JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
48, 3, pp. 551-566, Warsaw 2010

THE DYNAMIC MODEL OF A COMBAT TARGET HOMING SYSTEM OF AN UNMANNED AERIAL VEHICLE

Zbigniew Koruba
Kielce University of Technology, Faculty of Mechatronics and Machine Building, Kielce, Poland
e-mail: ksmzko@eden.tu.kielce.pl

Edyta Ładyżyńska-Kozdraś
Warsaw University of Technology, Faculty of Mechatronics, Warsaw, Poland
e-mail: e.ladyzynska@mchtr.pw.edu.pl

The work presents the concept of the application of an unmanned aerial vehicle (UAV) used in the process of direct reaching of ground targets (radio location stations, combat vehicles or even tanks). The kinematic model of UAV motion takes into consideration particular phases of the mission realised by the vessel, i.e. programmed flight during target search, follow flight after the encounter of the target as well as during the process of self directioning onto the target. Control laws for the automatic UAV combat pilot are presented and the dynamic model of automatically steered UAV is developed. In the examinations solutions of analytical mechanics for holonomic settings within the relative system tightly connected with the moving object are incorporated.

Key words: automatically steered aerial vehicle, control laws, controlled gyroscope

1. Introduction

All stages of UAV operation are characterised by great complexity. It requires various technological systems and solutions. The system of UAV control is of primary importance here. While UAV performs its mission, firstly, the measurement, evaluation and check of the flight path parameters and technical systems must be carried out. Secondly, it is necessary to properly control the flight and the seeking and illumination systems, which is achieved due to the identification and check of the above-mentioned parameters. The identification, check and control are all executed either directly by an operator or automatically.

The necessity of maintaining two-way (often continuous) communication with the ground command post is a distinct disadvantage to UAV operation as the post location might be revealed, although various means are employed to conceal such communication. In modern UAVs therefore, the autonomy of their systems in the task of ground target search and tracking plays the key role. It is required that it should be possible to introduce corrections during the programmed flight or even to change it completely depending on the situation, e.g. on target detection.

Modern ammunition of the so-called "precise" kind, like missiles, rockets and bombs (MRB) are controlled with semi-active homing methods. Such methods of control of the MRB flight path require target illumination, which is executed with radar beams or infrared radiation. The latter has been used more and more frequently because of well-known advantages it has.

Target illumination is usually performed from ground posts or from the air, from planes and helicopters. There are a number of disadvantages to this kind of illumination. It is necessary for the target to be visible. With illumination from ground posts, the target view can be blocked by natural obstructions. Moreover, the post can be easily detected and destroyed by the adversary. Manned planes and helicopters are used to carry out aerial illumination. As it is necessary to illuminate the target for a certain definite time, the flying vehicles face the risk of destruction. Such disadvantages are reduced to a large extent, if the illumination task is performed by a small-sized unmanned aerial vehicle. Manufactured according to "Stealth" technology, the vehicle is difficult to detect and kill. The problem is to execute its control in such a manner so that the vehicle would be able to fulfil the task of target illumination with sufficient accuracy.

On the modern battlefield, light small-sized UAVs perform the mission of ground target detection, tracking and illumination. Their modified versions, combat UAVs, are supposed not only to autonomously detect the target, but also destroy it with on-deck infrared homing missiles (tests on the combat model of Israeli Pioneer [1]). Alternatively, equipped with a warhead, they perform homing function in accordance with a specified homing algorithm (e.g. American vehicle called Lark [9]). The present paper puts forward a control algorithm for such combat UAVs, which on having autonomously detected targets, attack them (e.g. radar stations, combat vehicles or tanks) or illuminate them with a laser (Fig. 1).

Figure 2 presents the diagram of geometrical relationships holding between the kinematics of motion, in respect one to another, centres of mass of UAV and the target (points S, C) and point G (intersection of target detection and

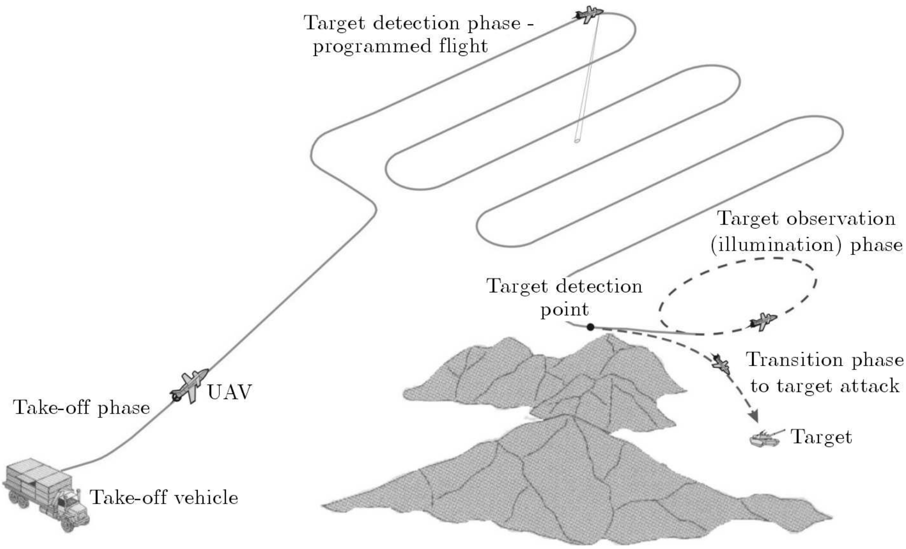

Fig. 1. General view of combat UAV mission performance

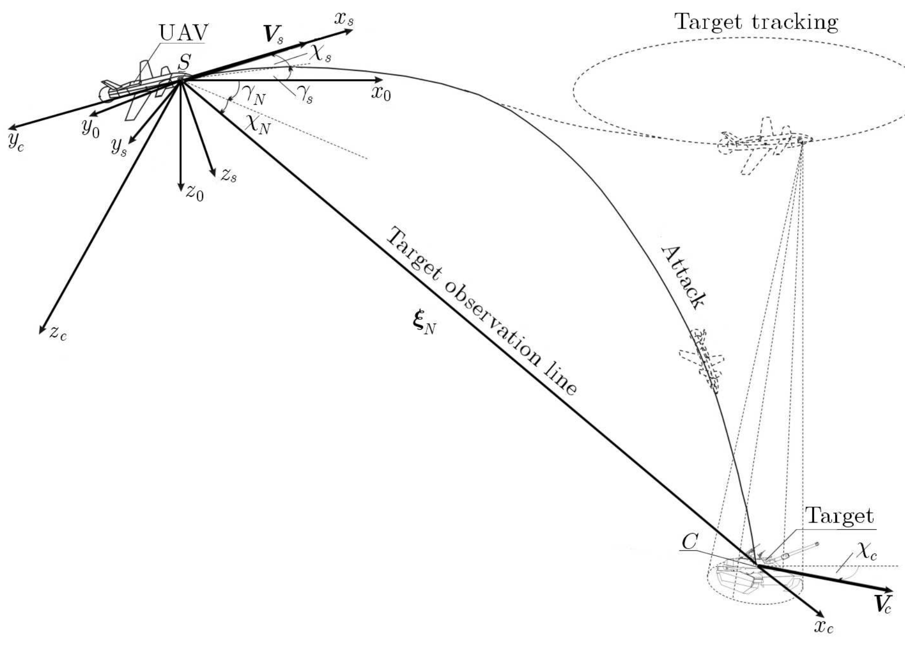

Fig. 2. Kinematics of combat UAV target homing guidance

observation line – LTSO with the Earth surface). On the basis of this diagram and following figures (Figs. 2-6), there kinematical equations of motion for UAV, LTSO, point G and the target were derived.

2. UAV navigation kinematics

2.1. Kinematic equation of UAV motion

Kinematics of UAV reciprocal motion and the ground target with the adopted co-ordinate systems shown in Fig. 2.

Projections of the left and the right hand of equation $(2.3)_1$ on the axes of the system $O_0x_{es}y_{es}z_{es}$ yield the following system of equations

$$\begin{aligned}
&\frac{d\boldsymbol{R}_{es}}{dt} = \boldsymbol{V}_s[\cos(\varphi_\chi^s - \chi_s)\cos\varphi_\gamma^s\cos\gamma_s + \sin\varphi_\gamma^s\sin\gamma_s]\\
&\frac{d\varphi_\chi^s}{dt}\boldsymbol{R}_{es}\cos\varphi_\gamma^s = -\boldsymbol{V}_s\sin(\varphi_\chi^s - \chi_s)\cos\gamma_s\\
&\frac{d\varphi_\gamma^s}{dt}\boldsymbol{R}_{es} = \boldsymbol{V}_s[\cos(\varphi_\chi^s - \chi_s)\sin\varphi_\gamma^s\cos\gamma_s - \cos\varphi_\gamma^s\sin\gamma_s]
\end{aligned} \tag{2.1}$$

where $\boldsymbol{R}_{es}$ is radius vectors centre of mass of UAV, $\boldsymbol{V}_s$ – vector of UAV flight, φ_γ^s, φ_χ^s – angles of inclination and deflection of vector $\boldsymbol{R}_{es}$, γ_s, χ_s – UAV flight angles and desired UAV flight angles.

The equations above show motion of the point S (UAV centre of mass) in relation to the motionless point O_0 (the origin of the terrestrial co-ordinate system). The path of UAV motion in the terrestrial co-ordinate system will be described by the following equations

$$\begin{aligned}
&x_{sx_0} = R_{es}\cos\varphi_\gamma^s\cos\varphi_\chi^s \qquad\qquad y_{sx_0} = R_{es}\cos\varphi_\gamma^s\sin\varphi_\chi^s\\
&z_{sz_0} = -R_{es}\sin\varphi_\gamma^s
\end{aligned} \tag{2.2}$$

2.2. The equation of motion of target detection and observation line (TDOL)

A procedure similar to that adopted for kinematic equations of UAV motion will give the following LTSO equation of motion

$$\frac{d\xi_N}{dt} = \Pi(t_0,t_w)(V_{sxn} - V_{gxn}) + [\Pi(t_w,t_s) + \Pi(t_s,t_k)](V_{sxn} - V_{cxn})$$

$$
\begin{aligned}
-\frac{d\chi_n}{dt}\xi_N\cos\gamma_n &= \Pi(t_0,t_w)(V_{syn}-V_{gyn}) + [\Pi(t_w,t_s)+\Pi(t_s,t_k)](V_{syn}-V_{cyn}) \\
\frac{d\gamma_n}{dt}\xi_N &= \Pi(t_0,t_w)(V_{szn}-V_{gzn}) + [\Pi(t_w,t_s)+\Pi(t_s,t_k)](V_{szn}-V_{czn})
\end{aligned}
\tag{2.3}
$$

where ξ_N is the distance from point G or C to S, γ_n, χ_n – angles of LTSO inclination and deflection, respectively, $\Pi(\cdot)$ – functions of rectangular impulse, t_0, t_w, t_s – instant of start of area penetration, target detection and start of target tracking and illumination, respectively, t_k – instant of target tracking completion (mission completion).

Components of the velocity vectors $\boldsymbol{V}_S$, $\boldsymbol{V}_G$ and $\boldsymbol{V}_C$ in the relative system $Sx_ny_nz_n$ take the following form

$$
\begin{aligned}
V_{sxn} &= V_s[\cos(\chi_n-\chi_s)\cos\gamma_n\cos\gamma_s - \sin\gamma_n\sin\gamma_s] \\
V_{syn} &= -V_s\sin(\chi_n-\chi_s)\cos\gamma_s \\
V_{szn} &= V_s[\cos(\chi_n-\chi_s)\sin\gamma_n\cos\gamma_s - \cos\gamma_n\sin\gamma_s] \\
V_{gxn} &= V_g[\cos(\chi_n-\chi_g)\cos\gamma_n\cos\gamma_g - \sin\gamma_n\sin\gamma_g] \\
V_{gyn} &= -V_g\sin(\chi_n-\chi_g)\cos\gamma_g \\
V_{gzn} &= V_g[\cos(\chi_n-\chi_g)\sin\gamma_n\cos\gamma_g - \cos\gamma_n\sin\gamma_g] \\
V_{cxn} &= V_c[\cos(\chi_n-\chi_c)\cos\gamma_n\cos\gamma_c - \sin\gamma_n\sin\gamma_c] \\
V_{cyn} &= -V_c\sin(\chi_n-\chi_c)\cos\gamma_c \\
V_{czn} &= V_c[\cos(\chi_n-\chi_c)\sin\gamma_n\cos\gamma_c - \cos\gamma_n\sin\gamma_c]
\end{aligned}
\tag{2.4}
$$

where γ_g, χ_g; γ_c, χ_c – angles of inclination and deflection of velocity vector in point G and of target velocity vector.

Trajectory of point G

$$
\begin{aligned}
\frac{dR_{eg}}{dt} &= \Pi(t_0,t_w)V_g\cos(\varphi_g-\chi_g) & x_{gx_0} &= R_{eg}\cos\varphi_g \\
\frac{d\varphi_g}{dt} &= \Pi(t_0,t_w)V_g\sin(\varphi_g-\chi_g) & y_{gy_0} &= R_{eg}\sin\varphi_g
\end{aligned}
\tag{2.5}
$$

Kinematcs of target motion

$$
\begin{aligned}
\frac{dR_{ec}}{dt} &= V_c[\cos(\varphi_\chi^c-\chi_c)\cos\varphi_\gamma^c\cos\gamma_c + \sin\varphi_\gamma^c\sin\gamma_c] \\
\frac{d\varphi_\chi^c}{dt}R_{ec}\cos\varphi_\gamma^c &= -V_c\sin(\varphi_\chi^c-\chi_s)\cos\gamma_c \\
\frac{d\varphi_\gamma^c}{dt}R_{ec} &= V_c[\cos(\varphi_\chi^c-\chi_c)\sin\varphi_\gamma^c\cos\gamma_c - \cos\varphi_\gamma^c\sin\gamma_c]
\end{aligned}
\tag{2.6}
$$

The target trajectory in the terrestrial co-ordinate system is described by the following equations

$$
\begin{aligned}
x_{cx0} &= R_{ec}\cos\varphi_\gamma^c\cos\varphi_\chi^c \qquad\qquad y_{sx0} = R_{ec}\cos\varphi_\gamma^c\sin\varphi_\chi^c \\
z_{sz0} &= -R_{ec}\sin\varphi_\gamma^c
\end{aligned}
\tag{2.7}
$$

where φ_γ^c, φ_χ^c are angles of inclination and deflection of vectors $\boldsymbol{R}_{ec}$.

3. Determination of desired UAV flight angles

The UAV flight angles χ_s and γ_s in the seeking phase and during attack on the detected target will be determined from the following relationship

$$
\chi_s^* = \Pi(t_0,t_w)\chi_s^p + \Pi(t_w,t_k)\chi_s^n \qquad\qquad \gamma_s^* = \Pi(t_0,t_w)\gamma_s^p + \Pi(t_w,t_k)\gamma_s^n \tag{3.1}
$$

Those above are distribution equations due to the functions of rectangular impulse $\Pi(\cdot)$, which occur in them. Thus they offer an option to describe changes in the UAV flight angles at its different stages.

The UAV flight angles χ_s and γ_s in the seeking, transition to tracking phase and laser illumination of the detected target have the following form

$$
\begin{aligned}
\chi_s^* &= \Pi(t_0,t_w)\chi_s^p + \Pi(t_w,t_s)\chi_s^t + \Pi(t_s,t_k)\chi_s^o \\
\gamma_s^* &= \Pi(t_0,t_w)\gamma_s^p + \Pi(t_w,t_s)\gamma_s^t + \Pi(t_s,t_k)\gamma_s^o
\end{aligned}
\tag{3.2}
$$

where γ_s^o, χ_s^o are UAV flight angles in target tracking and laser illumination, γ_s^t, χ_s^t – UAV flight angles in transition from programmed flight to target tracking flight.

Quantities χ_s^p and γ_s^p stand for pre-programmed UAV flight angles in the phase of the Earth surface patrolling (target seeking), therefore they are pre-set time functions

$$
\chi_s^p = \chi_s^p(t) \qquad\qquad \gamma_s^p = \gamma_s^p(t) \tag{3.3}
$$

Prior to the determination of UAV flight angles χ_s^o and γ_s^o, an assumption is made for the instance of seeking and simultaneous laser illumination of the detected target (Koruba, 2001).

For the sake of simplification, let us assume that UAV motion, both during the penetration and tracking, takes place in a horizontal plane at the pre-set altitude H_s, whereas the target and point G move in the terrestrial plane. Then, we will be able to assume that

$$\gamma_s^o = 0 \qquad \gamma_c = 0 \qquad \gamma_g = 0 \tag{3.4}$$

Furthermore, for the sake of convenience, a notation is introduced

$$r_N = \xi_N \cos\gamma_n \tag{3.5}$$

and the time derivative of this expression calculated

$$\frac{dr_N}{dt} = \frac{d\xi_N}{dt}\cos\gamma_n - \xi_N\frac{d\gamma_n}{dt}\sin\gamma_n \tag{3.6}$$

Taking into account (3.4)-(3.6), we can limit further considerations to planar motion in the horizontal plane and after inserting (3.4)-(3.6), equations (2.5) have the form

$$\begin{aligned}
\frac{dr_N}{dt} &= \Pi(t_0,t_w)[V_s\cos(\chi_n-\chi_s^p) - V_g\cos(\chi_n-\chi_g)] + \\
&\quad +\Pi(t_w,t_s)[V_s\cos(\chi_n-\chi_s^t) - V_c\cos(\chi_n-\chi_c)] + \\
&\quad \Pi(t_s,t_k)[V_s\cos(\chi_n-\chi_s^s) - V_c\cos(\chi_n-\chi_c)] \\
\frac{d\chi_n}{dt} &= \Pi(t_0,t_w)\frac{V_s\sin(\chi_n-\chi_g) - V_g\sin(\chi_n-\chi_s^p)}{r_N} + \\
&\quad +\Pi(t_w,t_s)\frac{V_s\sin(\chi_n-\chi_c) - V_c\sin(\chi_n-\chi_s^t)}{r_N} + \\
&\quad +\Pi(t_s,t_k)\frac{V_s\sin(\chi_n-\chi_c) - V_c\sin(\chi_n-\chi_s^s)}{r_N}
\end{aligned} \tag{3.7}$$

We can make a demand that at the instant of target detection, UAV should automatically start target tracking flight, which consists in the vehicle motion at the pre-set distance from the target $r_{N0} = \xi_{N0}\cos\gamma_n = \text{const}$ (in the horizontal plane at the constant altitude H_s).

Until the distance r_N between points S and C is different from r_{N0}, the program for the angle deflection $\chi_s = \chi_s^t$ and $\gamma_s = \gamma_s^t$, is determined from (Koruba, 1999)

$$\frac{d\chi_s^t}{dt} = a_\chi \operatorname{sgn}(r_{N0} - r_N)\frac{d\chi_n}{dt} \qquad \gamma_s^t = 0 \tag{3.8}$$

which makes UAV either approach or move away from the target (depending on the sign of the function $\operatorname{sgn}(r_{N0} - r_N)$), in accordance with the so-called proportional navigation method (Dubiel, 1980). When the condition $r_{N0} = r_N$, is satisfied, the program for the control of the angle χ_s^o is determined from equation $(3.7)_1$, which is rearranged to the form

$$V_s \cos(\chi_n - \chi_s^o) = V_c \cos(\chi_n - \chi_c) \tag{3.9}$$

Hence, on the assumption that UAV moves in the horizontal plane at the constant altitude H_s, UAV flight angles in laser illumination of the detected target χ_s^o and γ_s^o are determined from the relations

$$\chi_s^o = \chi_n - \arccos\Big[\frac{V_c}{V_s}\cos(\chi_n - \chi_c)\Big] \qquad \gamma_s^o = 0 \tag{3.10}$$

The UAV flight angles during attack on the detected target χ_s^n and γ_s^n, are determined from the relations describing the proportional approach (Dubiel, 1980)

$$\frac{d\chi_s^n}{dt} = a_\chi \frac{d\chi_n}{dt} \qquad \frac{d\gamma_s^n}{dt} = a_\gamma \frac{d\gamma_n}{dt} \tag{3.11}$$

where a_γ, a_χ are coefficients of proportional navigation.

The above determined angles χ_s^* and γ_s^* specify the desired position in space of the missile velocity vector. The discrepancy between the pre-set and actual angular position of UAV velocity vector becomes the displacement error. It is also called an incongruence parameter for the autopilot automatic regulation. The value and direction of the displacement error provides the basis to form a control signal, which after appropriate transformation is passed to executive organs. They deflect control surfaces in the lateral and longitudinal channel by the worked out angle values.

4. Combat UAV dynamics model

The description of UAV dynamics has been executed within the reference system fixed to the object. The following physical model assumptions have been adopted:

- UAV is treated as a rigid body with six degrees of freedom having movable but non-deformable steering modules,
- Vessel steering devices are weightless and their surfaces strictly control aerodynamic forces and moments,

- UAV steering takes place with the application of three channels: reclination through the sloping back of the height steering device as well as through the process of tilting through the process of shuttlecock steering,
- UAV masses and inertia moments are constant during flight,
- UAV possesses OSxz symmetry surface – geometrical, inertial and aerodynamical.

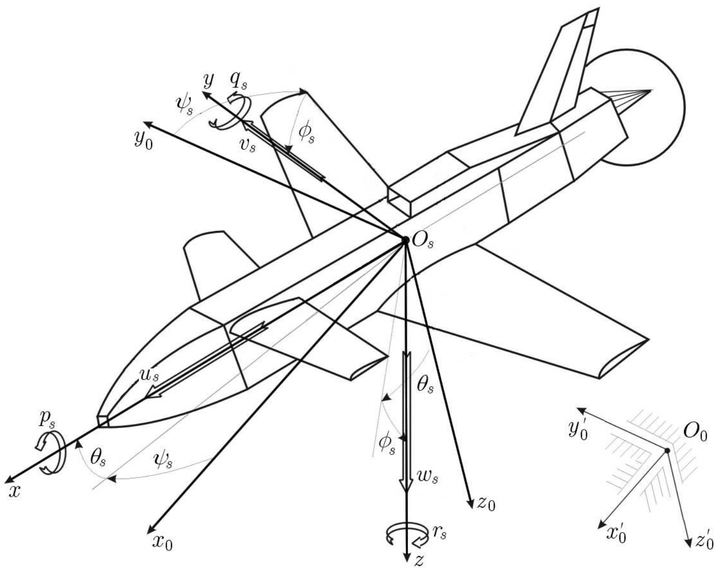

Fig. 3. The system of forces and moments affecting observation line during the flight

4.1. Dynamic equations of UAV motion

In the light of further analysis, description of UAV behaviour during spatial flight will be developed on the basis of co-ordinates ϕ_s, θ_s, ψ_s and quasi-velocites u_s, v_s, w_s, p_s, q_s, r_s with the application of Boltzmann-Hamel equations, which are convenient for an object with mechanical settings connected to it. These equations are generalised Lagrange equations of the II kind for non-inertial settings described within quasi-co-ordinates. In the general form, they are expressed as follows (Ładyżyńska-Kozdraś, 2004, 2008)

$$\frac{d}{dt}\frac{\partial T^*}{\partial \omega_\mu} - \frac{\partial T^*}{\partial \pi_\mu} + \sum_{r=1}^{k}\sum_{\alpha=1}^{k}\gamma_{\mu\alpha}^{r}\frac{\partial T^*}{\partial \omega_r}\omega_\alpha = Q_\mu^* \tag{4.1}$$

where: $\alpha, \mu, r = 1, 2, \ldots, k$; k is the the number of degrees of freedom, ω_μ – quasi-velocity, T^* – kinetic energy expressed in terms of quasi-velocities, π_μ – quasi-co-ordinates, Q^*_μ – generalised forces, $\gamma^r_{\alpha\mu}$ – three phase Boltzmann multipliers, determined from the following relation

$$\gamma^r_{\alpha\mu} = \sum_{\delta=1}^{k} \sum_{\lambda=1}^{k} \Big(\frac{\partial a_{r\delta}}{\partial q_\lambda} - \frac{\partial a_{r\lambda}}{\partial q_\delta} \Big) b_{\delta\mu} b_{\lambda\alpha} \tag{4.2}$$

The relations between quasi- and generalised velocities

$$\omega_\delta = \sum_{\alpha=1}^{k} a_{\delta\alpha} \dot{q}_\alpha \qquad\qquad \dot{q}_\delta = \sum_{\mu=1}^{k} b_{\delta\mu} \omega_\mu \tag{4.3}$$

where $\dot{q}_\delta$ are generalised velocities, q_k – generalised coefficients, $a_{\delta\alpha} = a_{\delta\alpha}(q_1, q_2, \ldots, q_k)$ as well as $b_{\delta\alpha} = b_{\delta\alpha}(q_1, q_2, \ldots, q_k)$ – coefficients which are functions of the generalised coordinates. Simultaneously, the following matrix expression $[a_{\delta\mu}] = [b_{\delta\mu}]^{-1}$ holds.

The researched UAV is described with the application of the following coefficient vectors and generalised velocities

$$\begin{aligned} \boldsymbol{q} &= [q_1, q_2, q_3, q_4, q_5, q_6]^\top = [x_{0s}, y_{0s}, z_{0s}, \phi_s, \theta_s, \psi_s]^\top \\ \dot{\boldsymbol{q}} &= [\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6]^\top = [\dot{x}_{0s}, \dot{y}_{0s}, \dot{z}_{0s}, \dot{\phi}_s, \dot{\theta}_s, \dot{\psi}_s]^\top \end{aligned} \tag{4.4}$$

where x_0, y_0, z_0 – location of UAV mass centre in terrestrial co-ordinate system, θ_s, ψ_s, ϕ_s – angles of UAV longitudinal axis inclination, deflection and tilt, respectively.

As well as quasi co-ordinates and quasi-velocities

$$\begin{aligned} \boldsymbol{\pi} &= [\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6]^\top = [\pi_u, \pi_v, \pi_w, \pi_p, \pi_q, \pi_r]^\top \\ \dot{\boldsymbol{\pi}} &= [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6]^\top = [u_s, v_s, w_s, p_s, q_s, r_s]^\top \end{aligned} \tag{4.5}$$

where $u_s, v_s, w_s;\ p_s, q_s, r_s]$ are components of vector of UAV flight linear velocity and angular velocity.

Relationships between quasi- and generalised velocities are expressed as (Ładyżyńska-Kozdraś, 2008)

$$
\begin{bmatrix} u_s \\ v_s \\ w_s \end{bmatrix} = \begin{bmatrix} \cos\psi_s\cos\theta_s & \sin\psi_s\cos\theta_s & -\sin\theta_s \\ \begin{array}{c}\sin\phi_s\cos\psi_s\sin\theta_s + \\ -\sin\psi_s\sin\phi_s\end{array} & \begin{array}{c}\sin\phi_s\sin\psi_s\sin\theta_s + \\ +\cos\psi_s\cos\phi_s\end{array} & \sin\phi_s\cos\theta_s \\ \begin{array}{c}\cos\phi_s\cos\psi_s\sin\theta_s + \\ +\sin\psi_s\sin\phi_s\end{array} & \begin{array}{c}\cos\phi_s\sin\psi_s\sin\theta_s + \\ -\cos\psi_s\sin\phi_s\end{array} & \cos\phi_s\cos\theta_s \end{bmatrix} \begin{bmatrix} \dot{x}_{0s} \\ \dot{y}_{0s} \\ \dot{z}_{0s} \end{bmatrix} \tag{4.6}
$$

$$
\begin{bmatrix} p_s \\ q_s \\ r_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta_s \\ 0 & \cos\phi_s & \sin\phi_s\cos\theta_s \\ 0 & -\sin\phi_s & \cos\phi_s\cos\theta_s \end{bmatrix} \begin{bmatrix} \dot{\phi}_s \\ \dot{\theta}_s \\ \dot{\psi}_s \end{bmatrix}
$$

The Boltzmann-Hamel equations, upon the calculation of Boltzmann multipliers as well determination of kinetic energy in terms of quasi-velocities, yield the following differential equations of the second kind which describe the behaviour of combat UAV upon the path during the process of target homing.

— Longitudinal motion

$$
m_s(\dot{u}_s + q_s w_s - r_s v_s) - S_x(q_s^2 + r_s^2) - S_y(\dot{r}_s - p_s q_s) + S_z(\dot{q}_s + p_s r_s) = X \tag{4.7}
$$

— Side motion

$$
(4.8) m_s(\dot{v}_s + r_s u_s - p_s w_s) + S_x(\dot{r}_s + p_s q_s) - S_y(p_s^2 + r_s^2) - S_z(\dot{p}_s - q_s r_s) = Y \tag{4.8}
$$

— Lift

$$
m_s(\dot{w}_s + p_s v_s - q_s u_s) - S_x(\dot{q}_s + p_s r_s) + S_y(\dot{p}_s + q_s r_s) - S_z(q_s^2 + p_s^2) = Z \tag{4.9}
$$

— Tilt

$$
\begin{aligned} & I_x\dot{p}_s - (I_y - I_z)q_s r_s - I_{xy}(\dot{q}_s - p_s r_s) - I_{xz}(\dot{r}_s + p_s q_s) - I_{yz}(q_s^2 - r_s^2) + \\ & \quad + S_y(\dot{w}_s + p_s v_s - q_s u_s) + S_z(p_s w_s - r_s u_s - \dot{v}_s) = L \end{aligned} \tag{4.10}
$$

— Reclination

$$
\begin{aligned} & I_y\dot{q}_s - (I_z - I_x)r_s p_s - I_{xy}(\dot{p}_s + q_s r_s) - I_{yz}(\dot{r}_s - p_s q_s) - I_{xz}(r_s^2 - p_s^2) + \\ & \quad - S_x(\dot{w}_s + p_s v_s - q_s u_s) + S_z(\dot{u}_s - r_s v_s + q_s w_s) = M \end{aligned} \tag{4.11}
$$

— Sloping away

$$
\begin{aligned} & I_z\dot{r}_s - (I_x - I_y)p_s q_s - I_{yz}(\dot{q}_s + p_s r_s) - I_{xz}(\dot{p}_s - r_s q_s) - I_{xy}(p_s^2 - q_s^2) + \\ & \quad + S_x(\dot{v}_s - p_s w_s + r_s u_s) - S_y(\dot{u}_s - r_s v_s + q_s w_s) = N \end{aligned} \tag{4.12}
$$

The aforementioned set of equations upon the determination of component forces X, Y, Z and moments L, M, N, which have been generalised, constitutes the general mathematic model of the described UAV dynamics. m_s is the mass of UAV, I_x, I_y, I_z – moments of inertia in relation to UAV individual axes, I_{xy}, I_{yz}, I_{zx} – moments of deviation of UAV, S_x, S_y, S_z – static moments in relation to UAV individual axes.

4.2. Forces and external moments influencing UAV motion

The vector of forces and the moment of external forces exerting influence upon the flying UAV, the components of which constitute the right-hand sides of equations of motion (4.7)-(4.12), constitutes the sum of the central influence, according to which the object moves. This vector is the resultant of the following forces: aerodynamic $\boldsymbol{Q}^a$, gravitation $\boldsymbol{Q}^g$ and steering forces (tilting of aerodynamic steering devices) $\boldsymbol{Q}^\delta$

$$\boldsymbol{Q}^* = \boldsymbol{Q}^a + \boldsymbol{Q}^g + \boldsymbol{Q}^\delta = \begin{bmatrix} X_s \\ Y_s \\ Z_s \\ L_s \\ M_s \\ N_s \end{bmatrix} = \begin{bmatrix} X^a \\ Y^a \\ Z^a \\ L^a \\ M^a \\ N^a \end{bmatrix} + \begin{bmatrix} X^g \\ Y^g \\ Z^g \\ L^g \\ M^g \\ N^g \end{bmatrix} + \begin{bmatrix} X^\delta \\ Y^\delta \\ Z^\delta \\ L^\delta \\ M^\delta \\ N^\delta \end{bmatrix} \tag{4.13}$$

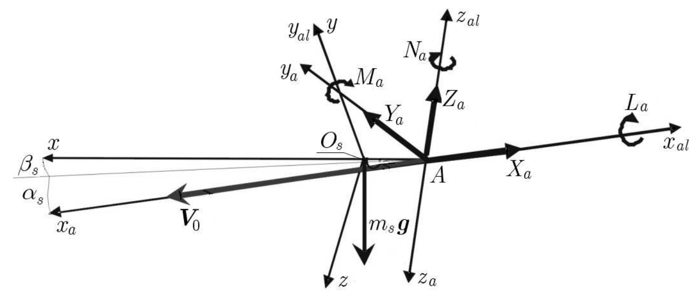

Fig. 4. Gravity force as well as aerodynamic forces and moments acting on UAV during flight

The matrix of external forces and gravity force (Fig. 4)

$$\boldsymbol{Q}^g = m_s g \begin{bmatrix} -\sin\theta_s \\ \cos\theta_s \sin\phi_s \\ \cos\theta_s \cos\phi_s \\ 0 \\ -x_c \cos\theta_s \cos\phi_s \\ x_c \cos\theta_s \sin\phi_s \end{bmatrix} = \begin{bmatrix} X^g \\ Y^g \\ Z^g \\ L^g \\ M^g \\ N^g \end{bmatrix} \tag{4.14}$$

The matrices of forces and aerodynamic moments (Fig. 4) are of the following form (Ładyżyńska-Kozdraś, 2004)

$$\boldsymbol{F}^a = \mathcal{A}\begin{bmatrix}\cos\alpha_s\cos\beta_s & \cos\alpha_s\sin\beta_s & -\sin\alpha_s\\ -\sin\beta_s & \cos\beta_s & 0\\ \sin\alpha_s\cos\beta_s & \sin\alpha_s\sin\beta_s & \cos\alpha_s\end{bmatrix}\begin{bmatrix}C_x\\ C_y\\ C_z\end{bmatrix} + \begin{bmatrix}X_q q\\ Y_p p + Y_r r\\ Z_q q\end{bmatrix} = \begin{bmatrix}X^a\\ Y^a\\ Z^a\end{bmatrix} \tag{4.15}$$

$$\begin{aligned}\boldsymbol{M}^a &= \mathcal{A}\begin{bmatrix}0 & -z_a & y_a\\ z_a & 0 & -x_a\\ -y_a & x_a & 0\end{bmatrix}\begin{bmatrix}\cos\alpha_s\cos\beta_s & \cos\alpha_s\sin\beta_s & -\sin\alpha_s\\ -\sin\beta_s & \cos\beta_s & 0\\ \sin\alpha_s\cos\beta_s & \sin\alpha_s\sin\beta_s & \cos\alpha_s\end{bmatrix}\begin{bmatrix}C_x\\ C_y\\ C_z\end{bmatrix} +\\ &+\mathcal{A}\begin{bmatrix}\cos\alpha_s\cos\beta_s & \cos\alpha_s\sin\beta_s & -\sin\alpha_s\\ -\sin\beta_s & \cos\beta_s & 0\\ \sin\alpha_s\cos\beta_s & \sin\alpha_s\sin\beta_s & \cos\alpha_s\end{bmatrix}\begin{bmatrix}C_l\\ C_m\\ C_n\end{bmatrix} + \begin{bmatrix}L_p p + L_r r\\ M_q q\\ N_p p + N_r r\end{bmatrix} = \begin{bmatrix}L^a\\ M^a\\ N^a\end{bmatrix}\end{aligned}$$

where

$$\mathcal{A} = \frac{1}{2}\rho S V_s^2$$

and $\rho(H_s)$ is the air density at the given height H_s, S – the surface of reference (surface of UAV wing), C_x, C_y, C_z – dimensionless aerodynamic coefficients of components of aerodynamic forces: resistance, side and carrier force, respectively, C_l, C_m, C_n – dimensionless aerodynamic coefficients of reclination, tilt and slumping away moments, respectively, and X_q, Y_p, Y_r, Z_q, L_p, L_r, M_q, N_p, N_r are derivatives of components of the aerodynamic forces and their resect to components of linear and angular speeds.

The location of the aerodynamic centre A with respect to the centre of mass O_s (Fig. 4) is given by

$$\boldsymbol{r}_A = x_a\boldsymbol{i} + y_a\boldsymbol{j} + z_a\boldsymbol{k} \tag{4.16}$$

UAV approach angle

$$\alpha_s = \arctan\frac{w_s}{u_s} \tag{4.17}$$

and slide angle

$$\beta_s = \arcsin\frac{v_s}{V_s} \tag{4.18}$$

During the flight, UAV is steered with the application of an automatic method. The process of steering takes place by making use of two tilt channels through controlling the height steering system δ_H and the tilt of the steering direction δ_V.

The matrix of forces and moments in the general method is presented as follows

$$\boldsymbol{Q}^{\delta} = \begin{bmatrix} X_{\delta H} & X_{\delta V} \\ 0 & Y_{\delta V} \\ Z_{\delta H} & 0 \\ 0 & L_{\delta V} \\ M_{\delta H} & 0 \\ 0 & N_{\delta V} \end{bmatrix} \begin{bmatrix} \delta_H \\ \delta_V \end{bmatrix} = \begin{bmatrix} X^{\delta} \\ Y^{\delta} \\ Z^{\delta} \\ L^{\delta} \\ M^{\delta} \\ N^{\delta} \end{bmatrix} \tag{4.19}$$

Through the application of Boltzmann-Hamel method for the holonomic constraints of UAV, equations of motion were determined. In the process of substitution into equations (4.7)-(4.12) the stipulated forces and moments of external forces acting upon UAV (4.13)-(4.19), the complete model of dynamics of UAV has been obtained.

4.3. UAV control

The control of UAV motion is realised by means of deflections of ailerons, rudder and elevator by angles δ_l, δ_m and δ_n, respectively.

The automatic pilot (AP) is responsible for UAV maintaining the desired flight path. On the basis of derived relations (3.1) and (3.2), it works out control signals for the control executive system.

Having taken into account the dynamics of rudder and elevator deflections, we arrive at the control formula for autopilot, which reads as follows

$$\begin{aligned} &\frac{d^2\delta_m}{dt^2} + h_{ms}\frac{d\delta_m}{dt} + k_{ms}\delta_m = k_m(\gamma_s - \gamma_s^*) + h_m\Big(\frac{d\gamma_s}{dt} - \frac{d\gamma_s^*}{dt}\Big) + b_m u_m \\ &\frac{d^2\delta_n}{dt^2} + h_{ns}\frac{d\delta_n}{dt} + k_{ns}\delta_n = k_n(\chi_s - \chi_s^*) + h_m\Big(\frac{d\chi_s}{dt} - \frac{d\chi_s^*}{dt}\Big) + b_n u_n \end{aligned} \tag{4.20}$$

In the course of its mission, a light UAV can be affected by various kinds of interference such as gusts of wind, vertical ascent and katabatic motion of air-masses or shock waves produced by missiles explosions nearby. At the instant of target detection, UAV automatically proceeds from the flight along programmed trajectory to the target tracking flight in accordance with the pre-set algorithm. In the case under consideration, the algorithm is meant to maintain a constant distance from the target. This way, the most favourable conditions for the target to be kept within the view area of the tracking system lens is provided. A sudden switch-over of the control system (from one flight phase to another) may disturb the UAV motion. Moreover, the dynamic effects, which result from the above-mentioned interference and control switch-over, change the flight quality and vehicle aerodynamic characteristics. The course of manoeuvres necessary to accomplish the assigned task suggests the occurrence of clearly non-linear characteristics of the controlled object (Koruba, 2001). It is therefore necessary to apply such an autopilot to UAV, so that the pre-set accuracy of the programmed and tracking flight would be guaranteed, and at the same time, UAV stability maintained.

5. Conclusions

The UAV navigation and control model presented in the paper fully describes the autonomous motion of a combat vehicle whose task is not only to detect and identify a ground target, but also to illuminate it with a laser or attack it. The operator intervention in the UAV control process can be limited to cases of total getting off of the pre-set path or target disappearance from the view area of the tracking system lens (wind gusts, missiles explosions, etc.). It is, therefore, necessary to provide means of sending information about such events to the control station so that the operator would be able to take over the UAV flight control if the need arises. Further theoretical investigations, calculations as well as simulation and experimental work should concentrate on: a) determination of the optimum UAV flight program, b) the algorithm for the Earth surface scanning to ensure the quickest target detection, c) the program for the minimum time of UAV transition from the programmed to target tracking flight or the detected target homing in accordance with the pre-set algorithm.

Acknowledgement

This work was supported by Polish Ministry of Science and Higher Education – project ON501 003534.

References

1. *Defence Procurement Analysis. Special Focus on UAVs*, Summer 1998
2. Dubiel S., 1980, *Rocket Construction. Part 1 – Overloads*, edition 2, Warsaw Military University of Technology Publishing House [in Polish]
3. Graffstein J., Krawczyk M., Maryniak J., 1997, Modelling the dynamics of automatically controlled unmanned aerial vehicle flight with the application

of non-holonomic constraints theory, *Scientific Papers of the Chair of Applied Mechanics*, **4**, XXXVI PTMTS Symposium "Modelling in Mechanics", Gliwice [in Polish]

4. Koruba Z., 1995, The options of terrain penetration and selected object tracking from unmanned aerial vehicle deck, *Scientific Papers of Rzeszów University of Technology, Mechanics*, **45**, 357-364 [in Polish]
5. Koruba Z., 1999, Unmanned aerial vehicle flight programme, ground surface scanning and laser target illumination, *Journal of Technical Physics*, **4**
6. Koruba Z., 2001, Dynamics and control of gyroscope on the deck of aerial vehicle, *Monographs, Studies, Dissertations No 25*, Kielce University of Technology, pp. 285
7. Ładyżyńska-Kozdraś E., 2004, Modelowanie niesymetrycznego odpalenia rakiety z manewrującego samolotu, *Zeszyty Naukowe Katedry Mechaniki Stosowanej Politechniki Śląskiej, Modelowanie w Mechanice*, **23**, 273-280
8. Ładyżyńska-Kozdraś E., 2008, Analiza dynamiki przestrzennego ruchu rakiety sterowanej automatycznie, *Mechanika w Lotnictwie, ML-XII 2008*, Wydawnictwo PTMTS, Warszawa
9. *Unmanned Vehicles*, **3**, 3, August-September 1998

Dynamiczny model naprowadzania bojowego bezpilotowego aparatu latającego

Streszczenie

W pracy przedstawiona została koncepcja zastosowania bezpilotowego aparatu latającego (BAL) do bezpośredniego rażenia celów naziemnych (stacje radiolokacyjne, wozy bojowe czy też czołgi). Model kinematyczny ruchu BAL uwzględnia poszczególne etapy realizowanej przez aparat misji, tj. lot programowy podczas wyszukiwania celu, lot śledzący po wykryciu celu oraz lot podczas samonaprowadzania na cel. Przytoczono prawa sterowania dla pilota automatycznego bojowego BAL. Opracowany został dynamiczny model ruchu automatycznie sterowanego BAL przy zastosowaniu równań mechaniki analitycznej dla układów holonomicznych w układzie odniesienia sztywno związanym z poruszającym się obiektem.

Manuscript received October 25, 2009; accepted for print December 3, 2009