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INTERFACIAL SHEAR AND PEELING STRESSES IN A TWO-PLATE STRUCTURE SUBJECTED TO MONOTANICALLY INCREASING THERMAL LOADING

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An approximate analytical model is developed for the evaluation of the interfacial shear and peeling stresses in a bi-material element composed of two elastic plates bonded together by an interface zero thickness material and subjected to monotonically increasing thermal loading. Thermal peeling stress is caused by thermal and elastic mismatch of a two-plate structure undergo a temperature change. The "peeling" stress can be determined from the evaluated interfacial shear stress and is proportional to deflections of the thinner plate of the structure, i.e. to its displacements with respect to the thicker plate. The interface is assumed to exhibit brittle failure at the critical shear stress value. The analytical solution qualitatively shows delamination and ultimate failure. The results are illustrated in figures and discussed.

Key words: monotonic thermal loading, delamination, interfacial shear, peeling stress

1. Introduction

The thermal response of the interface has an essential significance in the design of multilayered systems. Interfacial stresses of layered composite materials under thermal loading were analysed in numerous papers. In most of them, the analyses are based on linear fracture mechanics assumptions when both element cracking and interface debonding are treated as the mixed mode crack propagation with critical conditions expressed in terms of stress intensity factors, cf. Zhang (2000), Bleeck *et al.* (1998), Sorensen *et al.* (1998). The crack singularities at bi-material interfaces were analysed by Rizk and Erdogan (1989). Białas and Mróz (2006) analysed the progressive delamination and cracking of a thin layer bonded to a rigid or elastic substrate and subjected to thermal loading.

The approximate analytical model for the assessment of the interfacial stresses in a bi-material soldered assembly with a low-yield-stress of the bonding material, in bi-metal thermostats, bi-material assembly with a low-yield-stress bonding layer, in adhesively bonded and soldered assemblies and in a tri-material assembly were presented by Suhir (1986a,b, 1987, 1989, 2001, 2006).

Compared with the paper by Suhir (2006) in this study, we use the analytical shear lag analysis associated with damage constitutive equations to calculate the lengths of elastic and delamination zones, the interfacial shear stress and the critical temperature values cases at full debonding of the interface layer.

The present paper is aimed at the analysis of a two plate joint subjected to thermal loading. An approximate predictive model (Nikolova *et al.*, 2006; Nikolova, 2008) was developed for the evaluation of interfacial thermal stresses in a bi-material assembly. This material is considered linearly elastic at the stress level below the critical point and ideally plastic at higher stresses.

The delamination process is analysed for homogeneous plates with the stress free boundary. The plates are assumed as linear elastic and isotropic with different stiffness and thermal expansion moduli. The analytic solutions for the shear and peeling stresses are obtained. The main objective of the paper is to discuss the effect of the interface parameters and interfacial stresses on the delamination process.

2. Problem formulation

2.1. Two-plate structure model and basic equations

Let us consider two elastic plates A and B with different stiffness moduli and thermal expansion coefficients, bonded along the interface I and loaded by a thermal monotonic loading ΔT, where α_A, α_B denote thermal expansion coefficients (see Fig. 1).

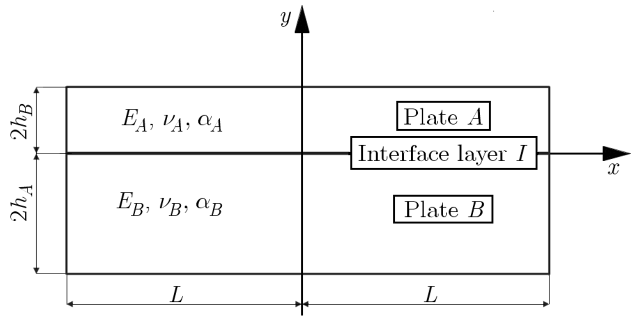

Fig. 1. Two-plate structure model A

The shear lag model is used and the plate bending is neglected. The neglecting of the bending effects in the shear-lag model results from the qualitative values of the stress-strain behavior. The interface is supposed to be with a negligible thickness and works only on shear. The axial stresses and strains are uniform over the cross section of each plate. At least one of the plates is thick and stiff enough, so that this component and the structure as a whole does not experience bending deformations.

In view of symmetry, only the solution for the half plate is derived taking the origin of the Cartesian coordinate at the centre of interface I. The plate length is denoted by L and thickness by $2h_A$, h_B. The Young moduli are E_A, E_B. The uniform temperature of plates is ΔT. The interface is simulated as the zero thickness layer with the shear response specified in terms of the displacement discontinuity on I (Nikolova *et al.*, 2006; Nikolova, 2008).

The following ordinary differential equations of plate equilibrium can be stated

$$\frac{d\sigma_A}{dx} = \frac{\tau^I}{2h_A} \qquad \frac{d\sigma_B}{dx} = -\frac{\tau^I}{2h_B} \tag{2.1}$$

where $\tau^I = \tau^I(x)$ is the interface shear stress. The elastic plate response is specified by the constitutive equations

$$\sigma_A = E_A(\varepsilon_A - \alpha_A \Delta T) \qquad \sigma_B = E_B(\varepsilon_B - \alpha_B \Delta T) \tag{2.2}$$

and the elastic interface response is specified by the relation

$$\tau^I = G^I w_I \tag{2.3}$$

where

$$w_I = \frac{u_A - u_B}{h_A + h_B} = \frac{u_I}{h_A + h_B} \qquad \varepsilon_A = \frac{du_A}{dx} \qquad \varepsilon_B = \frac{du_B}{dx} \tag{2.4}$$

and $u_A = u_A(x)$, $u_B = u_B(x)$ are the displacement fields in plates A and B, respectively, and G^I is the representative shear modulus of the adhesive layer.

Introduce now non-dimensional variables defined as follows

$$
\begin{aligned}
&\overline{x} = \frac{x}{h} \qquad &&\overline{u}_i = \frac{u_i}{h} \qquad &&\overline{\sigma}_i = \frac{\sigma_i}{E_B} \qquad &&\overline{\tau}^I = \frac{\tau^I}{E_B} \qquad &&\overline{G}^I = \frac{G^I}{E_B} \\
&\xi = \frac{h_A}{h_B} \qquad &&\eta = \frac{E_A}{E_B} \qquad &&i = A, B, I \qquad &&h = h_A + h_B
\end{aligned}
\tag{2.5}
$$

Equilibrium equations (2.1) now are

$$
\frac{d\overline{\sigma}_A}{d\overline{x}} = \frac{\overline{\tau}^I(1+\xi)}{2\xi} \qquad \frac{d\overline{\sigma}_B}{d\overline{x}} = -\frac{\overline{\tau}^I(1+\xi)}{2}
\tag{2.6}
$$

In subsequent derivation, the formulas will be expressed in terms of non-dimensional variables, but the dashes over the symbols will be deleted.

2.2. Elastic zone solution

Let us denote $u_I(x) = u_A(x) - u_B(x)$. In view of constitutive equations (2.2), we obtain for the elastic state

$$
\frac{d^2 u_I}{dx^2} = \lambda^2 u_I - \frac{d}{dx}[(\alpha_A - \alpha_B)\varDelta T]
\tag{2.7}
$$

where

$$
\lambda^2 = \frac{G^I(1+\xi)(1+\xi\eta)}{2\xi\eta}
\tag{2.8}
$$

Assuming uniform temperature fields in plates A and B and constant thermal expansion coefficients, equation (2.7) becomes

$$
\frac{d^2 u_I}{dx^2} = \lambda^2 u_I
\tag{2.9}
$$

and equilibrium equations (2.6) can be expressed as follows

$$
\frac{d^2 u_A}{dx^2} = \frac{\lambda^2}{1+\xi\eta} u_I \qquad \frac{d^2 u_B}{dx^2} = -\frac{\lambda^2}{1+\xi\eta}\xi\eta u_I
\tag{2.10}
$$

The general solution to equation (2.9) has the form

$$
u_I = A_1 \cosh(\lambda x) + A_2 \sinh(\lambda x)
\tag{2.11}
$$

where A_1, A_2 are integration constants.

Considering two homogeneous plates, we can state the boundary conditions

$$
\begin{aligned}
&u_I(0) = u_A(0) = u_B(0) = 0 \qquad &&\sigma_A^e(L) = \sigma_B^e(L) = 0 \\
&\varepsilon_A(L) = \alpha_A \varDelta T \qquad &&\varepsilon_B(L) = \alpha_B \varDelta T
\end{aligned}
\tag{2.12}
$$

Integrating Equations (2.10) and satisfying conditions (2.12), we obtain

$$
\tau^I(x) = G u_I(x) = \frac{G(\alpha_A - \alpha_B)\varDelta T}{\lambda}\frac{\sinh(\lambda x)}{\cosh(\lambda L)}
\tag{2.13}
$$

2.3. Length of the elastic zone

The length of the plastic zone l_e, which gives the magnitude of the brittle cracking along the interface layer can be calculated from (2.13) assuming that at $\tau^I(x) = \tau^{cr}$, $u_I(l_e) = u^{cr} = \tau^{cr}/G$. Then

$$\tau^I(x) = Gu_I(x) = \frac{G(\alpha_A - \alpha_B)\Delta T}{\lambda}\frac{\sinh(\lambda x)}{\cosh(\lambda L)} = \tau^{cr} \tag{2.14}$$

Using the substitution $\exp(\lambda l_e) = y$, we receive from (2.14) the following equation for y

$$y^2 - 2Ay - 1 = 0 \qquad\qquad A = \frac{\lambda\tau^{cr}\cosh(\lambda L)}{(\alpha_A - \alpha_B)\Delta T} \tag{2.15}$$

Then, two roots of (2.15) are available

$$y_{1,2} = A \pm \sqrt{A^2 + 1} \tag{2.16}$$

Now using the substitution $\exp(\lambda l_e) = y$, we obtain

$$l_{e_{1,2}} = \frac{1}{\lambda}\ln\Big(A \pm \sqrt{A^2 + 1}\Big)$$

Obviously, $A^2 + 1 > 0$.

Then, we have to choose the length of the debonding zone from the condition that this length has its maximum value, i.e.

$$l_e = \frac{1}{\lambda}\ln\Big(A + \sqrt{A^2 + 1}\Big) \tag{2.17}$$

2.4. Elasto-plastic zone solution

In the case when plastic strains occur in the bonding material, the following conditions must be fulfilled at the boundary $x = x_* = L - l_e$, between the "inner" (linearly elastic) and the "outer" (ideally plastic) zones

$$\tau^I(x_*) = \frac{G\Delta\alpha\Delta T}{\lambda}\frac{\sinh(\lambda x_*)}{\cosh(\lambda L)} = \frac{G\Delta\alpha\Delta T}{\lambda}\frac{\sinh[\lambda(L - l_e)]}{\cosh(\lambda L)} = \tau^{cr} \tag{2.18}$$

Condition (2.18) indicates that the shear stress at the boundary between the elastic and the plastic zones must be equal to the critical stress (Nikolova *et al.*, 2006; Nikolova, 2008).

2.5. Length of the plastic zone

The length of the plastic zone l_e can be calculated from (2.18) assuming that at $\tau^I(x_*) = \tau^{cr}$. Then

$$\tau^I(x_*) = \frac{G\Delta\alpha\Delta T}{\lambda}\frac{\sinh(\lambda x_*)}{\cosh(\lambda L)} = \tau^{cr}(x) \tag{2.19}$$

We obtain the following formula for the relative length of the plastic zone

$$x_* = L - l_e = L - \frac{1}{\lambda}\ln\Big(A + \sqrt{A^2 + 1}\Big) \tag{2.20}$$

3. Determination of the peeling stress

3.1. Basic equation of the peeling stress

The basic equation for the dimensional peeling stress $p(x)$, can be obtained using the following equation of equilibrium for the thinner plate A of the structure treated as an elongated thin plate (Suhir, 2006)

$$\int\limits_{-x_*}^{x}\int\limits_{-x_*}^{x} p(\varsigma)\,d\varsigma\,d\varsigma + D_1 w''(x) = \frac{h_A}{2}T(x) = \frac{h_A}{2}\Big(\int\limits_{-x_*}^{x} \tau^I(\varsigma)\,d\varsigma - \tau^{cr} l_e\Big) \tag{3.1}$$

where

$$D_A = \frac{E_A h_A^3}{12(1-\nu_A^2)} \tag{3.2}$$

is the flexural rigidity of the plate A, and $w(x)$ is the deflection function of this plate (with respect to the thicker plate that does not experience bending deformations)

$$T(x) = \int\limits_{-x_*}^{x} \tau(\varsigma)\,d\varsigma - \tau^{cr} l_e \tag{3.3}$$

are the thermally induced forces acting in the cross-sections of the two-plate structure, τ^{cr} is the critical stress of the interface, and l_e is the length of the elastic zone. The length l_e can be defined as $l_e = L - x_*$, where L is the half of the structure length. The origin 0 of the coordinate x is in the mid-cross-section of the structure (Suhir 1986a,b, 1989).

Equation (3.1) indicates that the "external" bending moment experienced by the plate 1, and expressed by the right part in equation (3.1) and the first term in the left part, should be equilibrated by the elastic bending moment, which is expressed by the second term in the left part of equation (3.1). This term is proportional, for small deflections, to the second derivative (curvature) of the deflection function $w(x)$.

The peeling stress $p(x)$, can be evaluated as

$$p(x) = Kw(x) \tag{3.4}$$

where K is the spring constant of the elastic foundation provided by the bonding layer (interface). In an approximate analysis, we assume that the spring constant of the elastic foundation can be put equal to Young's modulus of the bonding material.

Excluding the deflection function $w(x)$ from equations (3.1) and (3.4), we obtain the following integral equation for the peeling stress function $p(x)$

$$\int\limits_{-x_*}^{x}\int\limits_{-x_*}^{x} p(\varsigma)\,d\varsigma\,d\varsigma + \frac{D_A}{K}p''(x) = \frac{h_A}{2}T(x) \tag{3.5}$$

After differentiating this equation twice with respect to the coordinate x and considering relationship (3.3), we obtain the following basic equation for the peeling stress function

$$p^{IV}(x) + 4\beta^4 p(x) = 2\beta^4 h_A \tau'(x) \tag{3.6}$$

where

$$\beta = \sqrt[4]{\frac{K}{/4D_A}} \tag{3.7}$$

is the parameter of the peeling stress.

Equation (3.6) has form of the equation of bending of a beam lying on a continuous elastic foundation (see, for instance Suhir, 2006a,b) and loaded by a distributed load whose magnitude is proportional to the rate of changing in the interfacial shear stress along the assembly. Since the shear stress is constant outside the elastic region, the "peeling" stress is zero in this region. Note that in the close proximity to the boundary between the elastic and plastic zones, the peeling stress might change in a manner that violates its proportionality to the derivative of the shear stress. The behavior of the peeling stress in the proximity of this boundary might be different of what is described by equation (3.6).

In the case when plastic strains occur in the bonding material, the following conditions must be fulfilled at the boundary $x = x_*$ between the elastic and the ideally plastic zones:

$$\tau^I(x_*) = \tau^{cr} \qquad\qquad T(x_*) = -\tau^{cr} l_e \tag{3.8}$$

The first condition in (3.8) indicates that the shear stress at the boundary between the elastic and the plastic zones must be equal to the critical stress. The second condition follows from formula (3.3): the shear stress, $\tau^I(x)$ is self-equilibrated, and therefore the integral in (3.3) is zero for $x = x_*$. Physically, this condition is due to the fact that since the interfacial shear stress at the peripheral portions of the structure is constant (is equal to the critical stress τ^{cr}), the force $T(x)$ changes linearly at these portions from its value $\tau^{cr} l_e$ at the boundary of the elastic and the plastic zones to zero at the structure ends. The sign "minus" in front of the second boundary condition in (3.8) indicates that the force at the boundary should be compressive (negative) for the compressed plate of the structure. In the case of a purely elastic state of strain ($l_e = 0$), the following boundary condition should be fulfilled

$$T(L) = \int\limits_{-L}^{L} \tau^I(x)\, dx = 0 \tag{3.9}$$

This condition reflects the fact that there are no external longitudinal forces acting at the end cross-sections of the plate-structure.

The peeling stress should be self-equilibrated within the elastic region and, therefore, the following conditions of equilibrium with respect to the bending moments and the lateral forces should be fulfilled

$$\int\limits_{-x_*}^{x_*} \int\limits_{-x_*}^{x} p(\varsigma)\, d\varsigma\, d\varsigma = 0 \qquad\qquad \int\limits_{-x_*}^{x_*} p(\varsigma)\, d\varsigma = 0 \tag{3.10}$$

From (3.5), we find, by differentiation

$$\int\limits_{-x_*}^{x} p(\varsigma)\, d\varsigma + \frac{D_A}{K} p'''(x) = \frac{h_A}{2} \tau^I(x) \tag{3.11}$$

Relationships (3.5) and (3.11), with consideration to conditions (3.10), result in the following boundary conditions for the peeling stress function $p(x)$

$$p''(x_*) = -\frac{h_A K l_e \tau^{cr}}{4D_A} = -2\beta^4 h_A l_e \tau^{cr} \qquad\qquad p'''(x_*) = \frac{h_A K \tau^{cr}}{2D_A} = 2\beta^4 h_A \tau^{cr} \tag{3.12}$$

The peeling stress in the zones of plastic shear strains should be zero, as it follows from equation (3.6), the shear stress function is equal to the critical stress in these zones, and hence, does not change along the structure.

3.2. Solution to the peeling stress equation

Equation (3.6) has form of an equation of a beam lying on a continuous elastic foundation (Suhir, 2006a,b). We seek for the solution to this equation in form

$$p(x) = C_0V_0(\beta x) + C_1V_1(\beta x) + C_2V_2(\beta x) + C_3V_3(\beta x) + B\frac{\cosh(\lambda x)}{\cosh(\lambda x_*)} \tag{3.13}$$

This functions $p(x)$ is an odd function, i.e. $p(-x) = -p(x)$ and has its maximum value (zero derivative) at the origin and is symmetric with respect to the mid-cross-section of the assembly.

The final form of the solution to equation (3.6) is the following

$$p(x) = C_0V_0(\beta x) + C_2V_2(\beta x) + B\frac{\cosh(\lambda x)}{\cosh(\lambda x_*)} \tag{3.14}$$

where the functions $V_i(\beta x)$, $i = 0, 2$ are expressed as follows

$$V_0(\beta x) = \cosh(\beta x)\cos(\beta x) \qquad V_2(\beta x) = \sinh(\beta x)\sin(\beta x) \tag{3.15}$$

The first two terms in (3.14) provide the general solution to the homogeneous equation, which corresponds to non-homogeneous equation (3.6), and the third term is the particular solution to this equation.

Introducing this term into the equation (3.6), we obtain

$$B = \frac{2G^I E_B h_A \beta^4(\alpha_A - \alpha_B)\Delta T}{4\beta^4 + \lambda^4} \tag{3.16}$$

Using boundary conditions (3.12), we obtain the following algebraic equations for the constants C_0 and C_2 of integration

$$\begin{aligned} & -2\beta^3\sin(\beta x_*)\cosh(\beta x_*)C_0 - 2\beta^3\cos(\beta x_*)\sinh(\beta x_*)C_0 \\ & \quad -2\beta^3\sin(\beta x_*)\cosh(\beta x_*)C_2 + 2\beta^3\cos(\beta x_*)\sinh(\beta x_*)C_2 = 2\beta^4 h_A\tau^{cr} \\ & -\beta\sin(\beta x_*)\cosh(\beta x_*)C_0 + \beta\cos(\beta x_*)\sinh(\beta x_*)C_0 \\ & \quad +\beta\sin(\beta x_*)\cosh(\beta x_*)C_2 + \beta\cos(\beta x_*)\sinh(\beta x_*)C_2 = -2\beta^4 h_A l_e\tau^{cr} \end{aligned} \tag{3.17}$$

Algebraic equations (3.17) have the following solutions

$$\begin{aligned} C_0 &= \frac{\beta h_A[-\sin(\beta x_*)\cosh(\beta x_*)(2\beta^2 l_e - 1) + \sinh(\beta x_*)\cos(\beta x_*)(2\beta^2 l_e + 1)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)} \\ C_2 &= \frac{\beta h_A[\sinh(\beta x_*)\cos(\beta x_*)(2\beta^2 l_e - 1) + \sin(\beta x_*)\cosh(\beta x_*)(2\beta^2 l_e + 1)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)} \end{aligned} \tag{3.18}$$

Note that for long enough elastic zones, solution (3.14) can be simplified as follows

$$\begin{aligned} p(x) = & \frac{\beta h_A[\sin(\beta x_*)\cosh(\beta x_*)(1 - 2\beta^2 l_e)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)}\cosh(\beta x)\cos(\beta x) \\ & + \frac{\beta h_A[\sinh(\beta x_*)\cos(\beta x_*)(1 + 2\beta^2 l_e)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)}\cosh(\beta x)\cos(\beta x) \\ & - \frac{\beta h_A[\sinh(\beta x_*)\cos(\beta x_*)(1 - 2\beta^2 l_e)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)}\sinh(\beta x)\sin(\beta x) \\ & + \frac{\beta h_A[\sin(\beta x_*)\cosh(\beta x_*)(1 + 2\beta^2 l_e)]\tau^{cr}}{\cos(2\beta x_*) - \cosh(2\beta x_*)}\sinh(\beta x)\sin(\beta x) \\ & + \frac{2G^I E_B h_A \beta^4(\alpha_A - \alpha_B)\Delta T}{4\beta^4 + \lambda^4}\frac{\cosh(\lambda x)}{\cosh(\lambda x_*)} \end{aligned} \tag{3.19}$$

4. Numerical example

Let us consider two elastic plates A (from ZrO_2) and B (from Ti_6Al_4V) with finite lengths $2L = 120\,\mathrm{mm}$, and the thickness of the first plate h_A will be variable $2h_A \in [0.4\,\mathrm{mm}, 2\,\mathrm{mm}]$, $2h_B = 2\,\mathrm{mm}$, $h = h_A + h_B \in [1.2\,\mathrm{mm}, 2\,\mathrm{mm}]$, $\xi = h_A/h_B \in [0.2, 1]$ under monotonic temperature loading ΔT and

- ZrO_2 – elastic constant $E_A = 132.2\,\mathrm{GPa}$, coefficient of thermal expansion $\alpha_A = 13.3 \cdot 10^{-6}\,\mathrm{K}^{-1}$,
- Ti_6Al_4V – elastic constant $E_B = 122.7\,\mathrm{GPa}$, coefficient of thermal expansion $\alpha_B = 10.291 \cdot 10^{-6}\,\mathrm{K}^{-1}$,
- interface layer – elastic constant $E_0 = 2.1\,\mathrm{GPa}$, shear modulus $G^I = 800\,\mathrm{MPa}$, the critical shear stress $\tau^I = \tau^{cr} = 18\,\mathrm{MPa}$.

In Table 1, the variable parameters of the problem considered are presented

Table 1. Variable parameter

Name	Variable
Thickness of first plate A	h_A
Geometric parameter of the structure	$\xi = h_A/h_B$
Non-dimensional parameter	$\lambda = \sqrt{G^I(1+\xi)(1+\xi\eta)/(2\xi\eta)}$
Flexural rigidity of plate A	$D_A = E_A h_A^3/[12(1-\nu_A^2)]$
Parameter of the peeling stress	$\beta = \sqrt[4]{K/(4D_A)}$
Temperature monotonic loading	ΔT

The behaviour of the interfacial shear and the peeling stresses on x/h, peeling stresses $p(x)$ for different temperature monotonic loadings ∇T ($\nabla T \leqslant \nabla T^{cr}$) (for more details about calculation of the critical temperature ∇T^{cr}, see paper by Nikolova *et al.* (2006), Nikolova (2008) and the behaviour of peeling stresses $p(x)$ for different values of parameters $\xi = h_A/h_B$ are illustrated at the given below figures, respectively.

The calculated interfacial shear and the peeling stresses are plotted in Fig. 2. As it can be seen from the plot, one cannot simply truncate the diagram for the shear stress obtained on the basis of the elastic solution in order to evaluate the length of the plastic zone.

On the other hand, the plastic zone is about 30% from the elastic one, which results in the fact that the elastic approach cannot be used to assess the peeling stress either.

Figure 3 shows the behavior of the peeling stresses $p(x)$ for $\xi = h_A/h_B = 1$ and five different values of the temperature loading ΔT. The peeling stress increases with an increase in the temperature ΔT and reaches values exceeding the critical shear stress $\tau^I(x_*) = \tau^{cr}$. This initiates degradation of structural integrity and peeling of the interface. When the temperature loading ΔT tends towards ΔT^{cr} (Nikolova *et al.*, 2006;, Nikolova, 2008), the full delamination process occurs.

Figure 4 shows that the peeling stresses $p(x)$ depend on the geometry of the structure, including the parameter $\xi = h_A/h_B \in [0.2, 1]$, i.e. $p(x)$ increases with an decrease in the parameter ξ (very thin first plate A). This dependence is sensitive to the material properties and geometry of the bi-material structure. It can be seen that for given values of parameter ξ,

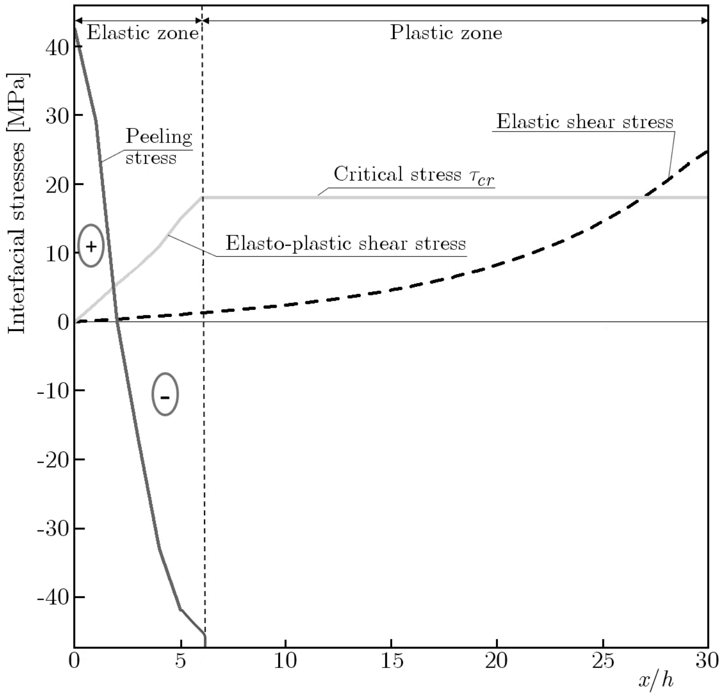

Fig. 2. Calculated interfacial stresses

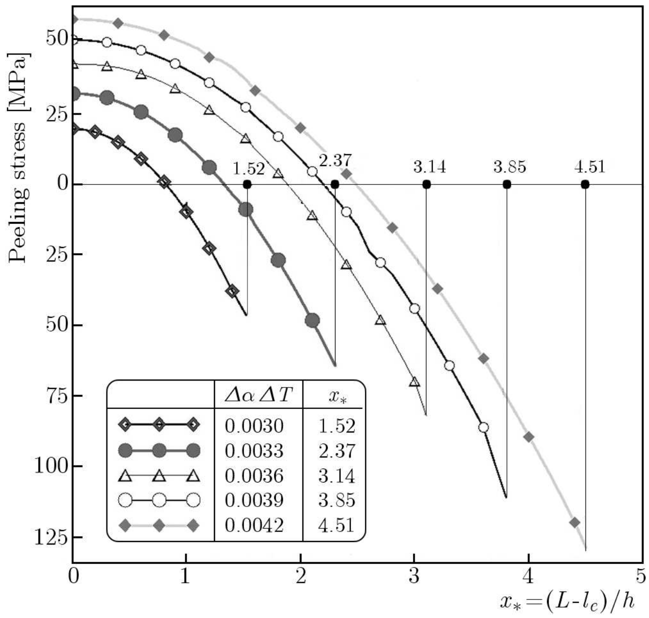

Fig. 3. Dependence of $p(x)$ on x_* for different temperature loadings ΔT

the thickness of the first plate and the material properties of the second plate play an important role.

When the thickness of the first plate is very small, i.e. the peeling stress grows extremely rapidly and induces large and irreversible damages to structural integrity, cracking, irregularities and peeling of the interface.

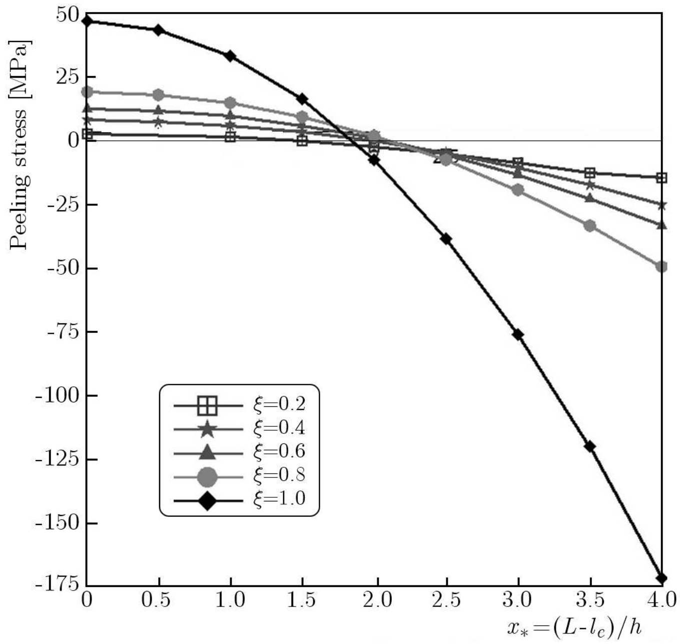

Fig. 4. Dependence of the peeling stresses $p(x)$ on x_* for five different values of the parameter ξ

5. Conclusions

In this paper, an analytical accurate and quick procedure based on the shear lag method and classical plate theory formulation for calculating the lengths of elastic and plastic (inelastic) zones, interfacial shear and peeling stresses of a two-plate structure subjected to monotonic thermal loading is presented.

The peeling stress is sensitive to material properties, geometry of the bi-material structure and applied thermal loading. The peeling stress reaches to values exceeding the critical shear stress at a very high temperature and for a structure with a very small thickness of the first plate, which induces large and irreversible damage to structural integrity, cracking, irregularities and peeling of the interface. Therefore, the thermally induced interfacial shear and peeling stresses in different ceramic-metal composites are preferable to be of low values.

The interfacial shear stress, peeling stress, and die cracking stress due to thermal and elastic mismatch in layered structures are one of the major reasons of the mechanical failure and delamination in multilayered structures. The results were illustrated in figures and discussed.

The results of the analysis can be used for the assessment of thermally induced stresses in different ceramic-metal composites and in manufacturing of new ideally designed ceramic-metal composites with optimal properties of both – the ceramic, such as high temperature resistance and hardness – and the metal, such as the ability to undergo plastic deformation.

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