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# CLOSED FORM SOLUTION FOR THE COLLAPSE OF POLYGONAL THIN-WALLED COLUMNS IN THE AXIAL CRUSHING CASE

Yamen Maalej
*LA2MP, University of Sfax, ENIS, University of Tunis El Manar, ENIT, Tunis, Tunisia*
*e-mail: yamen.maalej@hotmail.com*

Fahmi Chaari
*LAMIH, UMR 8201 – CNRS, University of Valenciennes, Valenciennes Cedex, France*
*e-mail: fahmi.chaari@univ-valenciennes.fr*

Bassem Zouari
*LA2MP, University of Sfax, ENIS, Sfax, Tunisia; e-mail: bzouari@yahoo.com*

Eric Markiewicz
*LAMIH, UMR 8201 – CNRS, University of Valenciennes, Valenciennes Cedex, France*
*e-mail: eric.markiewicz@univ-valenciennes.fr*

The main objective of this paper is to propose a new closed form solution, useful in the pre-design stage, that allows one to calculate the mean load in the case of post-collapse of polygonal thin-walled columns in the axial crushing case. This model gives a rapid and accurate evolution of the normalized mean load as function of the corner element angle as well as the ratio between the corner length and the column thickness. To identify the parameters of this model, numerical simulations with an explicit finite element software have been carried out and then compared to experimental results reported in the literature. Finally, all these results combined with the findings based on the known generalized mixed model developed by other researchers working on this topic enabled one to establish the closed form solution. This is a unified and continuous closed form solution, which is suitable for different columns shapes, even non-conventional shapes obtained thanks to the development of extrusion techniques.

*Keywords:* crushing, collapse, load prediction, polygonal, thin-walled column, strain rate

## 1. Introduction

The transportation safety as well as the reduction of energy consumption are the main concerns of automobile manufacturers. They aim to lightweight vehicle structures while maintaining at least or improving energy absorption capability during an accident. Numerical simulation is one way to achieve these purposes. Nowadays, we can run models of vehicle crash with millions of degrees of freedom. Except that such techniques are used once the entire structure is defined and designed. In addition, these models require a lot of computational time, which makes them unsuitable in the pre-design stage of structural parts of the vehicle. An alternative way is to develop analytical or semi-analytical models able to provide rapid estimation and more adapted to parametric studies. This has engendered models of axial post collapse of thin-walled multicorner sheet metal columns that were initially developed by Abramowicz and Wierzbicki (1989). Based on these pioneering works, other contributions have also been proposed. We can cite those of one of the present authors (Markiewicz *et al.*, 1996a; Drazetic *et al.*, 1995; Markiewicz, 1994) for multi-thickness and multi-cells columns.

In the last decades, several authors were interested in the energy absorption characteristics of thin-walled columns, depending on the sheet thickness and the angle of the corner element formed by two adjacent sheets of the column. Numerous literatures published concern the collapse of these columns under axial, transverse or oblique loading by using theoretical modeling analysis, experimental and numerical investigation (Zhang and Zhang, 2012; Han and Park, 1999; Yamashita *et al.*, 2003; Nia and Hamedani, 2010). However, a significant part of those research works is concerned with square or rectangular columns (Meguid *et al.*, 1996; Zhao and Abdennadher, 2004; Zhang *et al.*, 2007) and does not cover the behavior of polygonal cross-section columns. Indeed, corner elements with acute or obtuse angle are representative elements of prismatic columns and are always constitutive elements of a multi-cell column or honeycomb material, widely used in industry. Moreover, the development of extrusion techniques provided a wider range of columns shapes and thicknesses. Due to the complexity of the problem, few studies are interested in the influence of the angle of the corner element. The reference model widely used in the literature is that of Abramowicz and Wierzbicki (1989) who proposed a suitable deformation mechanism called the generalized mixed model. This model was subsequently taken by several authors to validate numerical models (Abramowicz, 2003; Meguid *et al.*, 1996; Zhang and Zhang, 2012; Zhang and Huh, 2010) or to develop a simplified crash modeling approach dedicated to the pre-design stage (Halgrin *et al.*, 1993; Markiewica *et al.*, 1996b; Cornette *et al.*, 1999; Markiewicz, 1994; Drazetic *et al.*, 1993).

The aim of this paper is to predict the mean load in the case of post-collapse of polygonal thin-walled columns in the axial crushing case, as function of various parameters of the cross-section geometry, and to propose a new closed form solution, useful in the pre-design stage.

This paper has the following outline. In Section 2, numerical simulations using the finite element method with an explicit resolution scheme are performed on a set of regular polygonal columns in order to identify the cross-section effect on the instantaneous post-collapse and mean crushing force. In Section 3, the analytical model based on the generalized mixed model developed by Abramowicz and Wierzbicki (1989) is presented. In Section 4, the results of the analytical model, combined with the simulation results enabled one to establish the closed form solution. Wierzbicki and Jones (1989) proposed in the past a closed form solution for the collapse of thin-walled columns but they were limited to square and hexagonal shapes with a separate equation for each section. This paper overcomes this limitation, since the proposed closed form is here unified and continuous. It can estimate the mean crushing force for different cross-sections taking into account the real shape and aspect ratio.

## 2. Finite element simulations

A set of regular polygonal columns with triangular, square, hexagonal and octagonal sections is considered in this FE study by using Abaqus/Explicit software. The purpose is to identify the cross-section effect on the instantaneous post-collapse and mean crushing force using a finite element method with an explicit resolution scheme. These results will be compared later with an analytical model, and will be useful to propose a closed form solution for the prediction of energy dissipation properties.

As shown in Fig. 1, numerical models are formed by the column and two rigid square plates representing the crushing plates. The upper rigid plate is moving downwards from top with a prescribed velocity $V = 2\,\mathrm{m/s}$ to compress the column in the axial direction. The lower plate is assumed to be clamped. A rough contact is defined between the ends of the column and the rigid plate. Self-contact with 0.2 Coulomb friction coefficient is also defined between all sides of the column in order to prevent interpenetration of faces during crushing and the formation of folding lines. Furthermore, depending on the column cross-section geometry to be crushed, imperfections are introduced in the initial meshed geometry in order to “privilege” the formation of anti-symmetric folding mode in the crushing process. These imperfections are introduced as a perturbation of some selected nodes with an aptitude of 0.05 mm. The location of these nodes is selected to be situated approximately in the half-wave length. The columns are meshed with a quadrangular shell (S4R) with a reduced integration in the plane and five integration points in the thickness and an element size of 2 mm. All the columns have the same length $L = 125$ mm. Different aspect ratios $C/t$ of column are considered, where $C$ denotes the width of the corner element and $t$ is the thickness. The constitutive material of the columns is assigned as steel, with the following mechanical properties: Young's modulus $E = 200$ GPa, Poisson's ratio $\nu = 0.3$ and yield stress $\sigma_0 = 350$ MPa. The steel is considered elasto-plastic with isotropic hardening and a tangent modulus of $E_t = 430$ MPa.

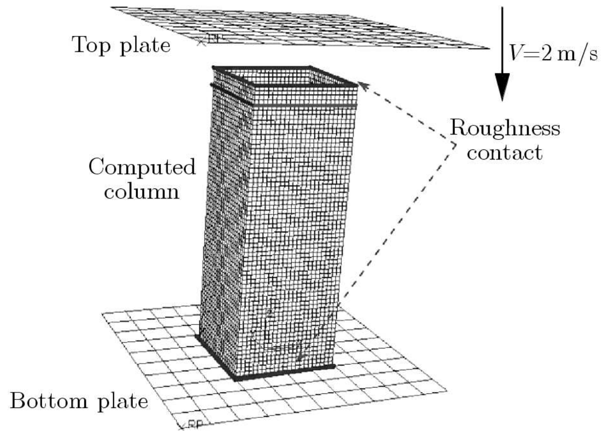


Fig. 1. FE Model description

To ensure the relevance of quasi-static simulation using an explicit finite element code, we firstly verified that the crushing force-displacement response must be independent of the applied velocity (considering that the constitutive material is not strain rate sensitive). Secondly, the total kinetic energy has been checked to remain negligible compared to the total internal energy during the whole crushing simulation. In addition, numerical instantaneous crushing forces for a square column cross-section column with the characteristics ($C = 47$ mm, $t = 0.9$ mm) have been compared to experimental results of Drazetic *et al.* (1995). As shown in Fig. 4, the numerical instantaneous crushing force is in agreement with the experimental results.

The deformed shapes of the columns, at different crushing distances are presented in Fig. 2. It can be found that all the columns deform in an asymmetric mode. Compared to the experimental tests performed by Zhang and Zhang (2012), it can be found that the deformed shapes of the numerical simulations are quite similar, which confirms the relevance of the numerical model.

Table 1 summarizes the results of the mean crushing force normalized by the number of corner elements $N$ of each cross-section geometry for different aspect ratios $C/t$. The results summarized in Table 1 are consistent with tendencies reported by Wierzbicki and Jones (1989) since the increasing of the number of corners leads to an increase in the crushing force per corner element. This tendency is valid for all the considered aspect ratios $C/t$.

**Table 1.** Mean crushing force per corner element for different aspect ratios and various cross--section geometry

| Cross-section | $N$ | $C/t = 30$ | $C/t = 52$ | $C/t = 80$ |
|---|---|---|---|---|
| Triangular | 3 | 6.55 | 2.95 | 1.70 |
| Square | 4 | 8.19 | 3.61 | 1.95 |
| Hexagonal | 6 | 9.53 | 4.19 | 2.04 |
| Octagonal | 8 | 10.10 | 4.27 | 2.09 |

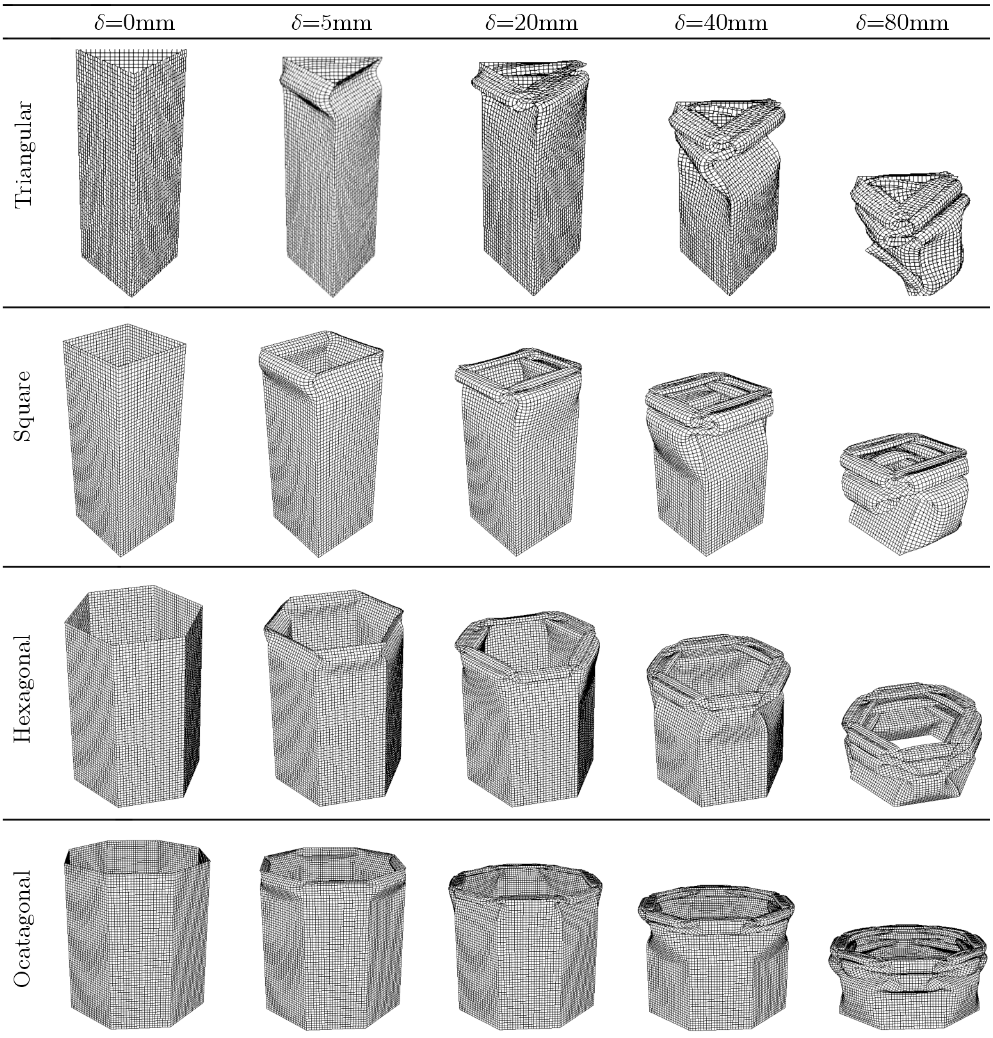


Fig. 2. Collapse modes versus crush distance for columns of triangular, square, hexagonal and octagonal cross-section geometry

The influence of the aspect ratio $C/t$ is also studied in this paper, and Table 1 shows, as expected and reported in previous studies, that for the same cross-section, increasing the aspect ratio causes a decrease in the crushing force.

## 3. Analytical model

### 3.1. Post-collapse mechanisms : generalized mixed model

The post-collapse phase of a thin-walled column is defined as the study of static deformation of a sheet surface and constitutes the basis of the generalized mixed model developed by Abramowicz and Wierzbicki (1989). The tube structure is broken down into basic corner elements with symmetry conditions. Each corner element is identified separately, $C$ denotes the width of the corner element, $t$ is the thickness of the sheet and $\varphi = \pi - 2\psi_0$ the angle between the two adjacent faces of the corner element (Fig. 3). $2\psi_0$ denotes the angle of intersection between lines $AD$ and $BC$. The constituent material is assumed to be rigid perfectly plastic, characterized by an energy equivalent flow yield stress $\sigma_0$. $M_0 = \sigma_0 t^2/4$ and $N_0 = \sigma_0 t$ designate respectively the bending moment and membrane stress per length unit at the limit of plastic flow $\sigma_0$.

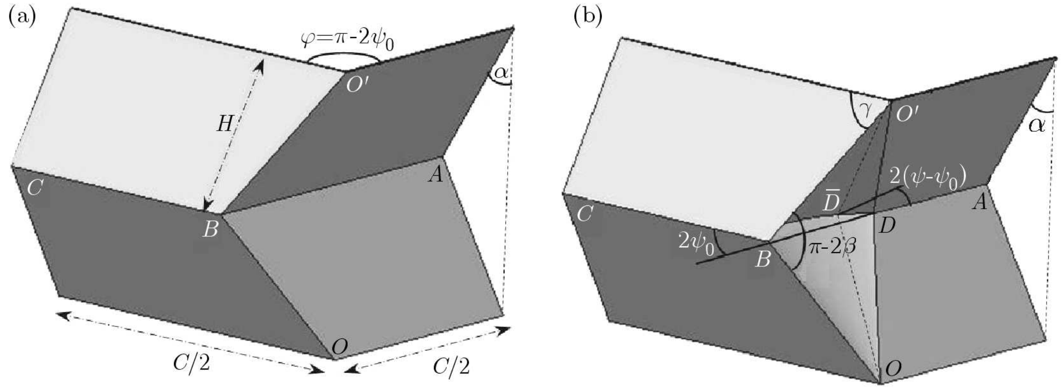


Fig. 3. Generalized mixed model; (a) phase I quasi inextensional, (b) phase II extensional

The idealized folding process, described by the rotation angle of the plates $\alpha$, is divided into two phases which are activated in series (Fig. 3). $H$ denotes the half-wavelength of plastic folding and $r$ the radius of the toroidal surface (point $B$ in Fig. 3).

- *Phase I, quasi inextensional (Fig. 3a):* characterized by three areas of energy dissipation and which persists until an intermediate configuration ($\alpha = \overline{\alpha}$):
  - a toroidal surface of radius $r$ (at point $B$ in Fig. 3) characterized by the energy dissipated rate $\dot{E}_1$

$$\dot{E}_1 = 4N_0 rH \cos\alpha \int\limits_0^{\beta(\alpha)} \frac{\dot{\alpha}}{\sqrt{\tan^2\psi_0 + \cos^2\phi}}\, d\phi \tag{3.1}$$

  - two horizontal stationary folding lines $AB$ and $BC$, each of them dissipating the energy rate $\dot{E}_2$

$$\dot{E}_2 = 2M_0 C\dot{\alpha} \tag{3.2}$$

  - two inclined moving hinge lines $OB$ and $O'B$ each of them dissipating the energy rate $\dot{E}_3$

$$\dot{E}_3 = 4M_0 \frac{H^2}{r} \tan^{-1}\psi_0 \cos\alpha \sqrt{\tan^2\psi_0 + \sin^2\alpha}\,\dot{\alpha} \tag{3.3}$$

- *Phase II, extensional (Fig. 3b):* from this intermediate configuration, the plastic inclined hinge lines $OBO'$ become stationary and split by rotating about the vertical axis $OO'$. So there is formation of two conical surfaces characteristics of the material extension and deflection of the line $ADB$ initially straight. The results in the extensional phase are also characterized by three areas of energy dissipation. These areas remain until the final configuration ($\alpha = \alpha_f$). The three dissipation areas are:
  - two conical surfaces $OBD$ and $O'BD$ each of them dissipating an energy rate $\dot{E}_4$

$$\dot{E}_4 = 4M_0 V_t \frac{H}{t} \tag{3.4}$$

  where $V_t$ is the tangential extension velocity at the cones

$$V_t = 2H\Big[\frac{\sin\overline{\alpha}\tan\psi_0 \sin 2\alpha}{2(\sin^2\overline{\alpha} + \tan^2\psi_0 \sin^2\alpha)} + (\psi - \psi_0)\cos\alpha\Big]\dot{\alpha} \tag{3.5}$$

- two horizontal stationary folding lines $AD$ and $BC$ each of them dissipating the same energy rate consumed in Phase I $\dot{E}_5 = \dot{E}_2$
- two inclined moving hinge lines $OB$ and $O'B$ each dissipating the energy rate $\dot{E}_6$

$$\dot{E}_6 = 2M_0 H \tan^{-1}\psi_0 \frac{\sin\overline{\alpha}(\sin^2\overline{\alpha} + \tan^2\psi_0)}{\sin^2\overline{\alpha} + \tan^2\psi_0 \sin^2\alpha}\dot{\alpha} \tag{3.6}$$

### 3.2. Theoretical model for the collapse crushing force

In this section, we present the methodology adopted to calculate the mean crushing force $P_m$ and the instantaneous post-collapse crushing force $P(\delta)$ according to the mechanisms described by the generalized mixed model. $\delta$ is the crushed distance that can be deducted from the angle rotation of the plates $\alpha$

$$\delta = 2H(1 - \cos\alpha) \tag{3.7}$$

The methodology consists in choosing a kinematic model of deformation defined by the vector of unknown parameters $\boldsymbol{\chi} = (r, H, \overline{\alpha})$. $r$ is the radius of toroidal surface, $H$ is the half-wavelength of plastic folding and $\overline{\alpha}$ is the switching angle between the quasi inextensional and extensional phase. The initial geometry is defined by the known parameters vector $\boldsymbol{\xi} = [C, t, 2\psi_0]$.

Assuming a rigid perfectly plastic material characterized by the energy equivalent flow stress $\sigma_0$, the principle of virtual power is then conducted. The instantaneous post-collapse crushing force $P(\delta)$ is calculated by summing the elementary efforts achieved for each corner element constituting the column. Applying the principle of virtual power, the equality between the internal and external rate of energies yields

$$\dot{E}_{int} = \dot{E}_{ext} = P(\delta)\dot{\delta} \tag{3.8}$$

where $\dot{\delta}$ is the crushing velocity.

Using the notation of Fig, 3, the total energy rate dissipated within the $j$-th corner element $\dot{E}^j_{int}$, subjected to a generalized mixed type deformation mechanism, reads

$$\dot{E}^j_{int} = \dot{E}^j_1 + 2\dot{E}^j_2 + 2\dot{E}^j_3 + 2\dot{E}^j_4 + 2\dot{E}^j_5 + 2\dot{E}^j_6 \tag{3.9}$$

$\dot{E}^j_i$ denotes the energy rate dissipated in the $i$-th dissipation area of the $j^{th}$ corner element.

Each component of the energy rate dissipation depends on the unknown variables $r$, $H$ and which will be determined later by means of minimizing the mean crushing force $P_m$. Given a column configuration constituted by an assembly of $N$ coin elements, the total internal energy rate can be computed by adding up the contributions of each corner element dissipation. Using the principle of virtual power, Eq. (3.8), the instantaneous post-collapse crushing force $P(\delta)$ yields

$$P(\delta) = \frac{1}{\dot{\delta}}\sum_{j=1}^{N}\dot{E}^j_{int} \tag{3.10}$$

The mean crushing force $P_m$ is then deduced depending on the unknown vector $\boldsymbol{\chi} = (r, H, \overline{\alpha})$ by integration of equation (3.10) between starting $(\alpha = 0)$ and final position $(\alpha = \alpha_f)$. For a single corner element, the mean crushing force $P_m$, normalized by the bending moment per unit length $M_0$, can be given as the sum of the following split terms

$$\frac{P_m(H, r, \overline{\alpha})}{M_0} = \Big[A_1\frac{r}{t} + (A_2 + A_5)\frac{C}{H} + A_3\frac{H}{r} + A_4\frac{H}{t} + A_6\Big]\frac{2H}{\delta_f} \tag{3.11}$$

where $A_i$ assigned to their respective coefficients denote the energy consumed during the collapse phase.

$A_1$, $A_2$ and $A_3$ are respectively the integration results of $\dot{E}_1$ (3.1), $\dot{E}_2$ (3.2) and $\dot{E}_3$ (3.3) between the starting $(\alpha = 0)$ and intermediate position $(\alpha = \overline{\alpha})$.

$A_4$, $A_5$ and $A_6$ are respectively the integration results of $\dot{E}_4$ (3.4), $\dot{E}_5$ (3.2) and $\dot{E}_6$ (3.6) between the intermediate $(\alpha = \overline{\alpha})$ and final position $(\alpha = \alpha_f)$.

$\delta_f$ is the effective final crushed distance associated with the final position $(\alpha = \alpha_f)$.

The unknown vector $\boldsymbol{\chi} = (r, H, \overline{\alpha})$ of the problem is then determined so as to minimize the mean crushing force $P_m(H, r, \overline{\alpha})$

$$\frac{\partial}{\partial \boldsymbol{\chi}} P_m(H, r, \overline{\alpha}) = 0 \tag{3.12}$$

The resolution of this problem is performed analytically using a computer algebra system (MAPLE). The unknown vector $\boldsymbol{\chi} = (r, H, \overline{\alpha})$ is then fed back into equation (2.10) so as to deduce the instantaneous post-collapse crushing force and substituted in equation (3.11) in order to estimate the normalized mean crushing force for a single corner element. The total mean crushing force for a column constituted by $N$ corner elements is then obtained by summing up the contributions of each corner element.

The analytical model allows us to compute the evolution of the instantaneous post-collapse crushing force $P(\delta)$ of the column by using equation (3.10). In this equation, we substitute the vector of unknowns $\boldsymbol{\chi} = (r, H, \overline{\alpha})$ by its value obtained by the minimizing of mean crushing force (3.12). Figure 4 shows the evolution of this instantaneous force obtained by the analytical model and by finite element computations on a square column with the characteristics: $C = 47$ mm, $t = 0.9$ mm, $E = 200$ GPa, $\nu = 0.3$, $\sigma_0 = 350$ MPa and $E_t = 430$ MPa.

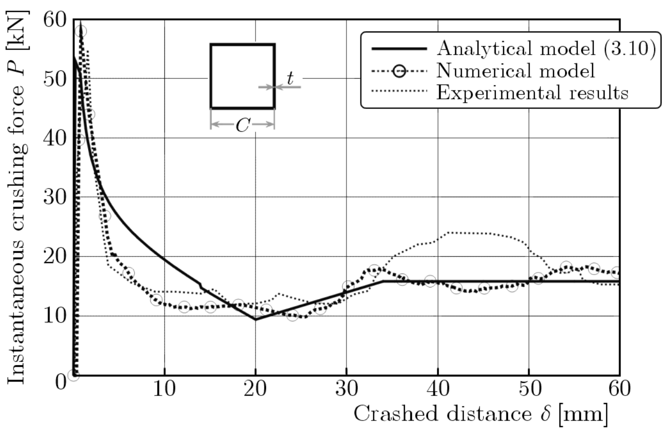


Fig. 4. Evolution of the instantaneous crushing force $P(\delta)$ for a square column using the analytical and numerical model and compared to the experimental results from Drazetic *et al.* (1995)

The analytical model provided the mean crushing force close to that obtained by the finite element calculations, however, the fall of the instantaneous crushing force effort at the beginning of the post-collapse phase is slower in the analytical model as compared to the simulation.

It is worth noting that the analytical model of the pre-collapse phase is not presented in this paper. Consequently, the analytical maximum crushing force is not defined. For the quasi-static and dynamic crushing behavior in the pre-collapse phase, we can refer to previous works done by one of the present author (Drazetic *et al.*, 1995; Markiewicz *et al.*, 1996a).

## 4. Closed form solution for the mean crushing force

By taking advantage of the developed analytical model and its programming on MAPLE software, we look to elaborate an empirical model traducing the variation of the mean crushing force versus the aspect ratio $C/t$ and the corner element angle $\varphi$. The purpose is to provide a closed form solution for the mean crushing force useful in the pre-design stage. To achieve this aim, the normalized mean crushing force $P_m/M_0$ is computed for different configurations of the aspect ratio $C/t$ in the case of a triangular $(\varphi = 60°)$, square $(\varphi = 90°)$, hexagonal $(\varphi = 120°)$ and octagonal $(\varphi = 135°)$ cross-section. Figure 5 shows the evolution of the normalized mean crushing force versus the aspect ratio for different cross-section geometries. In the same Fig. 5, results from numerical simulations of Section 2 (Table 1) are also presented. We propose to fit these curves by the following function

$$\frac{P_m}{M_0}\Big(\frac{C}{t},\varphi\Big) = A(\varphi)\Big(\frac{C}{t}\Big)^{B(\varphi)} \tag{4.1}$$

$A(\varphi)$ and $B(\varphi)$ are two parameters depending only on the corner element angle $\varphi$. The dependence function is achieved by interpolating the results of analytical calculation.

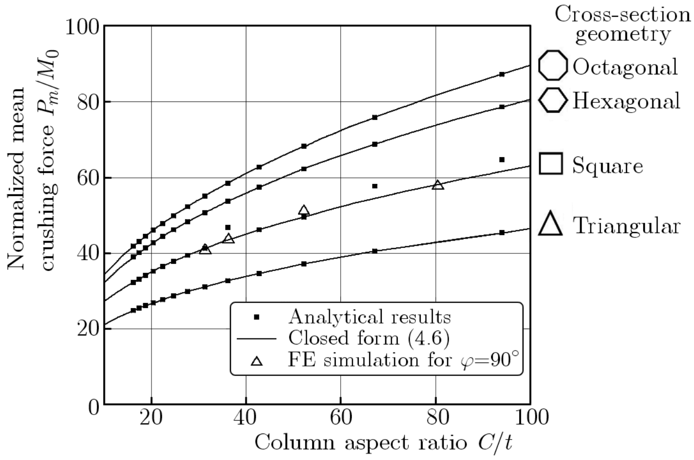


Fig. 5. Evolution of the normalized mean crushing force $P_m/M_0$, in terms of the aspect ratio $C/t$ and for various cross-sections of coin elements

The form of this function is in concordance with the empirical formula given by Abramowicz and Wierzbicki (1989) for the square and hexagonal cross-section geometry. The proposed model is wider since it provides an extrapolation for the corner angle ranging from 60° to 135°. Figure 6a represents the analytical coefficient $A(\varphi)$ for $\varphi$ angle ranging from 60° to 135°. A second order polynomial interpolation is well suitable

$$A(\varphi) = 2.13 + 9.44\varphi - 2\varphi^2 \tag{4.2}$$

Figure 6b represents the analytical exponent coefficient $B(\varphi)$ for $\varphi$ angle ranging from 60° to 135°. A first order polynomial interpolation is well appropriated for this exponent

$$B(\varphi) = \frac{1}{3} + 0.06\Big(\varphi - \frac{\pi}{3}\Big) \tag{4.3}$$

Finally, a closed form formula for the normalized mean crushing force for a corner element can be proposed

$$\frac{P_m}{M_0}\Big(\frac{C}{t},\varphi\Big) = (2.13 + 9.44\varphi - 2\varphi^2)\Big(\frac{C}{t}\Big)^{\frac{1}{3}+0.06\left(\varphi-\frac{\pi}{3}\right)} \tag{4.4}$$

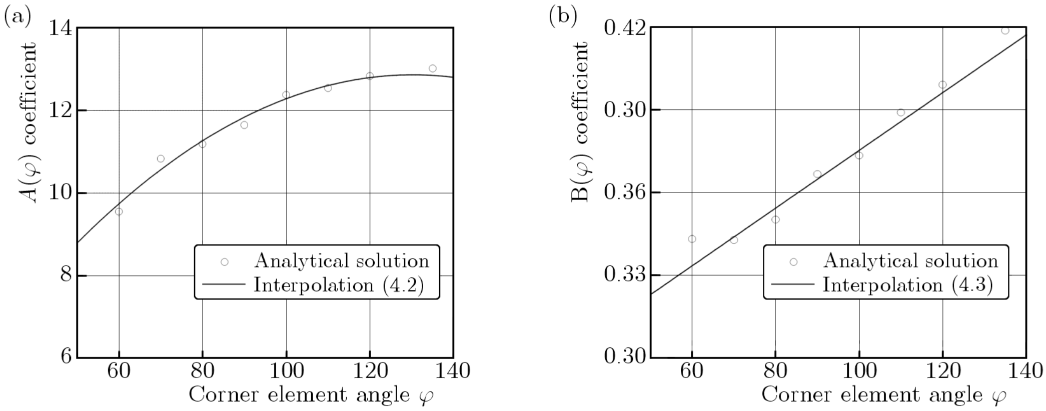


Fig. 6. Evolution of (a) coefficient $A(\varphi)$ and (b) exponent $B(\varphi)$ versus corner element angle $\varphi$

Wierzbicki and Jones (1989) proposed the following similar form with a separate equation for the mean crushing force of square and hexagonal cross-section geometry

$$\frac{P_m}{M_0}\Big(\frac{C}{t}\Big) = \begin{cases} 12.16\Big(\frac{C}{t}\Big)^{0.37} & \text{for a square cross-section and} \\ 13.49\Big(\frac{C}{t}\Big)^{0.4} & \text{for an hexagonal cross-section} \end{cases} \tag{4.5}$$

A comparison with these empirical results gives a small discrepancy summarized in Table 2.

**Table 2.** Discrepancy between closed form solution (4.4) and equation (4.6)

| Coss-section | $C/t = 30$ | $C/t = 52$ | $C/t = 80$ |
|---|---|---|---|
| Square | 1.89% | 2.18% | 2.40% |
| Hexagonal | 3.94% | 4.14% | 4.30% |

The proposed closed form is more general since it can estimate the mean crushing force for any cross-section geometry.

The proposed closed form can be more simplified by neglecting the influence of the corner element angle $\varphi$ in the exponent coefficient $B(\varphi)$ (4.3): $B(\varphi) \simeq 1/3$, in which case, the mean crushing force can be normalized by $\sqrt[3]{C/t}$ so as to obtain a simple and aspect ratio independent equation

$$\frac{P_m}{M_0\sqrt[3]{C/t}} = 2.13 + 9.44\varphi - 2\varphi^2 \tag{4.6}$$

To validate the results given by the analytical model and simplified empirical formula (4.6), the evolution of the normalized mean crushing force as function of the corner element angle $\varphi$ has been performed. These results are compared with those obtained experimentally (Abramowicz and Jones, 1984; Abramowicz and Wierzbicki, 1989) and numerically by the finite element simulation (Section 2, Table 1). Figure 7 illustrates the evolution of the normalized mean crushing force as function of the corner element angle $\varphi$. We can notice, as a general trend, that our purely analytical results recapitulated by useful formula (4.4) and the numerical simulations are in a good agreement with the experimental results (Abramowicz and Jones, 1984; Abramowicz and Wierzbicki, 1989) for $\varphi$ ranging from $60°$ to $120°$.

In crash application, the influence of the strain rate is necessary to be take into account. Indeed, in terms of the crash application, the good determination of historical crushing response requires consideration of inertia and the strain rate effect. For the post-collapse phase, subject

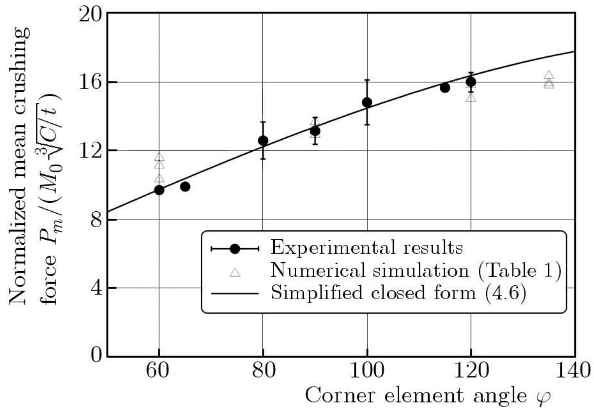


Fig. 7. Evolution of the normalized mean crushing force $P_m/(M_0\sqrt[3]{C/t})$ as function of the corner element angle $\varphi$ using simplified closed formula (4.6) and compared to numerical simulation (Table 1) and experimental results from Abramowicz and Jones (1984) and Abramowicz and Wierzbicki (1989)

of our study, many studies were conducted by Jones (1997), Abramowicz and Jones (1984), and Reid (1993). All considered that due to the large plastic deformation the inertia is negligible, and only the sensitivity of the material strain rate influenced the mean crushing dynamic force. The consideration of the strain rate effect can be done by using a dynamic correction laws type stress-strain rate. Several dynamic correction laws have been developed but the most widely used are the Johnson and Cook (1983) and Cowper and Symonds (1967) laws. The Cowper and Symonds (1967) dynamic correction model relates material static $\sigma_0$, and dynamic $\sigma_{0D}$ equivalent flow yield stresses to the mean strain rate $\bar{\dot{\epsilon}}$ according to

$$\sigma_{0D} = \sigma_0 \left(1 + \sqrt[p]{\frac{\bar{\dot{\epsilon}}}{D}}\right) \tag{4.7}$$

where $p$ and $D$ are respectively the rate of viscoplasticity and the sensitivity to the strain rate experimental parameters. They are fitted so as to well describe the material sensitivity to the strain rate in an axial crushing test.

## 5. Discussion and conclusion

The Analytical model and empirical formula for the determination of the mean crushing force have been presented in this paper. The studied polygonal columns were subjected to an axial crushing loading and were made of an elastic perfectly plastic material. Numerical simulations were conducted by using nonlinear explicit finite element software ABAQUS. The mean crushing forces of these polygonal columns derived by analytical model and numerical results were in good agreement with the experimental results and theoretical predictions identified in the literature.

The case of triangular cross-section presented in this paper is known to be unstable experimentally and is not usually used for energy absorption application. Nevertheless, a choice is made to consider it in the numerical and analytical parts since it is the lower "limit" border of the validity of the proposed closed form solution.

In the present study, the resolution of the governing equations of the column crushing performed analytically using a computer algebra system (MAPLE), unlike in previous contributions (Abramowicz and Wierzbicki, 1989; Wierzbicki and Jones, 1989; Drazetic *et al.*, 1995) incorporated numerical estimation. The developed model was used to define a closed form solution useful in the pre-design stage, which allows calculating the mean crushing axial force for polygonal thin-walled multicorner columns. Such a solution is much appreciated especially in the pre-design phase due to its rapidity compared to finite element models. The proposed here closed form is unified and continuous. It can estimate the mean crushing force for a large range of cross-sections geometries, taking into account the aspect ratio and even for non-conventional shapes obtained thanks to the development of extrusion techniques, but of course as long as we remain in the domain of validity of the analytical model.

Motivated by practical purposes, in particular for real crash events where the axial crushing mode of columns is always happening together with the bending collapse mode, an extension to oblique impact loading of mutlicorner thin-walled columns is the matter of on-going research.

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