GAS TURBINE RELIABILITY MODEL BASED ON TANGENT HYPERBOLIC RELIABILITY FUNCTION

Ahmed Zohair Djeddi, Ahmed Hafaifa

University of Djelfa, Faculty of Science and Technology, Djelfa, Algeria

e-mail: a_z_djeddi@univ-djelfa.dz; hafaifa.ahmed.dz@ieee.org

Abudura Salam

University of Médéa, Faculty of Science and Technology, Médéa, Algeria

e-mail: abudura.salam@univ-medea.dz

The present work deals with the exploration of a new model proposed for the reliability analysis of industrial production systems. This proposed model is mainly based on the tangent hyperbolic function, where the survival function is determined and used in the lifetime distribution modeling taking into account of estimation the parameters of the proposed function. On the other side, tests validation is performed using the real data of a gas turbine installation. The obtained results allow the modeling of damage effects, hence the prediction of the performance of the examined gas turbine using the proposed model gives good results in terms of validity.

Keywords: reliability estimation, reliability algorithms, lifetime distribution, Weibull distribution, availability

1. Introduction

Actually, the complexity of industrial plants and their equipment behavior led practically to very complex maintenance strategies that are containing a number of different tasks. Moreover, the random nature of degradation and failure that may occur makes the determination of the required strategy to fulfill the best decisions regarding their maintenance a very difficult practical task (Costa *et al.*, 2014; Lai, 1994; Guemana *et al.*, 2011; Rao *et al.*, 2005; Hasumi *et al.*, 2009). The main aim of the proposed model is to analyze and measure industrial system reliability parameters to achieve the best time determination for making decision on the maintenance actions. Whereas, in the present work, this study is based on a model of a gas turbine which is considered as the main important equipment of the compressor stations, gas pumps, oil pumps and petrol production. The reliability model proposed in this paper is based essentially on the elementary function of tangent hyperbolic which is proved in this paper to be theoretically and practically similar to the well know Weibull distribution which was introduced by the Swedish mathematician in 1951 (Weibull, 1951). The Weibull distribution is considered the first distribution used in reliability analysis due to its very big capacity to adapt a very large number of data sets. However, many lifetime distributions that have a bathtub-shaped hazard rate function have been introduced after Weibull distribution, in which the main aim was to improve the systems reliability. A survey presenting the state-of-the-art on the class of such distributions was presented by Moeini *et al.* (2013), Scott (1979), Lai *et al.* (1998, 2001, 2003).

This work proposes an exploration of a new model which will contribute to the enhancement of the industrial systems reliability. It is obvious that the hyperbolic tangent function has a curve similar to the cumulative function of the probability distribution. Hence, a new model based on a tangent hyperbolic function defined on $\Re^+$ can be proposed. It is very clear that

its form is similar to the form of Weibull's model. In this model, the survival function will be studied based on it.

For validation of the proposed model, a practical application has been made with real data. These data have been collected from a gas turbine operating in a natural gas transportation system which is used in the south of Algeria. The reliability approach developed in this paper allows the modeling of the effects of damage to predict the performance of the examined gas turbine operation and give good results in terms of validity compared with the Weibull approach.

2. Reliability model based on the tangent hyperbolic function

The availability of the industrial equipment control allows the industries to act positively on the conformity of production, the exploitation costs and the production successful competitiveness. Indeed, the life managing of the equipment that is being used in oil and gas facilities is based on taking into account the aspects of aging, the economic and regulatory factors in order to optimize the operation, maintenance and lifespan of the system structures and components. The main goal is to maintain the level of safety and reliability as well as to maximize the investment return on the overall lifespan. The reliability of industrial systems is always essential and attracts much attention among scientific researches. It is especially important for components whose failure can cause major problems in terms of maintainability, availability and security (Halimi *et al.*, 2014; Hafaifa *et al.*, 2013a,b; Sturges, 1926; Ruji, 1990; Trofimov *et al.*, 1978). As it is aforementioned, a new model based on the tangent hyperbolic function is proposed to be used for the analysis and the study of the survival function in the reliability modeling. The tangent hyperbolic function considered in this paper is defined as a function on $\Re^+$ (Fig. 1). It is expressed as follows

$$F(x) = \begin{cases} 0 & \text{for} \quad x \in \Re^+ \\ \tanh(x) & \text{for} \quad x \geqslant 0 \end{cases} \tag{2.1}$$
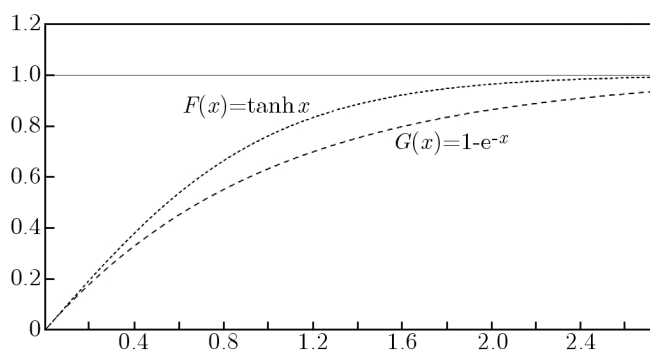

Fig. 1. Tangent hyperbolic function

To assess the impact of the aging effects on the equipment by annual frequency and to identify the sensitive equipment and the requested strategies for service priority to manage the risks that are associated with the age of the facilities, a cumulative distribution function is used. In this case it is the limit of $F(x)$ equal to 1 when x tends to +infinity and is equal to 0 when x tends to 0. The proposed function is a non-decreasing and a right-continuous function, so the proposed tangent hyperbolic function is fulfils these requirements and it presents a good choice as a candidate cumulative distribution function (Costa *et al.*, 2014; Guaily and Epstein, 2013)

$$\lim_{x \to 0} F(x) = 0 \qquad \lim_{x \to +\infty} F(x) = 1 \tag{2.2}$$

The distribution lifespan in the proposed tangent hyperbolic distribution form is deduced from the Weibull distribution. The survival function is represented as follows

$$R(t) = 1 - \tanh(\lambda t)^\beta \tag{2.3}$$

where β is the shape parameter and λ is the scale parameter. The cumulative distribution function is expressed as follows

$$F(t) = 1 - R(t) = \tanh(\lambda t)^\beta \tag{2.4}$$

Based on the derivative of the last function, the probability density function is obtained as follows:.

$$f(t) = \frac{dF(t)}{dt} = \frac{(\lambda t)^{\beta-1}\lambda\beta}{\cosh^2(\lambda t)^\beta} \tag{2.5}$$

The hazard function is determined in the flowing equations

$$P_r(t < T \leqslant t + \Delta t | T > t) = \frac{f(t)}{R(t)} \Delta t = h(t)\Delta t \tag{2.6}$$

where

$$h(t) = \frac{f(t)}{R(t)} = \frac{(\lambda t)^{\beta-1}\frac{\lambda\beta}{\cosh^2(\lambda t)^\beta}}{1 - \tanh(\lambda t)^\beta}$$

Using a variable converting the reliability $R(t)$ replaced by the hyperbolic function, $h(t)$ can be rewritten using exponential form as follows

$$h(t) = \frac{2\lambda\beta(\lambda t)^{\beta-1}}{1 + e^{-2(\lambda t)^\beta}} \tag{2.7}$$

2.1. Hazard and density functions

The hazard function in the proposed tangent hyperbolic model is expressed by equation (2.7) and is shown in Fig. 2. It is obvious that the curve of this function depends only on the parameter β. Therefore, the curve of this function can be discussed based on the value of β:

- In the case when $\beta = 1$, the hazard function tends quickly to 2λ

$$\lim_{t\to\infty} h(t) = 2\lambda \tag{2.8}$$

- In the case when $\beta = 2$, the hazard function becomes

$$h(t) = \frac{4t\lambda^2}{1 + e^{-2(\lambda t)^2}} \to 4t\lambda^2 \tag{2.9}$$

The hazard function $h(t)$ is a straight line with slope equal to $4\lambda^2$.

- In the case when $0 < \beta < 1$, the hazard function becomes

$$h(t) \to 2\lambda\beta(\lambda t)^{\beta-1} \tag{2.10}$$

The hazard function curve follows t^p form with $-1 < p < 0$.

- In the case when $\beta > 2$, the hazard function curve follows t^p with $p > 1$ (convex form).
- In the case when $1 < \beta < 2$, the hazard function curve follows t^p with $0 > p > 1$ (concave form).

It can be concluded that the curves of the hazard function have their Weibull equivalents curves. On the other side, the density function of the proposed tangent hyperbolic model presented in equation (2.5) is presented in Fig. 3 for different values of β.
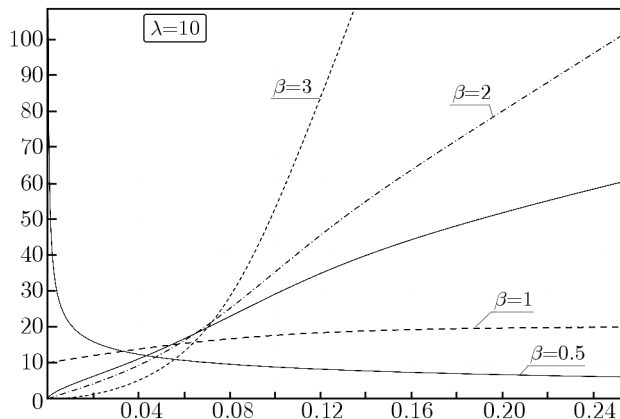
Fig. 2. Hazard function determined using the tangent hyperbolic algorithm
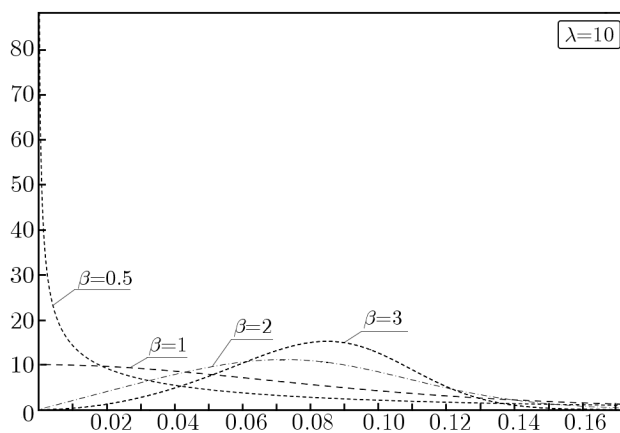

Fig. 3. Density function found using the hyperbolic model

3.   Application results

The data used in this paper are taken from the data history of a gas turbine (PGT10) used for natural gas transportation and installed in a gas plant located in the south of Algeria. Those data are presented in Table 1.

Table 1. Examined gas turbine data

N	1	2	3	4	5	6	7	8	9	10
TBF*	240	264	384	552	624	648	696	720	720	768
N	11	12	13	14	15	16	17	18	19	20
TBF	768	792	960	1272	1344	1464	1632	1680	1776	1944
N	21	22	23	24	25	26	27	28	29	30
TBF	1968	1992	2064	2136	2208	2376	2448	2448	2472	2664
N	31	32	33	34	35	36	37	38	39	40
TBF	2832	2880	3000	3600	3672	3720	4272	4656	5592	5856

∗ TBF is the Time Between Failures given in hours

3.1.   Esytimation of the model parameters

Using equation (2.4), the inverse of the tangent hyperbolic function can be obtained as follows

$$\tanh^{-1} y = (\lambda t)^{\beta} \tag{3.1}$$

The tangent hyperbolic can be eliminated from this equation based on the following well known expression

$$\tanh^{-1} y = \frac{1}{2} \log \frac{1+y}{1-y} \tag{3.2}$$

Furthermore, if the log is used to the two members of equation (3.1), the obtained expression is as follows

$$\log\left(\frac{1}{2} \log \frac{1+y}{1-y}\right) = \beta \log \lambda + \beta \log t \tag{3.3}$$

which can be transformed into a linear expression

$$Y = A + BX \tag{3.4}$$

where

$$Y = \log\left(\frac{1}{2} \log \frac{1+y}{1-y}\right) \qquad X = \log t \qquad A = \beta \log \lambda \qquad B = \beta$$

To estimate A and B, any graphical or analytical methods can be used, a possible approach is the simple regression analysis using (3.2). The estimation of the parameters based on the least--squares fit is shown in Fig. 4. The parameters of the examined gas turbine using the proposed approach are presented in Table 2. The obtained estimated parameters are $\lambda = 0.00034$ and $\beta = 1.24568$.

Table 2. The obtained parameters for the examined gas turbine

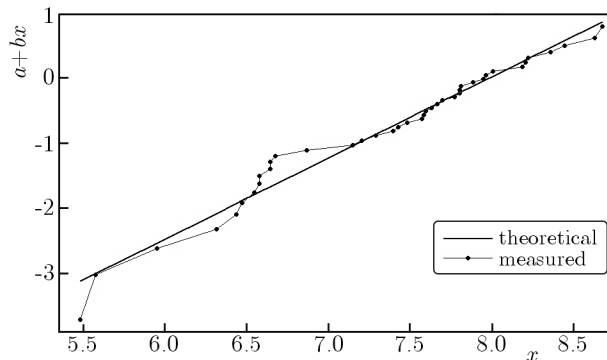|	Estimate	Std. Error	t value	$\Pr(> |t|)$
β	1.24569	0.03435	36.27	< 2E-16
$\beta \log \lambda$	−7.98420	0.02801	−285.07	< 2E-16

Fig. 4. The distribution function of the examined gas turbine determined using the hyperbolic algorithm

The plots of Y versus X which are related to the cumulative distribution function and time for the examined gas turbine determined using the proposed tangent hyperbolic algorithm are shown in Fig. 4, where the plotted data are scattered close to the fitted straight line. It can be said that this model is acceptable (Murthy *et al.*, 2004; Yang and Scott, 2013).

3.1.1. Cumulative distribution function

The cumulative distribution function of the examined gas turbine found using the tangent hyperbolic model is shown in Fig. 5. The plotted data are scattered close to the fitted curve. Following the cumulative distribution function expressed in equation (2.4), the parameters $\lambda = 0.00034$ and $\beta = 1.24568$.
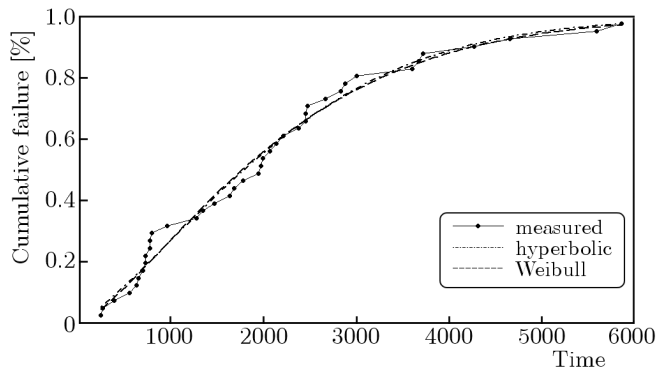
Fig. 5. The cumulative failure rate of the examined gas turbine using the hyperbolic model

3.1.2. Density and hazard functions

The density function and the hazard function of the examined gas turbine determined by using the tangent hyperbolic function model are shown respectively in Figs. 6 and 7. These functions are obtained by using equation (2.5) and taking into account the estimated parameter values $\lambda = 0.00034$ and $\beta = 1.24568$.
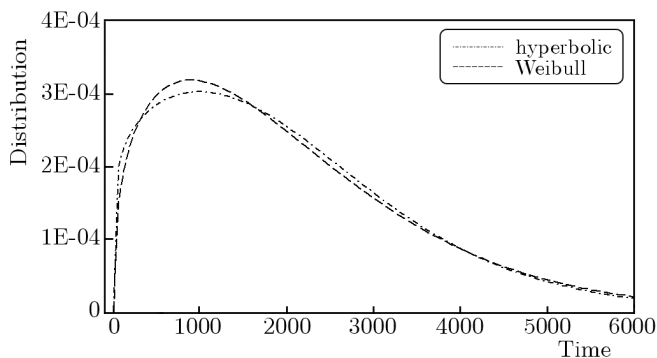

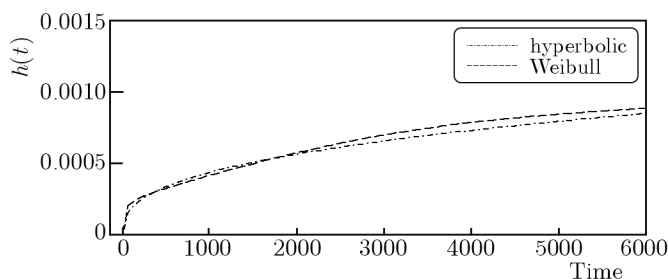Fig. 6. The distribution function of the model of the examined gas turbine using the hyperbolic model

Fig. 7. The hazard function of the examined gas turbine using the hyperbolic model

3.1.3. Mean time between failures

The Mean Time Between Failures (MTBF) is the predicted elapsed time between inherent failures of the system during exploitation (Yang and Scott, 2013) which is expressed as follows

$$\text{MTBF} = \int_0^{+\infty} t f(t) \, dt = \int_0^{+\infty} t \frac{(\lambda t)^{\beta-1} \lambda \beta}{\cosh^2 (\lambda t)^{\beta}} \, dt \tag{3.5}$$

It is worth noting that there is an alternative way for computing the expected value (Murthy *et al.*, 2004)

$$\text{MTBF} = \int\limits_0^{+\infty} S(t) \, dt = \int\limits_0^{+\infty} 1 - \tanh(\lambda t)^\beta \, dt \tag{3.6}$$

Table 3

Model	Weibull	Hyperbolic function
MTBF	2088.623 h	2075.40 h

4. Conclusion

The goal achieved in this paper is concerned with industrial reliability, which requires the specialists to be more competitive and more responsive to variable market conditions. It is obvious that to achieve better management of the facilities performance, it is necessary to improve the reliability. It has been shown that with the application of the proposed method based on the tangent hyperbolic function good results can be obtained. The main objective is to analyze and to measure the reliability parameters of the examined system in order to determine the best time to take maintenance action based on reliability analysis. The developed in this paper reliability approach allows the modeling of the damage effects to predict the performance of the examined gas turbine operation and to lead to good results in terms of validity compared with the Weibull approach. This model can be improved to have a bathtub-shaped hazard form.

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