ADAPTIVE FUZZY CONTROL FOR A CLASS OF CONSTRAINED NONLINEAR SYSTEMS WITH APPLICATION TO A SURFACE VESSEL

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In this paper, adaptive control for a class of uncertain nonlinear systems with input constraints is addressed. The main goal is to achieve a self-regulator PID controller whose coefficients are adjusted by using some adaptive fuzzy rules. The constraints on the control signal are taken into account as a saturation operator. The stability of the closed-loop system is analytically proved by using the Lyapunov stability theorem. The proposed method is then applied to a surface vessel with uncertain dynamic equations. The simulation results show the effectiveness of the proposed control strategy.

Keywords: self-regulator, fuzzy PID controller, constraint nonlinear systems, uncertainty, fuzzy estimation

1. Introduction

Dealing with the control problem of uncertain systems, various algorithms have been developed ensuring the robust stability and performance (Petersen and Tempo, 2014). Robust adaptive control has been formulated for a class of uncertain nonlinear systems by output feedback control (Xu and Huang, 2010; Lee, 2011). For nonlinear systems in the strict-feedback form with unknown static parameters, a robust adaptive control law was designed by Montaseri and Mohammad (2012), which guarantees the asymptotic output tracking despite matched and unmatched uncertainties. The neural-network-based robust control design, via an adaptive dynamic programming approach, was investigated in (Wang *et al.*, 2014) to obtain the optimal performance under a specified cost function. Some applications have been also introduced in the literature, in the presence of time-varying uncertainties and disturbances (Koofigar and Amelian, 2013). Nevertheless, taking the input constraint in the controller design procedure is still highly desired.

In the last decade, a considerable attention has been paid to robust control of nonlinear systems with input constraints (Chen *et al.*, 2010, 2014; Lu and Yao, 2014). In such cases, fuzzy logic and neural networks may be some alternative solutions. A direct adaptive fuzzy control approach has been presented for uncertain nonlinear systems in the presence of input saturation by incorporating a new auxiliary design system and Nussbaum gain functions (Li *et al.*, 2013). The problem of adaptive fuzzy tracking control for a class of pure-feedback nonlinear systems with input saturation was studied by Wang *et al.*, (2013a,b). Muñoz and Marquardt (2013) focused on the control design for input-output feedback linearizable nonlinear systems with bounded inputs and state constraints. An indirect adaptive fuzzy control scheme was developed for a wider class of nonlinear systems with the input constraint and unknown control direction by Wuxi *et al.* (2013) and Yongming *et al.* (2014). To this end, a barrier Lyapunov function and an auxiliary design system were employed.

From an application viewpoint, the surface vessels with uncertain nonlinear dynamics may be adopted to demonstrate the effectiveness of various control schemes. Nonlinear strategies

(Daly *et al.*, 2012), adaptive control (Fang *et al.*, 2004), and neural networks (Dai *et al.*, 2015) are samples of control algorithms in the previous investigations. Removing some drawbacks of such works, adaptive intelligent methods as adaptive neural networks, were presented by Li *et al.* (2015). In this study, an adaptive fuzzy algorithm is proposed to achieve the advantages of both intelligent and adaptive mechanisms for ensuring the robustness properties and taking the constraints into account.

Briefly discussing, there may exist some main restrictions in the previous investigations as, i) the fuzzy rules have been designed off-line and the stability and performance may be lost with changing the circumstances, ii) the stability analysis has not been presented in an analytical form, and iii) to ensure the stability of the closed-loop system, the initial value for the controller parameters must be set. To eliminate the aforementioned limitations, a self-regulator fuzzy PID controller is proposed in this paper, which guarantees the robustness properties against the system uncertainties and external disturbances.

This paper is organized as follows. In Section 2, the problem formulation and the constraints on input signal are introduced. In Section 3, an adaptive fuzzy controller is designed for a class of uncertain nonlinear systems with constrained input and the stability proof is given. The proposed method is applied to a surface vessel in Section 4 and the simulation results are presented. The concluding remarks are finally given in Section 5.

2.   Problem formulation

Consider a class of nonlinear systems, represented by the state-space description

$$
\begin{aligned}
&\dot{\mathbf{X}}_1 = \mathbf{X}_2 \\
&\dot{\mathbf{X}}_2 = \mathbf{X}_3 \\
&\vdots \\
&\dot{\mathbf{X}}_{n-1} = \mathbf{X}_n \\
&\dot{\mathbf{X}}_n = \mathbf{F}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n) + \mathbf{G}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n)\mathbf{p}(\mathbf{u}) + \mathbf{d}(t) \\
&\mathbf{Y} = \mathbf{X}_1
\end{aligned}
\tag{2.1}
$$

where $\mathbf{X} \in \mathbb{R}^{n \times m}$ denotes the vector of state variables, $d(t)$ represents the external disturbance, and $\mathbf{p}(\mathbf{u}) \in \mathbb{R}^m$ is the vector of constrained inputs.
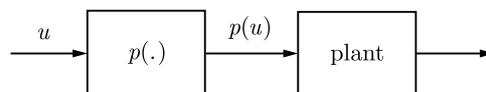

Fig. 1. Block diagram of $p(\mathbf{u})$

As schematically depicted in Fig. 1, the nonlinear operator $p(\mathbf{u})$ acts as a saturation constraint as

$$
p(u_i) = \begin{cases} \alpha u_u & \text{for} \quad u_i \geqslant u_u \\ \alpha u_i & \text{for} \quad u_l \leqslant u_i \leqslant u_u \\ \alpha u_l & \text{for} \quad u_i \leqslant u_l \end{cases} \qquad i = 1, 2, \ldots, m
\tag{2.2}
$$

where u_u, u_l and α denote the parameters of saturation operator.

The saturation operator $p(u_i)$ is described here as

$$
p(u_i) = a(u_i)u_i + b(u_i)
\tag{2.3}
$$

where $a(u_i)$ and $b(u_i)$ are given by

$$
a(u_i) = \begin{cases} 0 & \text{for} \quad u_i \geqslant u_u \\ \alpha & \text{for} \quad u_l \leqslant u_i \leqslant u_u \\ 0 & \text{for} \quad u_i \leqslant u_l \end{cases}
$$

$$
b(u_i) = \begin{cases} \alpha u_u & \text{for} \quad u_i \geqslant u_u \\ 0 & \text{for} \quad u_l \leqslant u_i \leqslant u_u \\ \alpha u_l & \text{for} \quad u_i \leqslant u_l \end{cases}
\tag{2.4}
$$

Incorporating description (2.3) into (2.1), yields

$$
\begin{aligned}
\dot{\mathbf{X}}_1 &= \mathbf{X}_2 \\
\dot{\mathbf{X}}_2 &= \mathbf{X}_3 \\
&\vdots \\
\dot{\mathbf{X}}_{n-1} &= \mathbf{X}_n \\
\dot{\mathbf{X}}_n &= \mathbf{F} + \mathbf{Gb(u)} + \mathbf{Ga(u)u} + \mathbf{d}(t) = \mathbf{F} + \mathbf{Gb(u)} + \widehat{\mathbf{G}}_u \mathbf{u} + \mathbf{d}(t) \\
\mathbf{Y} &= \mathbf{X}_1
\end{aligned}
\tag{2.5}
$$

Remark 1. The only information about the system model is that the invertible matrix $\widehat{\mathbf{G}}_u(\cdot)$, as an estimate of $\mathbf{G_u}(\cdot) = \mathbf{G}(\cdot)a(u)$, is available, see Mclain *et al.* (1999).

The control objective is to design the control input u such that $\mathbf{Y}$ tracks the smooth reference trajectory $\mathbf{Y}_d$. Define the tracking error vector $\mathbf{E} = [e_1, \ldots, e_m]^{\mathrm{T}}$ as

$$
\mathbf{E} = \mathbf{X}_1 - \mathbf{Y}_d = \mathbf{Y} - \mathbf{Y}_d
\tag{2.6}
$$

A PID control structure is adapted here as

$$
u_i = k_{Pi}e_i + k_{Ii}\int_0^t e_i(\tau)d\tau + k_{Di}\frac{de_i}{dt} \qquad i = 1, 2, \ldots, m
\tag{2.7}
$$

where e_i is the i-th component of the error vector $\mathbf{E}$, and k_{Pi}, k_{Ii} and k_{Di} denote respectively the proportional, integral and derivative coefficients.

3. Adaptive fuzzy controller design

3.1. Fuzzy estimation

In this Section, the l-th fuzzy rule of the fuzzy controller for estimating the unknown function $H(x)$ is formed by (Shaocheng *et al.*, 2000)

$$
R^l: \quad \text{if} \quad x_1 = A_1^l \quad \wedge \quad x_2 = A_2^l \quad \rightarrow \quad H(x) = \theta_l
\tag{3.1}
$$

where $\mathbf{x} = [x_1, x_2]^{\mathrm{T}}$ denotes the input vector, A_i^l is the membership function of each input. The fuzzy model for describing $H(\mathbf{x})$ is Mamdani, and the output of the fuzzy system can be obtained by

$$
H(\mathbf{x}) = \frac{\sum\limits_{l=1}^N \theta_l \prod\limits_{i=1}^2 \mu_{A_i^l}(x_i)}{\sum\limits_{l=1}^N \prod\limits_{i=1}^2 \mu_{A_i^l}(x_i)}
\tag{3.2}
$$

where $u_{F_i^l}$ is the fuzzy membership function and N is the number of rules. Now, form the unknown function as

$$H(\mathbf{x}) = \mathbf{\Phi}(\mathbf{x})^{\mathrm{T}}\boldsymbol{\theta} \tag{3.3}$$

where

$$\mathbf{\Phi}(\mathbf{x}) = [\phi_1(\mathbf{x}), \phi_2(\mathbf{x}), \dots, \phi_N(\mathbf{x})]^{\mathrm{T}} \qquad \phi_l(\mathbf{x}) = \frac{\prod\limits_{i=1}^{2} \mu_{A_i^l}(x_i)}{\sum\limits_{l=1}^{N} \prod\limits_{i=1}^{2} \mu_{A_i^l}(x_i)} \tag{3.4}$$

$$\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_N]^{\mathrm{T}}$$

3.2. Controller design

To facilitate the designing procedure, new state variables are defined here as

$$\mathbf{Z}_1 = \int\limits_0^t \mathbf{E}(\tau)\, d\tau$$

$$\mathbf{Z_2} = \mathbf{E}$$

$$\mathbf{Z}_3 = \frac{d\mathbf{E}}{dt} \tag{3.5}$$

$$\vdots$$

$$\mathbf{Z}_{n+1} = \frac{d^{n-1}\mathbf{E}}{dt^{n-1}}$$

by which the dynamic equations (2.1) may be rewritten as

$$\dot{\mathbf{Z}}_1 = \mathbf{Z}_2$$

$$\dot{\mathbf{Z}}_2 = \mathbf{Z}_3$$

$$\vdots \tag{3.6}$$

$$\dot{\mathbf{Z}}_n = \mathbf{Z}_{n+1}$$

$$\dot{\mathbf{Z}}_{n+1} = \mathbf{F}_t + \widehat{\mathbf{G}}_u \mathbf{u}$$

where

$$\mathbf{F}_t = \mathbf{F} - \frac{d^n \mathbf{Y}_d}{dt^n} + \mathbf{G}\mathbf{b}(\mathbf{u}) + (\mathbf{G}_u - \widehat{\mathbf{G}}_u)\mathbf{u} + \mathbf{d}(t) \tag{3.7}$$

Hence, input signal (2.7) may be given by

$$\mathbf{u} = \mathbf{K}_I \mathbf{Z}_1 + \mathbf{K}_P \mathbf{Z}_2 + \mathbf{K}_D \mathbf{Z}_3 \tag{3.8}$$

where

$$\mathbf{K}_P = \mathrm{diag}\begin{bmatrix} k_{P1} & k_{p2} & \cdots & k_{pm} \end{bmatrix}$$

$$\mathbf{K}_I = \mathrm{diag}\begin{bmatrix} k_{I1} & k_{I2} & \cdots & k_{Im} \end{bmatrix} \tag{3.9}$$

$$\mathbf{K}_D = \mathrm{diag}\begin{bmatrix} k_{D1} & k_{D2} & \cdots & k_{Dm} \end{bmatrix}$$

Now, define an ideal control signal $\mathbf{u}^*$ as

$$\mathbf{u}^* = \mathbf{\Phi}_T^{\mathrm{T}}\mathbf{\Theta}^* = \widehat{\mathbf{G}}_u^{-1}(-\mathbf{F}_t - \mathbf{K}_1 \mathbf{Z}_1 - \mathbf{K}_2 \mathbf{Z}_2 - \dots - \mathbf{K}_{n+1}\mathbf{Z}_{n+1}) \tag{3.10}$$

where $\mathbf{u}^*$ is obtained from the feedback linearization of system (3.6).

Remark 2. $\mathbf{K}_i, \quad i = 1, \ldots, n+1$, is chosen such that

$$\mathbf{A}_{cl} = \begin{bmatrix} 0 & I_m & 0 & \cdots & 0 \\ 0 & 0 & I_m & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I_m \\ -K_1 & -K_2 & \cdots & -K_n & -K_{n+1} \end{bmatrix} \tag{3.11}$$

is negative semi-definite.

The input signal u^* is not implementable, as $\mathbf{F}_t$ is unknown. Instead, an approximation of the ideal signal u^* is generated as

$$\widehat{\mathbf{u}} = \mathbf{\Phi}_T^{\mathrm{T}} \widehat{\mathbf{\Theta}} \tag{3.12}$$

where $\widehat{\mathbf{\Theta}}$ is an approximation of $\mathbf{\Theta}^*$.

Then, replacing (3.12) in (3.6), yields

$$\dot{\mathbf{Z}}_{n+1} = \mathbf{F}_t + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \widehat{\mathbf{\Theta}} \tag{3.13}$$

By adding and subtracting $\widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \mathbf{\Theta}^*$ in (3.13), one can write

$$\dot{\mathbf{Z}}_{n+1} = \mathbf{F}_t + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \widehat{\mathbf{\Theta}} - \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \mathbf{\Theta}^* + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \mathbf{\Theta}^* = \mathbf{F}_t + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \widetilde{\mathbf{\Theta}} + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \mathbf{\Theta}^* \tag{3.14}$$

where

$$\widetilde{\mathbf{\Theta}} = \widehat{\mathbf{\Theta}} - \mathbf{\Theta}^* \tag{3.15}$$

denotes the parameter estimation error. By substituting (3.10) into (3.14), one obtains

$$\dot{\mathbf{Z}}_{n+1} = -\mathbf{K}_1 \mathbf{Z}_1 - \mathbf{K}_2 \mathbf{Z}_2 - \ldots - \mathbf{K}_{n+1} \mathbf{Z}_{n+1} + \widehat{\mathbf{G}}_u \mathbf{\Phi}_T^{\mathrm{T}} \widetilde{\mathbf{\Theta}} \tag{3.16}$$

and

$$\dot{\mathbf{Z}} = \mathbf{A}_{cl} \mathbf{Z} + \mathbf{B}_{cl} \mathbf{\Phi}_T^{\mathrm{T}} \widetilde{\mathbf{\Theta}} \tag{3.17}$$

where

$$\mathbf{Z} = [\mathbf{Z}_1, \mathbf{Z}_2, \cdots, \mathbf{Z}_{n+1}]^{\mathrm{T}} \qquad \mathbf{B}_{cl} = [\mathbf{0}, \mathbf{0}, \ldots, \widehat{\mathbf{G}}_u]^{\mathrm{T}} \tag{3.18}$$

Remark 3. $\mathbf{A}_{cl}$ in (3.11) is a negative semi-definite matrix, so the positive definite symmetric matrix $\mathbf{P}$ can be found that satisfy the algebraic Lyapunov equation

$$\mathbf{A}_{cl}^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{A}_{cl} = -\mathbf{Q} \tag{3.19}$$

for any positive definite symmetric matrix $\mathbf{Q}$.

Theorem. Consider constrained nonlinear system (3.6). By applying the control input $\mathbf{u} = \mathbf{\Phi}_T^{\mathrm{T}} \mathbf{\Theta}$ and adaptive law $\dot{\widehat{\mathbf{\Theta}}} = -2\mathbf{\Gamma}^{\mathrm{T}} \mathbf{\Phi}_T \mathbf{B}_{cl}^{\mathrm{T}} \mathbf{P} \mathbf{Z}$, the closed loop stability and tracking performance are guaranteed.

More precisely

$$
\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{T1}^{\mathrm{T}} \boldsymbol{\Theta}_1 \\ \boldsymbol{\Phi}_{T2}^{\mathrm{T}} \boldsymbol{\Theta}_2 \\ \vdots \\ \boldsymbol{\Phi}_{Tm}^{\mathrm{T}} \boldsymbol{\Theta}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_{T1}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_{T2}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Phi}_{Tm}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Theta}_1 \\ \boldsymbol{\Theta}_2 \\ \vdots \\ \boldsymbol{\Theta}_m \end{bmatrix} = \boldsymbol{\Phi}_T^{\mathrm{T}} \boldsymbol{\Theta}
\tag{3.20}
$$

and

$$
u_i = \Phi_i^{\mathrm{T}} \theta_{pi} Z_{2i} + \Phi_i^{\mathrm{T}} \theta_{Ii} Z_{1i} + \Phi_i^{\mathrm{T}} \theta_{Di} Z_{3i} = \boldsymbol{\Phi_{Ti}}^{\mathrm{T}} \boldsymbol{\Theta}_i
\tag{3.21}
$$

in which

$$
\boldsymbol{\Phi}_{Ti} = [\Phi_i Z_{1i}, \Phi_i Z_{2i}, \Phi_i Z_{3i}]^{\mathrm{T}} \qquad \boldsymbol{\Theta}_i = [\theta_{Ii}, \theta_{pi}, \theta_{Di}]
\tag{3.22}
$$

and

$$
\begin{aligned}
k_{pi} &= g_{pi}\Big(e_i, \frac{de_i}{dt}\Big) = \Phi_i^{\mathrm{T}} \theta_{pi} \qquad k_{Ii} = g_{Ii}\Big(e_i, \frac{de_i}{dt}\Big) = \Phi_i^{\mathrm{T}} \theta_{Ii} \\
k_{Di} &= g_{Di}\Big(e_i, \frac{de_i}{dt}\Big) = \Phi_i^{\mathrm{T}} \theta_{Di}
\end{aligned}
\tag{3.23}
$$

Remark 4. The nonlinear functions $g_{pi}(\cdot)$, $g_{Ii}(\cdot)$ and $g_{Di}(\cdot)$ may be obtained by a formulation as $H(\mathbf{x})$ in (3.2).

Proof. Choose the Lyapunov function candidate

$$
V(\mathbf{Z}, \widetilde{\boldsymbol{\Theta}}) = \mathbf{Z}^{\mathrm{T}} \mathbf{P} \mathbf{Z} + \frac{1}{2} \widetilde{\boldsymbol{\Theta}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \widetilde{\boldsymbol{\Theta}} \qquad \boldsymbol{\Gamma} > 0
\tag{3.24}
$$

with $\mathbf{P} > 0$ and $\boldsymbol{\Gamma} > 0$. The time derivative of V is given by

$$
\dot{V}(\mathbf{Z}, \widetilde{\boldsymbol{\Theta}}) = \dot{\mathbf{Z}}^{\mathrm{T}} \mathbf{P} \mathbf{Z} + \mathbf{Z}^{\mathrm{T}} \mathbf{P} \dot{\mathbf{Z}} + \frac{1}{2} \dot{\widetilde{\boldsymbol{\Theta}}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \widetilde{\boldsymbol{\Theta}} + \frac{1}{2} \widetilde{\boldsymbol{\Theta}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \dot{\widetilde{\boldsymbol{\Theta}}}
\tag{3.25}
$$

By replacing (3.17) into (3.25) and some manipulations, one can obtain

$$
\begin{aligned}
\dot{V}(\mathbf{Z}, \widetilde{\boldsymbol{\Theta}}) &= \mathbf{Z}^{\mathrm{T}} (\mathbf{A}_{cl}^{\mathrm{T}} \mathbf{P} + \mathbf{P} \mathbf{A}_{cl}) \mathbf{Z} + 2 \mathbf{Z}^{\mathrm{T}} \mathbf{P} \mathbf{B}_{cl} \boldsymbol{\Phi}_T^{\mathrm{T}} \widetilde{\boldsymbol{\Theta}} + \dot{\widetilde{\boldsymbol{\Theta}}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1} \widetilde{\boldsymbol{\Theta}} \\
&= -\mathbf{Z}^{\mathrm{T}} \mathbf{Q} \mathbf{Z} + (2 \mathbf{Z}^{\mathrm{T}} \mathbf{P} \mathbf{B}_{cl} \boldsymbol{\Phi}_T^{\mathrm{T}} + \dot{\widetilde{\boldsymbol{\Theta}}}^{\mathrm{T}} \boldsymbol{\Gamma}^{-1}) \widetilde{\boldsymbol{\Theta}}
\end{aligned}
\tag{3.26}
$$

Then, by adopting the adaptation law

$$
\dot{\widetilde{\boldsymbol{\Theta}}} = -2 \boldsymbol{\Gamma}^{\mathrm{T}} \boldsymbol{\Phi}_T \mathbf{B}_{cl}^{\mathrm{T}} \mathbf{P} \mathbf{Z}
\tag{3.27}
$$

one can conclude

$$
\dot{V}(\mathbf{Z}, \widetilde{\boldsymbol{\Theta}}) = -\mathbf{Z}^{\mathrm{T}} \mathbf{Q} \mathbf{Z} < 0
\tag{3.28}
$$

Thus, Barbalat's Lemma (Sastry and Shankar, 1999; Åström and Wittenmark, 2013) ensures that the vector $\mathbf{Z}$ is asymptotically converged to zero.
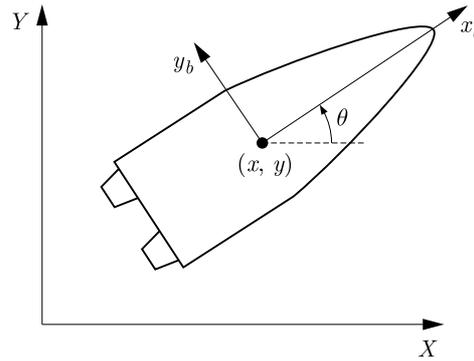
Fig. 2. Surface vessel in the inertial fixed and body fixed frames

4. Simulation

In this Section, the performance of the controller is evaluated in two situations, and the proposed method is applied to a surface vessel schematically shown in Fig. 2.

Such a three-input three-output system may be described by (Fang *et al.*, 2004)

$$
\begin{aligned}
m_{11}\dot{v}_x + d_{11}v_x &= \tau_1 \\
m_{22}\dot{v}_y + m_{23}\dot{w} + d_{22}v_v + d_{23}w &= \tau_2 \\
m_{33}\dot{w} + m_{23}\dot{v}_y + d_{23}v_y + d_{33}w &= \tau_3
\end{aligned}
\tag{4.1}
$$

in which (x, y) and θ are respectively the surface vessel position and yaw angle in the inertial coordinate system and (v_x, v_y), and w denote respectively the surface vessel speed and rotational speed in the body coordinate system.

Dynamical equations (4.1) with using a set of simple mathematical operations can be rewritten in the form

$$
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}, \dot{\mathbf{q}}) = \tau^*
\tag{4.2}
$$

where

$$
\mathbf{q} = [x, y, \theta]^{\mathrm{T}}
$$

$$
\mathbf{M}(\mathbf{q}) = \begin{bmatrix} m_{11}\cos^2\theta + m_{22}\sin^2\theta & -m_d\cos\theta\sin\theta & -m_{23}\sin\theta \\ -m_d\cos\theta\sin\theta & m_{22}\cos^2\theta + m_{11}\sin^2\theta & m_{23}\cos\theta \\ -m_{23}\sin\theta & m_{23}\cos\theta & m_{33} \end{bmatrix}
$$

$$
\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} \dot{\theta}(m_d\cos\theta\sin\theta) & \dot{\theta}(m_{11}\cos^2\theta + m_{22}\sin^2\theta) & 0 \\ -\dot{\theta}(m_{22}\cos^2\theta + m_{11}\sin^2\theta) & -\dot{\theta}(m_d\cos\theta\sin\theta) & 0 \\ -\dot{\theta}(m_{23}\cos\theta) & -\dot{\theta}(m_{23}\sin\theta) & 0 \end{bmatrix}
\tag{4.3}
$$

$$
\mathbf{G}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{K}(\mathbf{q})\dot{\mathbf{q}}
$$

$$
\mathbf{K}(\mathbf{q}) = \begin{bmatrix} d_{11}\cos^2\theta + d_{22}\sin^2\theta & -d_d\cos\theta\sin\theta & -d_{23}\sin\theta \\ -d_d\cos\theta\sin\theta & d_{22}\cos^2\theta + d_{11}\sin^2\theta & d_{23}\cos\theta \\ -d_{23}\sin\theta & d_{23}\cos\theta & d_{33} \end{bmatrix}
$$

and

$$
\boldsymbol{\tau}^* = [\tau_1, \tau_2, \tau_3]
\tag{4.4}
$$

To facilitate the designing procedure, choose the state variables as

$$
\mathbf{X}_1 = \mathbf{q} \qquad \mathbf{X}_2 = \dot{\mathbf{q}} \qquad \mathbf{X}_1, \mathbf{X}_2 \in \mathbb{R}^3
\tag{4.5}
$$

The state space representation may be as

$$\dot{\mathbf{X}}_1 = \mathbf{X}_2 \qquad \dot{\mathbf{X}}_2 = \mathbf{F}(\mathbf{X}_1, \mathbf{X}_2) + \mathbf{G}_u(\mathbf{X}_1, \mathbf{X}_2)\mathbf{u} \tag{4.6}$$

where

$$\mathbf{F}(\mathbf{X}_1, \mathbf{X}_2) = -\mathbf{M}^{-1}(\mathbf{X}_1)[\mathbf{C}(\mathbf{X}_1, \mathbf{X}_2)\mathbf{X}_2 + \mathbf{G}(\mathbf{X}_1, \mathbf{X}_2)] \qquad \mathbf{G}_u(\mathbf{X}_1, \mathbf{X}_2) = \mathbf{M}^{-1}(\mathbf{X}_1) \tag{4.7}$$

and

$$\mathbf{u} = \boldsymbol{\tau}^* \qquad \mathbf{u} \in \mathbb{R}^3 \tag{4.8}$$

The numerical values of the model parameters in equation (4.1) are given in Table 1, as given by Fang *et al.* (2004).

Table 1. Model parameter values for the surface vessel

Parameter	Value	Parameter	Value	Parameter	Value
m_{11} [kg]	1.0852	m_{33} [kg]	0.2153	d_{11} [kg/s]	0.08656
m_{22} [kg]	2.0575	d_{11} [kg]	0.08656	d_{22} [kg/s]	0.0762
d_{33} [kg/s]	0.0031	d_{23} [kg/s]	0.151	d_{32} [kg/s]	0.0151

The initial values and eigenvalues of the matrix $\mathbf{A}_{cl} \in \mathbb{R}^{9 \times 9}$ are selected as

$$\begin{aligned} \mathbf{X}_{10} &= [1, -1, 0.3]^{\mathrm{T}} \qquad \mathbf{X}_{20} = [0, 0, 0]^{\mathrm{T}} \\ \boldsymbol{\lambda} &= [-1, -1, -1, -1, -1, -1, -1, -1, -1] \end{aligned} \tag{4.9}$$

The matrix $\boldsymbol{\Gamma}$, constant scalars γ_i, $i = 1, 2, 3$ and membership functions are chosen here as

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_1 I_L & 0 & 0 \\ 0 & \gamma_2 I_L & 0 \\ 0 & 0 & \gamma_3 I_L \end{bmatrix} \qquad \gamma_1 = \gamma_2 = 10 \qquad \gamma_3 = 10^3 \tag{4.10}$$
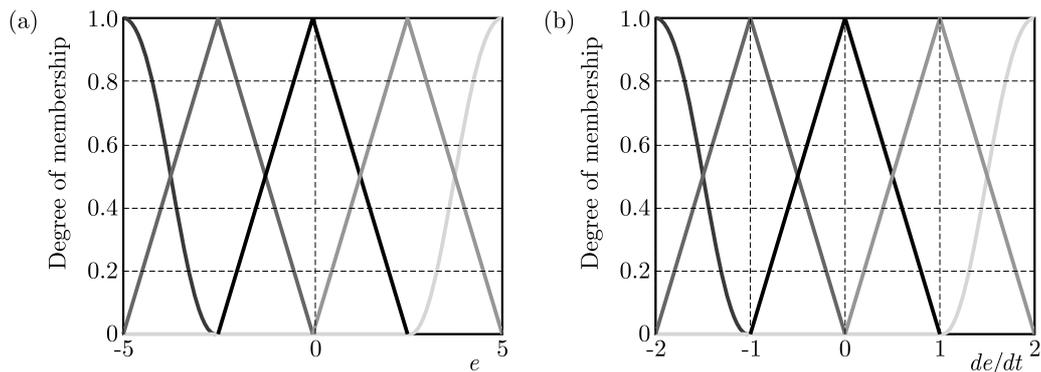

Fig. 3. (a) Membership function of the tracking error, (b) membership function of the derivative of the tracking error

Case *I*. The tracking performance of the proposed constrained control scheme is evaluated here and compared with that of the existing sliding mode method, see Zeinali and Leila (2010). Assume the reference position and the saturation limits are respectively given by $[x_d(t), y_d(t), \theta_d(t)]^{\mathrm{T}} = [3.5\,\mathrm{m}, 2\,\mathrm{m}, 0\,\mathrm{rad}]^{\mathrm{T}}$ and $-2 < \tau_i < 2$, $i = 1, 2, 3$. The external disturbance $\mathbf{d}(t) = (\sin(t) + 1)[1, 1, 1]^{\mathrm{T}}$ also perturb the system at time $t = 5\,\mathrm{s}$.

Figures 4a and 4b show that the tracking of the reference positions for x and y is obtained in the presence of disturbance. Figure 5 shows the capability of the proposed scheme in disturbance rejection compared with the sliding mode control by Zeinali and Notash (2010). The convergence of the controller coefficients K_p, K_D, K_I for tracking $x_d(t)$, $y_d(t)$ and $\theta_d(t)$ are demonstrated in Figs. 6 and 7. The control efforts in the proposed adaptive fuzzy method and the existing sliding controller are illustrated in Fig. 8.
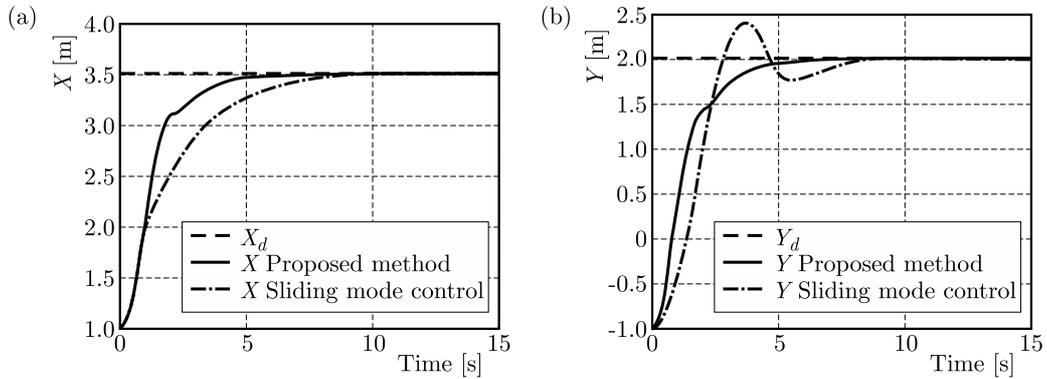
Fig. 4. (a) X direction and (b) Y direction tracking of the surface vessel
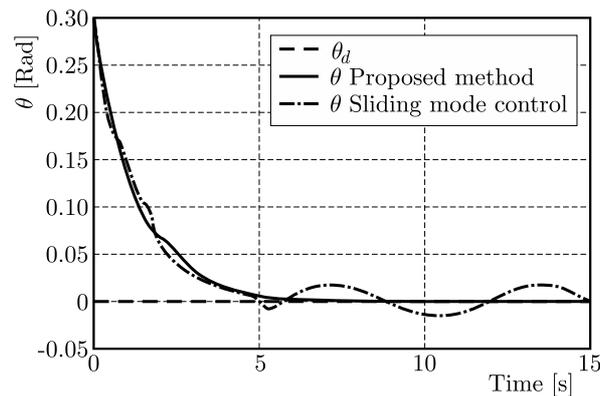
Fig. 5. Tracking of the pitch θ of the surface vessel
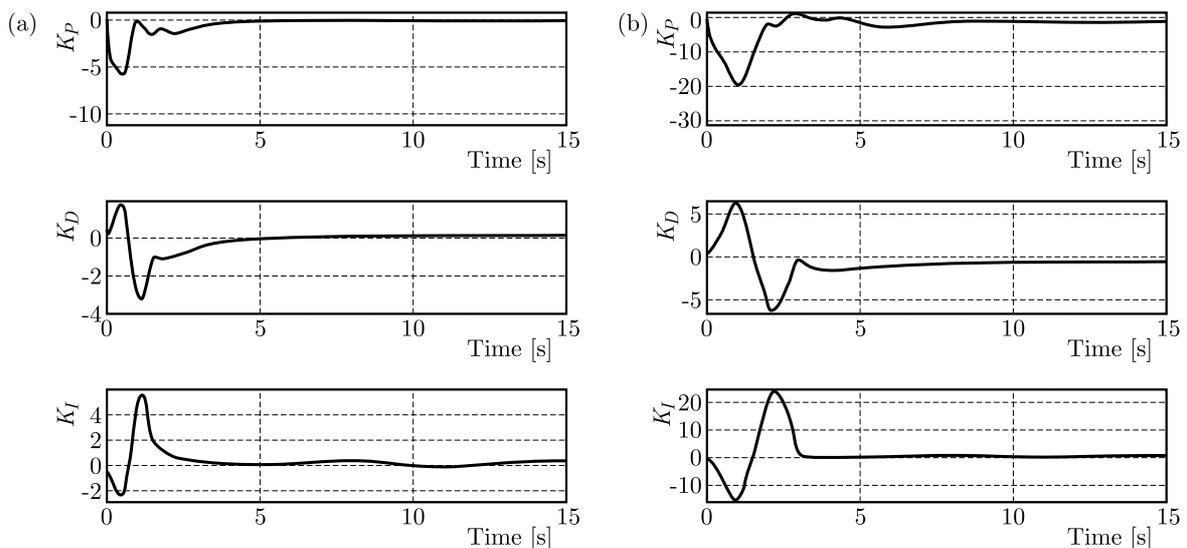
Fig. 6. Convergence of the controller parameters in (a) the x direction, (b) the y direction
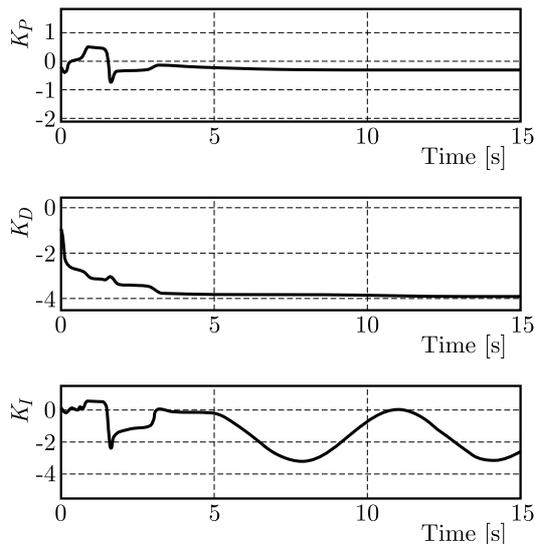
Fig. 7. Convergence of K_p, K_D, K_I for the yaw controller
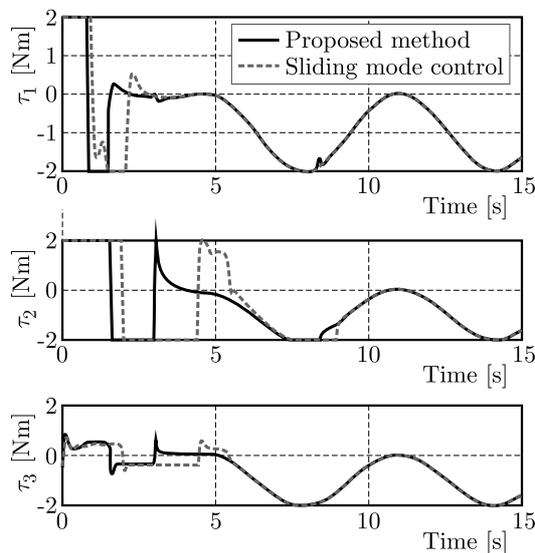
Fig. 8. Control signals in the proposed algorithm and the sliding control

To make a comparison between the designed adaptive fuzzy controller and the existing sliding control (Zeinali and Leila 2010), consider a cost function as

$$J = \int_0^{t_f} \left(\|e(t)\|^2 + \|u(t)\|^2 \right) dt \tag{4.11}$$

The lower cost of the proposed controller, as reported in Table 2, shows the advantage of the proposed approach.

Table 2. The costs of controllers in Case I

Controller	Sliding mode	Proposed method
J	117.1378	61.3327

<u>Case *II*.</u> In this case, the reference signal and the saturation operator parameters are considered respectively as

$$\begin{bmatrix} x_d(t) \\ y_d(t) \\ \theta_d(t) \end{bmatrix} = \begin{bmatrix} \sin(0.5t) \\ \cos(0.5t) \\ 0 \end{bmatrix} \qquad \text{and} \qquad -2 < \tau_i < 2 \qquad i = 1, 2, 3$$

The simulation results, illustrated in Figs. 9 and 10, show that the proposed method gives smoother responses with less tracking error, compared with the sliding mode control (Zeinali and Leila, 2010). In the tracking of the reference output on the channel y, the sliding mode algorithm is unstable, while the proposed method is stable and the tracking error is converged to zero. Figure 11 shows that the control effort of the proposed method is much lower than that in the other method. Unlike the sliding mode, the control signal is zero in the steady state for the proposed method. Comparing the results may be also possible by adopting cost function (4.11), as numerically reported in Table 3.
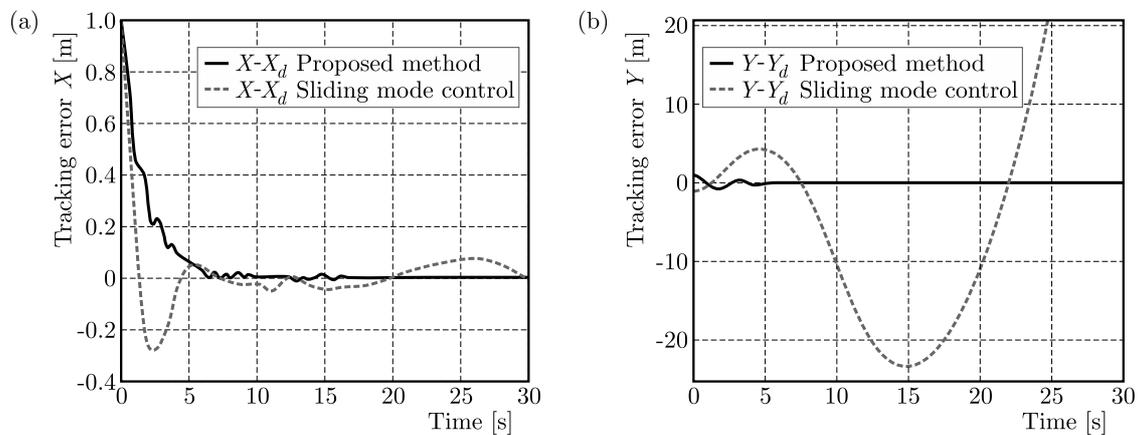

Fig. 9. Tracking error of the (a) the x direction, (b) the y direction
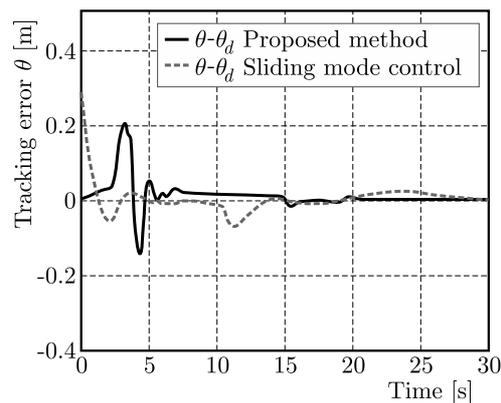
Fig. 10. Tracking error of pitch θ

Table 3. The cost of controllers in Case *II*

| Proposed method	Sliding mode	Controller
J	16933.4	77.8
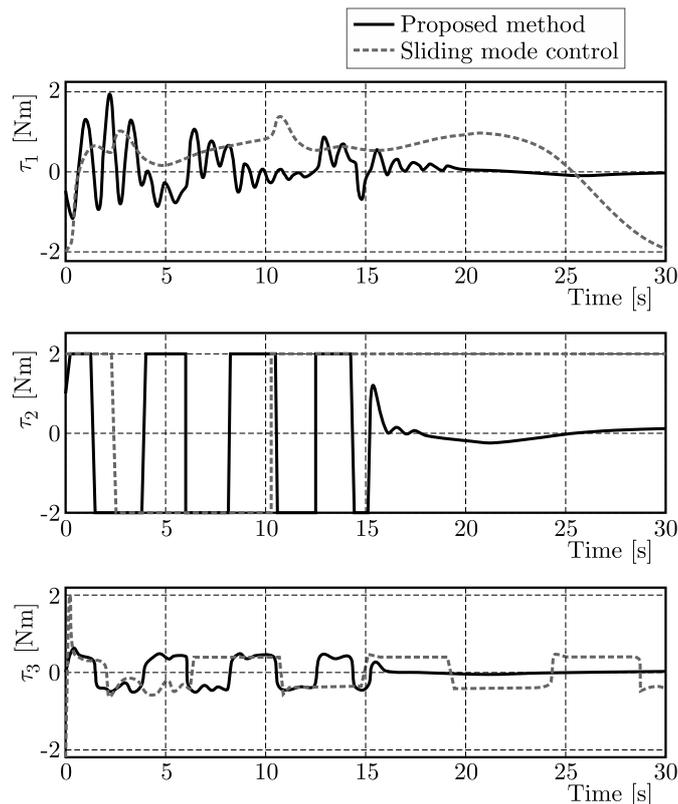
Fig. 11. Control signals

5. Conclusion

Focusing on the constraints on the inputs of nonlinear systems, the problem of robust tracking is investigated here. To solve the problem, an adaptive fuzzy algorithm is proposed for which the robust stability is proved using the Lyapunov stability theorem. As a practical situation, the problem is formulated for a surface vessel, taking the limitations on the control input into account. The designed controller is applied and the simulation results are presented to show the benefits of the method. The existing sliding control is also applied to the vessel and a cost function is defined to compare the results with the proposed scheme. In addition to demonstrations, a cost function is defined, and a numerical comparison is also made to show the benefits of the adaptive fuzzy algorithm.

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