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A NONLINEAR MODEL FOR THE ELASTOPLASTIC ANALYSIS OF 2D FRAMES ACCOUNTING FOR DAMAGE

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A simple and efficient procedure for non-linear analysis of frames is presented, under the hypothesis that the non-linear effects, if appear, are concentrated in the beam-ends. We consider a damage model based on Continuum Damage Mechanics, but affecting the cross-section as a whole. The elastoplastic behaviour is included formulating the tangent elastoplastic stiffness matrix in such a way that the yield function, in terms of internal forces (axial, shear and bending moment), is affected by the damage in each plastic cross-section. After the verification of the model, an example of application is solved for different assumptions on the yield function (depending on the internal forces considered) with the damage being taken into account or disregarded. The differences on the collapse load, for each case, are shown and some conclusions obtained, among them that the method can evaluate in a more accurate way the load that causes the collapse of frames under increasing loading, considering a fully plastic non-linear analysis.

Key words: plastic methods, structural analysis, material nonlinearities, elastoplastic stiffness matrix, Bonora damage model

1. Introduction

In the civil and structural engineering, there are several approaches to deal with damage. The structural damage can be quantified through a damage index, which is the value of damage normalized to the failure level of the structure: a value equal to 1 corresponds to the complete structural failure (Faleiro *et al.*, 2008), so the structure can not withstand further loadings.

In this paper, the damage index is derived from Continuum Mechanics and Ductile Fracture theories applied to metallic materials. Using standard stress-strain relationships in elastoplasticity together with thermodynamic laws for irreversible processes, and assuming that fracture takes place at a certain rate of plastic deformation, after several mathematical manipulations it is possible to couple general plasticity theory with damage theory through the hypothesis of strain equivalence (Lemaitre, 1985) to relate equivalent plastic deformation with damage.

The Continuum Damage Mechanics (CDM) approach, initially proposed by Lemaitre, takes into account the effects associated to a given damage state through the definition of an internal state variable. The set of constitutive equations for the damaged material is then derived within a thermodynamic framework. Many authors have modified Lemaitre's linear damage accumulation law in order to be able to incorporate experimental damage measurements with different types of materials fitting in it. A nonlinear CDM model, recently proposed by Bonora (Bonora, 1997, 1998; Bonora *et al.*, 2005) is able to precisely describe the damage evolution for different types of metals and has been used by other authors (Bobiński and Tejchman, 2006; Chaboche, 1984; Chandrakanth and Pandey, 1993, 1995a; Tai and Yang, 1986; Tai, 1990).

The aim of this paper is to develop a general, accurate, efficient and simple procedure for solving the fully non-linear problem of framed structures, using elastoplastic beam finite elements and considering material nonlinearities and the loss of rigidity due to the increase of damage in the cross-section and using an explicit form of the tangent stiffness matrix, called the *elastoplastic damaged stiffness matrix* (Ibijola, 2002; Yingchun, 2004). The basis of this method is a direct combination of existing formulations (Navier-Bernoulli's beam theory and Bonora's CDM damage model) to determine in a more accurate way the collapse load of standard frames.

2. Damage model for cross-sections of beams based on CDM

From a general point of view, damage can be defined as a progressive loss of load carrying capability as a result of some irreversible processes that occur in the material microstructure during the deformation process (Lemaitre, 1985). Assuming that micro-cracks and micro-voids have a uniformly distributed orientation, the scalar D can be defined in terms of the relative reduction of the cross-section (Lemaitre, 1984)

$$D = 1 - \frac{A_{\mathit{eff}}}{A_0} \tag{2.1}$$

where A_0 is the initial section and A_{eff} is the effective area: $A_{\mathit{eff}} = A_0(1-D)$.

For every value of $D \in [0,1)$, the effective stress and strain for uniaxial behaviour can be defined (Simo and Ju, 1987)

$$\text{Effective stress:} \qquad \overline{\sigma} = \frac{\sigma}{(1-D)} = \frac{F}{A_{\mathit{eff}}} \tag{2.2}$$

$$\text{Effective strain:} \qquad \overline{\varepsilon} = (1-D)\varepsilon \tag{2.3}$$

where ε and σ are the usual strain and stress Cauchy tensors. For a virgin material, $D = D_0 \approx 0$ and for a exhausted state $D = D_{cr} < 1$, where D_0 is the initial amount of damage and D_{cr} is the critical damage.

Then, assuming the hypothesis of strain and stress equivalence, the material behaviour for a damaged material can be written as

$$\varepsilon = \frac{\overline{\sigma}}{E} = \frac{\sigma}{(1-D)E} \tag{2.4}$$

and now it is necessary to show the evolution of D from its initial value D_0 (usually 0) to D_{cr}, value less than or equal to 1 from which the former expressions are not considered valid.

For the Bonora (1997) model assumed, D depends only, for each material and temperature T, on the amount of equivalent plastic strain through the following expressions

$$\begin{aligned} \phi &= F_p(\sigma_{eq}, R, \sigma_y) + \phi^*(Y, D, \dot{p}, T) \\ Y &= -\frac{\sigma_{eq}^2}{2E(1-D)^2}\Big[\frac{2}{3}(1+\nu) + 3(1-2\nu)\Big(\frac{\sigma_m}{\sigma_{eq}}\Big)^2\Big] \end{aligned} \tag{2.5}$$

where ϕ is the total dissipation potential (in function of the equivalent stress σ_{eq}, material hardening R and yield stress σ_y), ϕ^* is the damage dissipation potential and Y is the damage energy release rate. F_p is the dissipation potential associated with plastic deformation, $\dot{p}$ is the accumulated effective plastic strain, σ_m is the hydrostatic stress, σ_{eq} is the Von Mises equivalent stress, ν is the Poisson ratio, E is the Young modulus and the relation σ_m/σ_{eq} is called the *triaxiality ratio* or *stress rigidity parameter* (Lebedev *et al.*, 2001).

Bonora proposed the following expression for the damage dissipation potential

$$\phi^* = \Big[\frac{1}{2}\Big(-\frac{Y}{S_o}\Big)^2 \frac{S_o}{1-D}\Big] \frac{(D_{cr}-D)^{\alpha-1/\alpha}}{p^{(2+n)/n}} \tag{2.6}$$

where S_o is a material constant, n is the Ramberg-Osgood material exponent, α is the damage exponent that determines the shape of the damage evolution and p is the accumulated plastic strain.

Assuming that the rate of the plastic multiplier $\dot{\lambda}$ is proportional to the rate of the effective accumulated plastic strain $\dot{p}$

$$\dot{\lambda} = \dot{p}(1-D) \tag{2.7}$$

and that for proportional loading the kinetic law, according to Lemaitre's model, is

$$\dot{D} = -\dot{\lambda}\frac{\partial \phi^*}{\partial Y} \tag{2.8}$$

the relationship between damage and effective plastic strain is, finally

$$D = D_0 + (D_{cr} - D_0)\left\{1 - \left[1 - \frac{\ln \frac{p}{p_{th}}}{\ln \frac{p_{cr}}{p_{th}}}\right]^\alpha\right\} \tag{2.9}$$

where p_{th} is the plastic threshold value and p_{cr} is the critical plastic value corresponding to D_{cr} (Bonora, 1997).

3. Elastoplastic stiffness matrix considering damage

Assuming standard elastoplastic behaviour (Deierlein *et al.*, 2001) for the beam element, with additive decomposition of displacements du^{ep} at the ends of the element into elastic du^e and plastic du^p components

$$\{du^{ep}\} = \{du^e\} + \{du^p\} \tag{3.1}$$

and that plastic deformation takes place only on the beam-ends (concentrated plasticity), and hence also damage, the resulting beam model is represented in Fig. 1, where the initial and deformed configurations are shown (note that the length of small segments at the beam-ends should be infinitesimal).

This concentrated plasticity model does not account for the spreading of plasticity from outer fibers inwards. This behaviour could be considered using more advanced models like the layered approach (Chandrakanth and Pandey,

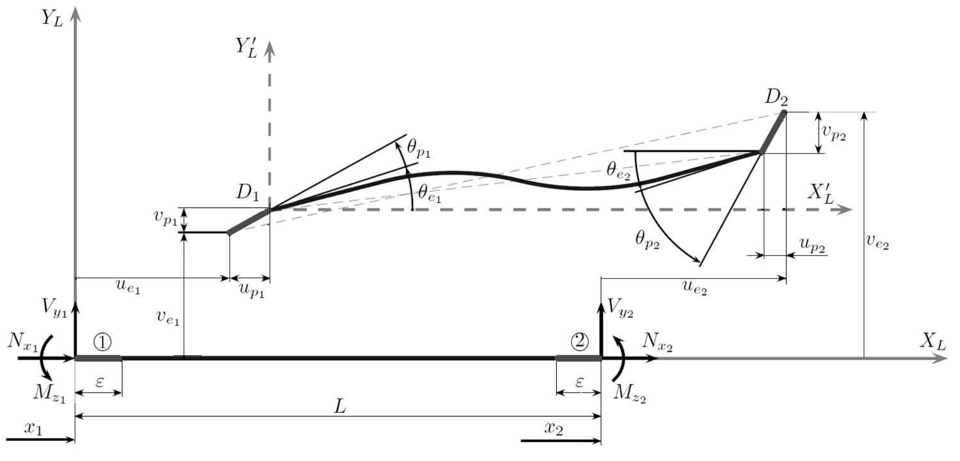

Fig. 1. Beam element with plasticity and damage at its ends

1997) but, in engineering practice, the distributed plasticity models are less frequently used than frame theories with concentrated plastic hinges (Inglessis *et al.*, 1999).

The linear elastic response is governed by

$$\{dF\} = [K]\{du^e\} \tag{3.2}$$

where $[K]$ is the standard elastic stiffness matrix of a beam element and $\{dF\}$ is the beam-end force vector which for the 2D case presented in this paper is $\{dF\}^\top = \{dN_{x1}, dV_{y1}, dM_{z1}, dN_{x2}, dV_{y2}, dM_{z2}\}^\top$, and the displacement vector is $\{du^e\}^\top = \{u_{x1}, u_{y1}, \theta_1, u_{x2}, u_{y2}, \theta_2\}^\top$.

In a similar way, it is necessary to determine the relationship between the increment of force and the increment of elastoplastic displacement $\{du^{ep}\}$

$$\{dF\} = [K^{ep}]\{du^{ep}\} \tag{3.3}$$

From Eqs. (3.1) and (3.2)

$$\{dF\} = [K]\Big(\{du^{ep}\} - \{du^p\}\Big) \tag{3.4}$$

so the increment of plastic displacement $\{du^p\}$, assuming associated flow rule, can be expressed as

$$\{du^p\} = \{d\lambda\}\left\{\frac{dZ}{dF}\right\} \tag{3.5}$$

where Z is the yield function for the beam element and $\{d\lambda\}$ is the vector of so-called plastic multipliers $d\lambda_1, d\lambda_2$ in each beam-end. Using the plastic consistency condition

$$d\lambda \begin{cases} = 0 & \text{if} \quad Z < 0 \quad \text{or} \quad p < p_{th} \\ > 0 & \text{if} \quad Z = 0 \quad \text{or} \quad p \geqslant p_{th} \end{cases} \tag{3.6}$$

together with Eq. (2.8) for the elastoplastic beam element

$$\{dD\} = \{d\lambda\} \left\{ \frac{-\partial \phi^*}{\partial Y} \right\} \tag{3.7}$$

and the plastic flow rule condition

$$\dot{Z} = \left\{ \frac{\partial Z}{\partial F} \right\} \{dF\} + \left\{ \frac{\partial Z}{\partial D} \right\} \{dD\} = 0 \tag{3.8}$$

and substituting Eqs. (3.4) and (3.7) into Eq. (3.8), the following expression for $\{d\lambda\}$ can be found

$$\{d\lambda\} = \frac{\left\{ \frac{\partial Z}{\partial F} \right\} [K] \{du^{ep}\}}{\left\{ \frac{\partial Z}{\partial F} \right\} [K] \left\{ \frac{\partial Z}{\partial F} \right\} + \left\{ \frac{\partial Z}{\partial D} \right\} \left\{ \frac{\partial \phi^*}{\partial Y} \right\}} \tag{3.9}$$

and taking former equations to Eq. (3.3), the final (Chica *et al.*, 2010) relationship between forces and displacements for the elastoplastic beam element is

$$\{dF\} = [K] \left(1 - \frac{[K] \left\{ \frac{\partial Z}{\partial F} \right\} \left\{ \frac{\partial Z}{\partial F} \right\}}{\left\{ \frac{\partial Z}{\partial F} \right\} [K] \left\{ \frac{\partial Z}{\partial F} \right\} + \left\{ \frac{\partial Z}{\partial D} \right\} \left\{ \frac{\partial \phi^*}{\partial Y} \right\}} \right) \{du^{ep}\} = [K^{ep}] \{du^{ep}\} \tag{3.10}$$

Now it is necessary to relate the term $\{\partial\phi^*/\partial Y\}$ with known parameters for the beam element. Using Eq. (2.6)

$$\frac{\partial \phi^*}{\partial Y} = \frac{Y}{S_o} \frac{(D_{cr} - D)^{\alpha - 1/\alpha}}{p^{(2+n)/n}} \frac{1}{1 - D} \tag{3.11}$$

and substituting the expression for Y given in Eq. (2.5) in Eq. (3.11)

$$\frac{\partial \phi^*}{\partial Y} = - \frac{\sigma_{eq}^2}{(1 - D)^2} f\Big(\frac{\sigma_m}{\sigma_{eq}} \Big) \frac{1}{2ES_o} \frac{(D_{cr} - D)^{\alpha - 1/\alpha}}{p^{(2+n)/n}} \frac{1}{1 - D} \tag{3.12}$$

and using Von Mises plastic criterion for ductile materials, together with the Ramberg-Osgood (Ramber and Osgood, 1943) power law, the effective equivalent stress can be given as a function of the accumulated plastic strain as

$$\frac{\sigma_{eq}}{1 - D} = \kappa p^{1/n} \tag{3.13}$$

where κ is a material constant. Then, substituting Eq. (3.13) into Eq. (3.12), n vanishes, resulting

$$\frac{\partial \phi^*}{\partial Y} = -f\Big(\frac{\sigma_m}{\sigma_{eq}}\Big)\frac{\kappa^2}{2ES_o}\frac{(D_{cr}-D)^{\alpha-1/\alpha}}{p}\frac{1}{1-D} \tag{3.14}$$

To determine the term $f(\sigma_m/\sigma_{eq})[\kappa^2/(2ES_o)]$, the following procedure is used. Equations (2.7) and (2.8), together with Eq. (3.14) leads to

$$\dot{D} = \frac{\kappa^2}{2ES_o}(D_{cr}-D)^{\alpha-1/\alpha} f\Big(\frac{\sigma_m}{\sigma_{eq}}\Big)\frac{\dot{p}}{p} \tag{3.15}$$

and integrating between D_0 and D_{cr}

$$(D_{cr}-D_0)^{1/\alpha} = \frac{1}{\alpha}\frac{\kappa^2}{2ES_o}\ln\frac{p_{cr}}{p_{th}} f\Big(\frac{\sigma_m}{\sigma_{eq}}\Big) \tag{3.16}$$

where it is possible to identify

$$\frac{\kappa^2}{2ES_o} f\Big(\frac{\sigma_m}{\sigma_{eq}}\Big) = \alpha\frac{(D_{cr}-D_0)^{1/\alpha}}{\ln\frac{p_{cr}}{p_{th}}} \tag{3.17}$$

so that finally the referred factor in Eq. (3.10) is now known

$$\Big\{\frac{\partial \phi^*}{\partial Y}\Big\} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \tag{3.18}$$

with

$$A_i = -\alpha\frac{(D_{cr}-D_0)^{1/\alpha}}{\ln\frac{p_{cr}}{p_{th}}}\frac{(D_{cr}-D_i)^{\alpha-1/\alpha}}{p_i}\frac{1}{1-D_i}$$

and only the terms involving Z are not yet identified. Z is the so-called yield function, which includes damage, meaning, for any cross-section, the values of damage, axial and shear forces and bending moment from which plastic and damage levels can increase, according to the flow rule.

For simplicity but without loss of generality, we present the derivation of the yield function for a rectangular $b \times h$ cross-section in a 2D beam, assuming the Von Mises yield criterion, associated flow rule and damage as defined previously. According to the CDM and neglecting plastic hardening, the yield criterion is expressed in terms of the effective stress as

$$Z = \frac{\sigma_{eq}}{1-D} - \sigma_y \leqslant 0 \tag{3.19}$$

where σ_y is the elastic limit of the material, and σ_{eq} is given, according to the von Mises yield criterion, by

$$\sigma_{eq} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \tag{3.20}$$

where σ_x is the normal stress in the beam due to the axial force and bending moment and τ_{xy} is the shear stress due to the shear force. Although real materials exhibit some kind of hardening, its effects can be neglected for some ductile steels as the one used in this paper (S-1015).

As it is common for undamaged materials, the values of the plastic bending moment M_p, plastic axial force N_p and plastic shear force V_p that cause the full yielding of the cross-section of the beam are (Krenk *et al.*, 1999; Neal, 1985; Olsen, 1999)

$$M_p = \frac{\sigma_y bh^2}{4} \qquad N_p = \sigma_y bh \qquad V_p = \frac{2\sigma_y bh}{3\sqrt{3}} \tag{3.21}$$

and including the hypothesis of strain equivalence (Lemaitre, 1985), these expressions change

$$M_p = \frac{\sigma_y bh^2}{4(1-D)} \qquad N_p = \frac{\sigma_y bh}{1-D} \qquad V_p = \frac{2\sigma_y bh}{3\sqrt{3}(1-D)} \tag{3.22}$$

In the case of a section simultaneously subjected to the bending moment M_z, axial N_x and shear forces V_y, for the usual case in which plasticity first appears in the outer part of the cross-section due to M_z, and considering that the section is fully plastic when the shear stress reaches its maximum value $\sigma_y/\sqrt{3}$ in any internal point of the section, the resulting equation is

$$M_z = \frac{\sigma_y bh^2}{4} - \frac{N_x^2}{4b\sigma_y} - \frac{9}{16}\frac{V_y^2}{b\sigma_y(1-D)^2} \tag{3.23}$$

Substituting the former expressions into Eq. (3.19), the yield function Z is obtained and shown in Fig. 2 for different values of D

$$Z = \frac{|M_z|}{M_p} + \Big(\frac{N_x}{N_p}\Big)^2 \frac{1}{1-D} + \frac{1}{3}\Big(\frac{V_y}{V_p}\Big)^2 \frac{1}{(1-D)^3} - (1-D) = 0 \tag{3.24}$$

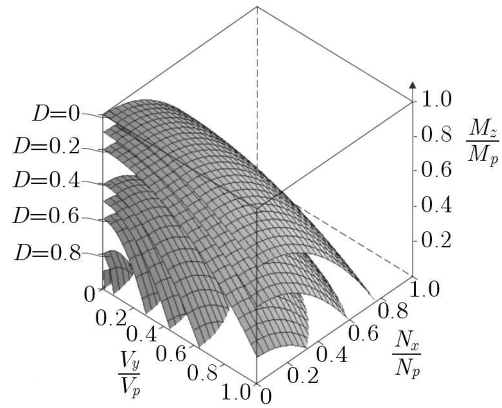

Fig. 2. Yield function for different values of D

Finally, once Z is known, the following factors that appear in Eq. (3.10) can be determined. Substituting Eq. (3.24) into Eq. (3.17) and Eq. (3.18)

$$\left\{\frac{\partial Z}{\partial F}\right\} = \begin{bmatrix} A_1 & B_1 & \dfrac{1}{M_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_2 & B_2 & \dfrac{1}{M_p} \end{bmatrix}^\top$$

$$\left\{\frac{\partial Z}{\partial D}\right\} = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \tag{3.25}$$

where

$$A_i = \frac{2N_{xi}}{N_p^2(1-D_i)} \qquad\qquad B_i = \frac{2}{3}\frac{V_{yi}}{V_p^2(1-D_1 i^3}$$
$$C_i = \Big(\frac{N_{xi}}{N_p}\Big)^2 \frac{1}{(1-D_i)^2} + \Big(\frac{V_{yi}}{V_p}\Big)^2 \frac{1}{(1-D_i)^4} + 1$$

and so, the elastoplastic damaged stiffness matrix is completely defined. All the former expressions are put together in a standard incremental algorithm and implemented in a computer code. In each increment, iterations are needed to ensure that in any plastic (and damaged) cross-section all the conditions are fulfilled. The code is checked using a test problem (Fig. 3) and applied to a standard building frame as the one shown in Fig. 5.

4. Validation

The method was used to solve the problem shown in Fig. 3, for which the data was available in the literature (Inglessis *et al.*, 1999) or could be obtained by experimental or statistical methods (Rucka and Wilde, 2010; Rinaldi *et al.*, 2006).

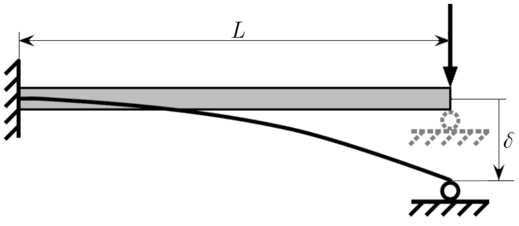

Fig. 3. Test on steel member: specimen and loading

The load is applied by increasing the value of δ in the free end. The reaction F in this point is plotted vs. δ in Fig. 4 where a comparison between the experimental and numerical results obtained by Inglessis (Inglessis *et al.*, 1999) and the results using the proposed method is shown.

The parameters for the simulation were $L = 665\,\mathrm{mm}$, $E \cdot I = 1.906 \cdot 10^7\,\mathrm{N\,mm^2}$, $p_{cr} = 1.4$, $p_{th} = 0.259$, $\alpha = 0.2175$, $D_0 = 0$ and $D_{cr} = 1$. In spite of the simplicity of the proposed method, the results are accurate enough even for this demanding test, where the damage value in the clamped end reaches the value of 0.520.

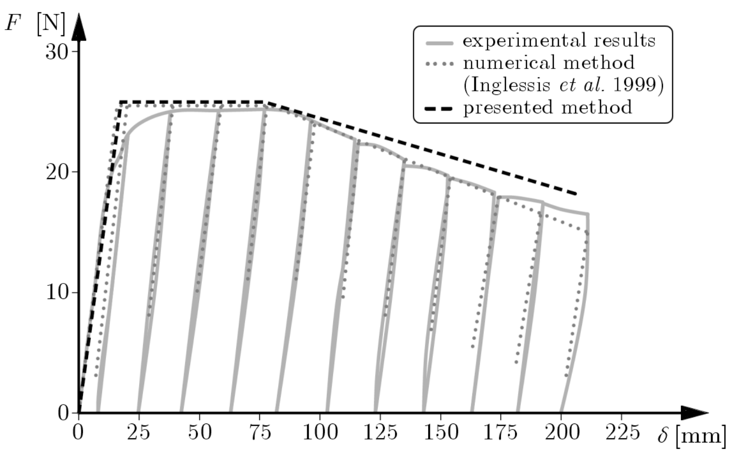

Fig. 4. Experimental vs. numerical results

5. Example

After validation, we use the method to compare the collapse load of the 2D frame shown in Fig. 5 under different yielding assumptions. The frame is clamped on the bottom of its two columns and subjected to a horizontal load in node 4 of magnitude $P = 62.5\,\mathrm{kN}$ and vertical loads in nodes 2, 3 and 4 of the same magnitude, which are proportionally increased using the parameter λ. The assumed properties are: $L = 1\,\mathrm{m}$, $E = 200\,\mathrm{GPa}$, $A = 0.1 \times 0.1\,\mathrm{m}^2$, $\sigma_y = 250\,\mathrm{MPa}$ (yield stress). The material is a Steel-1015 and its parameters of evolution of damage are reported in the literature (Le Roy, 1981; La Rosa *et al.*, 2001) so that $p_{cr} = 1.4$, $p_{th} = 0.259$ and $\alpha = 0.28$.

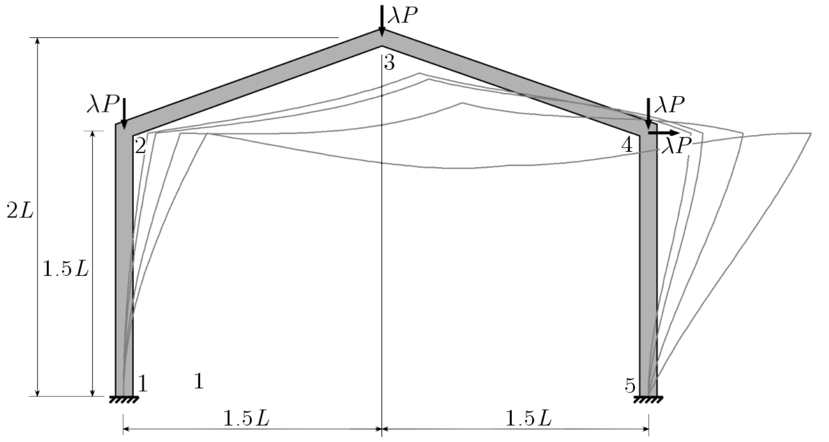

Fig. 5. Progressive collapse

Three different considerations for the yield function are considered. In the first one (1), we use the classic plastic method so that plastic hinges can appear only due to a bending moment. In the second one (2), the axial and shear forces and bending moment are considered in the yield function, but damage is not. Finally, in the third case (3), all effects are taken into account. For all the three cases, the order of appearance of the plastic hinges (1) or the plastic sections (2) and (3) is $5 \to 4 \to 3 \to 1$. The response curves, for λ vs. horizontal displacement of node 4, are shown in Fig. 6.

For case 1, in which only the bending moment is considered, the response follows a polygonal curve of decreasing slope. When N and V are considered, together with M (case 2), the response is a continuous curve that is below the previous polygonal one. When, in addition, damage is considered (case 3), the

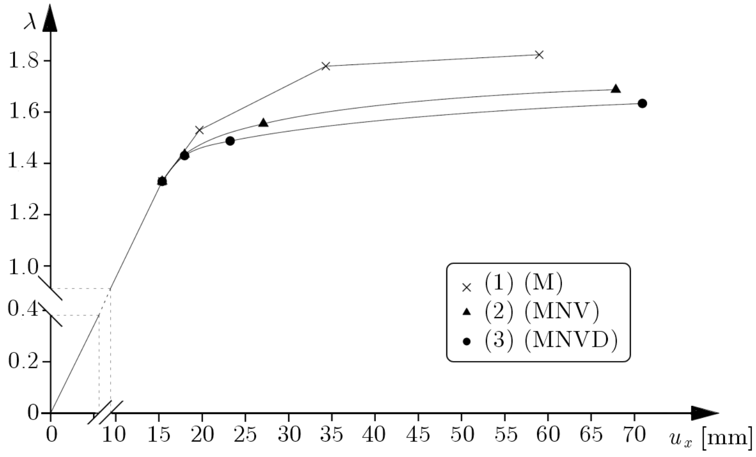

Fig. 6. Load factor λ vs. horizontal displacement of node 4

response is even lower, showing that the stiffness of the frame decreases when more sophisticated models are taken into account.

The deformed shape, amplified $\times 25$, is shown in Fig. 5 for the loads 1.348, 1.433, 1.486 and 1.633, corresponding to the formation of plastic sections at 5, 4, 3 and 1, respectively, for case 3 (1.348, 1.436, 1.541 and 1.672 for case 2, and 1.348, 1.531, 1.765 and 1.833 for case 1).

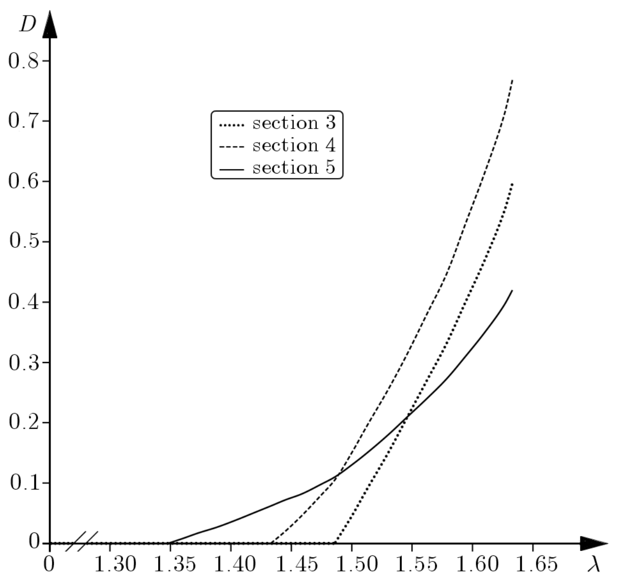

Fig. 7. Evolution of the damage with the load

For case 3, the evolution of the damage with the load is shown in Fig. 7. Note that the analysis fails to converge once the plastic state is reached in section 1, so damage can not be evaluated in this section.

6. Conclusions

Using Continuum Damage Mechanics assumptions, a simple and efficient procedure for the analysis of frames has been developed. One-dimensional finite elements (elastoplastic beams) are formulated and non-linear effects (plasticity and damage) are supposed to be concentrated in the beam-ends. The resultant numerical method is incremental and iterations are needed in each increment to ensure that all the beam-ends would be balanced and comply with plastic conditions for each level of damage. The stiffness matrix depends on geometry and on material properties, as usual, but also on the yield function Z, plastic deformation and damage in the beam-ends.

Under increasing loading, once plastic deformation appears in any cross--section, damage increases and the stiffness of the beam decreases, and hence the frame becomes more flexible. More plastic and damaged cross-sections can appear and, eventually, for some loading factor, convergence would not be achieved: it has reached the collapse state. The more effects are included in Z (internal forces, damage), the less the collapse load is. For the simplest case (Z depending only on the bending moment) the results obtained coincide with the standard plastic analysis based on plastic hinges.

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Nieliniowy model do sprężysto-plastycznej analizy problemu uszkodzeń dwuwymiarowych ram

Streszczenie

W pracy zaprezentowano prostą i skuteczną metodę nieliniowej analizy dwuwymiarowych ram przy założeniu hipotezy, że efekty nieliniowe, jeśli występują, są skoncentrowane na końcach belek tworzących układ ramy. Rozważono kontynualny model procesu zniszczenia obejmujący przekrój belki jako całość. Właściwości elasto-sprężyste materiału ujęto poprzez zdefiniowanie macierzy stycznej sztywności sprężysto-plastycznej w taki sposób, że funkcja uplastycznienia wyrażona w kategoriach obciążeń wewnętrznych (sił osiowych, tnących oraz momentu gnącego) zależy od stanu zniszczenia w każdym uplastycznionym przekroju. Po zweryfikowaniu modelu, rozwiązano przykład zastosowania analizy dla różnych założeń narzuconych na funkcję uplastycznienia (w zależności od wziętych pod uwagę obciążeń wewnętrznych) z uwzględnieniem zniszczenia lub bez. Dla każdego przypadku pokazano różnice w wartościach obciążenia zewnętrznego prowadzącego do wyboczenia ramy oraz sformułowano wnioski. Wykazano, że przedstawiona metoda nieliniowej analizy uplastycznienia pozwala na bardziej precyzyjne określenie krytycznych obciążeń prowadzących do zniszczenia konstrukcji.

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