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RESULTS OF A RESEARCH PREDICTING THE POSITION OF AN AIRCRAFT DURING APPROACH AND LANDING USING THE BESSEL FUNCTION

Marek Grzegorzewski
Air Force Academy in Deblin, Poland
e-mail: marekgrzegorzewski@wp.pl

The article presents theoretical foundations of the position prediction functionality of a GPS receiver to be used by a pilot in the event of an instantaneous lack of position data occurring due to various reasons.

Key words: satellite technology, aviation, predication position

1. Navigating an aircraft in space by means of a position potential

The number and character of factors which exert an influence on obtaining the complete navigation information result in the fact that observation data do not always constitute the complete information regarding the precise assessment of the location of a given aircraft. Even when the satellite system is fully available and its configuration proper, some factors (e.g. abrupt changes of flight parameters or maneuvers of the aircraft) may prevent its utilization. The above-mentioned factors necessitate working out additional methods which supplement the process of navigating an aircraft. Mathematical methods are one of the most frequent ways of finding solutions to such problems. In the presented work, an alternative filter has been developed which relies mostly on the following parameters related to motion of an aircraft: position, velocity and position error statistics. In reality, a cluster of observed position fixes contains all vital kinematic information of the aircraft. The assumptions of the program use the concept of the "position potential" based on the data received from the navigation services. According to Newton's law of universal gravitation, a free mass particle is attracted by another mass. The acceleration of a particle that has a given mass is directly proportional to the gravitation constant G and inversely proportional to the square of the distance from the attracted mass. Per analogiam, let us regard the statistical confidence regions (position error ellipsoids) of the most probable position fix as a "source of force", which should "attract" a trajectory of a particle passing through them. The potential field of such a single particle, which in our case is an aircraft, should reflect the observed position fix and the required force to be exerted on the particle, which should monotonically decrease when the particle approaches the assumed position fix. When the particle enters the position error ellipsoid, it will be attracted with a force whose magnitude will be proportional to the intensity of the potential. The potential monotonically decreases with the decreasing distance between the particle and the assumed fix. Furthermore, in order to allow the particle to continue moving after the position fix appears, the potential of the attracting source will be dissipated exponentially in time. If we select the position density function ("position potential"), which contains the dissipation exponent α and the parameter G (positioning uncertainty, corresponding to Newton's gravitation constant), we assume that the trajectory of the particle will represent the real trajectory of the vehicle.

The first attempt at developing a navigation filter that was based on the above assumptions was made by Inzinga and Vanicek (1985). Their study assumes that the force of the position potential field affecting the particle is connected with the probability which describes the fact that the position fix is included in a two-dimensional error ellipse. Since the equation of motion of the particle is extremely difficult to be treated analytically, the following model of operation was selected:

- selecting the function of the position potential,
- formulation of temporal dissipation of the potential field,
- describing the equation of motion,
- solving the equation of motion,
- determining parameters α and G,
- finding the final solution to the navigational problem.

First of all, we have to select the proper potential function. Using the selected function of position density, we can establish the time-related position potential field for a sequence of position fixes. Subsequently, we set up a model of motion of an individual particle and find the solution to its equation of motion. In order to reflect the changing navigational environment, the potential function contains certain variable parameters α and G. When the parameters α and G as well as the initial conditions are not known, the number of possible positions of the particle is infinite. Because of that, further data is necessary in order to determine the proper trajectory of the particle. Therefore, we have to set up a "self-learning" procedure for the filter so that it will determine parameters and conditions based on earlier observations. Parameters and initial motion conditions in the motion model are related to previous observations by means of "equations by motion". At this stage, we solve the equation by motion with the assumption that the initial conditions are known. Then, we have to determine the parameters α and G and in order to optimize them we will use the least-squares method. The estimation of a given future position of the particle (aircraft), i.e. making a prediction, is possible owing to the model of motion having the determined parameters and on the basis of the present position of the particle. Error estimation is possible at the very moment of obtaining the data concerning a new position fix, by using the values of the differences between the predicted and the actual positions, i.e. checking whether the new position fix is located within the position error ellipsoid.

1.1. Navigation model

We assume that the density function (Plucińska and Pluciński, 2000) for a three-dimensional vector of random position is

$$\varPhi_{r_0} = \frac{1}{K}\exp\Big[-\frac{1}{2}(\mathbf{r}-\mathbf{r}_0)^{\mathrm{T}}\mathbf{C}^{-1}(\mathbf{r}-\mathbf{r}_0)\Big] \tag{1.1}$$

where $\mathbf{r} = [x,y,z]^{\mathrm{T}}$ is the particle position vector at the present instant t, $\mathbf{r}_0 = [x_0,y_0,z_0]^{\mathrm{T}}$ – vector of particle position at the instant set t_0, x,y,z are coordinates of the position vector of the particle (the aircraft), $\mathbf{C}$ – symmetrical covariance matrix (Smirnov, 1967)

$$\mathbf{C} = \begin{bmatrix} D^2X & \operatorname{cov}(X,Y) & \operatorname{cov}(X,Z) \\ \operatorname{cov}(Y,X) & D^2Y & \operatorname{cov}(Y,Z) \\ \operatorname{cov}(Z,X) & \operatorname{cov}(Z,Y) & D^2Z \end{bmatrix} \tag{1.2}$$

and

$$K = \sqrt{(2\pi)^3 \det \mathbf{C}} \tag{1.3}$$

Let us assume that the particle potential U_i is the basis for the determination of its position fix

$$U_i(t) = G(\mathbf{r}-\mathbf{r}_{0i})^{\mathrm{T}}\mathbf{C}_i^{-1}(\mathbf{r}-\mathbf{r}_{0i})\mathrm{e}^{-\alpha(t-t_i)} \tag{1.4}$$

$(\mathbf{r}-\mathbf{r}_{0i})^{\mathrm{T}}\mathbf{C}_i^{-1}(\mathbf{r}-\mathbf{r}_{0i})$ is the quadratic form of the error ellipsoid at the instant t_i, time $t \geqslant t_i$. The points located on the error ellipsoid have identical probability distribution density. $\mathbf{C}_i$ is the positive covariance matrix of the i-th particle, whereas positive parameters α and G (their values are to be determined) represent, respectively; α – dissipation parameter, with the increasing value of α parameter, the potential of the particle decreases exponentially; G – uncertainty of the position fix determination. It is a counterpart of the gravitational constant in Newton's attraction.

The potential of the particle is directly proportional to G parameter.

In relation to the potential determined in (1.4), the time-varying t potential created by n position fixes of a particular particle is expressed as

$$U=\sum_{i=1}^{n}U_i=G\mathrm{e}^{-\alpha t}\sum_{i=1}^{n}(\mathbf{r}-\mathbf{r}_{0i})^{\mathrm{T}}\mathbf{C}_i^{-1}(\mathbf{r}-\mathbf{r}_{0i})\mathrm{e}^{-\alpha t_i} \tag{1.5}$$

where $t \geqslant t_n$.

It should be noted that the filter (potential) keeps pace with the kinematics of the particle (aircraft). The time-varying potential field is constantly updated by each new position fix of the particle in order to incorporate the most recent information as soon as possible.

Local disturbances (e.g. wind velocity, weather conditions) do not significantly influence the principal directions of the error ellipsoid. Therefore, we may assume that the principal axes of the error ellipsoid are parallel to the coordinate axes (X,Y,Z). Consequently, random variables X, Y, Z are independent and uncorrelated, and the covariance matrix is diagonal

$$\mathrm{cov}\,(X,Z)=\mathrm{cov}\,(Y,Z)=\mathrm{cov}\,(X,Y)=0 \tag{1.6}$$

Equation of motion (1.4), after adopting the following notation

$$\mathbf{A}=2\sum_{i=1}^{n}\mathrm{e}^{-\alpha t_i}\mathbf{C}_i^{-1} \qquad\qquad \mathbf{B}=2\sum_{i=1}^{n}\mathrm{e}^{-\alpha t_i}\mathbf{C}_i^{-1}\mathbf{r}_{0i} \tag{1.7}$$

where $\mathbf{A}$ – matrix (3×3), $\mathbf{B}$ – vector, is expressed in the coordinates as follows $(t \geqslant t_n)$

$$\begin{aligned}
&\ddot{x}(t)=-GA_x\Big(x-\frac{B_x}{A_x}\Big)\mathrm{e}^{-\alpha t} \qquad\qquad \ddot{y}(t)=-GA_y\Big(y-\frac{B_y}{A_y}\Big)\mathrm{e}^{-\alpha t}\\
&\ddot{z}(t)=-GA_z\Big(z-\frac{B_z}{A_z}\Big)\mathrm{e}^{-\alpha t}
\end{aligned} \tag{1.8}$$

which is a system of inhomogeneous linear differential Bessel equations of the second order (see Kącki and Siewierski, 2002).

The solution to system of equations (1.8) will be the following

$$\begin{aligned}
x(t)&=\frac{B_x}{A_x}+a_1J_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_x}\Big)+a_2N_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_x}\Big)\\
y(t)&=\frac{B_x}{A_x}+b_1J_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_y}\Big)+b_2N_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_y}\Big)\\
z(t)&=\frac{B_x}{A_x}+c_1J_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_z}\Big)+c_2N_0\Big(\frac{2}{\alpha}\mathrm{e}^{-\frac{\alpha}{2}t}\sqrt{GA_z}\Big)
\end{aligned} \tag{1.9}$$

where the first element of the solution is to be regarded as a particular solution to an inhomogeneous differential equation, and both remaining elements, containing the constants, as general solutions to a homogeneous differential equation.

In order to determine the integration constants a_1, a_2, b_1, b_2, c_1, c_2, the initial conditions have to be used, i.e. they must be added to system of equations (1.8) (formulation of the Cauchy problem).

Equations (1.9) and (1.10) with known (to be determined) parameters G and α are basic equations of our study see Grzegorzewski (2011).

Using the initial condition, for $t_n = 0$

$$\boldsymbol{r}(0) = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} \qquad \dot{\boldsymbol{r}}(0) = \begin{bmatrix} \dot{x}_n \\ \dot{y}_n \\ \dot{z}_n \end{bmatrix} \tag{1.10}$$

we compute the constants a_1, a_2, b_1, b_2, c_1, c_2

$$\begin{aligned}
a_1 &= -\frac{\pi}{\alpha}\Big[\Big(x_n - \frac{B_x}{A_x}\Big)\sqrt{GA_x}N_1\Big(\frac{2}{\alpha}\sqrt{GA_x}\Big) - \dot{x}_n N_0\Big(\frac{2}{\alpha}\sqrt{GA_x}\Big)\Big] \\
a_2 &= \frac{\pi}{\alpha}\Big[\Big(x_n - \frac{B_x}{A_x}\Big)\sqrt{GA_x}J_1\Big(\frac{2}{\alpha}\sqrt{GA_x}\Big) - \dot{x}_n J_0\Big(\frac{2}{\alpha}\sqrt{GA_x}\Big)\Big] \\
b_1 &= -\frac{\pi}{\alpha}\Big[\Big(y_n - \frac{B_y}{A_y}\Big)\sqrt{GA_y}N_1\Big(\frac{2}{\alpha}\sqrt{GA_y}\Big) - \dot{x}_n N_0\Big(\frac{2}{\alpha}\sqrt{GA_y}\Big)\Big] \\
b_2 &= \frac{\pi}{\alpha}\Big[\Big(y_n - \frac{B_y}{A_y}\Big)\sqrt{GA_y}J_1\Big(\frac{2}{\alpha}\sqrt{GA_y}\Big) - \dot{x}_n J_0\Big(\frac{2}{\alpha}\sqrt{GA_y}\Big)\Big] \\
c_1 &= -\frac{\pi}{\alpha}\Big[\Big(z_n - \frac{B_z}{A_z}\Big)\sqrt{GA_z}N_1\Big(\frac{2}{\alpha}\sqrt{GA_z}\Big) - \dot{x}_n N_0\Big(\frac{2}{\alpha}\sqrt{GA_z}\Big)\Big] \\
c_2 &= \frac{\pi}{\alpha}\Big[\Big(z_n - \frac{B_z}{A_z}\Big)\sqrt{GA_z}J_1\Big(\frac{2}{\alpha}\sqrt{GA_z}\Big) - \dot{x}_n J_0\Big(\frac{2}{\alpha}\sqrt{GA_z}\Big)\Big]
\end{aligned} \tag{1.11}$$

Because of initial condition (1.10), the computed integration constants a_1, a_2, b_1, b_2, c_1, c_2 represented by (1.11) exist within the time interval $[t_n, t_{n+1}]$ and $[x, y, z]$ – vector of the particle (aircraft) position, [m]; $[\dot{x}, \dot{y}, \dot{z}]$ – vector of the particle (aircraft) velocity, [m s^{-1}].

1.2. Optimization of α and G parameters

In order to optimize α and G parameters, let us analyze the following problem, consisting in solving the equation of motion $(T \geqslant t_n)$ (Grzegorzewski, 2011)

$$\begin{aligned}
&\ddot{x}(t) = -G(A_x x - B_x)\mathrm{e}^{-\alpha t} \qquad \ddot{y}(t) = -G(A_y y - B_y)\mathrm{e}^{-\alpha t} \\
&\ddot{z}(t) = -G(A_z z - B_z)\mathrm{e}^{-\alpha t}
\end{aligned} \tag{1.12}$$

where the vector $\mathbf{Ar}$ is represented by

$$A_x = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{xi} \qquad A_y = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{yi} \qquad A_z = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{zi} \tag{1.13}$$

and the coordinates of the vector $\mathbf{B}$ are represented as

$$\begin{aligned}
&B_x = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{xi} x_{0i} \qquad B_y = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{yi} y_{0i} \\
&B_z = 2\sum_{i=1}^{n} \mathrm{e}^{\alpha(t_i - t_n)} p_{zi} z_{0i}
\end{aligned} \tag{1.14}$$

with the initial conditions set for $t_n = 0$ as in Eq. (1.10).

The vector of the particle position is described by Eqs. (1.9) (Grzegorzewski, 2011).

Our objective is to optimize the parameters α and G so that the function of two variables

$$f(\alpha,G)=\sum_{i=1}^{n}[\mathbf{r}(t_i)-\mathbf{r}_{0i}]^2=\sum_{i=1}^{n}\Big\{[x(t_i)-x_{0i}]^2+[y(t_i)-y_{0i}]^2+[z(t_i)-z_{0i}]^2\Big\}=\min \tag{1.15}$$

where: x_{0i}, y_{0i}, z_{0i} is the position of the particle (aircraft) at the instance t_i (data obtained from a satellite), $x(t_i), y(t_i), z(t_i)$ – position of the particle (aircraft) at the instance t_i computed from (1.9).

By substituting the optimized parameters α and G into equations (1.9) and (Grzegorzewski, 2011) we will obtain from the mathematical model the vector of the particle (aircraft) position and the vector of its velocity at the instant t.

2. Comparing various methods of predicting the aircraft position

Our task was to create an algorithm, which may be implemented and then used in practice, in order to predict subsequent positions of an aircraft.

The results of our work are the following:

- Three different algorithms, written as MS Excel macros, which predict the position of an aircraft at the subsequent instances on the basis of its positions at the previous instances.
- Comparison of the efficiency of the created algorithms when using various values of the parameters.
- Suggestions concerning the proper selection of an algorithm and its implementation.

2.1. Methods under comparison

The starting point was the method described by Grzegorzewski (2011). Its main advantage is the fact that it has been created on the basis of real physical assumptions. This method is henceforth called αGn method – from its two key parameters. In our studies, implementation of other methods was also taken into consideration. Having analyzed their theoretical correctness, we carried out their preliminary tests on the basis of the real data contained in the MS Excel sheet, data.xls. Two methods proved to be worth further testing. They represent two different approaches:

a) Establishing the relationship between the coordinates of the aircraft position fix and time.
b) Establishing the relationship between the aircraft position at a given instant of time and its position at the previous time instances.

In the next part of this report we provide a brief discussion of the three methods and the results of the tests conducted in order to assess their usefulness.

2.1.1. αGn method

Technically, its essence lies in expressing the solution of an ordinary second order linear differential equation as a function of two parameters α and G, and, subsequently, selecting the optimum values of those parameters which will minimize the error function F_0. Knowing those values facilitates the determination of the predicted values. The solution of the equation contains Bessel special functions.

Function $f(\alpha,G)$ represents the sum of squares of the distances between the theoretical points and the observed points at n previous time instances. Initial terms of the series expansion of the Bessel function were used for the approximation of that function. Due to high complexity of the formulae describing the function $f(\alpha,G)$, applying traditional optimization methods was not possible. Therefore, we decided to use for its optimization a certain version of an evolutionary algorithm developed by ourselves. The implementation of that algorithm is, however, time-consuming, and increasing the number of previous time instances n causes a considerable increase of duration of the algorithm operation. The parameters affecting the course of action of the evolutionary algorithm are: population size, number of discarded candidate solutions (individuals), width (step) of the local method, number of steps, and finally ranges: α, G.

2.1.2. lin n method

According to our experience and the data that we obtained, predicting the height h is of crucial importance whereas predicting the remaining coordinates X and Y is considerably less important. Therefore, both this method and the next are being discussed in the context of predicting the height, but the discussion may, if necessary, be repeated separately for X and Y. This is one of the simplest methods. It assumes a linear relationship between the height and time

$$h(t) = b_0 + b_i t \tag{2.1}$$

The coefficients b_0, b_i are determined with the least squares method on the basis of the course of the flight so far (n measurements, $n > 2$), and they are used for making a prediction. After preliminary tests, other methods of determining a functional relationship between the height and time were discarded as being substantially less effective.

2.1.3. baryc n method

This method is in fact one of the versions of the $ARIMA(p, d, q)$ method, namely $A(1, 1, 0)$, and its description is expressed as: $h_i = \beta_1 h_{i-1} + \beta_2 h_{i-2}$ with $\beta_1 + \beta_2 = 1$.

The coefficients β_1, β_2 are determined with the least squares method on the basis of the course of the flight so far (n measurements, $n > 2$) and used for making a prediction. After preliminary tests, other methods representing the $ARIMA$ type were discarded as being substantially less effective. For similar reasons, two other methods of a similar type were discarded after preliminary tests.

2.1.4. In-flight test results

Results representative for the data set included in MS Excel sheet, data.xls (flight of the Cessna aircraft) are presented in Fig. 1a. The data was used for comparing the methods mentioned above and for optimizing the α and G parameters.

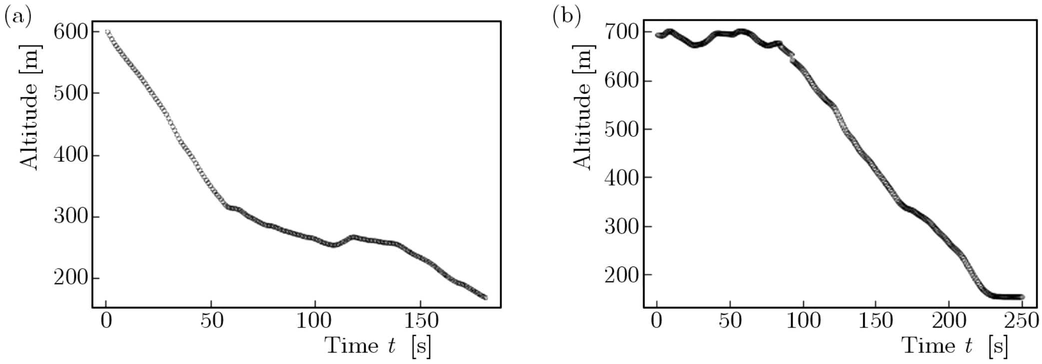


Fig. 1. (a) Height in the data.xls sheet; (b) height in the 4 minutes 92.xls sheet

Table 1 contains the results for the data set from MS Excel sheet, 4 minutes 92.xls (Fig. 1b). Predictions were examined from 10th to 240th second, i.e. at 231 instants. Similar calculations were performed for data.xls (Fig. 1a).

Table 1. Parameters *alpha* and G determined for αG2 and αG3 methods

	α	G
αG2 method		
Mean	13.01	0.97
Stand. deviation	5.78	1.00
Minimum	1.00	0.010003
Maximum	19.97	2.993
αG2 method		
Mean	14.22	0.12
Stand. deviation	5.25	0.22
Minimum	1.19	0.010003
Maximum	19.98	1.95

Numerical experiments were performed in the actual flight, shown in Fig. 1, using the following methods: αGn, *lin n*, and *baryc n*. The last two measurements were taken into consideration (in the case of αGn method, this means that the memory of the potential was reduced to 2 seconds), and the predicted position of the aircraft was calculated for 2 or 3 seconds forward (due to the complexity of calculations – Bessel functions). At the same time, particular methods were compared and analyzed for the number of times that a given method was the best. On the basis of the numerical experiments, it is believed that the following methods should be used in further in-flight tests: $\alpha G2$, $\alpha G3$, and *lin 2*. In the event of substantial flight disturbances, $\alpha G2$ and $\alpha G3$ methods "adapts to the actual flight trajectory" faster.

3. Position prediction – results

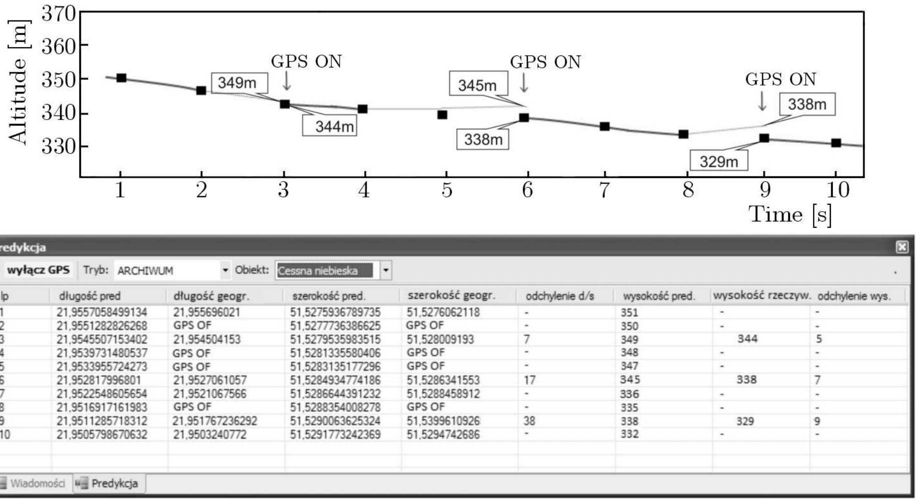
Experiments concerning the position prediction were conducted during flights designated as Flight 12, and Flight 22.

3.1. Flight 12

Flight 12 was performed on 12 September 2011, between 9:02:40 s and 9:04:00 s. On the basis of the SV arrangement, PDOP = 2.25. Waypoints of the glide path which is shown in Fig. 1a were determined once a second. The following points were planned for the analysis:

- reference point 3,
- reference point 6,
- reference point 9.

Figure 2 shows the parameters of the glide path in Flight 12. The aim of the graph is to show the trajectory of the glide path of an aircraft (coordinates recorded once every 10 seconds). In the three points that were planned to be shown in the graph, the GPS system stops working and the position prediction system takes over. The comparison of the position prediction result at point 3 (3rd second of the aircraft flight on the glide path) with the position obtained from the GPS shows that the real position of the aircraft was 5 meters higher than the position computed by means of prediction. The directional deviation from the glide path was also recorded – 7 meters to the right.



lp	długość pred	długość geogr.	szerokość pred.	szerokość geogr.	odchylenie d/s	wysokość pred.	wysokość rzeczyw.	odchylenie wys.
1	21,9557058499134	21,955696021	51,5275936789735	51,5276062118	-	351	-	-
2	21,9551282826268	GPS OF	51,5277736386625	GPS OF	-	350	-	-
3	21,9545507153402	21,954504153	51,5279535983515	51,528009193	7	349	344	5
4	21,9539731480537	GPS OF	51,5281335580406	GPS OF	-	348	-	-
5	21,9533955724273	GPS OF	51,5283135177296	GPS OF	-	347	-	-
6	21,952817996801	21,9527061057	51,5284934774186	51,5286341553	17	345	338	7
7	21,9522548605654	21,9521067566	51,5286644391232	51,5288458912	-	336	-	-
8	21,9516917161983	GPS OF	51,5288354008278	GPS OF	-	335	-	-
9	21,9511285718312	21,951767236292	51,5290063625324	51,5399610926	38	338	329	9
10	21,9505798670632	21,9503240772	51,5291773242369	51,5294742686	-	332	-	-

Fig. 2. Glide path graph – Flight 12

After the subsequent second of flight, the GPS is deactivated again. At point 6 (6th second of flight), the comparison of the position prediction result with the position obtained from the GPS shows a 7-metre height difference between the real position of the aircraft and the position computed by means of prediction. The position prediction system maintains the tendency to compute the parameters of the subsequent positions above the real height. Taking into consideration the safety of flight and landing, such a tendency guarantees a safe landing.

Table 2. Geographical coordinates – Flight 12

Point	GPS coordinates	Air-speed [km/h]	GPS heading	Point coordinates – prediction with the heading	A	Point coordinates – linear prediction (″)	B
4	51°34′25.607″N, 21°53′51.036″E	227	263°				
5	51°34′22.008″N, 21°53′21.443″E	228	256°	51°34′23.159″N, 21°53′20.508″E	54 m	51°34′26.543″N, 21°53′17.987″E	156 m
6	51°34′14.735″N, 21°52′51.528″E	226	245°	51°34′16.68″N, 21°52′22.979″E	72 m	51°34′23.087″N, 21°52′42.672″E	320 m
7	51°34′0.984″N, 21°52′49.979″E	224	190°				

A – Prediction with the heading – value of deviation (′)
B – Linear prediction value of deviation (″)

Table 2 contains the coordinates of waypoints in the aircraft trajectory in the turn into the landing heading. Control points 5 and 6 were selected for research purposes. The point coordinates are computed every 10 seconds of flight. Analysis of the flight during the turn into the landing heading is particularly significant from the point of view of ensuring the safety of the approach to landing at the destination aerodrome. Correct execution of this flight phase in the IFR conditions guarantees safe landing of the aircraft. The commencement of that flight phase at the exactly prescribed point in the air and maintaining flight parameters during the turn, i.e. airspeed, bank angle and the turn rate, have a considerable influence on the final result of executing the turn.

Figure 3 illustrates a segment of the trajectory during the turn into the landing heading. At point 4, the GPS system is deactivated and the position prediction system takes over. After

10 seconds of flight to point 5, the difference between the real position of the aircraft and its position obtained from "linear prediction" (black color, point $5''$) is 155 meters. In the system that has memorized the tendencies of heading changes during the flight and has taken into account those changes in the position prediction with the heading, the difference between position $5'$ (blue color) and the real position (point 5) is only 40 meters. Continuation of the flight maintaining the same parameters results in further deviation between the aircraft computed position from its real trajectory. The distance between point $6''$ and point 6 is 309 meters whereas the distance between points $6'$ and 6 is 67 meters. On the basis of the assessment of the position prediction system efficiency in the turn into the landing heading in IMC (Instrument Meteorological Conditions) flights, a conclusion should be made that "prediction with the heading" facilitates establishing the aircraft into the runway heading.

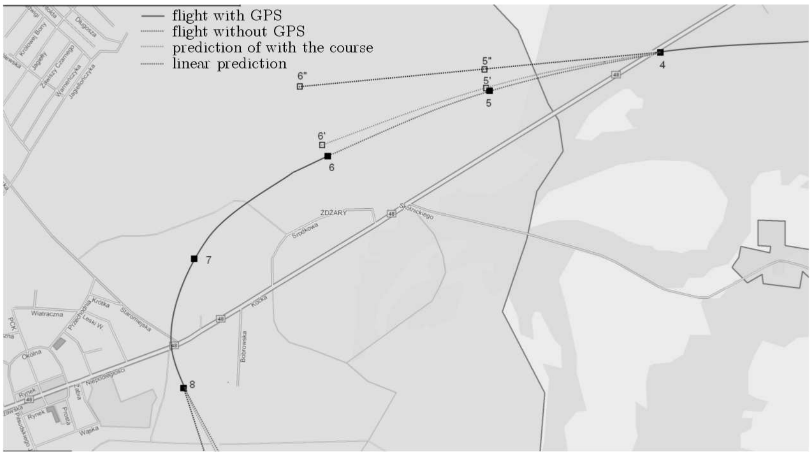


Fig. 3. Flight trajectory during the turn into landing heading – Flight 12

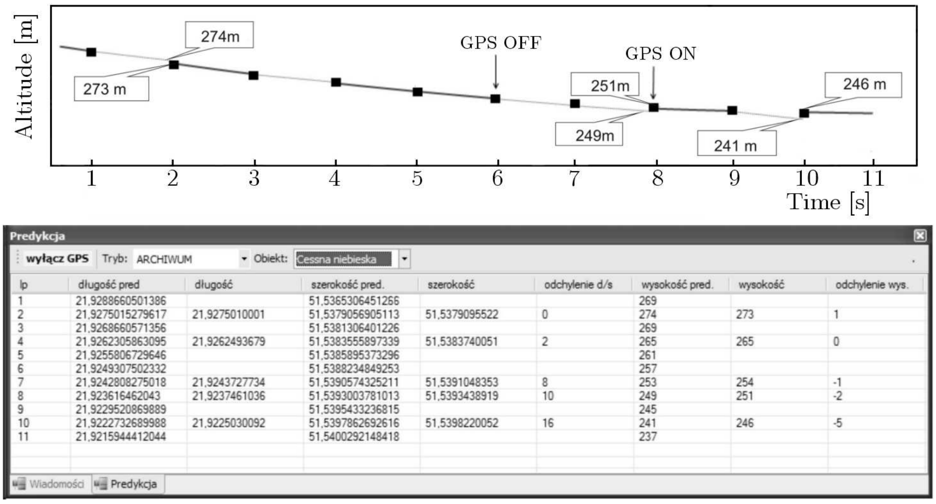
3.2. Flight 22

Flight 22 was performed on 22 September 2011, between13:15:05 s and 13:16:45 s. On the basis of the SV arrangement, PDOP = 2.1. The following points were planned:

- reference point 2,
- reference point 4,
- reference points 7 and 8,
- reference point 10.

The parameters of the glide path in Flight 22 are shown in Fig. 4. The aim of the graph is to show the trajectory of the glide path of an aircraft (coordinates recorded once a second) and present the results of the position prediction system. In the four points that were planned to be shown in the graph, the GPS system stops working and the position prediction system takes over. The comparison of the position prediction result at point 4 (4th second of the aircraft flight on the glide path) with the position obtained from the GPS shows that the real position of the aircraft was identical with the position computed by means of prediction. the directional deviation from the glide path was also recorded – 2 meters to the right of the runway heading. In the 6th second of flight, the GPS is deactivated again. At point 8 (8th second of flight), the height difference between the position obtained by use of the position prediction and the position obtained from the GPS was −2 meters.

Figure 5 illustrates a segment of the trajectory during the turn into the landing heading. At point 1, the GPS system is deactivated and the position prediction system takes over. After 10 seconds of flight to point 2, the difference between the real position of the aircraft and its



lp	długość pred	długość	szerokość pred.	szerokość	odchylenie d/s	wysokość pred.	wysokość	odchylenie wys.
1	21,9288660501386		51,5365306451266			269		
2	21,9275015279617	21,9275010001	51,5379056905113	51,5379095522	0	274	273	1
3	21,9268660571356		51,5381306401226			269		
4	21,9262305863095	21,9262493679	51,5383555897339	51,5383740051	2	265	265	0
5	21,9255806729646		51,5385895373296			261		
6	21,9249307502332		51,5388234849253			257		
7	21,9242808275018	21,9243727734	51,5390574325211	51,5391048353	8	253	254	-1
8	21,923616462043	21,9237461036	51,5393003781013	51,5393438919	10	249	251	-2
9	21,9229520869889		51,5395433236815			245		
10	21,9222732689988	21,9225030092	51,5397862692616	51,5398220052	16	241	246	-5
11	21,9215944412044		51,5400292148418			237		

Fig. 4. Glide path graph – Flight 22

Table 3. Geographical coordinates – Flight 22

Point	GPS coordinates	Air-speed [km/h]	GPS heading	Point coordinates – prediction with the heading	A	Point coordinates – linear prediction (″)	B
1	51°32′26.735″N, 21°51′36.144″E	222	267°				
2	51°32′25.583″N, 21°50′58.271″E	223	275°	51°32′25.367″N, 21°50′58.163″E	7 m	51°32′24.719″N, 21°50′58.127″E	27 m
3	51°32′28.608″N, 21°50′20.651″E	221	302°				
4	51°32′45.599″N, 21°49′53.436″E	224	28°	51°32′43.007″N, 21°49′45.084″E	178 m	51°32′36.383″N, 21°49′39.215″E	393 m
5	51°33′8.208″N, 21°50′4.956″E	225	55°				
6	51°33′24.335″N, 21°50′27.743″E	225	74°				
7	51°33′34.848″N, 21°51′4.571″E	223	91°	51°33′36.72″N, 21°51′4.14″E	59 m	51°33′40.319″N, 21°51′0.36″E	188 m
8	51°33′34.56″N, 21°51′45.143″E	220	100°	51°33′40.319″N, 21°51′44.495″E	179 m	51°33′57.167″N, 21°51′39.276″E	710 m
9	51°33′57.167″N, 21°51′39.276″E	190	107°				
10	51°33′17.856″N, 21°52′52.284″E	180	106°	51°33′19.115″N, 21°52′52.608″E	40 m	51°33′20.159″N, 21°52′53.327″E	74 m

A – Prediction with the heading – value of deviation (′)
B – Linear prediction value of deviation (″)

position obtained from "linear prediction" (black color, point 2") is 27 meters. In the system that has memorized the tendencies of heading changes during the flight and has taken into account those changes in the position prediction with the heading, the difference between position 2′ (blue color) and the real position (point 2) is only 7 meters. After that the flight is GPS-controlled again. At point 3, the GPS system is deactivated and the position prediction system takes over. The accuracy of prediction is assessed at point 4. The distance between point 4″ and point 4 is

393 meters whereas the distance between points $4'$ and 4 is 178 meters. After exiting the turn to establish on landing heading, point $8''$ is 710 meters form point 8 whereas point $8'$ is 179 meters from 8. After exiting the turn to establish on landing heading at point 10, point $10''$ is 74 meters form point 10 whereas point $10'$ is 40 meters from 10.

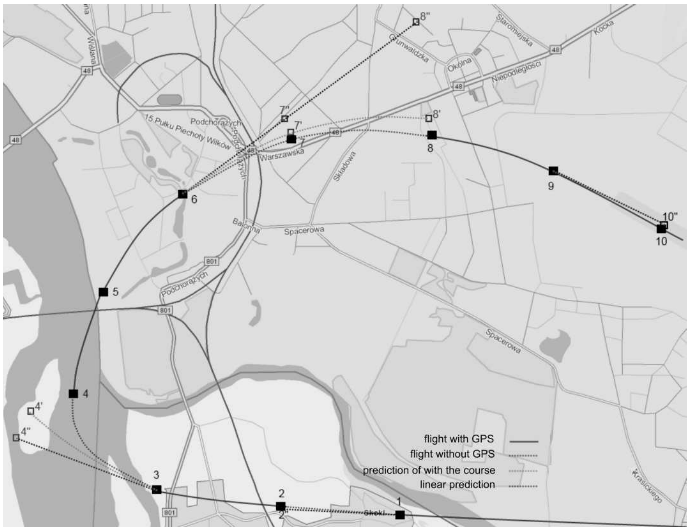


Fig. 5. Flight trajectory during the turn into landing heading – Flight 22

4. Summary and conclusions

On the basis of the conducted numerical experiments, we hold the view that the methods to be implemented in further in-flight tests are:

a) $\alpha G2$ and $\alpha G3$,

b) *linear 2*.

When there are no sudden changes in the course of the flight, the accuracy of the methods mentioned above is considerably higher than the accuracy of the remaining methods. However, only after implementing them in a real instrument, a GPS receiver, the real operational time efficiency of each of them may be assessed. According to our predictions, the *linear 2* method is a little faster than the $\alpha G2$, which in turn is faster than $\alpha G3$. In the event of substantial flight disturbances, none of those methods, or any other method known to us, may be used without running a great risk of making a serious error. During the flight, making the position prediction ought to be repeated once every second on the basis of the known position fix of the aircraft in the previous 2 or 3 seconds. The prediction may be used as necessary (lack of GPS data) provided that there are no sudden changes in the flight parameters. In the event of such changes, the prediction should not be utilized.

The accuracy of the prediction decreases with time. Therefore, the pilot ought to be warned after a given time (we recommend 3 seconds) that the position of the aircraft being shown may differ considerably from its real position. Glide paths in Figs. 2 and 4 show the differences between real and predicted positions of the aircraft in the vertical plane. The differences in the horizontal plane are illustrated in Figs. 3 and 5, allowing the determination of the magnitude of the corrections necessary to be put in the flight path in order to obtain the heading corresponding to the runway axis.

Glide paths in Figs. 2 and 4 show that the magnitude of corrections to be made to the aircraft altitude, meets the requirements of Category I and II (according to ICAO – Requirements for Navigation Systems in the Field of Reliability, Availability and Accuracy).

The presented solution facilitates the performance of the aircraft landing within Category II (ICAO). There is still a need, however, to continue research on the use of position prediction.

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