

MEASUREMENT OF FLOW RATE IN SQUARE-SECTIONED DUCT BEND

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In this paper the authors describe an attempt to utilise installed square-sectioned elbows in order to measure the fluid flow rate. In order to practically accomplish the measurement of the volumetric flow rate of the air, a special research stand has been built, and square shaped elbows have been installed (80×80 mm in dimension). The numerical computations were carried out using the software package FLUENT 6.2. The obtained results were compared to corresponding ones coming from orifice measurements and from experimental work available in the literature. The comparative analysis of the obtained numerical and experimental results evidenced a high degree of their conformity.

Key words: measurement of flow rate, inverse coefficient problem

1. Introduction

Standardized methods of fluid flow rate measurement are based on the integration of the velocity profile. They are basically the only methods that can be applied to a rectangular cross section according to Bean (1971) and Spitzer (2001). The above-mentioned methods are characterised by multi-discrete measurement of local velocities at points that have been appropriately distributed on a lateral stream cross section. In order to obtain a satisfactory result of measurement, accuracy is indispensable in providing a large number of measuring points. Consequently, such a method is time-consuming and may be quite tedious. Constant registration of the flow intensity is also impeded.

In the case when a classical measuring orifice is used to measure the flow rate, it is necessary to install long pipeline sections with a circular cross section as described by Bean (1971), Spitzer (2001) and in compliance with PN-EN ISO (2004). It appears that circular-shaped elbow flow meters, which have been installed on the pipeline, enable one to obtain a signal that corresponds to the entire volumetric rate of the fluid flow as suggested by Bean (1971), Spitzer (2001) and Rup and Malinowski (2005, 2006). The principle that pertains to operation of the elbow flow meter is based on the usage of the ratio of volumetric rate of flow to the difference between pressures measured at extreme points of the elbow arc secant. In practice, engineers sometimes use a solution where the pressure is measured through pulse apertures that are situated along the plane, which is not aligned with the secant. In this case, the pressure difference that is measured attains lower values according to Bean (1971), Spitzer (2001) and Rup and Malinowski (2005, 2006).

The standard measurement is carried out in the following way: the pressure difference is measured and, subsequently, the relevant volumetric rate of flow value is assigned based on the previously worked out characteristic of the instrument. At times, the instrument characteristics are substituted by simple algebra dependencies, which have been devised on the basis of experimental results.

Recently, a new measurement method for usage of elbow flow meters in circular-sectioned pipes has been elaborated, see Rup and Malinowski (2005, 2006). The idea of this particular method is based on selecting such a numeric solution of fluid equations of motion (including turbulence model equations) which fulfills the given accuracy, the balance condition that has been measured and calculated at extreme points of elbow flow meter secant. This brand new method eliminates the necessity to calibrate elbow flow meters.

The main goal of this paper is an extension of the method of identification or control of the volumetric flow rate of the fluid inside circular sectioned pipes, elaborated earlier by Rup and Malinowski (2005, 2006), for channels where quadratic arc elbows have been installed.

2. Test stand

To determine the value of the measured pressure difference (Δp_{exp}) necessary to carry out the described method, an appropriate test stand was built. A sample section of the stand is shown in Fig. 1. The measuring set displayed in Fig. 1 consists of a rectangular pipeline with the following dimensions: $a \times a =$

= 80 mm × 80 mm, installed elbow arc (90°) (1), with the average radius equal $R_s = 160$ mm. The length of the straight line section of the pipeline with a quadratic cross section, that was placed in front of the elbow arc equals $L_3 = 20D_h = 1.6$ m, whilst the length of the appropriate section beyond the arc equals $L_4 = 25D_h = 2.0$ m. For control purposes, a measuring orifice was also installed at station (2). The inner diameter of the measuring orifice is equal to $d_{orif} = 78$ mm. The measurement of pressure decrease on the orifice was carried out using a water manometer (3). In order to obtain a correct measurement of the volumetric flow rate via the orifice, a straight line section of the pipeline (L_1, L_2) was installed in front and behind the orifice as suggested by Spitzer (2001) and according to PN-EN ISO (2004).

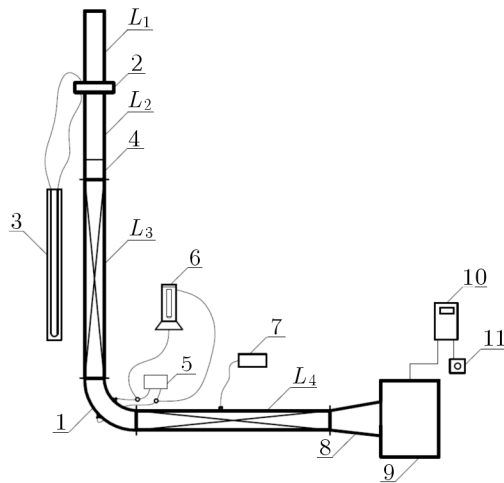


Fig. 1. Test stand; 1 – elbow arc with quadratic cross section, 2 – measuring orifice, 3 – water manometer, 4 – transition from circular to square-section pipe, 5 – pressure converter and digital meter, 6 – Askania differential micro-manometer, 7 – temperature sensor, 8 – sheet metal cone, 9 – centrifugal fan, 10 – thyristor frequency converter, 11 – frequency regulator

Between the circular and square-section pipe, transition segment (4) was installed. Two pulse apertures have been drilled at the elbow arc secant in order to measure the pressure difference. The mentioned pressure difference was measured with the use of the pressure converter and digital meter (5). For control purposes, Askania micro-manometer (6) has been attached diagonally to the ends of pressure receivers. The temperature of flowing air was measured using resistance-type temperature sensor (7). Based on the temperature of the flowing air that has been measured, it was possible to determine thermo-physical properties of the fluid. As shown in Fig. 1, the pipeline was placed on

the suction side of the fan so that flow disturbances originating from the fan could be eliminated. The tail end of the pipeline was linked with suction flange of MPB500T (9) fan by applying metal sheet cone (8). The desired volumetric stream flow was regulated by thyristor frequency converter (10) which enabled smooth control of the fan RPM.

3. Determination of the fluid flow rate according to the idea of the applied method

Considering the turbulent nature of the examined flows, the time averaged Navier Stokes equations (RANS) with continuity equation were used. Assuming that the flows were incompressible, in statistically stationary flow the time averaged equations resulting from the momentum and mass conservation equations have the following form

$$\begin{aligned} \frac{\partial \bar{v}_i}{\partial x_i} &= 0 \\ \rho \frac{\partial}{\partial x_j} (\bar{v}_i \bar{v}_j) &= \frac{\partial}{\partial x_j} \left[-\bar{p} \delta_{ij} + \mu \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i} \right) - \overline{\rho v'_i v'_j} \right] \end{aligned} \quad (3.1)$$

The last term on the right side of equation (3.1) expresses the components of the Reynolds stress tensor

$$\bar{r}_{ij}^{(t)} = -\overline{\rho v'_i v'_j} \quad (3.2)$$

The appearance of the additional term in equation of turbulent flow (3.2) causes that the system of equations (3.1) becomes an opened one. For a unique description of fluid motion, additional relations determining the components of turbulent stress tensor (3.2) must be taken into consideration. To determine these components the 7-equation turbulence model (RSM) available in FLUENT was used as described in Fluent User Manual (2008). The RSM model is recommended especially in the case of the fluid flow in curved channels. To obtain a unique description of the investigated flow, the following boundary conditions were formulated: given value of static pressure on the inlet of the duct, zeroing of the velocity vector and turbulent kinetic energy on the walls, assumed value of mean flow velocity, turbulence intensity Tu_0 on the inlet of the duct and length scale l . The turbulence intensity and the length scale were defined by the following relations

$$Tu_0 = 0.16Re^{-1/8} \quad l = 0.07D_h \quad (3.3)$$

The mentioned uniqueness of the description of the examined flow requires that a precise geometrical model is created. This model contains an exact 3D geometry of the bend along with corresponding straight ducts on the inlet and outlet as presented in Fig. 2. The model was meshed in GAMBIT, a FLUENT pre-processor.

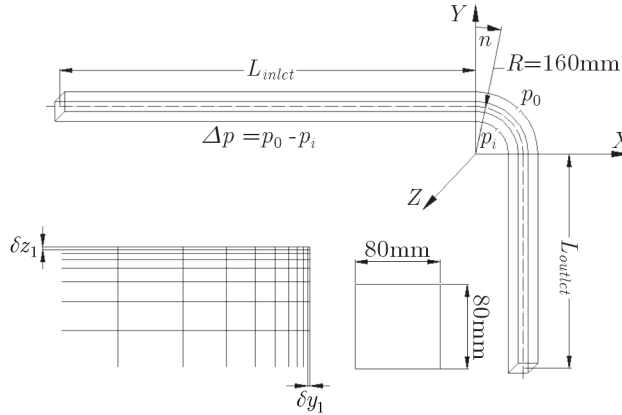


Fig. 2. Geometry dimensions and mesh in the boundary layer of the volume

To achieve high accuracy of the results, a fine mesh was created, especially in areas where large velocity gradients exist, that is near the duct walls. The ratio of the smallest element adjacent to the wall to the hydraulic diameter $\delta y_1/D_h = \delta z_1/D_h = 0.00075$ and the growth factor (ratio of the preceding element to the next element) is 1.3 for 16 elements. The size of the last element is $\delta y_{16}/D_h = \delta z_{16}/D_h = 3.992/80 = 0.04991$.

The longitudinal dimensions of the finite volumes increased from $\delta x'_1/D_h = 4/80 = 0.05$ at the inlet of the duct ($x'/D_h = -1600/80$) to $\delta x'_{28}/D_h = 8/80 = 0.10$ at two hydraulic diameters downstream from the inlet ($x'/D_h = -1440/80$) and then remained constant ($\delta x/D_h = 0.10$) to two hydraulic diameters from the bend ($x'/D_h = -160/80$). From this point, the length of the volumes decreased to a value of $\delta x'_{216}/D_h = 4/80 = 0.05$. The length of the volumes within the bend was $\Delta\varphi' = 4/80 = 0.05$ on the outer wall of the bend and decreased towards the inner wall. The finite volumes in the outlet section of the duct increased from $\delta x/D_h = 0.05$ to $\delta x/D_h = 0.10$ at four hydraulic diameters downstream from the bend. The lengths of the remaining volumes were constant ($\delta x/D_h = 0.10$). The total number of segmented control volumes in the model shown in Fig. 2 is 1034775, while the number of grid nodes is 1083392. The obtained grid utilises only prismatic

elements based on squares. Figure 2 presents a sample mesh at the inlet section of the duct.

It is worth mentioning that the generated mesh is characterised by a very small skewness coefficient of the finite volumes. The maximum value of the skewness coefficient in the presented examples does not exceed 0.5.

The simulation was carried out in FLUENT, a commercially available solver. The obtained results were compared with the corresponding results found in the reference elaborated by Sudo *et al.* (2001). In this case, the geometrical model was such that the parameters describing the fluid flow and its physical properties were identical as the ones described by Sudo *et al.* (2001).

In the reference, mentioned above, the development of steady turbulent flow of air was examined in a curved $80\text{ mm} \times 80\text{ mm}$ square duct. The mean bend radius was $R_s = R/D_h = 2.0$. To measure the longitudinal components of velocity a hot-wire anemometer was used.

Figure 3 represents the longitudinal component of average velocity profile of the air flow in the section of the curved square duct described by the following coordinates: $x'/D_h = -1.0$ and crosswise $z'/D_h = 0$. The Reynolds number: $Re = 40000$. The points connected with lines represent numerical simulation values for RSM model. The discrete points represent experimental values obtained by Sudo *et al.* (2001).

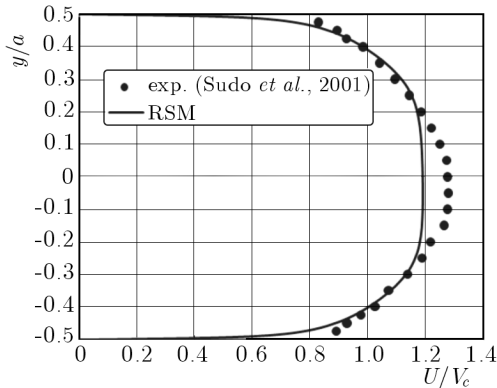


Fig. 3. Comparison of velocity profiles obtained with RSM model with experimental results for $x'/D_h = -1.0$, $z'/D_h = 0$

Figures 4 and 5 represent similar profiles measured for $\varphi = 60^\circ$ and $z/D_h = 0$ and for $x/D_h = 10.0$ and $z/D_h = 0$, respectively. It can be clearly seen in Figures 3 to 5 that numerical results and experimental results show a deformation of the velocity profile caused by the curvature of the duct as presented by Sudo *et al.* (2001).

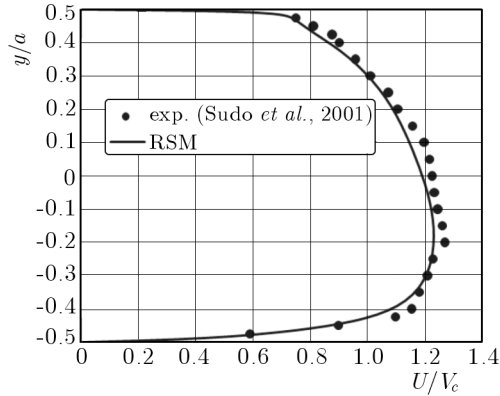


Fig. 4. Comparison of velocity profiles obtained with RSM model with experimental results for $\varphi = 60^\circ$ ($z/D_h = 0$)

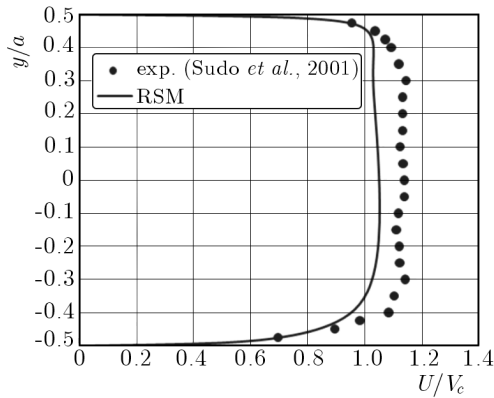


Fig. 5. Comparison of velocity profiles obtained with RSM model with experimental results for $x/D_h = 10.0$, $z/D_h = 0$

This paper concerns a method designed to measure the flow rate of fluid in square-sectioned bends installed in ducts. The method of indirect measurement of the flow rate in circular-sectioned pipes, applied in the study, was earlier developed by Rup and Malinowski (2005).

The essence of this method is based on the measurement of pressure difference in stationary flow conditions on two opposite walls of the bend angle secant. In the next phase of the measurement, a comparison of the value of measured difference ($\Delta p_{exp} = p_i - p_0$) with the corresponding value obtained through numerical solution (Δp_{num}) is carried out. The numerical value (Δp_{num}) is obtained through solution of mass and momentum equations and equations describing the selected turbulence model; the examined value of vo-

lumetric (mass) flow is adjusted so as to satisfy the assumed accuracy of the condition of equality of both values being compared

$$F(\text{Re}) = |\Delta p_{exp} - \Delta p_{num}| \leq \varepsilon \quad (3.4)$$

where ε is a given value which equals in the example to the inaccuracy of pressure measurement.

In numerical computations, an exact geometry of the flow space is taken into consideration along with the measured temperature and pressure of fluid on the base of which its thermo-physical properties are described. In the first step of computations, two extreme values of the Reynolds number (Re_1 and Re_2) are assumed corresponding to the minimum and maximum values of volumetric flow rate being measured. In the next phase of computations, the unknown value of the the volumetric flow rate is numerically determined by minimizing the expression of the difference between measured value (Δp_{exp}) and the corresponding calculated value (Δp_{num}) (3.4). The process of minimizing of the mentioned pressure difference (measured and calculated values) is carried out by the method of secants. In other words, the principal of this method is to select such a value of the Reynolds number – used as a coefficient in the partial differential equations of fluid motion – for which the computed pressure difference (Δp_{num}) satisfies condition (3.4). The field of velocities and pressures in the fluid is being determined numerically for the assumed Re number values. The numerically found values of the pressure drop Δp_{num} differ in both extreme cases from the pressure drop measured in the same points $\Delta p_{exp} = p_0 - p_i$ (Figs. 1, 2). It is evident that the assumed extremely different values of the Reynolds number Re_1 and Re_2 do not meet condition (3.4) in both cases, thus they cannot constitute its radicals (solutions). In order to find the next approximation of condition (3.4), the mentioned secant method is being applied. Following the idea of the method of secants described by Taler and Duda (2006), the next n-th approximation of the radical of condition (3.4) is being determined as follows

$$\text{Re}_n = \text{Re}_{n-1} - \frac{F(\text{Re}_{n-1})(\text{Re}_{n-1} - \text{Re}_{n-2})}{F(\text{Re}_{n-1}) - F(\text{Re}_{n-2})} \quad \text{for } n > 2 \quad (3.5)$$

The process ends when condition (3.4) is satisfied. The volumetric rate of flow of the elbow flow meter in question is determined from the formula

$$Q = \frac{\pi D_h \nu}{4} \text{Re} \quad (3.6)$$

where Re is the last calculated value of the Reynolds number, ν – the kinematic viscosity.

Because of the lack of optimizing procedures in the FLUENT package, it was necessary to elaborate a suitable procedure coupling that package with the mentioned numerical procedure which implements the idea of the method of secants. That procedure was written in C++ language. The consecutive iterations used journal files of the package FLUENT according to Fluent User Manual (2008).

Table 1. Tabulation of measured and calculated values

Δp_{orf} [Pa]	Q_{orf} [m ³ /s]	Δp_{exp} [Pa]	Δp_{num} [Pa]	Re_{num} [-]	Q_{elb} [m ³ /s]	$\frac{Q_{orf}-Q_{elb}}{Q_{orf}}$ [-]
58.7	0.03584	20.7	20.6	29484.4	0.03446	0.038
166.4	0.05965	68.0	67.9	53526.1	0.06255	-0.049
362.2	0.08735	147.0	146.9	78736.1	0.09201	-0.053
440.6	0.09615	178.5	178.6	86814.2	0.10145	-0.055
587.4	0.11070	236.6	236.7	99941.1	0.11679	-0.055
704.9	0.12103	275.5	275.4	107805.3	0.12598	-0.041
802.8	0.12897	322.5	322.5	116662.2	0.13633	-0.057
969.2	0.14138	394.5	394.4	129010.4	0.15076	-0.066
1096.5	0.15013	444.3	444.2	136925.9	0.16001	-0.066
1292.3	0.16261	532.3	532.4	149890.3	0.17516	-0.077

At the end of calculations for the last value of Re , the procedure compares the values of pressure differences, and then starts next calculations assuming the newly found value of the Reynolds number or stops iteration when condition (3.4) is satisfied. The maximum number of iteration steps carried out in those investigations did not exceed 10. The values of the flow rates found by using the orifice Q_{orf} are given in Table 1. The measurement results and the Reynolds number Re determined according to the idea of the applied method are given in Table 1 as well. The relative deviations of experimentally determined values of Q_{orf} and the corresponding ones determined by implementing the applied method (Q_{elb}) shown in Table 1 are small.

The maximum relative deviation of the flow rate in the discussed elbow flow meter is $\Delta = 7.7\%$, and concerns the greatest rate of flow in that elbow flow meter. It is to be stressed that the mentioned relative deviation can be considerably minimized by using ducts with a lesser size or elbows with a smaller curvature radius in the construction of elbow flow meters for smaller flow rates. On the other hand, in the case of flows with a Reynolds number $Re > 200\,000$, it is advisable to apply flow meter constructions with greater size of ducts in order to eliminate, among other things, intense vibrations

of the duct which affect the accuracy of the measurement of the pressure difference Δp_{exp} .

4. Conclusions

The indirect method used to measure the volumetric flow rate of a fluid is characterised by high accuracy and repeatability. The high accuracy is possible due to a very realistic mathematical model of the complex flow in the curved duct. The indirect method eliminates the necessity of frequent calibration of the flow meter. The discussed elbow flow meter, implementing the extended intermediate measuring method, can be applied to determine the flow rate of gases as well as liquids and their suspensions. In the performed calculations, the fluid was treated as incompressible due to the fact that the maximum value of flow average velocity in experimental measurements was lower than 40 m/s.

The experimental investigations carried out indicate that the measured pressure differences Δp_{exp} at the end points of the secant of the elbow are nearly 3 times smaller than the corresponding pressure differences measured on the orifice at the same flow rate.

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Pomiar strumienia płynu w kanale z łukiem kolana o przekroju kwadratowym

Streszczenie

W pracy podjęto próbę wykorzystania zainstalowanych w kanałach przepływowych łuków kolana o przekroju kwadratowym do pomiaru strumienia objętości płynu. W tym celu wykorzystano opracowaną wcześniej metodę pomiaru pośredniego dla rurociągów o przekroju kołowym. Dla praktycznej realizacji pomiaru wspomnianego strumienia przepływu zbudowano stanowisko badawcze za łukiem kolana o przekroju poprzecznym w kształcie kwadratu (80×80 mm). Obliczenia wykonano za pomocą pakietu FLUENT 6.2. Uzyskane rezultaty porównano z odpowiednimi zmierzonymi za pomocą kryzy pomiarowej oraz z innymi wynikami doświadczalnymi dostępnymi w literaturze. Z analizy porównawczej wynika wysoki stopień zgodności otrzymanych rezultatów pomiaru pośredniego.

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