OPTIMAL POSITIONS OF TUNABLE TRANSLATIONAL AND ROTATIONAL DYNAMIC ABSORBERS IN GLOBAL VIBRATION CONTROL IN BEAMS

Waldemar Łatas

Cracow University of Technology, Institute of Applied Mechanics, Kraków, Poland

e-mail: latas@mech.pk.edu.pl

The paper discusses the problem of vibration of an Euler-Bernoulli beam with translational and rotational dynamic absorbers attached. The beam is subjected to concentrated and distributed harmonic forces. The equation of motion is solved using the Fourier method. The time Laplace transformation allows one to determine the amplitude-frequency characteristics of the beam deflection. The aim of the study is to examine the influence of positions of the translational and rotational dynamic absorbers on vibration suppression in the global control problem in beams. Numerical examples present reduction of kinetic energy of the cantilever beam with tunable absorbers over a wide range of frequencies. Optimal positions of the absorbers are obtained.

Keywords: dynamic vibration absorber, beam vibration, vibration reduction

1. Introduction

The main purpose of dynamic vibration absorbers (DVA) is suppression of motion at the point of attachment (Harris and Piersol, 2002; Korenev and Reznikov, 1993; Mead 1999). The most common DVAs attached to the vibrating structure subjected to harmonic excitation are passive tuned mass dampers (TMD). They are used to attenuate both the longitudinal and torsional vibration. Many works have been devoted to the optimization of dynamic absorbers parameters in both the linear and non-linear problems (Bisegna and Caruso, 2012; Krenk and Høgsberg, 2008; Lee *et al.*, 2006; Mohtat and Dehghan-Niri, 2011; Rüdinger 2006; Sgobba and Marano, 2010; Tigli, 2012).

In civil engineering, TMDs are often used in structures susceptible to vibration induced by wind flow or seismic ground motion: suspension and cable-stayed bridges (Abdel-Rohman and Mariam, 2006; Chen and Cai, 2004; Chen and Wu, 2008), high-rise buildings (Bekdas and Nigdeli, 2011; Liu *et al.*, 2008; Moon, 2011; Nagarajaiah and Varadarajan, 2005; Wang and Lin, 2007], chimneys (Ricciardelli, 2001; Brownjohn *et al.*, 2010), masts, wind turbine towers (Łatas and Martynowicz, 2012). Tuned mass dampers are used in road and railway bridges (Li *et al.*, 2005; Luu *et al.*, 2012; Shi and Cai, 2008; Yau and Yang 2004a,b), footbridges (Caetano *et al.*, 2010; Li *et al.*, 2010) where the types of loading depend on the traffic of vehicles and pedestrians.

Because of multiple possible applications, a lot of attention has been put to the choice of parameters of dynamic absorbers in beams (Brennan and Dayou, 2000; Esmalizadeh and Jalili, 1998; Yang and Sedaghati, 2009; Younesian *et al.*, 2006). In continuous structures such as beams, usually the best absorber locations are the points of loading application, but it may be technically difficult to do so. The problem is more difficult when non-collocated control and global vibration control are considered. In such cases or when the distributed loading is assumed, the main task is to find the appropriate positioning of damping devices on the structure (Brennan and Dayou, 2000; Cheung and Wong, 2008).

The main drawbacks of passive TMDs are their limitations to narrow frequency bands only, ineffectiveness for non-stationary vibration and sensitivity for inaccurate tuning. To overcome these limitations, an active force is introduced between the damper and the structure (Lim, 2008; Wang and Lin, 2007). A disadvantage of active systems is, in turn, high energy consumption and dependence on the fault-free energy delivery. Their use can also cause instability of the structure. Semi-active systems which can alter stiffness or damping (Keye et al., 2009; Kim and Kang, 2012; Nagarajaiah and Varadarajan, 2005; Ricciardelli et al., 2000) in real time, improve the efficiency of passive systems, do not require large amounts of energy and cannot destabilize the system.

To improve the efficiency of vibration reduction, there are systems of mass dampers tuned to a single (Li and Ni, 2007; Li et al., 2005; Yau and Yang, 2004a,b) or several resonant frequencies for broadband excitation (Caetano et al., 2010; Luu et al., 2012). A special type of these systems are continuous absorbers (Thompson, 2008).

In this article, a model of the beam subjected to concentrated and distributed harmonic forces is given, with a system of translational and rotational dynamic vibration absorbers attached. The effect of rotational absorbers for the effectiveness of vibration reduction is investigated. Optimization of positions of the tunable absorbers in a cantilever beam subjected to a uniform distributed harmonic force with kinetic energy taken as the criterion is presented.

2. Theoretical model

Figure 1 presents a structure considered in the paper – a beam subjected to concentrated and distributed harmonic forces, with a system of translational and rotational dynamic vibration absorbers attached. The beam is of length L, with the following constant parameters: mass density ρ, cross-section area A, geometrical moment of inertia I, Young's modulus E.
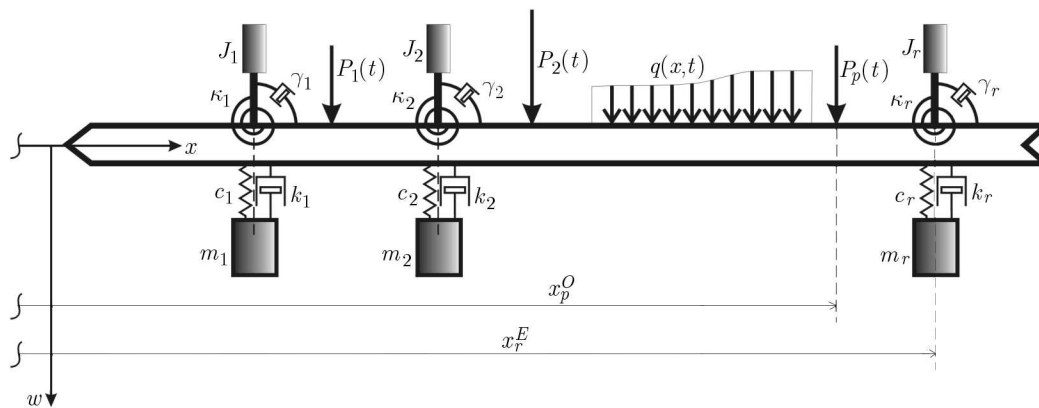

Fig. 1. Beam with a system of translational-rotational vibration absorbers

The equation of motion of the beam presented in Fig. 1 – assuming the Euler-Bernoulli theory and internal damping described by the Voigt-Kelvin rheological model (parameter α) – is given as (Cheung and Wong, 2008)

$$
\rho A \frac{\partial^2 w}{\partial t^2} + EI\alpha \frac{\partial^5 w}{\partial x^4 \partial t} + EI \frac{\partial^4 w}{\partial x^4} = q(x,t) + \sum_{j=1}^{p} P_j(t)\delta(x - x_j^O)
$$
$$
+ \sum_{j=1}^{r} F_j(t)\delta(x - x_j^E) + \sum_{j=1}^{r} \frac{\partial M_j(t)\delta(x - x_j^E)}{\partial x}
$$

(2.1)

where: $q(x, t)$ – distributed force; $P_j(t)$ – j-th concentrated force applied at the point x_j^O; $F_j(t)$ – j-th concentrated force applied at the point x_j^E, acting on the beam from the translational dynamic vibration absorber; $M_j(t)$ – j-th concentrated torque applied at the point x_j^E, acting on the beam from the rotational dynamic vibration absorber; m_j, c_j, k_j – mass, damping and stiffness coefficients of the j-th translational dynamic vibration absorber; J_j, γ_j, κ_j – moment of inertia, damping and stiffness coefficients of the j-th rotational dynamic vibration absorber; p – number of concentrated forces; r – number of translational-rotational vibration absorbers.

The term translational-rotational absorber represents two dampers, one rotational and one translational, mounted in the same place.

Solution to equation (2.1) is assumed in form of the series

$$w(x, t) = \sum_{i=1}^{\infty} q_i(t)\varphi_i(x) \tag{2.2}$$

where $\varphi_i(x)$ are the modes of vibration of the beam without absorbers attached, which are dependent on the specific boundary conditions. The functions $q_i(t)$ are time-dependent generalized co-ordinates that should be determined. It is assumed that the distributed loading has the form: $q(x, t) = h(t)g(x)$.

Substituting series (2.4) into equation (2.1) and performing the time Laplace transformation (with initial conditions equal to zero) leads to

$$\sum_{i=1}^{\infty} \Big[\rho A s^2 Q_i(s) + EI\alpha\beta_i^4 s Q_i(s) + EI\beta_i^4 Q_i(s) - a_i H(s) - \sum_{j=1}^{p} d_{ji} P_j(s)$$
$$- \sum_{j=1}^{r} b_{ji} F_j(s) - \sum_{j=1}^{r} e_{ji} M_j(s)\Big]\varphi_i(x) = 0 \tag{2.3}$$

Assuming orthogonality of the functions $\varphi_i(x)$ with the weight function $\eta(x)$, the following values of the coefficients appearing in equation (2.3) are obtained

$$a_i = \frac{1}{K_i^2}\int_0^l g(x)\varphi_i(x)\,dx \qquad d_{ji} = \frac{\varphi_i(x_j^O)}{K_i^2} \qquad b_{ji} = \frac{\varphi_i(x_j^E)}{K_i^2} \qquad e_{ji} = \frac{-\varphi_i'(x_j^E)}{K_i^2} \tag{2.4}$$

where: $K_i^2 = \int_0^L \eta(x)\varphi_i^2(x)dx$, and additionally, in equation (2.3) the notation is introduced $\beta_i^4 = [\rho A/(EI)]\omega_i^2$; ω_i is the i-th natural frequency of the beam without the vibration absorbers attached and with the internal damping neglected ($\alpha = 0$). In equation (2.3), the symbols: $Q_i(s)$, $H(s)$, $P_j(s)$, $F_j(s)$, $M_j(s)$ denote the Laplace transforms of the: $q_i(t)$, $h(t)$, $P_j(t)$, $F_j(t)$, $M_j(t)$, respectively.

Taking into account the linear independence of the functions $\varphi_i(x)$, from equation (2.3) an expression for the Laplace transform $W(x, s)$ of the beam deflection $w(x, t)$ can be obtained

$$W(x, s) = \sum_{i=1}^{\infty} \frac{a_i H(s) + \sum_{j=1}^{p} d_{ji} P_j(s) + \sum_{j=1}^{r} b_{ji} F_j(s) + \sum_{j=1}^{r} e_{ji} M_j(s)}{\rho A s^2 + EI(1+\alpha s)\beta_i^4}\varphi_i(x) \tag{2.5}$$

and its derivative with respect to x

$$\frac{\partial W(x, s)}{\partial x} = \sum_{i=1}^{\infty} \frac{a_i H(s) + \sum_{j=1}^{p} d_{ji} P_j(s) + \sum_{j=1}^{r} b_{ji} F_j(s) + \sum_{j=1}^{r} e_{ji} M_j(s)}{\rho A s^2 + EI(1+\alpha s)\beta_i^4}\varphi_i'(x) \tag{2.6}$$

The transforms of the force $F_j(s)$ and torque $M_j(s)$ acting on the beam from the j-th translational-rotational vibration absorber attached at the point x_j^E are given by the expressions (Cheung and Wong, 2008)

$$F_j(s) = -W(x_j^E, s)\frac{(c_j s + k_j)m_j s^2}{m_j s^2 + c_j s + k_j} \qquad M_j(s) = -\Theta(x_j^E, s)\frac{(\gamma_j s + \kappa_j)J_j s^2}{J_j s^2 + \gamma_j s + \kappa_j} \qquad (2.7)$$

where the notation $\Theta(x, s) = -\partial W(x, s)/\partial x$ is introduced.

The transforms given by formulas (2.7) should be inserted into expressions (2.5) and (2.6). The resulting transforms of the beam deflection and slope should be satisfied at the points where the absorbers are attached to the beam. These conditions yield a system of linear algebraic equations to determine the unknowns $W(x_k^E, s)$ and $\Theta(x_k^E, s)$ $(k = 1, 2, \ldots, r)$.

To simplify the expressions, the following notations are introduced

$$
\begin{aligned}
W(x_j^E, s) &= W_j & \Theta(x_j^E, s) &= \Theta_j \\
\varphi_i(x_j^E) &= \varphi_{ij} & \varphi_i'(x_j^E) &= \varepsilon_{ij} \\
a_i H(s) + \sum_{j=1}^{p} d_{ji} P_j(s) &= A_i & \rho A s^2 + EI(1 + \alpha s)\beta_i^4 &= B_i \\
\frac{(c_j s + k_j)m_j s^2}{m_j s^2 + c_j s + k_j} b_{ji} &= D_{ji} & \frac{(\gamma_j s + \kappa_j)J_j s^2}{J_j s^2 + \gamma_j s + \kappa_j} e_{ji} &= E_{ji}
\end{aligned}
\qquad (2.8)
$$

The system of $2r$ linear equations for the unknowns W_k, Θ_k $(k = 1, 2, \ldots, r)$ takes the form

$$
\begin{aligned}
W_k\left(1 + \sum_{i=1}^{\infty} D_{ki}\frac{\varphi_{ik}}{B_i}\right) + \sum_{j=1, j\neq k}^{r} \sum_{i=1}^{\infty} W_j D_{ji}\frac{\varphi_{ik}}{B_i} + \sum_{j=1}^{r}\sum_{i=1}^{\infty} \Theta_j E_{ji}\frac{\varphi_{ik}}{B_i} &= \sum_{i=1}^{\infty} A_i\frac{\varphi_{ik}}{B_i} \\
\Theta_k\left(\sum_{i=1}^{\infty} D_{ki}\frac{\varepsilon_{ik}}{B_i} - 1\right) + \sum_{j=1, j\neq k}^{r} \sum_{i=1}^{\infty} \Theta_j E_{ji}\frac{\varepsilon_{ik}}{B_i} + \sum_{j=1}^{r}\sum_{i=1}^{\infty} W_j D_{ji}\frac{\varepsilon_{ik}}{B_i} &= \sum_{i=1}^{\infty} A_i\frac{\varepsilon_{ik}}{B_i}
\end{aligned}
\qquad (2.9)
$$

Having solved the system (2.9), the transforms of the forces $F_j(s)$ and torques $M_j(s)$ may be calculated from expressions (2.7) and used to calculate from formulas (2.5) and (2.6) the transforms of the deflection and slope of the beam. Assuming steady-state vibration, after substituting $s = j\omega$ $(j = \sqrt{-1})$, the expressions for the deflection and slope of the beam in the frequency domain may be obtained.

3. Numerical results – optimization of position of the tunable vibration absorbers

The numerical algorithm allows one to calculate in the s-domain the transforms of the beam deflection and slope for arbitrary boundary conditions. For a harmonic excitation, the amplitude--frequency characteristics of the bending moment, transverse force, time-averaged kinetic energy of the whole beam or its part may be further calculated.

3.1. Optimization of the position of the tunable translational-rotational vibration absorber

A cantilever steel beam is excited by a uniform distributed harmonic force (Fig. 2). The beam is of length $l = 1.0$ m, mass density $\rho = 7800 \, \text{kg/m}^3$, Young's modulus $E = 2.1 \cdot 10^{11} \, \text{N/m}^2$, with rectangular cross-section of width $b = 0.05$ m and height $h = 0.005$ m. The internal damping of the beam is neglected. There is only one translational-rotational absorber attached at the point x_1^E. The aim of the absorber is to attenuate vibration of the beam. The time-averaged kinetic energy of the whole beam is used as the global measure of vibration.
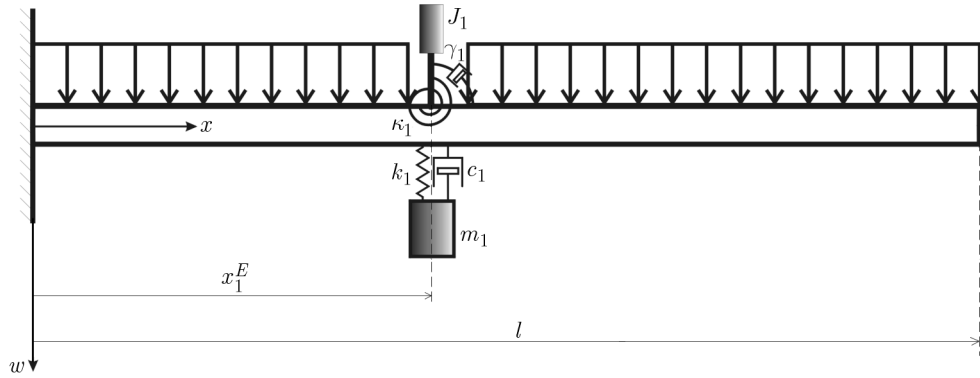
Fig. 2. Cantilever beam of length l excited by a uniform distributed harmonic force with a dynamic translational-rotational vibration absorber attached

From the practical point of view, it is preferable to use tunable dampers (Brennan and Dayou, 2000; Dayou and Brennan, 2002) because a simple control algorithm can be used. It is assumed in further calculations that the absorbers attached are tuned so that they are resonant at each single frequency and do not have energy dissipating appliances ($c_1 = 0, \gamma_1 = 0$). The first four natural frequencies of the presented beam are: $f_1 = 4.19\,\mathrm{Hz}$, $f_2 = 26.26\,\mathrm{Hz}$, $f_3 = 73.54\,\mathrm{Hz}$, $f_4 = 144.11\,\mathrm{Hz}$.

The main aim is to find the optimal position of the tunable translational-rotational vibration absorber for a given frequency band. For comparison, in Fig. 3 the calculated time-averaged kinetic energy for different positions of the single translational absorber ($J_1 = 0$) in the range of frequency $\langle 0.0\,\mathrm{Hz}, 30.0\,\mathrm{Hz}\rangle$ is shown.
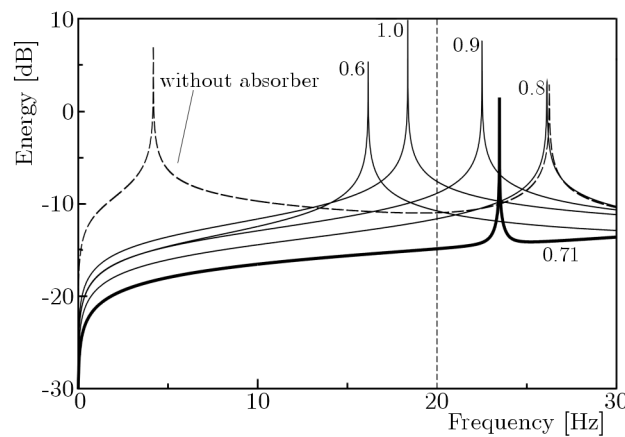

Fig. 3. Kinetic energy of the cantilever beam: without any absorber; with a single translational absorber placed at different points of the beam. The absorber attached is tuned to be resonant at each frequency. The numbers denote the dimensionless position of the absorber x_1^E/l

The best location of the single translational absorber in the range $\langle 0.0\,\mathrm{Hz}, 20.0\,\mathrm{Hz}\rangle$ is $x_{1OPT}^E = 0.71l$.

For other locations, the results obtained are worse. The calculations performed for the translational-rotational absorber (translational and rotational absorbers are attached at the same location) give the same optimal position in the range $\langle 0.0\,\mathrm{Hz}, 40.0\,\mathrm{Hz}\rangle$ equal $x_{1OPT}^E = 0.71l$. In Fig. 4, plots of kinetic energy calculated for the optimal locations of the translational and translational-rotational absorbers are shown for comparison.

The drawback of the tunable absorbers is that they cause an increase in the global vibration at the new natural frequencies of the resulting structure (Brennan and Dayou, 2000). It is clearly seen in Figs. 3 and 4. In the case of a single translational absorber, there is an additional
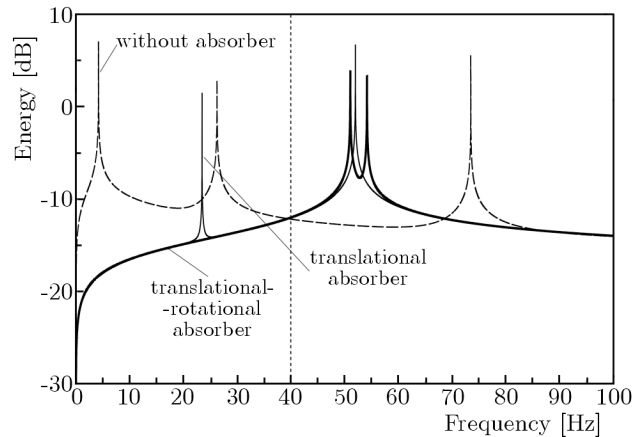
Fig. 4. Kinetic energy of the cantilever beam: without any absorber; with the translational and translational-rotational absorbers placed at the optimal position $x^E_{1OPT} = 0.71l$. The absorbers attached are tuned to be resonant at each frequency

resonant frequency equal to 23.49 Hz. The addition of the rotational absorber enhances vibration attenuation. There is no additional resonance at the frequency equal to 23.49 Hz and the translational-rotational absorber works efficiently up to frequency 40 Hz.

In both cases, the optimal solutions are very sensitive to the accuracy of the absorber location. A relatively small change in it can significantly decrease the effectiveness of vibration suppression. A further improvement may be obtained by installing a few absorbers in different places of the beam.

In Sections 3.2 and 3.3 situations with two absorbers located at different places are presented. Referring to graphs in Fig. 4, the aim of optimization is to find the positions of the absorbers, which give the best vibration reduction efficiency in the given range $\langle 0.0\,\text{Hz}, 40.0\,\text{Hz}\rangle$.

3.2. Optimization of positions of the tunable translational and rotational vibration absorbers

Figure 5 presents the analyzed beam with two dynamic vibration absorbers without damping: one translational placed at the point x^E_1 and one rotational placed at the point x^E_2. The aim is to find the optimal locations of the absorbers x^{*E}_{1OPT}, x^{*E}_{2OPT}, which can give the best vibration reduction in the range $\langle 0.0\,\text{Hz}, 40.0\,\text{Hz}\rangle$. The absorbers attached are tuned to be resonant at each frequency. The results of numerical calculations are summarized in Section 3.4.
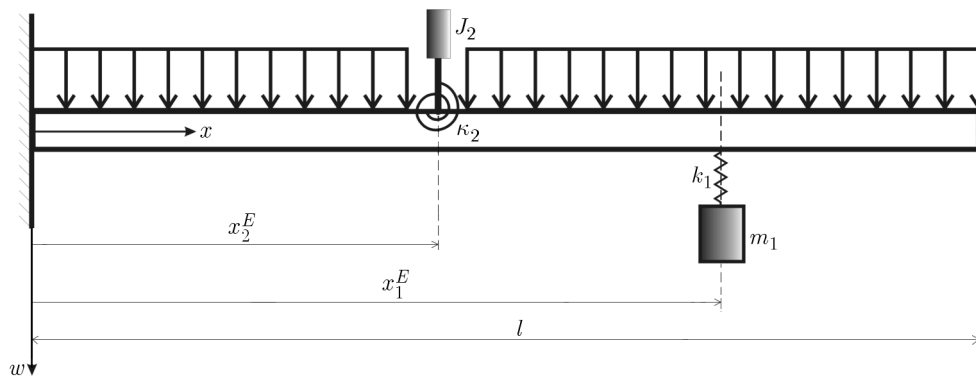

Fig. 5. Cantilever beam of length l excited by a uniform distributed harmonic force with tunable dynamic translational and rotational vibration absorbers attached at different locations

3.3. Optimization of positions of the two tunable translational vibration absorbers

Figure 6 presents the studied beam with two dynamic translational vibration absorbers without damping placed at the points x_1^E, x_2^E. The aim is to find the optimal locations of the absorbers x_{1OPT}^{**E}, x_{2OPT}^{**E}, which give the best vibration reduction in the range $\langle 0.0\,\text{Hz}, 40.0\,\text{Hz} \rangle$. The absorbers attached are tuned to be resonant at each frequency. The results of numerical calculations are gathered in Section 3.4.
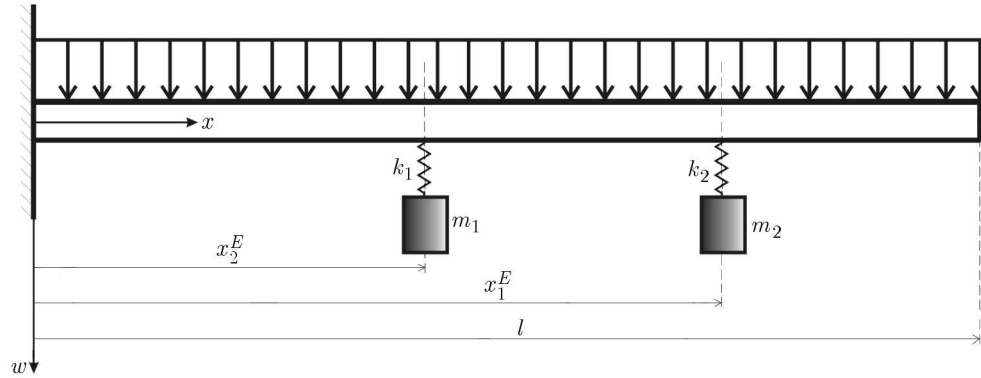

Fig. 6. Cantilever beam of length l excited by a uniform distributed harmonic force with two tunable dynamic translational vibration absorbers attached

3.4. Summary of the results of numerical calculations

The calculated optimum positions of the absorbers for the cases presented in Section 3.2 and 3.3 are as follows:
– two absorbers; one translational of position x_1^E and one rotational of position x_2^E (Fig. 5)

$$x_{1OPT}^{*E} = 0.60l \qquad x_{2OPT}^{*E} = 0.80l$$

– two translational absorbers of positions x_1^E, x_2^E (Fig. 6)

$$x_{1OPT}^{**E} = 0.41l \qquad x_{2OPT}^{**E} = 0.83l$$

In Fig. 7, plots of kinetic energy for the given above optimal positions of the absorbers in the two investigated cases are shown. For comparison, the kinetic energy for the optimal position of the translational-rotational absorber ($x_{1OPT}^E = 0.71l$, Fig. 2) is also presented.

The option to use several absorbers in different locations greatly increases the efficiency of vibration suppression. The calculations performed show that the use of two translational absorbers gives a better result than the use of one translational and one rotational absorber. But if there is only one place to locate the absorber, the addition of the rotational absorber to the translational one may improve vibration attenuation.

The results presented are valid for a harmonic force uniformly distributed over the length of the beam. For another type of loading, the optimization can give different results, as for the other frequency bands. Further improvement of thr performance may be obtained by adding damping or de-tuning the absorbers (Brennan and Dayou, 2000).

4. Conclusions

The model presented in the paper can be used in local and global problems of optimal choice of positions and physical parameters of translational and rotational vibrations absorbers in beams.
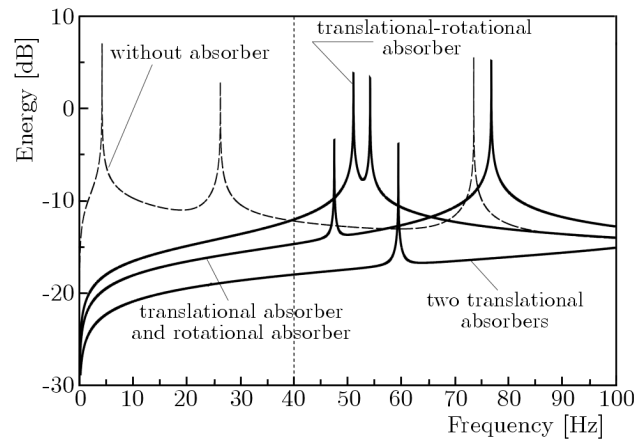
Fig. 7. Kinetic energy of the cantilever beam: without any absorber; with the translational-rotational absorber; with one translational and one rotational absorber; with two translational absorbers. The absorbers attached are placed at the optimal positions and are tuned to be resonant at each frequency

Theoretical calculations are illustrated by optimization of positions of the tunable absorbers in the global control of kinetic energy of the beam. The results of numerical calculations show that by adding a rotational absorber, the effectiveness of vibration reduction can be improved, as it can absorb rotational motion of the beam by applying torque to the beam. The addition of rotational absorbers to translational ones allows one to move resonances and expands the frequency band of effective vibration reduction. This effect can be also used in other structures such as frames, curved beams and pipes.

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