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INFLUENCE OF ATMOSPHERIC TURBULENCE ON BOMB RELEASE

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A model of the bomb release dynamics is presented in the paper, and how it is influenced by a wind field. The applied way of describing this field is discussed with account taken of its stochastic nature. Exemplary results of a numerical simulation of bombing are submitted and concluded.

Key words: atmospheric turbulence, bombing

1. Introduction

Any bomb release should be carried out in such a way that the target is hit with the maximum possible degree of accuracy. Release conditions can vary significantly. Different usually are: air speed at bomb release, bomb release altitude and the angle of bomb release. Depending on these parameters, a different point on the Earth's surface is reached. Wind is another factor that can influence the bomb trajectory. If the wind exhibits a constant speed and direction, then the situation is simple. However, each time the influence of atmospheric turbulence on the bomb release can be different because of its stochastic nature. Therefore, even if initial conditions of bomb release are the same, bomb trajectories are different.

The aim of this study is to estimate the influence of the stochastic wind field on flight of a small training bomb. A series of numerical simulations were carried out with a six-degrees-of-freedom model of motion applied. The model describes the bomb gliding in the three-dimensional space. To determine the wind field, Shinozuka's method has been applied. This method is usually used to model stochastic processes.

2. Mathematical description of bomb motion

2.1. Assumptions for a physical model

To analyse the bomb flight dynamics, the following assumptions were made that allow formulation of a mathematical description of motion:

1. The bomb is a rigid body of constant mass, constant moments of inertia and a constant position of the centre of mass.
2. The bomb has two symmetry planes. These are the Oxz and Oxy planes (Fig. 1) that are planes of geometric, mass, and aerodynamic symmetries.

2.2. Coordinates systems

To formulate a mathematical model of the bomb, the following orthogonal coordinate systems were used:

$Oxyz$ – bomb-fixed system with its origin at the bomb centre of mass
$Ox_ay_az_a$ – air-trajectory reference frame
$Ox_gy_gz_g$ – Earth-referenced system with its origin at the bomb centre of mass.

These systems are related to each other by means of the following angles:

— $Oxyz$ and $Ox_gy_gz_g$ systems: with the angle of yaw $\varPsi$, the angle of pitch $\varTheta$ and the angle of roll $\varPhi$

— $Oxyz$ and $Ox_ay_az_a$ systems: with the angle of sideslip β and the angle of attack α.

Subsequent turns by angles $\varPsi$, $\varTheta$ and $\varPhi$ about the coordinate axes can result in finding the matrix of transition from the $Ox_gy_gz_g$ system to the $Oxyz$ system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{L}_{s/g} \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix} \tag{2.1}$$

where the $\mathbf{L}_{s/g}$ matrix is

$$\mathbf{L}_{s/g} = \begin{bmatrix} \cos\varPsi\cos\varTheta & \sin\varPsi\cos\varTheta & -\sin\varTheta \\ \cos\varPsi\sin\varTheta\sin\varPhi - \sin\varPsi\cos\varPhi & \sin\varPsi\sin\varTheta\sin\varPhi + \cos\varPsi\cos\varPhi & \cos\varTheta\sin\varPhi \\ \cos\varPsi\sin\varTheta\cos\varPhi + \sin\varPsi\sin\varPhi & \sin\varPsi\sin\varTheta\cos\varPhi - \cos\varPsi\sin\varPhi & \cos\varTheta\cos\varPhi \end{bmatrix} \tag{2.2}$$

Further turns by angles β and α result in finding the matrix of transition from the $Ox_ay_az_a$ system to the $Oxyz$ system

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{L}_{s/a} \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix} \tag{2.3}$$

where the $\mathbf{L}_{s/g}$ matrix has elements

$$\mathbf{L}_{s/a} = \begin{bmatrix} \cos\alpha\cos\beta & -\cos\alpha\sin\beta & -\sin\alpha \\ \sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha \end{bmatrix} \tag{2.4}$$

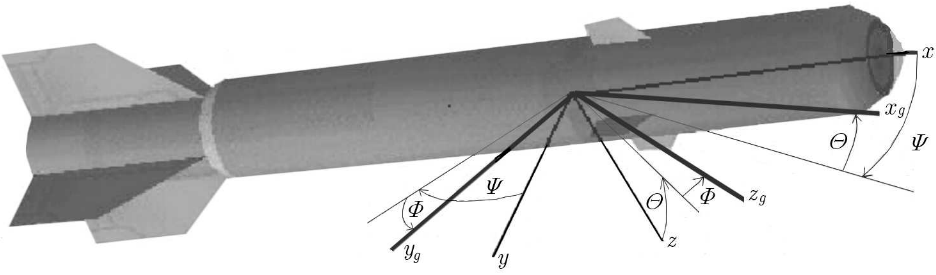

Fig. 1. Coordinate systems with angles of transition

2.3. Equations of motion of the bomb

2.3.1. A general form of the equations of motion

Since tunnel measurements of aerodynamic forces are usually taken in the air-trajectory reference frame $Ox_ay_az_a$, equations of equilibrium of forces will be determined in this system. However, equations of equilibrium of moments will be determined in the bomb-fixed coordinate system $Oxyz$, because the tensor of moments of inertia is independent of time in this system.

- The vector equation of motion of the bomb centre of mass takes the following form

$$\frac{d(m\boldsymbol{V})}{dt} = \frac{\partial(m\boldsymbol{V})}{\partial t} + \boldsymbol{\Omega} \times (m\boldsymbol{V}) = \boldsymbol{F} \tag{2.5}$$

and can be rewritten as three scalar equations in any rectangular moving system of coordinates

$$
\begin{aligned}
&m(\dot{U}+QW-RV)=X \qquad\qquad m(\dot{V}+RU-PW)=Y\\
&m(\dot{W}+PV-QU)=Z
\end{aligned}
\tag{2.6}
$$

where

m	–	mass of a bomb
$\boldsymbol{V}$	–	velocity vector with components $\boldsymbol{V}=[U,V,W]^\top$ in the moving system of coordinates
$\boldsymbol{\Omega}$	–	vector of angular velocity of the moving system against the inertial reference frame with components $\boldsymbol{\Omega}=[P,Q,R]^\top$ in the moving system of coordinates
$\boldsymbol{F}$	–	resultant vector of forces acting on the bomb with components $[X,Y,Z]^\top$ in the moving system of coordinates.

In the air-trajectory reference frame $Ox_ay_az_a$, the velocity vector has only one component $U_a=V$ (which should not be mistaken for the second component of the vector $\boldsymbol{V}$ according to the designations above). Equations (2.6) take the following form

$$
m\dot{V}=X_a \qquad mR_aV=Y_a \qquad -mQ_aV=Z_a \tag{2.7}
$$

Assuming that both the angular velocity of the bomb-fixed system $Oxyz$ against the inertial reference frame $\boldsymbol{\Omega}_s$ and the velocity of the system $Oxyz$ against the $Ox_ay_az_a$ system are known, the vector of the angular velocity of the $Ox_ay_az_a$ system against the inertial reference frame can be determined

$$
\boldsymbol{\Omega}_a=\boldsymbol{\Omega}_s+\boldsymbol{\Omega}_{s/a}=\boldsymbol{\Omega}_s+\dot{\boldsymbol{\beta}}-\dot{\boldsymbol{\alpha}} \tag{2.8}
$$

In the $Oxyz$ frame, the $\boldsymbol{\Omega}_s$ vector has the following components: $\boldsymbol{\Omega}_s=[P,Q,R]^\top$; in the $Ox_ay_az_a$ coordinate system, the $\dot{\boldsymbol{\beta}}$ vector has the following components: $\dot{\boldsymbol{\beta}}=[0,0,\dot{\beta}]^\top$; whereas in the $Oxyz$ frame, the $\dot{\boldsymbol{\alpha}}$ vector has the components: $\dot{\boldsymbol{\alpha}}=[0,\dot{\alpha},0]^\top$. Taking the foregoing into account, and using transition matrix (2.4), on the basis of (2.8), we receive

$$
\begin{aligned}
P_a&=P\cos\alpha\cos\beta+(Q-\dot{\alpha})\sin\beta+R\sin\alpha\cos\beta\\
Q_a&=-P\cos\alpha\sin\beta+(Q-\dot{\alpha})\cos\beta-R\sin\alpha\sin\beta\\
R_a&=-P\sin\alpha+R\cos\alpha+\dot{\beta}
\end{aligned}
\tag{2.9}
$$

Having applied (2.9) to equations (2.7), after some transformations, the following system of equations is arrived at

$$\begin{aligned} &\dot{V} = \frac{1}{m}X_a \qquad \dot{\beta} = \frac{1}{mV}Y_a + P\sin\alpha - R\cos\alpha \\ &\dot{\alpha} = \frac{1}{\cos\beta}\Big[\frac{Z_a}{mV} + Q\cos\beta - (P\cos\alpha + R\sin\alpha)\sin\beta\Big] \end{aligned} \tag{2.10}$$

- The vector equation of moments of force equilibrium has the following form

$$\frac{d(\boldsymbol{K})}{dt} = \frac{\partial(\boldsymbol{K})}{\partial t} + \boldsymbol{\Omega} \times \boldsymbol{K} = \boldsymbol{M} \tag{2.11}$$

where $\boldsymbol{M}$ is the resultant moment of forces acting on the bomb with components $\boldsymbol{M} = [L, M, N]^\top$ in the moving system of coordinates.

The vector of the bomb angular momentum is

$$\boldsymbol{K} = \mathbf{I}\boldsymbol{\Omega} \tag{2.12}$$

where the inertia tensor (moments and products of inertia) $\mathbf{I}$ is determined as

$$\mathbf{I} = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zx} & I_z \end{bmatrix} \tag{2.13}$$

As said above, equations (2.11) will be written down in the bomb-fixed system $Oxyz$. The stability of mass characteristics of the bomb in this system makes all derivatives of components of the moment-of-inertia tensor against time equal to zero. It means that

$$\frac{\partial \boldsymbol{K}}{\partial t} = \frac{\partial(\mathbf{I}\boldsymbol{\Omega}_s)}{\partial t} = \frac{\partial \mathbf{I}}{\partial t}\boldsymbol{\Omega}_s + \mathbf{I}\frac{\partial \boldsymbol{\Omega}_s}{\partial t} = \mathbf{I}\frac{\partial \boldsymbol{\Omega}_s}{\partial t} \tag{2.14}$$

What is arrived at after some transformations, on the basis of (2.11) and with (2.14) applied, is a system of three scalar equations that describe angular motion of the bomb in the moving bomb-fixed system of coordinates $Oxyz$. It takes the following form

$$\begin{aligned} &I_x\dot{P} - I_{yz}(Q^2 - R^2) - I_{zx}(\dot{R} + PQ) - I_{xy}(\dot{Q} - RP) - (I_y - I_z)QR = L \\ &I_y\dot{Q} - I_{zx}(R^2 - P^2) - I_{xy}(\dot{P} + QR) - I_{yz}(\dot{R} - PQ) - (I_z - I_x)RP = M \\ &I_z\dot{R} - I_{xy}(P^2 - Q^2) - I_{yz}(\dot{Q} + RP) - I_{zx}(\dot{P} - QR) - (I_x - I_y)PQ = N \end{aligned} \tag{2.15}$$

Since the Oxz and Oxy planes are the bomb planes of symmetry, the following dependences occur

$$I_{xy} = I_{yx} = I_{zy} = I_{yz} = 0 \tag{2.16}$$

Therefore, system of equations (2.15) is reduced to

$$\begin{aligned} & I_x\dot{P} - (I_y - I_z)QR = L \qquad I_y\dot{Q} - (I_z - I_x)RP = M \\ & I_z\dot{R} - (I_x - I_y)PQ = N \end{aligned} \tag{2.17}$$

Finally, after some elementary transformations, system (2.17) takes the form

$$\begin{aligned} & \dot{P} = \frac{1}{I_x}[L + (I_y - I_z)QR] \qquad \dot{Q} = \frac{1}{I_y}[M + (I_z - I_x)RP] \\ & \dot{R} = \frac{1}{I_xI_z}[L + (I_y - I_z)QR] \end{aligned} \tag{2.18}$$

Complementary to systems (2.10) and (2.18) are kinematic relations that permit determination of the rates of changes of the angles Ψ, Θ and Φ, if the angular velocities P, Q, R are known

$$\begin{aligned} & \dot{\Phi} = P + (R\cos\Phi + Q\sin\Phi)\tan\Theta \qquad \dot{\Theta} = Q\cos\Phi - R\sin\Phi \\ & \dot{\Psi} = \frac{1}{\cos\Theta}(R\cos\Phi + Q\sin\Phi) \end{aligned} \tag{2.19}$$

Furthermore, with relationships (2.1) and (2.3) applied, the velocity vector of the bomb centre of mass in the $Ox_gy_gz_g$ system can be determined

$$\begin{bmatrix} U_g \\ V_g \\ W_g \end{bmatrix} = \begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{z}_g \end{bmatrix} = \mathbf{L}_{s/g}^{-1}\mathbf{L}_{s/a}\begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} \tag{2.20}$$

Particular components are as follows:

$$\begin{aligned} \dot{x}_g &= V[\cos\alpha\cos\beta\cos\Theta\cos\Psi + \sin\beta(\sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi) + \\ &\quad + \sin\alpha\cos\beta(\cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi)] \\ \dot{y}_g &= V[\cos\alpha\cos\beta\cos\Theta\sin\Psi + \sin\beta(\sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi) + \\ &\quad + \sin\alpha\cos\beta(\cos\Phi\sin\Theta\sin\Psi + \sin\Phi\cos\Psi)] \\ \dot{z}_g &= V[-\cos\alpha\cos\beta\sin\Theta + \sin\beta\sin\Phi\cos\Theta + \sin\alpha\cos\beta\cos\Phi\cos\Theta] \end{aligned} \tag{2.21}$$

Equations (2.10), (2.18), (2.19), and (2.21) compose a system of 12 ordinary differential equations that describe spatial motion of the bomb treated as a rigid body. It can be written down in the following form

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{F}(t, \boldsymbol{X}, \boldsymbol{S}) \tag{2.22}$$

where

$\boldsymbol{X}$	–	twelve-element vector of the bomb flight parameters, $\boldsymbol{X} = [V, \alpha, \beta, P, Q, R, \Phi, \Theta, \Psi, x_g, y_g, z_g]^\top$
V	–	velocity of the bomb (absolute value of the bomb velocity vector)
α	–	angle of attack
β	–	angle of sideslip
P, Q, R	–	angular velocities of rolling, pitching and yawing in the $Oxyz$ system of coordinates
Θ, Φ, Ψ	–	bomb angles of pitch, roll and yaw.

2.3.2. General expressions to describe forces and moments acting on the bomb

Forces acting on the bomb

The right-hand side of equation (2.5) represents forces that act on the bomb

$$\boldsymbol{F} = \boldsymbol{Q} + \boldsymbol{R} \tag{2.23}$$

According to designations in equations (2.7), there are the following components

$$\begin{aligned} X_a &= Q_{x_a} + R_{x_a} \qquad & Y_a &= Q_{y_a} + R_{y_a} \\ Z_a &= Q_{z_a} + R_{z_a} \end{aligned} \tag{2.24}$$

Particular components in expression (2.24) are determined below. They are as follows:

- the bomb weight $\boldsymbol{Q}$ that has only one component $\boldsymbol{Q} = [0, 0, mg]^\top$ in the $Ox_g y_g z_g$ system. Using dependences (2.1) and (2.3), one can calculate components of the vector $\boldsymbol{Q}$ in the $Ox_a y_a z_a$ system

$$\begin{bmatrix} Q_{x_a} \\ Q_{y_a} \\ Q_{z_a} \end{bmatrix} = \mathbf{L}_{s/a}^{-1} \mathbf{L}_{s/g} \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{2.25}$$

We get

$$Q_{x_a} = mg(-\cos\alpha\cos\beta\sin\Theta + \sin\beta\cos\Theta\sin\Phi + \sin\alpha\cos\beta\cos\Theta\cos\Phi)$$
$$Q_{y_a} = mg(\cos\alpha\sin\beta\sin\Theta + \cos\beta\cos\Theta\sin\Phi - \sin\alpha\sin\beta\cos\Theta\cos\Phi)$$
$$Q_{z_a} = mg(\sin\alpha\sin\Theta + \cos\alpha\cos\Theta\cos\Phi) \tag{2.26}$$

- the aerodynamic force $\boldsymbol{R}$ that has the following components in the $Ox_ay_az_a$ system

$$R_{x_a} = -P_{xa} = -C_{xa}\frac{\rho V^2}{2}S \qquad R_{y_a} = P_{ya} = -C_{ya}\frac{\rho V^2}{2}S$$
$$R_{z_a} = -P_{za} = -C_{za}\frac{\rho V^2}{2}S \tag{2.27}$$

where

C_{xa}, C_{ya}, C_{za}	–	coefficients of aerodynamic drag, side and lift forces
S	–	cross-sectional area of the bomb
ρ	–	air density.

Moments of forces acting on the bomb

The right-hand side of system of equations (2.17) contains the vector $\boldsymbol{M} = [L, M, N]^\top$ that is a resultant vector of moments of forces acting on the bomb. Since equations (2.18) are determined within the system of the bomb principal axes of inertia with its origin at the bomb centre of mass, the aerodynamic moments are the only moments acting on the bomb. Therefore, particular components are as follows

$$L = C_l\frac{\rho V^2}{2}Sd \qquad M = C_m\frac{\rho V^2}{2}Sd \qquad N = C_n\frac{\rho V^2}{2}Sd \tag{2.28}$$

where

C_l, C_m, C_n	–	coefficients of rolling, pitching and yawing moments, respectively
d	–	diameter of the bomb.

2.4. Aerodynamic forces and moments acting on the bomb

Aerodynamic forces and moments acting on the bomb, described with expressions (2.27) and (2.28), are determined according to their aerodynamic coefficients. These coefficients depend on many factors such as the bomb shape, angle of attack, angle of sideslip, Mach number, Reynolds number, and angular velocities. There are no general methods for finding these characteristics for any spatial position of the bomb. Therefore, various methods are used depending on the problem discussed, availability of source data on the bomb, and research apparatus.

The most widely used method is wind-tunnel testing. Both real objects and models thereof can be tested. While conducting the model testing, account should be taken of the so-called similarity criteria. These should be met to provide highly reliable results. Another way of determining the aerodynamic coefficients is to apply methods of numerical fluid mechanics. These methods, based on equations that describe continuous-media flows, allow for numerical calculations of basic aerodynamic characteristics of different kinds of objects. There are also methods of the so-called identification: characteristics are found in effect of several flight tests. Various parameters are measured during these tests to enable identification of the bomb aerodynamic characteristics.

In the case of both aerodynamic forces and moments, it is assumed that the total aerodynamic coefficient is the sum of the static component and components effected by the bomb non-zero angular velocities. The superposition principle understood in this way can be applied in the following general form

$$C_a = C_{a\,static}(\alpha,\beta) + C_a^p P + C_a^q Q + C_a^r R + C_a^{pq} PQ + C_a^{pr} PR + C_a^{qr} QR \tag{2.29}$$

For individual aerodynamic coefficients, some components above are equal to zero or are so small that can be omitted.

A detailed description of the way of determining all aerodynamic coefficients can be found in Lebedew (1973) and Ostalawskij (1957). Static aerodynamic characteristics obtained from the tunnel testing have been used in these calculations. They are shown in Figs. 2, 3, and 4.

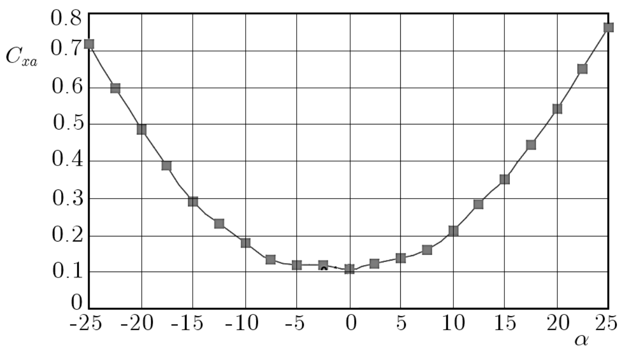

Fig. 2. Aerodynamic drag coefficient $C_{xa}(\alpha)$

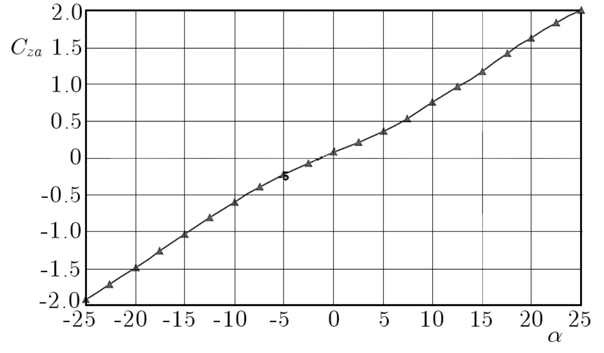

Fig. 3. Aerodynamic lift and side force coefficients $C_{za}(\alpha)$, $(C_{ya}(\beta))$

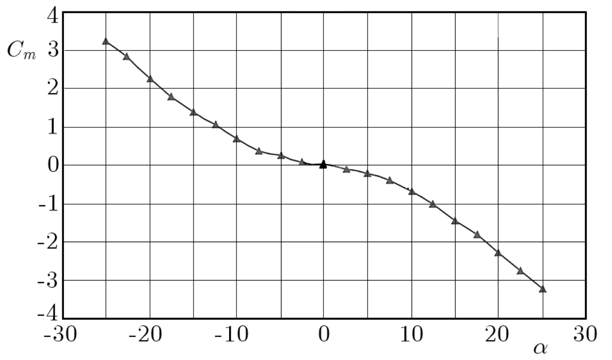

Fig. 4. Coefficient of aerodynamic pitching (yawing) moment $C_m(\alpha)$, $(C_n(\beta))$

Dynamic derivatives have been found using dependences taken from Ostalawskij (1957). The most important are derivatives that determine damping moments. They are as follows

$$C_m^q = -C_{zaH}^{\alpha_H} \frac{S_H L_H^2}{Sd} \tag{2.30}$$

where

$C_{zaH}^{\alpha_H}$	–	derivative of aerodynamic lift coefficient as related to the angle of attack, $C_{zaH}^{\alpha_H} = \partial C_{zaH}/\partial \alpha_H$
S_H	–	bomb 'tail-plane' area
L_H	–	distance between control surfaces of the bomb and its centre of mass.

Because of the symmetry, these two are equal

$$C_n^r = C_m^q \tag{2.31}$$

While determining the aerodynamic drag coefficient, account was taken of the effect of compressibility of the air on the change in value of this coefficient at the zero angle of attack. It was assumed that this coefficient remains constant up to the Mach number reaching its critical value. Above this critical value, the aerodynamic drag coefficient increases to reach the maximum value for the Mach number 1.1. The calculations resulted in the following

$$C_{xa0} = \begin{cases} 0.11 & \text{for} \quad Ma < Ma_{kr} \approx 0.7 \\ 0.5 & \text{for} \quad Ma = 1.1 \end{cases} \tag{2.32}$$

For the Mach number ranging from 0.7 to 1.1, this coefficient increases according to the following dependence

$$C_{xa0} = 0.11 + 0.39\Big[\cos\Big(3\pi\frac{Ma-0.7}{0.4}\Big) - 9\cos\Big(\pi\frac{Ma-0.7}{0.4}\Big) + 8\Big] \tag{2.33}$$

3. A model of the wind field

In studies on the effect of wind upon flying objects, the wind field is described with functional dependences that represent actual atmospheric phenomena only with a limited accuracy (Holbit, 1988). The reason is that the wind actually shows the variable stochastic nature. Therefore, a reliable representation of the effect of wind upon any flying object (such as a bomb) should allow for this feature of the wind field (Mnitowski, 2006; Shinozuka, 1971; Shinozuka and Jan, 1972).

The most general description requires that the wind is assumed to be variable against time and space

$$\boldsymbol{V}_w = \boldsymbol{V}_w(t, x_g, y_g, z_g) \tag{3.1}$$

For the sake of simplification, it is quite often assumed that turbulence is independent of time. However, the dependence on spatial coordinates means that for sufficiently big objects the turbulence at various points on the surface of a moving object can be different. It fundamentally changes local angles of attack and it has to be taken into account in the analysis. If the object is small in size as compared to the characteristic dimensions of gusts that affect it, then it can be assumed that the wind changes the angle of attack and/or the angle of sideslip by exactly the same value for the whole object. The modified angles can be found if the vector of velocity against the air is known (Fig. 5). This velocity, marked as $\boldsymbol{V}_*$, equals to the difference between the object velocity against the Earth $\boldsymbol{V} = [u, v, w]^\top$ and that of the wind against the Earth $\boldsymbol{V}_w = [u_w, v_w, w_w]^\top$

$$\boldsymbol{V}_* = \boldsymbol{V} - \boldsymbol{V}_w \tag{3.2}$$

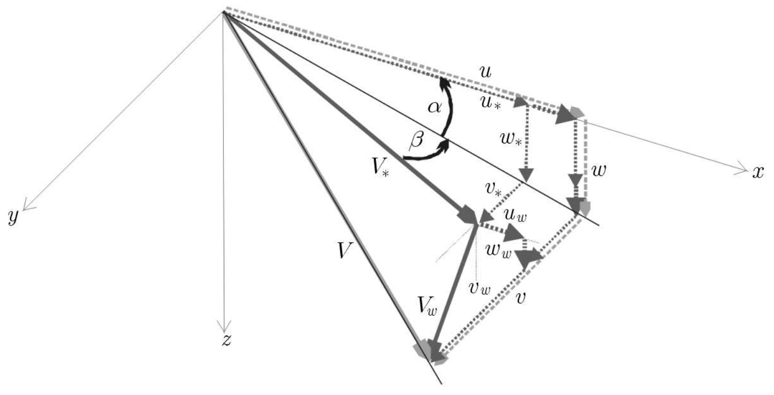

Fig. 5. Determination of the angles of sideslip and attack

3.1. Impact of wind on the angle of attack and sideslip

Hence, the angle of attack and the angle of sideslip are, respectively, as follows

$$\begin{aligned} \alpha &= \arctan\frac{w_*}{u_*} = \arctan\frac{w - w_w}{u - u_w} \\ \beta &= \arctan\frac{v_*}{\sqrt{u_*^2 + w_*^2}} = \arctan\frac{v - v_w}{\sqrt{(u - u_w)^2 + (w - w_w)^2}} \end{aligned} \tag{3.3}$$

Components of the vectors $\boldsymbol{V}$ and $\mathbf{V}_w$ to be found above refer to the object/bomb-fixed coordinate system.

3.2. Shinozuka's method

While analysing the effect of atmospheric turbulence on the bomb flight, it becomes crucial to reconstruct the stochastic structure of the wind field. The method proposed by Shinozuka proves very useful in this case (Shinozuka, 1971; Shinozuka and Jan, 1972), since it facilitates numerical simulation of stochastic processes. This method is based on the assumption that any stochastic process is a sum of periodic courses, the amplitudes of which depend on the so-called Power Spectral Density $\boldsymbol{\Phi}$, whereas phases thereof are random functions of the "white noise" type. The basic expression to calculate the wind field in terms of a stochastic process takes the following form (see Mnitowski, 2006)

$$\upsilon_i(\boldsymbol{r}) = \sum_{j=1}^{i} \sum_{l=1}^{L} |H_{ij}(\boldsymbol{\Omega}_l)| \sqrt{2\Delta\boldsymbol{\Omega}} \cos(\boldsymbol{\Omega}_l' \boldsymbol{r} + \phi_{jl}) \tag{3.4}$$

where: $\boldsymbol{\Omega}$ is a perturbed vector of "spatial" frequency; $\boldsymbol{r}$ – vector that determines the position of the point under consideration; ϕ_{jl} – mutually independent and stochastically variable phase displacements of values 0-2π; $\mathbf{H}$ – lower triangular matrix of amplitudes related to the matrix of Power Spectral Density $\boldsymbol{\Phi}$ by means of the following dependence

$$\boldsymbol{\Phi}(\boldsymbol{\Omega}) = \mathbf{H}(\boldsymbol{\Omega})\mathbf{H}^{\top}(\boldsymbol{\Omega}) \tag{3.5}$$

In a general case, if components of the matrix $\boldsymbol{\Phi}(\boldsymbol{\Omega})$ are known, then the matrix components $\mathbf{H}(\boldsymbol{\Omega})$ can be determined using the following dependences

$$\begin{aligned} H_{11} &= \sqrt{\Phi_{11}} & H_{21} &= \frac{\Phi_{21}}{H_{11}} \\ H_{22} &= \sqrt{\Phi_{22} - (H_{21})^2} & H_{31} &= \frac{\Phi_{31}}{H_{11}} \\ H_{32} &= \frac{\Phi_{32} - H_{31}H_{21}}{H_{22}} & H_{33} &= \sqrt{\Phi_{33} - (H_{31})^2 - (H_{32})^2} \end{aligned} \tag{3.6}$$

Expression (3.4) provides capability to calculate components of the wind in the Earth-related system. What has been assumed in this study is that characteristics of the wind depend on the xg coordinate. It means that the expression takes the following form

$$\begin{aligned} u_{wg\,t}(x_g) &= \sum_{l_x=1}^{L_x} |H_{11}(\Omega_{xl_x})| \sqrt{2\Delta\Omega_x} \cos(\Omega_{xl_x}' x_g + \phi_{1l_x}) \\ v_{wg\,t}(x_g) &= \sum_{l_x=1}^{L_x} |H_{21}(\Omega_{xl_x})| \sqrt{2\Delta\Omega_x} \cos(\Omega_{xl_x}' x_g + \phi_{1l_x}) + \\ &+ \sum_{l_x=1}^{L_x} |H_{22}(\Omega_{xl_x})| \sqrt{2\Delta\Omega_x} \cos(\Omega_{xl_x}' x_g + \phi_{2l_x}) \end{aligned} \tag{3.7}$$

$$
\begin{aligned}
w_{wg\,t}(x_g) = & \sum_{l_x=1}^{L_x} |H_{31}(\Omega_{xl_x})|\sqrt{2\Delta\Omega_x}\cos(\Omega'_{xl_x}x_g + \phi_{1l_x}) + \\
& + \sum_{l_x=1}^{L_x} |H_{32}(\Omega_{xl_x})|\sqrt{2\Delta\Omega_x}\cos(\Omega'_{xl_x}x_g + \phi_{2l_x}) + \\
& + \sum_{l_x=1}^{L_x} |H_{33}(\Omega_{xl_x})|\sqrt{2\Delta\Omega_x}\cos(\Omega'_{xl_x}x_g + \phi_{3l_x})
\end{aligned}
$$

To carry out calculations, boundary values of the frequency are assumed, with "analytical frequencies" in between

$$
\Omega_{x\,lower} \leqslant \Omega_x \leqslant \Omega_{x\,upper} \tag{3.8}
$$

Each interval is subdivided into L_i subintervals of the following length

$$
\Delta\Omega_x = \frac{\Omega_{x\,upper} - \Omega_{x\,lower}}{L_x} \tag{3.9}
$$

The frequencies that occur in formulae (3.7) as arguments of the matrix components $\mathbf{H}(\boldsymbol{\Omega})$ are as follows

$$
\Omega_{xl_x} = \Omega_{x\,lower} + (l_x - 1)\Delta\Omega_x \tag{3.10}
$$

On the other hand, the frequencies occurring in the arguments of the cosine function, additionally denoted with $(\cdot)'$, are as follows

$$
\Omega'_{xl_x} = \Omega_{xl_x} + \delta\Omega_{xl_x} \tag{3.11}
$$

The perturbations $\delta\Omega$ are randomly generated and then added to avoid periodicity of the simulated gust. They satisfy the following inequalities

$$
-\frac{\Delta'\Omega_x}{2} \leqslant \delta\Omega_{xl_x} \leqslant \frac{\Delta'\Omega_x}{2} \tag{3.12}
$$

where

$$
\Delta'\Omega_x \ll \Delta\Omega_x \tag{3.13}
$$

The phase displacements ϕ_{jl_x} $(j = 1, 2, 3)$ are mutually independent, randomly variable, and included in the range of 0-2π.

3.3. Dryden's power spectrum

According to what has been stated above, the effective determination of a stochastic process requires the Power Spectral Density to be known. Such a spectrum can be found immediately on the basis of available measurements of wind field fluctuations, or using specific expressions presented in the literature. The Dryden spectrum used in flight mechanics has been applied to studies on bomb release. Expressions (3.7) require that a one-dimensional spectrum is known. The spectrum has the following form

$$\boldsymbol{\Phi}(\Omega_x) = \frac{L_w}{2\pi} \frac{\sigma^2}{(1+L_w^2\Omega_x^2)^2} \begin{bmatrix} 2(1+L_w^2\Omega_x^2) & 0 & 0 \\ 0 & 1+3L_w^2\Omega_x^2 & 0 \\ 0 & 0 & 1+3L_w^2\Omega_x^2 \end{bmatrix} \tag{3.14}$$

where: L_w is the scale of turbulence, σ – standard deviation.

Figures 6 and 7 show exemplary spectra.

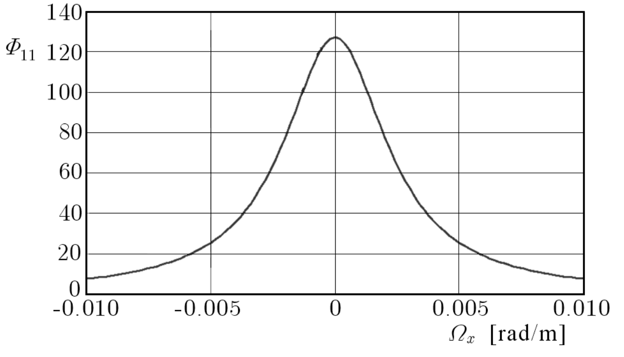

Fig. 6. Two-sided one-dimensional spectrum $\Phi_{11}(\Omega_x)$

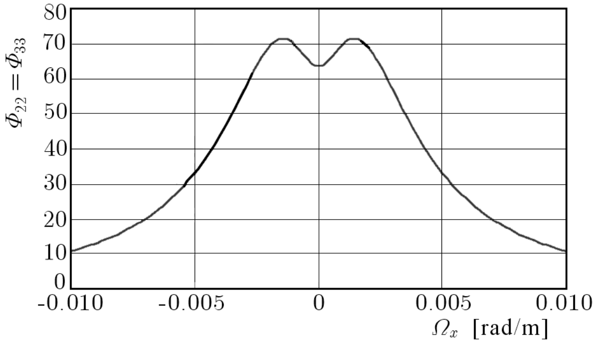

Fig. 7. Two-sided one-dimensional spectrum $\Phi_{22}(\Omega_x) = \Phi_{33}(\Omega_x)$

4. Simulation of bomb release in atmospheric turbulence

The release of a small training bomb in turbulent atmosphere was simulated. The total of 213 'bomb releases' were carried out. It was assumed that a bomb of 15 kg is released from the altitude of 2000 metres. An aircraft keeps flying at the speed of 236 m/s (850 km/h). The standard deviation is the same for each wind component and amounts to 10 m/s; the wind field is described with the above-discussed relationships.

The results shown below refer to only one randomly chosen effect of the simulation. Figures 8-10 present particular components of the wind field which were acting on the bomb and hence changing its angles of attack and sideslip, and immediately affecting aerodynamic forces and moments. It is evident that the wind components show neither periodicity nor regularity - they are random in their nature, as assumed.

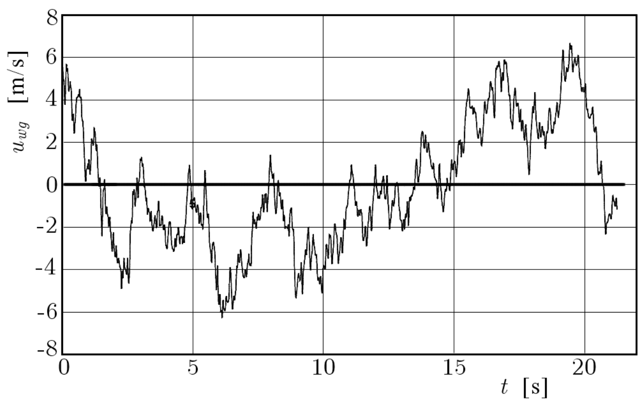

Fig. 8. Diagram of wind component u_{wg}

Subsequent figures present exemplary flight parameters against time. For the sake of comparison, each diagram also shows results on the 'no-wind-effect' bomb release (i.e. bomb release with no atmospheric turbulence affecting it). These are represented with the bold line. The plot of flight velocity of a bomb generally does not differ from that of bomb velocity under 'no-wind-effect' conditions (Fig. 11). The only difference is that it is only in the initial phase of the bomb release that the decrease in velocity is greater. The reason is that high angles of attack appear (Fig. 12), which results in an increase in the aerodynamic drag and deceleration of the bomb in the initial, relatively flat, part of trajectory. The bomb angles of attack and sideslip (Fig. 13) keep oscillating all the time, yet these angles do not increase because the bomb

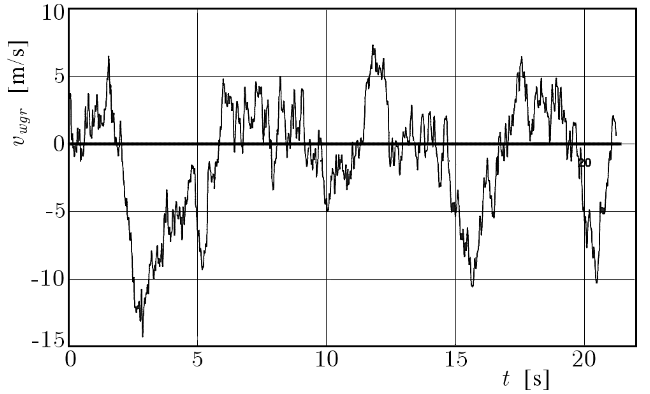

Fig. 9. Diagram of wind component v_{wg}

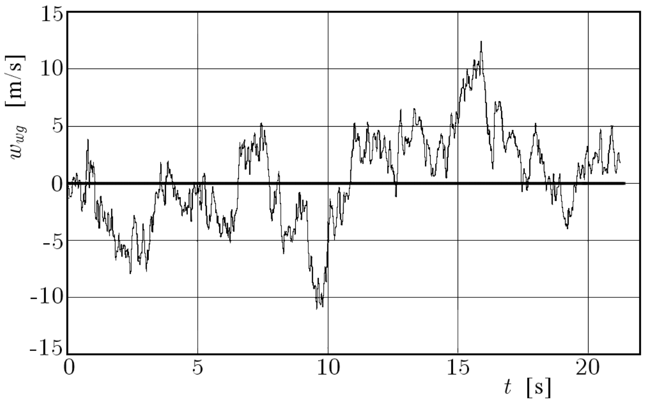

Fig. 10. Diagram of wind component w_{wg}

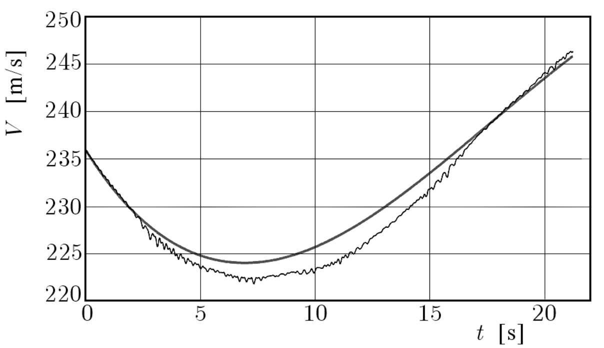

Fig. 11. Flight speed of the bomb

under consideration remains statically and dynamically stable. Changes in the angle of attack under 'no-wind-effect' conditions also prove stability of the bomb – fading oscillations of this parameter can be observed.

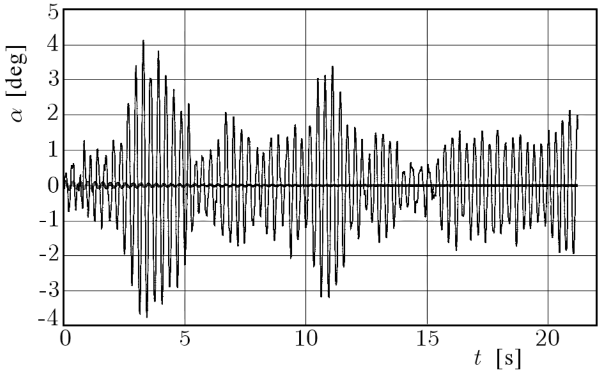

Fig. 12. Angle of attack of the bomb

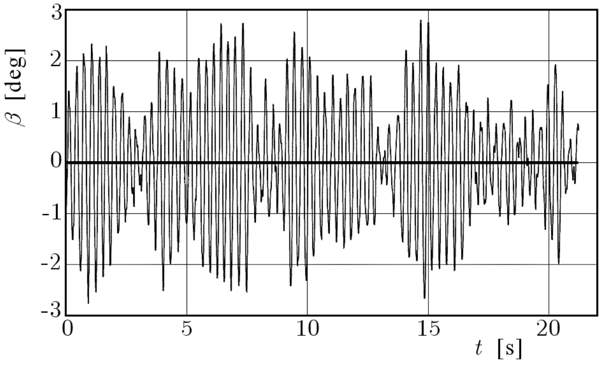

Fig. 13. Angle of sideslip of the bomb

The flightpath angle (Fig. 14) under 'no-wind-effect' conditions (i.e. with no turbulence) decreases almost linearly. It means that the bomb follows the trajectory very much like a parabola (the first-order derivative equals to the flightpath angle, which – at the angle of attack close to zero – equals to the bomb angle of pitch). With the wind/turbulence present, the bomb angle of pitch decreases and at the same time oscillates against the 'no-wind-effect'-related plot.

Wind-effected is also the oscillation of the bomb angle of yaw (Fig. 15), which in turn results in the bomb flightpath deviation from the vertical plane, in which the trajectory for the 'no-wind-effect' condition is to be found. This

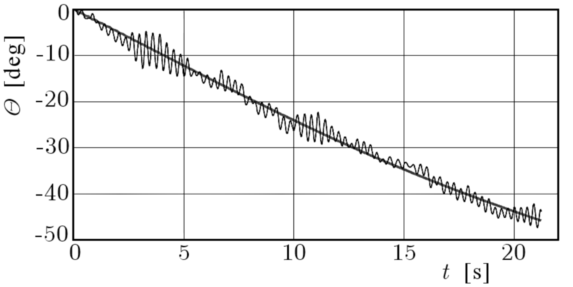

Fig. 14. Angle of pitch of the bomb

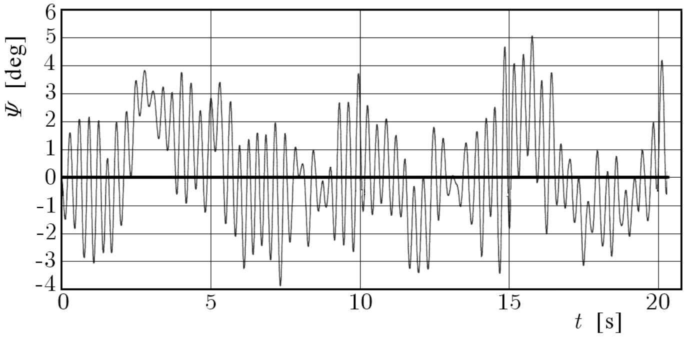

Fig. 15. Angle of yaw of the bomb

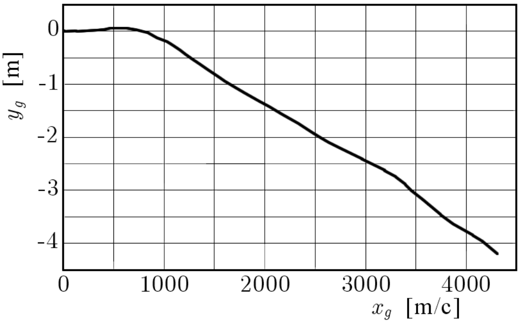

Fig. 16. Bomb flightpath projected on the horizontal plane Ox_gy_g

deviation is shown in Fig. 16. It is evident that the bomb deviates to the left. The comparison of trajectories projected onto a vertical plane (Fig. 17) proves impossible since the changes in coordinates are too large.

Calculations show that the wind considerably affects the accuracy of the bomb release. It is evident from Fig. 18, which shows the point of the bomb fall

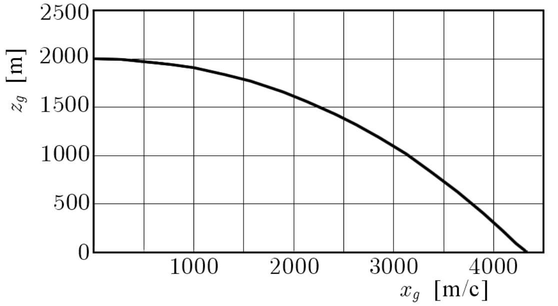

Fig. 17. Bomb flightpath projected on the vertical plane Ox_gz_g

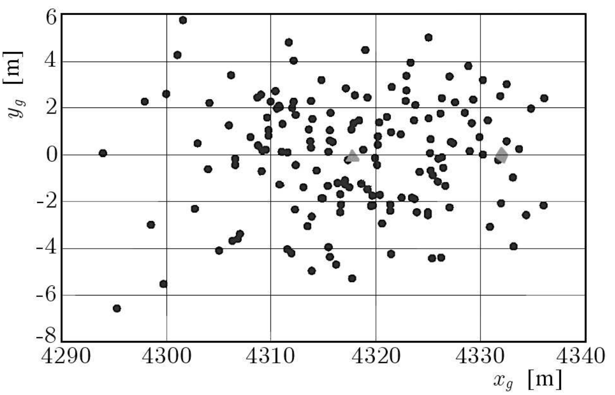

Fig. 18. Points of the bomb fall on the horizontal plane Ox_gy_g

on the Earth's surface. In this figure, the point showing the bomb fall under 'no-wind-effect' conditions is marked with a square. Coordinates of this point are (4332;0). The majority of points that represent the drop of the bomb with the wind are much closer to the initial point. It means that the wind reduces the range of the bomb drop. The reason is that the mean aerodynamic-drag coefficient increases because of the increased angles of attack (Fig. 19).

These points of the bomb fall compose then a set subjected to analysis. With average values of coordinates of these points calculated, the position of the so-called "averaged point of the bomb fall" could be determined. It is marked in Fig. 18 with a triangle. The coordinates thereof are $(4317.66; -0.07)$. This point is situated 14.34 m closer than that reached under 'no-wind-effect' conditions. This proves that the range of the bomb drop is reduced when the effect of the wind is taken into account.

Variances have also been determined and basing thereon, standard deviations of the point-of-fall coordinates have been found. These are as follows:

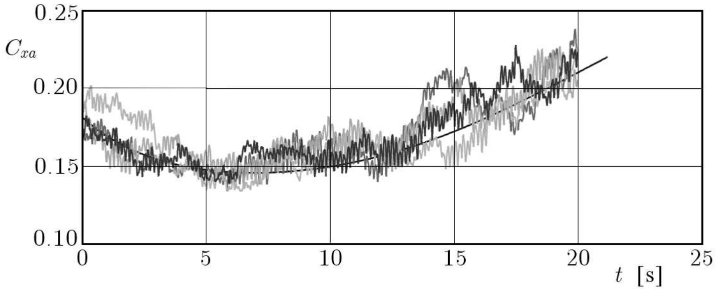

Fig. 19. Aerodynamic drag coefficient during the bomb drop

9.145 m for the x_g coordinate, and 2.436 m for the y_g coordinate. It means that if there is a stochastic wind acting on the bomb,then the points of fall are situated within an ellipse with the semimajor axis along the Ox_g axis.

5. Conclusions

The conducted analysis has proved that the effect of wind on the accuracy of bomb release is essential and should be taken into account when planning the bombing. The stochastic nature of atmospheric turbulence results in a random distribution of points of fall. Gusts increase angles of attack during the bomb flight and result in reduction of the range of drop. Further studies will cover the question of determination of final parameters of the bomb flight in the wind, depending on the bomb release altitude, angle of release, weight, and parameters that describe the wind field.

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Wpływ turbulencji atmosferycznych na zrzut bomby

Streszczenie

W pracy przedstawiono model dynamiki zrzutu bomby. Pokazano wpływ pola wiatru na szybowanie bomby. Strukturę wiatru opisano, uwzględniając jego stochastyczny charakter. Pokazano wyniki symulacji numerycznej zrzutu oraz poddano je analizie.

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