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TWO CALCULATION SCHEMES FOR DETERMINATION OF THERMAL STRESSES DUE TO FRICTIONAL HEATING DURING BRAKING

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Thermal stresses in three-element (disc/pad/metal slice) and two-element (disc/pad) models of frictional heating during braking are analysed. Evolutions and distributions of the lateral normal stresses in depth from the surface of friction are studied for the following frictional system materials: cast iron/ metal ceramics/steel and metal ceramics/cast iron.

Key words: braking, friction, heat generation, temperature, thermal stresses

1. Introduction

The plane and axi-symmetric transient temperature field and thermal stresses in brake systems were analysed by Ling (1973), Burton (1980), Barber *et al.* (1985), Barber and Comminou (1989) and other authors. The temperatures and stresses from a solution to a one-dimensional contact problem with transient frictional heat generation were studied by Fasekas (1953), Ho *et al.* (1974), Chichinadze *et al.* (1979), Olesiak *et al.* (1997).

The heating on a friction surface during braking leads to a temperature shock that generates surface cracks (Mackin *et al.*, 2002; Yamabe *et al.*, 2003). Cleavage of the material in the process of thermal cracking results from tensile stresses which occur when the friction elements are heated by the moving heat flow. When the stresses exceed the tensile strength of the material, then cracks arise on the contact surface. Low thermal conductivity of friction pad materials is the reason why considerable thermal stresses are generated in a thin subsurface layer during braking. As a result, destruction of the friction surface can take place both during heating at braking, and during cooling after a stop. An analysis of surface temperatures and stresses in the frictional elements, using microscopic examination and the finite elements method, showed that the dominant stresses in these components are thermal stresses (Kennedy and Karpe, 1982). The mathematical model of thermal splitting of homogeneous and piece-wise homogeneous bodies during laser or frictional heating of their surfaces by a heat flow of known intensity was proposed in articles by Yevtushenko *et al.* (2007), Yevtushenko and Kuciej (2006).

The solutions to two heat conduction problems with frictional heating during braking with a uniform retardation were obtained in a paper by Yevtushenko and Kuciej (2010) for both a tribosystem consisting of a half-space sliding on a surface of a strip deposited on a semi-infinite foundation (three-element system) and for a two-element system consisting of a plane-parallel strip and a half-space. Using the non-stationary temperature field obtained by Yevtushenko and Kuciej (2010), we find the solution to distributions of quasi-static thermal stresses. The solution determines the thermal stresses in the tribosystems, both in the heating phase during braking and in the cooling phase after a stop.

The numerical results are obtained for a metal-ceramic pad and a cast-iron disc. The metal-ceramic frictional materials are now extensively used in brake systems because of their high thermal stability and wear resistance (Blau, 2001; Daehn and Breslin, 2006).

We note that the contact surfaces of the friction elements are generally rough and the heat contact is imperfect. In this paper, we consider the limiting case of perfect contact between these surfaces, when temperatures on the surface of contact are equal. The wear of the surfaces in contact is negligible.

2. Three-element tribosystem

2.1. Temperature

The problem of contact interaction of two semi-infinite bodies is under consideration. One of them is a homogeneous half-space (the disc) and the other is a half-space (the foundation), with a surface covered by a strip of thickness d (the pad) (Fig. 1). It is supposed that a constant pressure p_0 between the strip and the foundation takes place. The upper half-space slides with the velocity

$$V(t) = V_0\Big(1 - \frac{t}{t_s}\Big)H(t_s - t) \qquad\qquad t \geqslant 0 \tag{2.1}$$

in the direction of the y-axis on the strip surface, where V_0 is the initial velocity of sliding, t is time, t_s is braking time; $H(\cdot)$ is Heaviside's step function. Due to friction the heat is generated on the contact plane $z = 0$. It is assumed that the sum of intensities of the frictional heat fluxes directed into each component of the friction pair is equal to the specific friction power

$$q(t) = q_0 q^*(t) \qquad\qquad t \geqslant 0 \tag{2.2}$$

where, taking equation (2.1) into account, we have

$$q_0 = fV_0p_0 \qquad\qquad q^*(t) = \Big(1 - \frac{t}{t_s}\Big)H(t_s - t) \qquad\qquad t \geqslant 0 \tag{2.3}$$

f is the coefficient of friction.

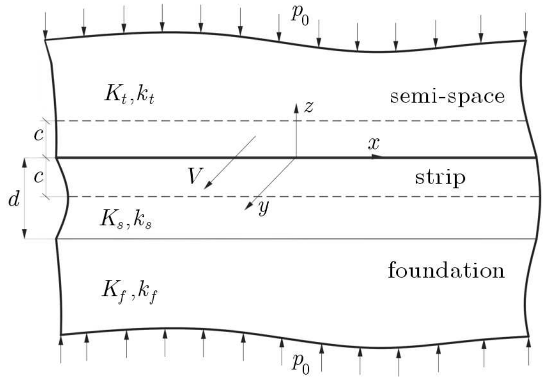

Fig. 1. Scheme of the problem

On such assumptions, the corresponding boundary-value problem of heat conduction can be written as:
— for $\tau > 0$

$$\frac{\partial^2 T^*(\zeta,\tau)}{\partial \zeta^2} = \begin{cases} \dfrac{1}{k_t^*}\dfrac{\partial T^*(\zeta,\tau)}{\partial \tau} & 0 < \zeta < \infty \\ \dfrac{\partial T^*(\zeta,\tau)}{\partial \tau} & -1 < \zeta < 0 \\ \dfrac{1}{k_f^*}\dfrac{\partial T^*(\zeta,\tau)}{\partial \tau} & -\infty < \zeta < -1 \end{cases} \tag{2.4}$$

— for $\tau \geqslant 0$

$$
\begin{aligned}
&T^*(0^-,\tau) = T^*(0^+,\tau) \qquad && \frac{\partial T^*}{\partial \zeta}\Big|_{\zeta=0^-} - K_t^*\frac{\partial T^*}{\partial \zeta}\Big|_{\zeta=0^+} = q^*(\tau)\\
&T^*(-1,\tau) = T^*(-1,\tau) \qquad && \frac{\partial T^*}{\partial \zeta}\Big|_{\zeta=-1^+} = K_f^*\frac{\partial T^*}{\partial \zeta}\Big|_{\zeta=-1^-}
\end{aligned}
\tag{2.5}
$$

and

$$
\begin{aligned}
&T^*(\zeta,\tau) \to 0 \qquad && |\zeta| \to \infty, \quad \tau \geqslant 0\\
&T^*(\zeta,0) = 0 \qquad && |\zeta| < \infty
\end{aligned}
\tag{2.6}
$$

where

$$
\begin{aligned}
&q^*(\tau) = \Big(1 - \frac{\tau}{\tau_s}\Big)H(\tau_s - \tau) \qquad \tau \geqslant 0\\
&\zeta = \frac{z}{d} \qquad \tau = \frac{k_s t}{d^2} \qquad \tau_s = \frac{k_s t_s}{d^2} \qquad T^* = \frac{T}{T_0}\\
&T_0 = \frac{q_0 d}{K_s} \qquad K_{t,f}^* = \frac{K_{t,f}}{K_s} \qquad k_{t,f}^* = \frac{k_{t,f}}{k_s}
\end{aligned}
\tag{2.7}
$$

T is temperature, K, k are the thermal conductivity and diffusivity, respectively. Here and henceforth, all values and the parameters concerning the top half-space, strip and foundation will have lower indices t, s and f, respectively.

The solution to boundary-value problem (2.4)-(2.6) takes the form (Yevtushenko and Kuciej, 2010), $\tau \geqslant 0$

$$
T^*(\zeta,\tau) = [T^{(0)*}(\zeta,\tau) - T^{(1)*}(\zeta,\tau)]H(\tau) + T^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s) \tag{2.8}
$$

where

$$
T^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{1+\varepsilon_t}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty} \Lambda^n T_n^{(k)*}(\zeta,\tau) \qquad k = 0,1
$$

$$
T_n^{(k)*}(\zeta,\tau) = \begin{cases}
F^{(k)}\Big(\dfrac{2n\sqrt{k_t^*}+\zeta}{2\sqrt{k_t^*\tau}}\Big) + \lambda_f F_k\Big[\dfrac{(2n+2)\sqrt{k_t^*}+\zeta}{2\sqrt{k_t^*\tau}}\Big] & 0 \leqslant \zeta < \infty\\[2ex]
F^{(k)}\Big(\dfrac{2n-\zeta}{2\sqrt{\tau}}\Big) + \lambda_f F_k\Big(\dfrac{2n+2+\zeta}{2\sqrt{\tau}}\Big) & -1 \leqslant \zeta \leqslant 0\\[2ex]
\dfrac{2}{(1+\varepsilon_f)}F^{(k)}\Big[\dfrac{(2n+1)\sqrt{k_f^*}-(1+\zeta)}{2\sqrt{k_f^*\tau}}\Big] & -\infty < \zeta \leqslant -1\\
& n = 0,1,2,\ldots
\end{cases}
\tag{2.9}
$$

$$
\begin{aligned}
&F^{(0)}(x) = \frac{1}{\sqrt{\pi}}\exp(-x^2) - x\ \mathrm{erfc}\,(x)\\
&F^{(1)}(x) = 3^{-1}[2(1+x^2)F_0(x) - x\,\mathrm{erfc}\,(x)]
\end{aligned}
$$

and

$$\Lambda = \begin{cases} \lambda \equiv \lambda_t \lambda_f & 0 \leqslant \lambda < 1 \\ |\lambda| & -1 < \lambda < 0 \end{cases} \tag{2.10}$$

$$\begin{aligned} \lambda_t &= \frac{1-\varepsilon_t}{1+\varepsilon_t} & \lambda_f &= \frac{1-\varepsilon_f}{1+\varepsilon_f} \\ \varepsilon_t &= \frac{K_t^*}{\sqrt{k_t^*}} & \varepsilon_f &= \frac{K_f^*}{\sqrt{k_f^*}} \end{aligned} \tag{2.11}$$

$\mathrm{erfc}\,(x) = 1 - \mathrm{erf}\,(x)$, $\mathrm{erf}\,(x)$ is the Gauss error function.

2.2. Thermal stresses

Experimental and FEM examinations of the surface of frictional elements of brakes proved that among the three normal components of the stress tensor – lateral σ_x, longitudinal σ_y and in the direction of heating σ_z – the σ_x component exerts the strongest influence on thermal cracking (Dostanko *et al.*, 2002; Choi and Lee, 2004). It results in the material cracking along the direction of sliding (the heat flow motion). If the lateral component σ_x is sufficiently greater than the tensile strength of materials, then the micro-cracks oriented at various angles to the direction of cracking are created and a divergence between the line of cracking and the direction of heat flow motion occurs. The normal component of the stress tensor σ_z has no essential meaning in the one-dimensional problem.

On the base of these data, the dimensionless quasi-static lateral $\sigma_x^* = \sigma_x/\sigma_0$ stress in the tribosystem induced by non-stationary temperature field (2.8)-(2.11) can be determined from the equations which describe thermal bending of a thick plate of the thickness c with free ends (Fig. 1) (Timoshenko and Goodier, 1970; Yevtushenko and Matysiak, 2005)

$$\sigma^*(\zeta,\tau) = [\sigma^{(0)*}(\zeta,\tau) - \sigma^{(1)*}(\zeta,\tau)]H(\tau) + \sigma^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s) \tag{2.12}$$

$|\zeta| \leqslant c^*$, $\tau \geqslant 0$ and

$$\begin{aligned} \sigma^{(k)*}(\zeta,\tau) &= \varepsilon^{(k)*}(\zeta,\tau) - T^{(k)*}(\zeta,\tau) \qquad\qquad k = 0,1 \\ \varepsilon^{(k)*}(\zeta,\tau) &= \frac{1}{1+\varepsilon_t}\sum_{n=0}^{\infty} \Lambda^n \varepsilon_n^{(k)*}(\zeta,\tau) \\ \varepsilon_n^{(k)*}(\zeta,\tau) &= Q_n^{(k)}(\tau) - \frac{\zeta}{c^*}R_n^{(k)}(\tau) \end{aligned} \tag{2.13}$$

where

$$
\begin{aligned}
Q_n^{(k)}(\tau) &= 4I_n^{(k)}(\tau) - 6J_n^{(k)}(\tau)\\
R_n^{(k)}(\tau) &= 6I_n^{(k)}(\tau) - 12J_n^{(k)}(\tau)\\
I_n^{(k)}(\tau) &= \frac{1}{c^*}\int\limits_0^{c^*} T_n^{(k)*}(\pm\zeta,\tau)\,d\zeta\\
J_n^{(k)}(\tau) &= \frac{1}{c^{*2}}\int\limits_0^{c^*} \zeta T_n^{(k)*}(\pm\zeta,\tau)\,d\zeta
\end{aligned}
\tag{2.14}
$$

$n = 0,1,2,\ldots$, $c^* = c/d$, $\sigma_0 = \alpha_t ET_0/(1-\nu)$, E is Young's modulus, ν is Poisson's ratio, α_t is linear thermal expansion coefficient. In equations (2.12), $(2.13)_1$ $\sigma^{(k)*}$, $k=0,1$ denote the dimensionless stresses corresponding to the dimensionless temperatures $T^{(k)*}$, $k=0,1$ $(2.9)_1$. The upper sign (plus) in equations $(2.14)_{3,4}$ and further, we shall choose for the top half-space, and the lower sign (minus) – for the strip.

From formulas $(2.13)_3$ it follows that the dimensionless normal deformations $\varepsilon^{(k)*}$, $k=0,1$ are linearly dependent on the dimensionless distance ζ from the surface of friction, and that the dimensionless stresses $\sigma^{(k)*}$, $k=0,1$ $(2.13)_1$ are proportional to the difference of these deformations and the dimensionless temperature $T^{(k)*}(\zeta,\tau)$.

By substituting in the right-hand side of formulas $(2.14)_{3,4}$ the functions $T_n^{(k)*}(\zeta,\tau)$, $k=0,1$ $(2.9)_2$, and then integrating, we find

$$
\begin{aligned}
I_n^{(k)}(\tau) &= \frac{4\sqrt{\kappa}\,\tau}{c^*}\Big(\frac{\tau}{\tau_s}\Big)^k [L_{2n}^{(k)}(\tau,\kappa,c^*) \pm \lambda_f L_{2n+2}^{(k)}(\tau,\kappa,\pm c^*)]\\
J_n^{(k)}(\tau) &= \frac{4\kappa\tau}{(c^*)^2}\Big(\frac{\tau}{\tau_s}\Big)^k \Big\{[2\sqrt{\tau}M_{2n}^{(k)}(\tau,\kappa,c^*) - 2nL_{2n}^{(k)}(\tau,\kappa,c^*)] +\\
&\quad +\lambda_f[2\sqrt{\tau}M_{2n+2}^{(k)}(\tau,\kappa,\pm c^*) - (2n+2)L_{2n+2}^{(k)}(\tau,\kappa,\pm c^*)]\Big\}
\end{aligned}
\tag{2.15}
$$

and

$$
\begin{aligned}
L_n^{(k)}(\tau,\kappa,c^*) &= L^{(k)}\Big(\frac{n\sqrt{\kappa}+c^*}{2\sqrt{\kappa\tau}}\Big) - L^{(k)}\Big(\frac{n}{2\sqrt{\tau}}\Big)\\
M_n^{(k)}(\tau,\kappa,c^*) &= M^{(k)}\Big(\frac{n\sqrt{\kappa}+c^*}{2\sqrt{\kappa\tau}}\Big) - M^{(k)}\Big(\frac{n}{2\sqrt{\tau}}\Big)\\
L^{(0)}(x) &\equiv \int\limits_0^x F^{(0)}(s)\,ds = \frac{1}{4} + \frac{x}{2\sqrt{\pi}}\exp(-x^2) - \frac{1+2x^2}{4}\,\mathrm{erfc}\,(x)
\end{aligned}
$$

$$M^{(0)}(x) \equiv \int\limits_0^x sF^{(0)}(s)\,ds = \frac{1}{6\sqrt{\pi}} - \frac{1-2x^2}{6\sqrt{\pi}}\exp(-x^2) - \frac{x^3}{3}\,\mathrm{erfc}\,(x) \qquad (2.16)$$

$$L^{(1)}(x) \equiv \int\limits_0^x F^{(1)}(s)\,ds = \frac{1}{8} + \frac{5x+2x^2}{12\sqrt{\pi}}\exp(-x^2) - \frac{3+12x^2+4x^4}{24}\,\mathrm{erfc}\,(x)$$

$$M^{(1)}(x) \equiv \int\limits_0^x sF^{(1)}(s)\,ds = \frac{1}{15\sqrt{\pi}} - \frac{1-4x^2-2x^4}{15\sqrt{\pi}}\exp(-x^2) + \\ - \frac{x^3(5+2x^2)}{15}\,\mathrm{erfc}\,(x)$$

where the functions $F^{(k)}(x)$, $k = 0,1$ have form $(2.9)_{3,4}$ and the parameter $\kappa \equiv k_t^*$ for the top half-space and $\kappa \equiv 1$ for the strip.

2.3. Numerical analysis

The numerical results have been computed for the commercial friction pair ChNMKh cast iron disc (the upper half-space with $K_t = 51\,\mathrm{Wm^{-1}K^{-1}}$, $k_t = 14 \cdot 10^{-6}\,\mathrm{m^2s^{-1}}$) and the metal-ceramics FMK-11 frictional element of the pad (the strip with $K_s = 34.3\,\mathrm{Wm^{-1}K^{-1}}$, $k_s = 15.2 \cdot 10^{-6}\,\mathrm{m^2s^{-1}}$) on 30KhGSA steel foundation with $K_f = 37.2\,\mathrm{Wm^{-1}K^{-1}}$, $k_f = 10.3 \cdot 10^{-6}\,\mathrm{m^2s^{-1}}$ (Balakin and Sergienko, 1999). The friction conditions were (Chichinadze and Braun, 1979): $p_0 = 1\,\mathrm{MPa}$, $V_0 = 30\,\mathrm{ms^{-1}}$, $f = 0.7$, $t_s = 3.44\,\mathrm{s}$. The initial temperature was equal to $20°\mathrm{C}$.

The isolines of the dimensionless normal stresses σ^* (2.12) are presented in Fig. 2. The distributions of stresses in the top half-space and the strip are nearly identical. Such symmetry shows that the coefficients of thermal activity of the materials $\varepsilon_t = 1.55$ and $\varepsilon_f = 1.31$ slightly differ from each other. It is observed that in the time interval $0 < t \leqslant 2\,\mathrm{s}$ the regions of compressive stress occur near the surface of friction $\zeta = 0$. Inside the strip and the top-half-space tensile stresses are generated. Between the regions with compressive and tensile stresses, there are two isolines of the zero level. The third line of zero stresses, which "descends" from the surface $\zeta = 0$, appears when the heating time is equal $t_c \approx 2.2\,\mathrm{s}$ and it is shorter than the time of braking $t_s = 3.44\,\mathrm{s}$. It means that during the relaxation phase at $t > t_s$, when there is no more heating, the sign of stresses does not change and the region of tensile stresses expands further from the heated surface – the line of zero stresses moves in time parallel to the surface of friction.

The initiation of superficial cracks generation is accompanied with a monotonic increase in tensile normal stresses. During heating of frictional elements

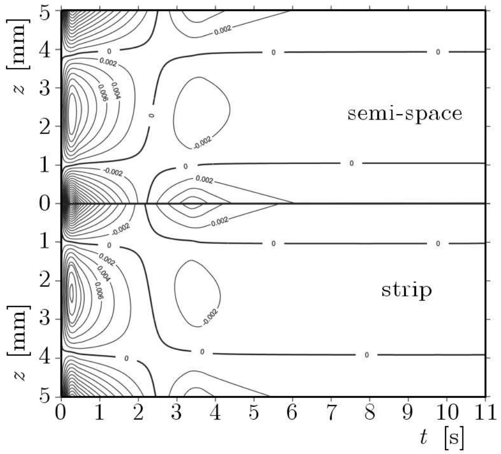

Fig. 2. Isolines of the dimensionless lateral stress σ^* in FMK-11 pad and cast iron disc

while braking, it is the change of sign of superficial stresses that plays the key role in forecasting the initiation and preventive maintenance of thermal cracking (Yevtushenko *et al.*, 2007; Yevtushenko and Kuciej, 2006). The time, connected with the change of type of stresses from compressive to tensile, for the FMK-11 pad and for the cast iron disc is equal $t_c \approx 2.2\,\mathrm{s}$ (Fig. 3).

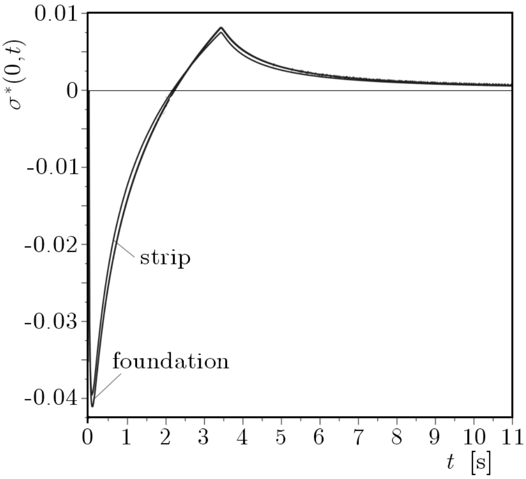

Fig. 3. Evolution of the dimensionless lateral stress σ^* on the surface of friction

With the increase in the braking time t_s, the time t_c of change of the sign of normal stresses increases almost linearly. Irrespective of the duration of braking, t_c is almost identical for the strip (the tribosystem FMK-11) and for the top half-space (cast iron) (Fig. 4).

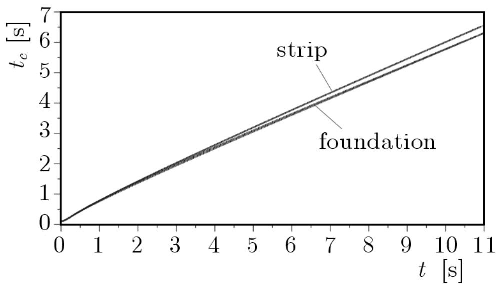

Fig. 4. Time t_c of the sign change of the lateral stress σ_x on the surface of friction $z = 0$ versus duration t_s of the braking for the strip thickness $d = 5\,\mathrm{mm}$

With the increase of thickness d of the pad, the time t_c of initiation of tensile stresses on the contact surface increases (Fig. 5).

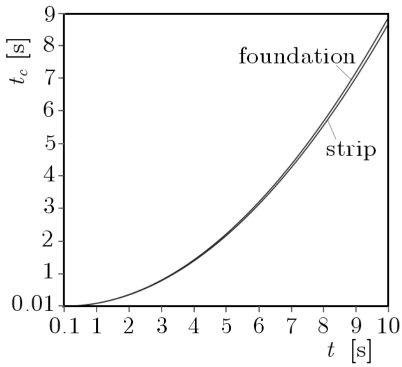

Fig. 5. Time t_c of the sign change of the lateral stress σ_x on the surface of friction $z = 0$ versus thickness d of the strip for a fixed value of the stop time $t_s = 3.44\,\mathrm{s}$

3. Two-elements tribosystem

3.1. Temperature

The problem of contact interaction of a plane-parallel strip (the pad) and a half-space (the disc) is under consideration. The upper surface of the strip and the foundation are subjected in infinity to the constant pressure p_0. The strip slides with velocity (2.1) over the surface of the half-space along the y-axis of the Cartesian coordinate system $Oxyz$ on the plane of contact (Fig. 6).

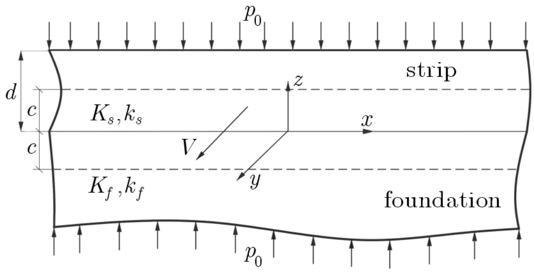

Fig. 6. Scheme of the problem

The transient dimensionless temperature fields in the strip and in the foundation can be found from the solution to the heat conduction problem of friction during braking:
— for $\tau > 0$

$$\frac{\partial^2 T^*(\zeta,\tau)}{\partial\zeta^2} = \begin{cases} \dfrac{\partial T^*(\zeta,\tau)}{\partial\tau} & 0 < \zeta < 1 \\ \dfrac{1}{k_f^*}\dfrac{\partial T^*(\zeta,\tau)}{\partial\tau} & -\infty < \zeta < 0 \end{cases} \tag{3.1}$$

— for $\tau \geqslant 0$

$$\begin{array}{ll} T^*(0^-,\tau) = T^*(0^+,\tau) & K_f^*\dfrac{\partial T^*}{\partial\zeta}\Big|_{\zeta=0^-} - \dfrac{\partial T^*}{\partial\zeta}\Big|_{\zeta=0^+} = q^*(\tau) \\ T^*(1,\tau) = 0 & T^*(\zeta,\tau) \to 0 \qquad \zeta \to -\infty \\ T^*(\zeta,0) = 0 \qquad -\infty < \zeta \leqslant 1 & \end{array} \tag{3.2}$$

where assumptions (2.7) are taken into account.

The solution to boundary-value problem (3.1) and (3.2) may be written in form (2.8), where the dimensionless temperatures $T^{(k)*}(\zeta,\tau)$, $k = 0,1$ are given by formulae (Yevtushenko and Kuciej, 2010)

$$T^{(k)*}(\zeta,\tau) = \frac{2\sqrt{\tau}}{1+\varepsilon_f}\Big(\frac{\tau}{\tau_s}\Big)^k \sum_{n=0}^{\infty} \Lambda^n T_n^{(k)*}(\zeta,\tau) \qquad \begin{array}{l} -\infty < \zeta \leqslant 1 \\ \tau \geqslant 0 \\ k = 0,1 \end{array}$$

$$T_n^{(k)*}(\zeta,\tau) = \begin{cases} F^{(k)}\Big(\dfrac{2n+\zeta}{2\sqrt{\tau}}\Big) - F^{(k)}\Big(\dfrac{2n+2-\zeta}{2\sqrt{\tau}}\Big) & 0 < \zeta \leqslant 1 \\ F^{(k)}\Big(\dfrac{2n\sqrt{k_f^*}-\zeta}{2\sqrt{k_f^*\tau}}\Big) - F^{(k)}\Big[\dfrac{(2n+2)\sqrt{k_f^*}-\zeta}{2\sqrt{k_f^*\tau}}\Big] & -\infty < \zeta \leqslant 0 \end{cases} \tag{3.3}$$

where the functions $F^{(k)}(x)$, $k=0,1$ have form $(2.9)_{3,4}$ and the coefficient Λ is given by Eq. (2.10) at $\lambda=\lambda_f$ (2.11).

If the upper surface $\zeta=1$ of the strip is thermally insulated, then the corresponding solution may be represented by means of equations (2.8) and $(3.3)_1$, in which (Yevtushenko and Kuciej, 2010)

$$T_n^{(k)*}(\zeta,\tau)=\begin{cases} F^{(k)}\Big(\dfrac{2n+\zeta}{2\sqrt{\tau}}\Big)+F^{(k)}\Big(\dfrac{2n+2-\zeta}{2\sqrt{\tau}}\Big) & 0<\zeta\leqslant 1 \\ F^{(k)}\Big(\dfrac{2n\sqrt{k_f^*}-\zeta}{2\sqrt{k_f^*\tau}}\Big)+F^{(k)}\Big[\dfrac{(2n+2)\sqrt{k_f^*}-\zeta}{2\sqrt{k_f^*\tau}}\Big] & -\infty<\zeta\leqslant 0 \end{cases} \tag{3.4}$$

3.2. Thermal stresses

Similarly, like in the three-element problem, the quasi-static longitudinal stress σ_x in the two-element tribosystem induced by the non-stationary temperature field $T^*(\zeta,\tau)$, (2.8), (3.3) and (3.4) can be determined from the equations (2.12)-(2.14) in which

$$\begin{aligned} I_n^{(k)}(\tau)&=\frac{4\sqrt{\kappa\tau}}{c^*}\Big(\frac{\tau}{\tau_s}\Big)^k[L_{2n}^{(k)}(\tau,\kappa,c^*)\pm L_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)] \\ J_n^{(k)}(\tau)&=\frac{4\kappa\tau}{(c^*)^2}\Big(\frac{\tau}{\tau_s}\Big)^k\Big\{[2\sqrt{\tau}M_{2n}^{(k)}(\tau,\kappa,c^*)-2nL_{2n}^{(k)}(\tau,\kappa,c^*)]- \\ &\quad -[2\sqrt{\tau}M_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)-(2n+2)L_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)]\Big\} \end{aligned} \tag{3.5}$$

for zero temperature on the upper surface $\zeta=1$ of the strip and

$$\begin{aligned} I_n^{(k)}(\tau)&=\frac{4\sqrt{\kappa\tau}}{c^*}\Big(\frac{\tau}{\tau_s}\Big)^k[L_{2n}^{(k)}(\tau,\kappa,c^*)\mp L_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)] \\ J_n^{(k)}(\tau)&=\frac{4\kappa\tau}{(c^*)^2}\Big(\frac{\tau}{\tau_s}\Big)^k\Big\{[2\sqrt{\tau}M_{2n}^{(k)}(\tau,\kappa,c^*)-2nL_{2n}^{(k)}(\tau,\kappa,c^*)]+ \\ &\quad +[2\sqrt{\tau}M_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)-(2n+2)L_{2n+2}^{(k)}(\tau,\kappa,\mp c^*)]\Big\} \end{aligned} \tag{3.6}$$

if this surface is thermally insulated. The functions $L_n^{(k)}(\tau,\kappa,c^*)$ and $M_n^{(k)}(\tau,\kappa,c^*)$ in equations (3.5) and (3.6) are determined from formulae (2.16).

3.3. Numerical analysis

The numerical results have been obtained for the friction couple of cast iron (the foundation) and FMK-11 metal-ceramics strip (the frictional element of the pad). The material properties and friction conditions for calculations were the same as in the previous section of the paper. All the results presented in Figs. 7-9 were obtained for two limiting types of boundary conditions on the upper surface $z = d$ ($\zeta = 1$) of the strip: at zero temperature and for thermal insulation.

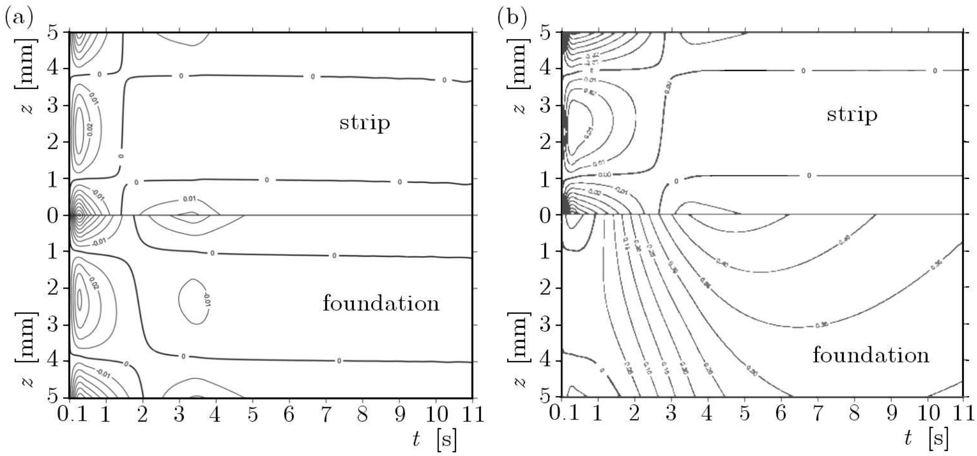

Fig. 7. Isolines of the dimensionless lateral stress σ^* in FMK-11 pad and cast iron disc; (a) zero temperature, (b) thermally insulated

The isolines of dimensionless lateral stresses σ^* (2.12)-(2.14), (3.5) and (3.6) are presented in Fig. 7. The distributions of these stresses in the FMK-11 pad and the cast iron disc are nearly identical, when temperature on the upper surface of the strip is equal to zero (Fig. 7a). It is possible to explain such symmetry by the fact that the coefficients of thermal conductivity of the material slightly differ from each other. It is observed that in the time interval $0 < t < 1.5$ s, the regions of compressive stress occur near the surface of friction $\zeta = 0$ (Fig. 7a). During the same time, inside the pad and disc tensile stresses are generated. Between the regions with compressive and tensile stresses, there are two isolines with zero stresses. The third line of the zero level, which begins from the surface $\zeta = 0$, appears when the heating time is equal $t \equiv t_c \approx 1.5$ s for the pad and $t_c \approx 1.8$ s for the disc. These values of time are lower than the time of braking $t_s = 3.44$ s. It means that during the cooling phase at $t > t_s$, when there is no more heating, the sign of stresses does not change and the region of tensile stresses expands further from the heated surface –

the line of zero stresses moves parallel to the surface of friction (Fig. 7a). A different distribution of dimensionless lateral stress σ^* is observed in the case of thermal insulation of the upper surface of the strip (Fig. 7b). In the heating phase within the time interval $0 < t < 2.8$ s for the pad and $0 < t < 0.9$ s for the disc, compressive superficial stresses occur. In this case, the level of tensile stresses in the disc is greater than in the case of zero temperature on the upper surface of the strip.

In the case of thermal insulation of the upper surface of the strip, the magnitude of surface tensile stresses in the cast iron disc is much greater than in the case of zero temperature on this surface (Fig. 8). The time t_c, connected with the change of the type of stresses from compressive to tensile, is the greatest for the FMK-11 pad, and the least – for the cast iron disc.

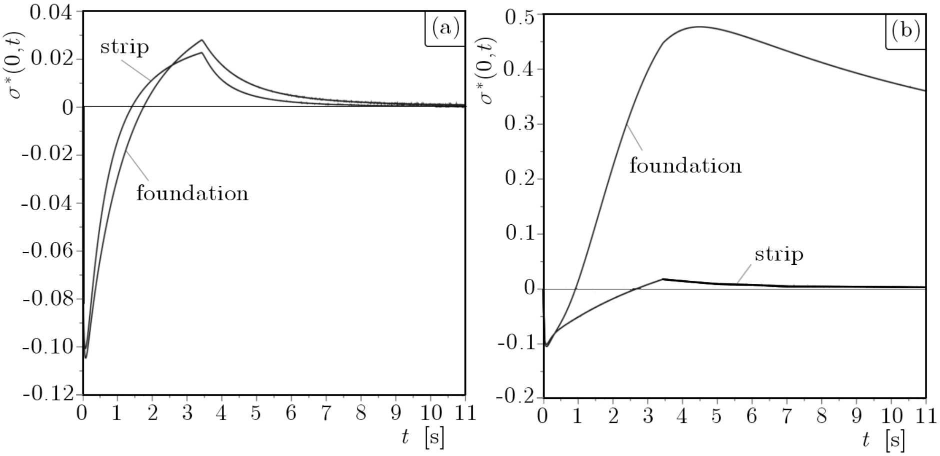

Fig. 8. Evolution of the dimensionless lateral stress σ^* on the contact surface; (a) zero temperature, (b) thermally insulated

With an increase in the braking time t_s, the time t_c of change of the sign of lateral stress increases (Fig. 9). Whereas the upper surface of the strip is maintained at zero temperature (is thermally insulated) at a fixed time of braking, the time of change of the sign of normal stress for the FMK-11 pad is less (greater) than the same time for the cast-iron disc (Fig. 9a). The greatest difference between the times of occurrence of tensile stresses on the friction surface is observed in the case of the thermal insulation of the upper surface of the strip (Fig. 9b).

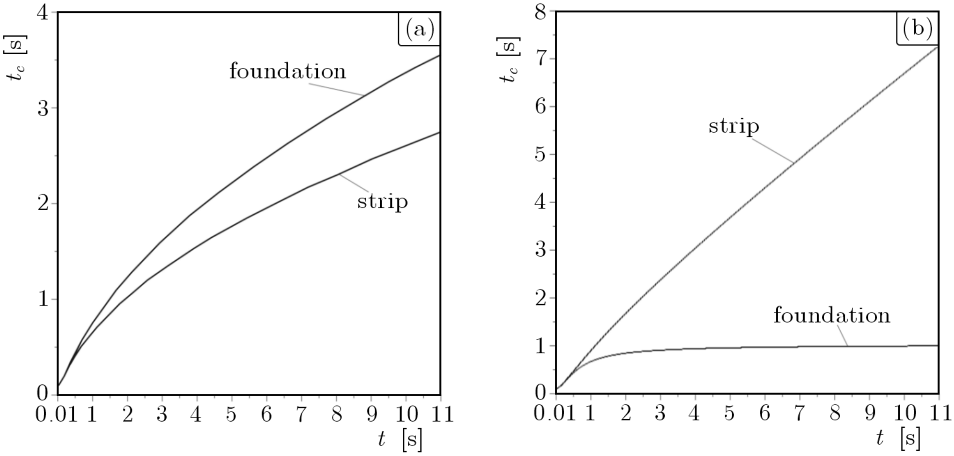

Fig. 9. Time t_c of the sign change of the lateral stress σ_x on the surface of friction $z = 0$ versus duration t_s of the braking; (a) zero temperature, (b) thermally insulated

4. Conclusions

Two calculation models for determination of thermal stresses during braking have been proposed. The first model consists of three elements: a disc (half-space), pad (strip) and a steel slice (half-space). The second one includes only two elements: the pad (strip) and the disc (half-space). The three-element model allows one to consider the influence of physical properties of the steel slice material, on which the frictional element (pad) is put, on the distribution of thermal stresses. In the two-element scheme, the influence of cooling (boundary conditions) on the free top surface of the pad is taken into account.

The quasi-static thermal stresses have been obtained by the Timoshenko approximate model of thermal bending of a thick plate with no fixed edges. Thus, the one-dimensional transient temperature field, which we received earlier, for both tribosystems was used. Unlike other solutions, this one determines thermal stresses in each element of the brake, both in the heating phase at braking and in the cooling phase, when the brake is stopped. Numerical calculations were carried out for the cast iron disc and the metal ceramic pad on the steel slice.

Analysis of the evolution of thermal stresses in the frictional elements during braking proves that when it is heated, considerable lateral compressive stresses occur near the contact surface. The value of these stresses decreases with an increase of time, and after some time t_c, the sign changes – which means that tensile stresses take place. The moment when it happens increases monotonously with the braking time t_s.

The influence of boundary conditions on the upper surface of the pad on the distribution of lateral thermal stresses has been investigated, too. It has been established that in the case when the initial temperature is supported on the top surface of the pad during braking, the evolution of these stresses on the contact surface is approximate as in the case of the three-element model. Significant values of tensile stresses in the disc appear when the top surface of the pad is thermally isolated. In this case, initiation of superficial cracks in the disc seems to be most possible. The possible initiation of these cracks during braking can be described as a series of the following steps:

- due to local intensive frictional heating near the contact surface, a field of normal stresses is formed;
- after the beginning of braking, at the time instant t_c, tensile normal stresses occur near the subsurface region;
- when the stresses exceed the tensile strength of the material, the surface may crack;
- development of the crack into the material is limited by regions of compressive stress which occur near the contact surface.

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Dwa schematy obliczeń do opisania naprężeń termicznych wywołanych nagrzewaniem tarciowym podczas hamowania

Streszczenie

W pracy przeanalizowano rozkład naprężeń termicznych w trzy- (tarcza/nakładka/zacisk) oraz dwu- (tarcza/nakładka) elementowych układach nagrzewanych tarciowo podczas hamowania. Dla materiałów tarciowych: żeliwo/metalo-ceramika/stal oraz żeliwo/metalo-ceramika, zbadano ewolucję oraz rozkład normalnych naprężeń termicznych.

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