B}[\widetilde{\mathbf{W}}_f^\top \boldsymbol{S}_f(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \widetilde{\mathbf{W}}_G^\top \boldsymbol{S}_\Delta(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})] + \boldsymbol{B}[R_f + R_G] \tag{3.10}$$

where: $R_f = \varepsilon_f(\alpha,\beta,\gamma)$, $R_G = \varepsilon_G(\alpha,\beta,\gamma)u$, $\boldsymbol{S}_\Delta(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) = u \otimes \boldsymbol{S}_G(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})$.

The stability of the system was checked according to the Lyapunov stability criterion. It is known that the dynamic system will be stable if a Lyapunov function exists for it (Giergiel *et al.*, 2000, 2002).

A function has been assumed in the form

$$\boldsymbol{V} = \frac{1}{2}\widetilde{\boldsymbol{\alpha}}^\top \mathbf{P}\widetilde{\boldsymbol{\alpha}} + \frac{1}{2}\operatorname{tr}\widetilde{\mathbf{W}}_f^\top \mathbf{F}_f^{-1}\widetilde{\mathbf{W}}_f + \frac{1}{2}\operatorname{tr}\widetilde{\mathbf{W}}_G^\top \mathbf{F}_G^{-1}\widetilde{\mathbf{W}}_G \tag{3.11}$$

If this function is to be the Lyapunov function, its derivative has to be negative

$$
\begin{aligned}
\dot{\boldsymbol{V}} &= -\widetilde{\boldsymbol{\alpha}}^\top \mathbf{Q}\widetilde{\boldsymbol{\alpha}} + \widetilde{\boldsymbol{\alpha}}^\top \mathbf{P}\boldsymbol{B}[\widetilde{\mathbf{W}}_f^\top \boldsymbol{S}_f(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \widetilde{\mathbf{W}}_G^\top \boldsymbol{S}_\Delta(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma}) + \\
&\quad + R_f + R_G] + \operatorname{tr} \widetilde{\mathbf{W}}_f^\top \mathbf{F}_f^{-1} \dot{\widetilde{\mathbf{W}}}_f + \operatorname{tr} \widetilde{\mathbf{W}}_G^\top \mathbf{F}_G^{-1} \dot{\widetilde{\mathbf{W}}}_G
\end{aligned}
\tag{3.12}
$$

The training of the neural network weights has been carried out according to the formula

$$
\begin{aligned}
\dot{\widetilde{\mathbf{W}}}_f &= -\mathbf{F}_f \boldsymbol{S}_f(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})\widetilde{\boldsymbol{\alpha}}^\top \mathbf{P}\boldsymbol{B} \\
\dot{\widetilde{\mathbf{W}}}_G &= -\mathbf{F}_G \boldsymbol{S}_\Delta(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})\widetilde{\boldsymbol{\alpha}}^\top \mathbf{P}\boldsymbol{B}
\end{aligned}
\tag{3.13}
$$

From the matrix form of the Lyapunov equation

$$
\mathbf{E}^\top \mathbf{P} + \mathbf{P}\mathbf{E} = -\mathbf{Q} = -\mathbf{I} \tag{3.14}
$$

a Hertmitian matrix was determined as

$$
\mathbf{P} = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \tag{3.15}
$$

by solving the equation

$$
\begin{bmatrix} e_{11} & e_{21} \\ e_{12} & e_{22} \end{bmatrix} \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} + \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \tag{3.16}
$$

A denotation has been assumed

$$
\boldsymbol{h} = \mathbf{P}\boldsymbol{B} \tag{3.17}
$$

where

$$
\boldsymbol{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{3.18}
$$

Finally, the weight training algorithm for (3.13) has the form

$$
\begin{aligned}
\dot{\widehat{\mathbf{W}}}_f &= \mathbf{F}_f \boldsymbol{S}_f(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})\widetilde{\boldsymbol{\alpha}}^\top \boldsymbol{h} \\
\dot{\widehat{\mathbf{W}}}_G &= \mathbf{F}_G \boldsymbol{S}_G(\alpha,\beta,\gamma,\widehat{\beta},\widehat{\gamma})\widetilde{\boldsymbol{\alpha}}^\top \boldsymbol{h}
\end{aligned}
\tag{3.19}
$$

The identification of the mathematical model of the inspection robot was carried out according to this procedure.

4. The simulation and verification using fuzzy logic

The verification was carried out on a prototype of the inspection robot. We may expect that the estimated model will be different from the mathematical model (Giergiel *et al.*, 2002).

To determine functions $\widehat{f}(\alpha, \widehat{\beta}, \widehat{\gamma})$, $\widehat{G}(\alpha, \widehat{\beta}, \widehat{\gamma})$, fuzzy logic systems were created in the application Matlab$^{\mathrm{TM}}$ (Fig. 3), which makes it possible to create models of fuzzy logic (*fuzzy logic toolbox*) (Buratowski and Żylski, 3003; [11]).

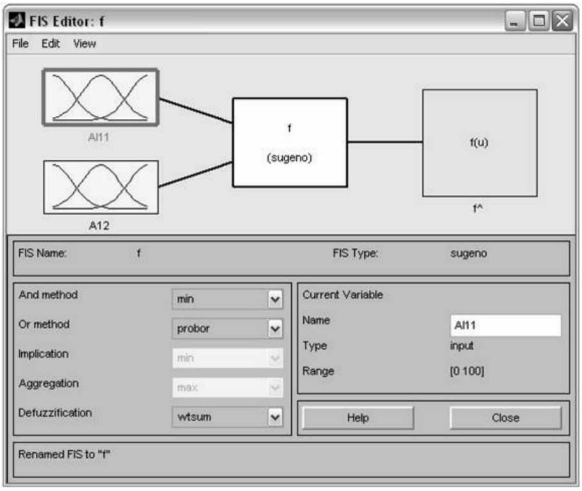

Fig. 3. Model of fuzzy logic approximated non-linear functions

The task of the fuzzy logic system is to determine functions $\widehat{f}(\alpha, \widehat{\beta}, \widehat{\gamma})$, $\widehat{G}(\alpha, \widehat{\beta}, \widehat{\gamma})$ in such a way, that an error $\widetilde{\boldsymbol{\alpha}}$ between the state vector $\boldsymbol{\alpha}$ of the computing model and the estimated state vector $\widehat{\boldsymbol{\alpha}}$ is the smallest. Takagi-Sugeno's model was applied in the designing phase (Buratowski and Żylski, 3003; Rutkowska *et al.*, 1997; [11]). The fuzzification block transforms the input space in form $X = [\dot{\alpha}_{1a}, \dot{\alpha}_{1b}] \times [\dot{\alpha}_{2a}, \dot{\alpha}_{2b}] \subset R^n$ into a fuzzy set $A \in X$ characterised by the membership function $\mu_A(x) : X \to [0, 1]$, which assigns a degree of affiliation into fuzzy sets. In Fig. 4, the membership functions are presented in the form of Gauss' function (gaussmf) according to the input range: $\dot{\alpha}_1 \in [0, 100]$, $\dot{\alpha}_2 \in [0, 10]$.

The base of rules for the model description was accepted as in Fig. 5. Three membership functions were accepted for the inputs of the fuzzy system and 9 rules of inferring were created. A principle was offered: every rule from one input with every rule of the other input, since the information about each output from the fuzzy systems is missing.

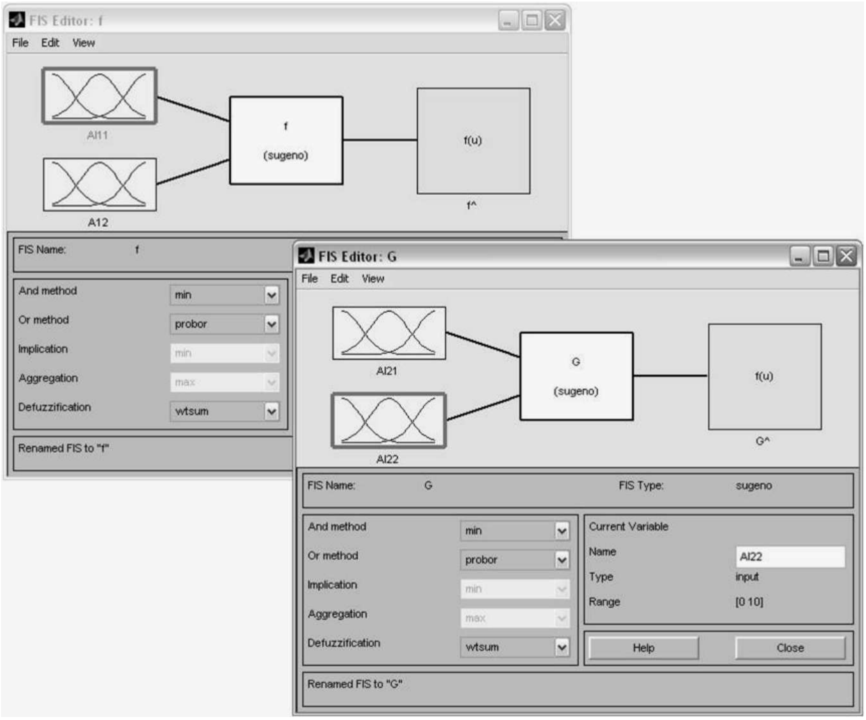

Fig. 4. Functions of affiliation and intervals of variability

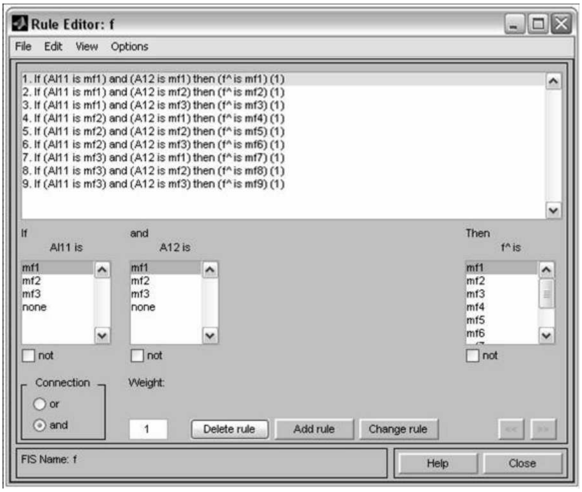

Fig. 5. Base of rules for the accepted set

The set A was accepted on the input with T-norm (Osowski, 1996) of the minimum type

$$\mu_{A_1^j \times \ldots \times A_n^j}(x) = \min[\mu_{A_1^j}, \ldots, \mu_{A_n^j}] \tag{4.1}$$

On the output of the Takagi-Sugeno model presented in Fig. 6, a signal was received

$$y(x) = \frac{\sum\limits_{j=1}^{M} \overline{y}_j \tau_j}{\sum\limits_{j=1}^{M} \tau_j} \tag{4.2}$$

where

$$\tau_j = \prod_{i=1}^{n} \mu_{A_i^j}(x_i) \tag{4.3}$$

is the ignition level of the j-th rule.

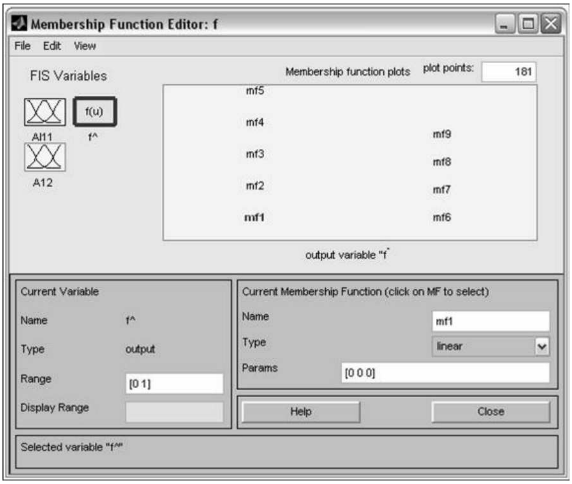

Fig. 6. Exit of the fuzzy logic system

The described fuzzy logic systems were applied for approximation of non-linear functions (2.8) and they were modeled in the form Fig. 7.

Fuzzy Logic sets f and G are responsible for approximation of non-linear function (2.8). All fuzzy sets use numerical information which explicitly connects the input and output signals. In order to check the proposed solution, a verification was carried out.

At the beginning, the simulation and experimental results were compared (Fig. 8).

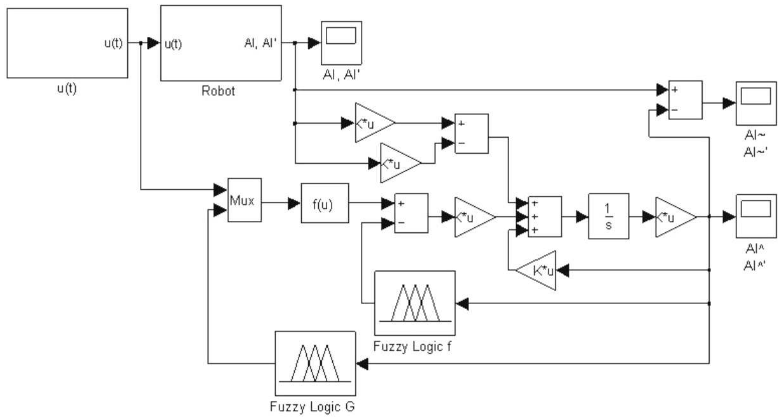

Fig. 7. Structure of identification with fuzzy logic

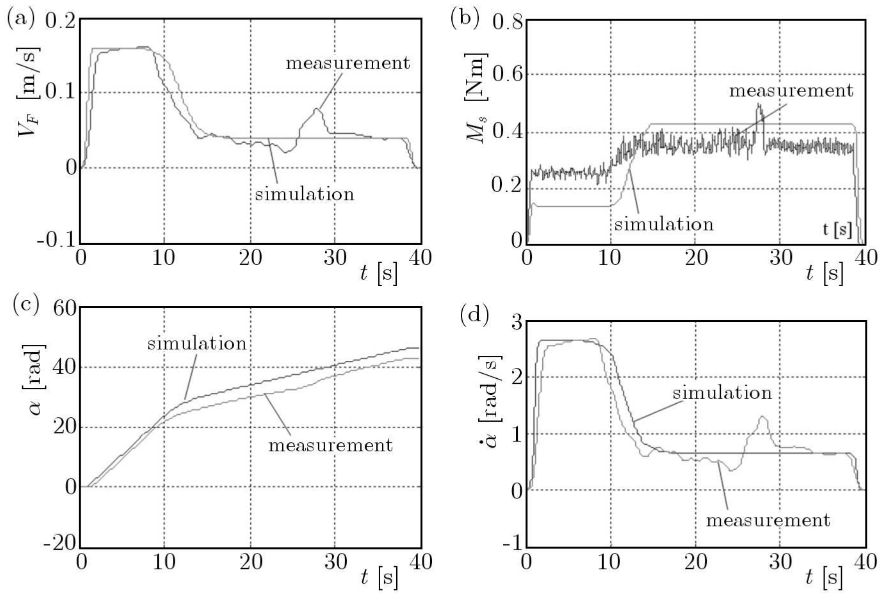

Fig. 8. Diagrams from simulation and measurements; (a) velocities of the F point of the robot, (b) torque of the motor, (c), (d) variables

In the next stage, the parameter identification of the inspection robot was carried out according to the structure (Fig. 7) designed in Matlab$^{\mathrm{TM}}$-Simulink software, taking as the input function $u(t)$ the torque of the motor (Fig. 8b and Fig. 9a).

The torque moment on the motor shaft received from measurements was taken as an the input function (Fig. 8b and Fig. 9a), and then fuzzy identification of the inspection robot parameters was carried out according to remarks

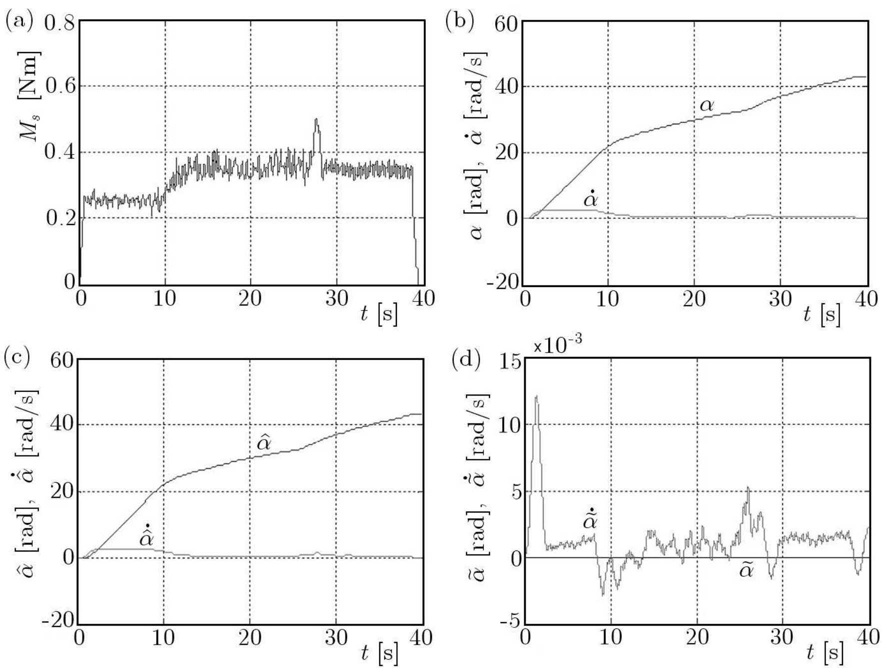

Fig. 9. Results of identification; (a) input signal, (b) angle of rotation and angular velocity on the shaft driving motor, (c) parameters estimator, (d) errors estimator

given in Section 2. The estimated parameters were the angle of rotation and angular velocity (Fig. 9c) of the motor shaft, which were compared with the parameters obtained during measurements (Fig. 9b). Subtracting them, the angle estimation error and the angular velocity estimation error were obtained as (2.10) (Fig. 9d). It can be seen that the estimation error for the angle of rotation of the motor shaft $\tilde{\alpha}$ is equal zero (Fig. 9d) but there is a small error in the estimation of the angular velocity $\dot{\tilde{\alpha}}$. The obtained solutions of fuzzy logic identification are limited, and the proposed procedure enables identification of non-linear systems by applying fuzzy logic systems.

5. Simulation and verification using neural networks

In the next stage, the identification of the robot parameters was done with the application of neural networks according to the structure (Fig. 10) given for the input function $u(t)$ in form of the torque of the motor (Fig. 8b and Fig. 11a).

On the schema (Fig. 10), "Neural network f" and "Neural network G" are the models describen in Section 3.

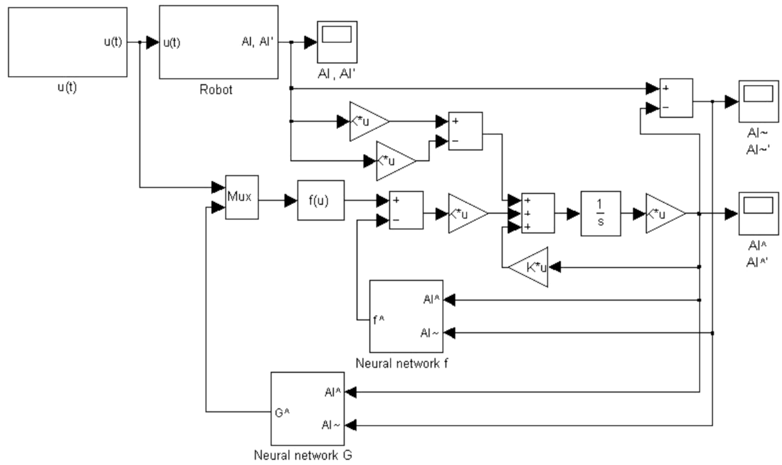

Fig. 10. Structure of identification with neural networks

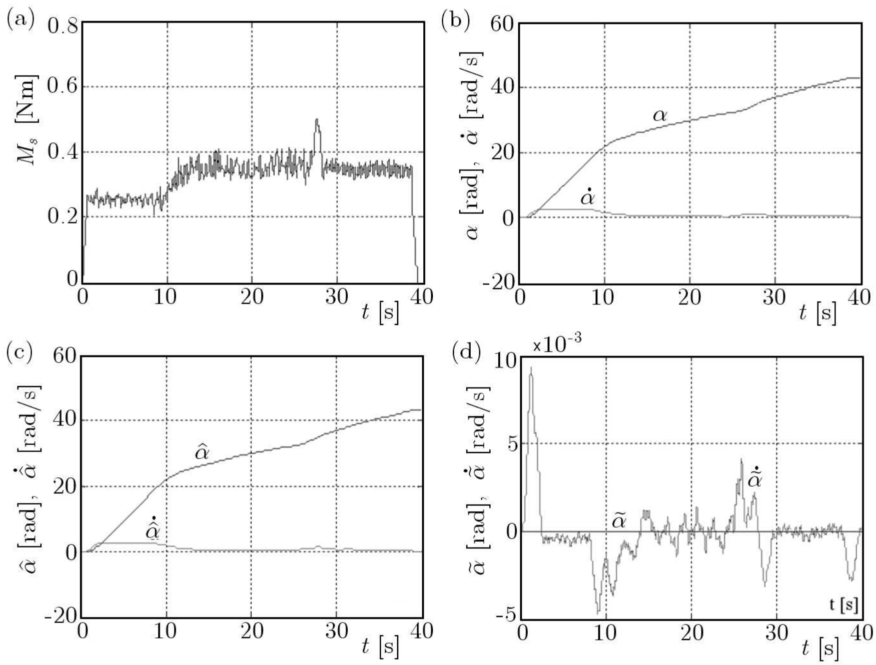

Fig. 11. Results of identification; (a) input signal, (b) angle of rotation and angular velocity on the shaft driving motor, (c) parameters estimator, (d) errors estimator

Taking the torque on the motor shaft from the measurements as the input function (Fig. 8b and Fig. 11a), the neural identification of the inspection robot parameters was carried out in accordance with the procedure given in Section 3. The estimated parameters are the angle of rotation and angular velocity (Fig. 11c) of the motor shaft. They were compared to the parameters obtained during measurements (Fig. 11b). Subtracting them, the angle estimation error and the angular velocity estimation error were obtained as (2.10) (Fig. 11d). It can be seen that the estimation error for the angle of rotation of the motor shaft $\widetilde{\alpha}$ is equal zero (Fig. 11d), but there is an error in the estimation of the angular velocity of the motor shaft $\dot{\widetilde{\alpha}}$. The obtained solutions are limited, and the proposed procedure enables identification of non-linear systems by applying neural networks.

6. Comparison of applied methods

The identification with neural networks and with fuzzy logic were compared according to the structure (Fig. 12).

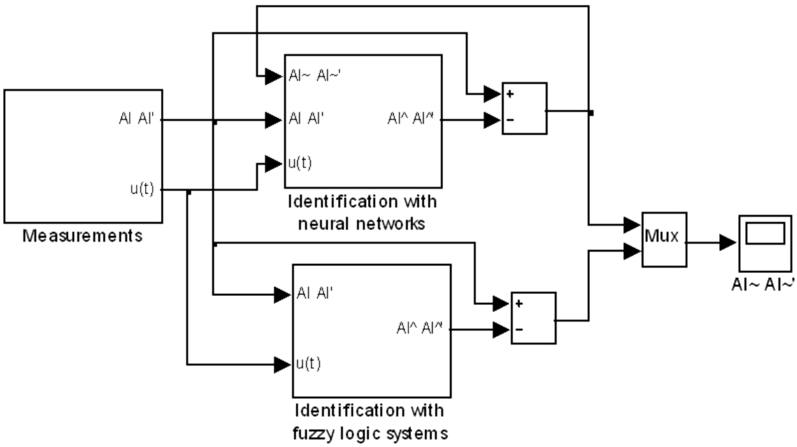

Fig. 12. Structure of the comparison of identification with neural networks and fuzzy logic systems

On the schema (Fig. 12), "Identification with neural networks" is representing the model described in Swction 3, and "Identification with fuzzy logic systems" the model from Section 2.

Three experiments were carried out in order to compare the two methods of identification, whose errors are presented for the estimator (Fig. 13-15) according to formula (2.10).

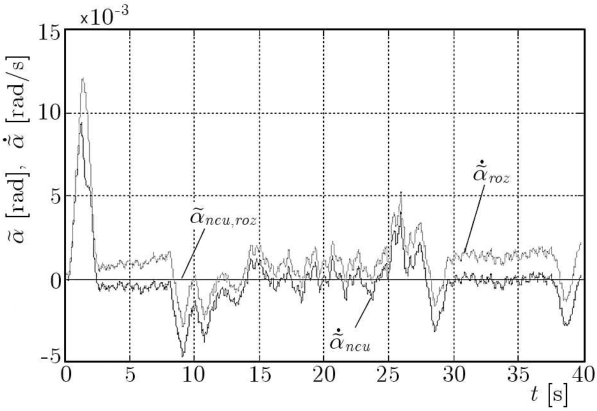

Fig. 13. Errors estimation, experiment 1

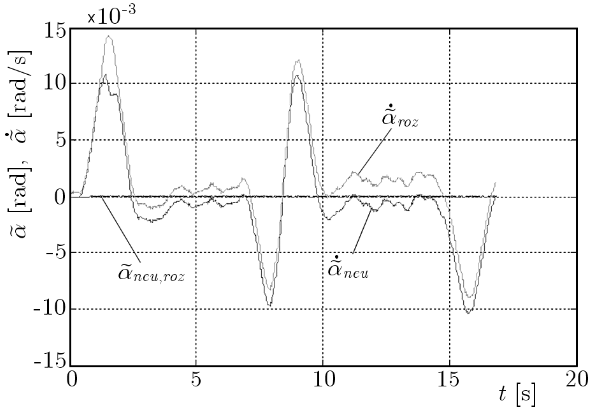

Fig. 14. Errors estimation, experiment 2

The conducted comparison shows (Fig. 13-15) that the estimation errors for the angle of rotation of the motor shaft from neural identification $\tilde{\alpha}_{neu}$ and from fuzzy identification $\tilde{\alpha}_{roz}$ are both zero. There are some errors of the angular velocity estimation from the identification with neural networks $\dot{\tilde{\alpha}}_{neu}$ and from the identification with fuzzy logic $\dot{\tilde{\alpha}}_{roz}$. The mean error of the angular velocity estimation for neural networks is slightly smaller than for fuzzy logic. The errors of the experiment are kept within 0.2-1% of the recorded values.

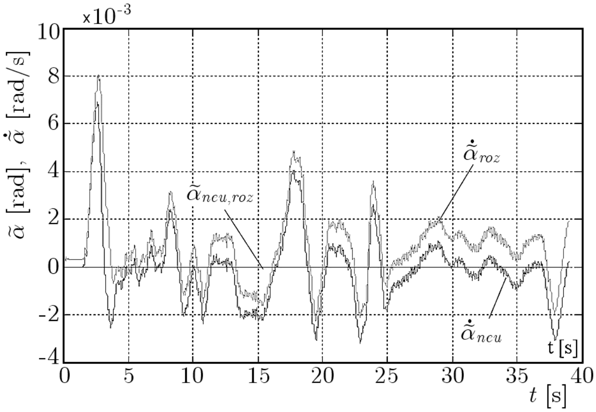

Fig. 15. Errors estimation, experiment 3

7. Conclusions

During the verification stage, a comparison between for the test rig and simulation results was conducted. It seems that the results obtained from measurements in the real robot are satisfactory and slightly differ from simulation runs, which confirms the adequacy of the design and simulation stage. Visible differences are an effect of many factors appearing during the simulations (inaccuracies of the estimation of the model parameters, missing description of physical phenomena "incomplete modelling") as well as during the measurements (parametric interferences – changes in diameter of the pipeline).

Having carried out the stages of identification with neural networks and with fuzzy logic, it is possible to assume that these methods can be successfully applied for the identification of dynamical equations of motion and actual parameters as well as for monitoring dynamic loads and detection of damage.

Since the results are similar, there is a question arising, which method should be applied for the identification of non-linear systems? To resolve this problem, one may ask which method can be more easily and fartly implemented? And the answer is simple: fuzzy logic systems.

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Identyfikacja modelu matematycznego mobilnego robota inspekcyjnego układami z logiką rozmytą i sieciami neuronowymi

Streszczenie

W artykule przedstawiono porównanie identyfikacji modelu matematycznego mobilnego robota inspekcyjnego układami z logiką rozmytą i sieciami neuronowymi. Rozwiązanie problemu zostało przeprowadzone na drodze numerycznej i doświadczalnej.

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