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ACOUSTIC INTENSITY VECTOR GENERATED BY VIBRATING SET OF SMALL AREAS WITH RANDOM AMPLITUDES

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The paper presents a generalisation of the hybrid method of estimation of sound radiated by vibrating surfaces, formulated previously for the deterministic case of random vibrations. The analysis is made for random amplitudes of vibrations in a narrow frequency band. The results show complexity of the analysis in comparison with the deterministic case. Therefore, the method does not seem to be efficient, like the deterministic one, in engineering applications.

Key words: acoustic radiation, random vibrations, sound intensity vector, structural noise

1. Introduction

For several years, the author have been dealing with the problem of acoustic radiation of vibtrating plates and shallow shells in deterministic and random cases (Kozień and Nizioł, 2005, 2006, 2007; Kozień and Saltarski, 2007, Nizioł and Kozień, 2000, 2001).

A combination of the method of analysis of acoustic radiation by a harmonically vibrating small plane element (Kwiek, 1968) and the method of estimation of the sound intensity vector by knowledge of their amplitudes (Mann *et al.*, 1987) result in a new method of estimation, proposed by the author and called the hybrid method (Kozień, 2005, 2006). The method was previously formulated for a deterministic case of structural vibrations.

In the presented paper, a generalisation of the method for the case of randomly vibrating system of small elements is discussed. The assumption is that the amplitudes of vibrations of the elements are random processes described by a probability density function.

Due to assumptions, the analysis is valid for a relatively narrow frequency band.

2. Theoretical background of the hybrid method in the deterministic case

The analysis is provided for a monochromatic wave for the given frequency ω and the analysis is performed in the complex space.

Analysis of an acoustic field generated by vibrating surfaces is based on determination of the resultant acoustic intensity vector $\boldsymbol{I}$ in a chosen control point in the acoustic volume. The vibrating area is previously divided into the sub-areas.

Each vibrating element is the source of radiated sound for the given frequency ω. The assumption of the method is that every sub-area is a small surface element. The "smallness" of the element is interpreted here with respect to the wavelength (associated with the wave frequency), in accordance with (2.1), where r_0 is the radius of the sub-domain or the greatest distance between the sub-domain centre and its boundary points (Kwiek, 1968)

$$r_0 \ll \frac{\lambda}{4\pi} \tag{2.1}$$

The well-known relationships between the angular frequency ω, wavenumber k, wavelength λ and speed of sound in an acoustic medium c are given as

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \tag{2.2}$$

The analysis is described in the Cartesian co-ordinate system $Oxyz$. The position of the chosen control point $P(x,y,z)$ is given by the vector $\boldsymbol{R} = \overline{OP}$, and the position of the center of the i-th sub-area $Q_i(x_i,y_i,z_i)$ by the vector $\boldsymbol{\rho}_i = \overline{OQ_i}$ (Fig. 1). The following relationship between the mentioned vectors is valid

$$\boldsymbol{r}_i = \boldsymbol{R} - \boldsymbol{\rho}_i \tag{2.3}$$

Hence, the distance between the center of the i-th sub-area and the control point P can be obtained basing on the relationship

$$r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} \tag{2.4}$$

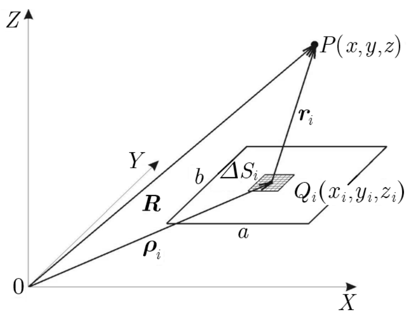

Fig. 1. Geometry of the sub-area and the control point P

For such a case, the acoustic pressure and the partial velocity vector generated by the i-th sub-area in the control point $P(x,y,z)$, can be obtained basing on the following formulas (Kwiek, 1968)

$$\begin{aligned} p_i &= p_i(\boldsymbol{r}_i,\boldsymbol{R}) = -\frac{1}{2\pi}\Delta S_i\omega^2\rho_0\frac{A_i}{r_i}\mathrm{e}^{\mathrm{i}(\omega t-kr_i)} \\ \boldsymbol{v}_i &= \boldsymbol{v}_i(\boldsymbol{r}_i,\boldsymbol{R}) = \boldsymbol{v}_{r_i}(\boldsymbol{r}_i,\boldsymbol{R}) = A_i\Delta S_i\Big(-\frac{\omega^2}{2\pi c}\frac{1}{r_i}+\mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big)\mathrm{e}^{\mathrm{i}(\omega t-kr_i)}\frac{\boldsymbol{r}_i}{r_i} \end{aligned} \tag{2.5}$$

where ΔS_i is the area of the i-th sub-area, and A_i is the amplitude of its vibrations.

Formula $(2.5)_2$ can be written in the Cartesian co-ordinate system in form of three scalar relationships on the components of the partial velocity vector $\boldsymbol{v}$ which is parallel to the vector $\boldsymbol{r}_i$ as in the following

$$\begin{aligned} v_{i_x}(x_i,y_i,z_i,x,y,z) &= A_i\Delta S_i\Big(-\frac{\omega^2}{2\pi c}\frac{1}{r_i}+\mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big)\frac{x-x_i}{r_i}\mathrm{e}^{\mathrm{i}(\omega t-kr_i)} \\ v_{i_y}(x_i,y_i,z_i,x,y,z) &= A_i\Delta S_i\Big(-\frac{\omega^2}{2\pi c}\frac{1}{r_i}+\mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big)\frac{y-y_i}{r_i}\mathrm{e}^{\mathrm{i}(\omega t-kr_i)} \\ v_{i_z}(x_i,y_i,z_i,x,y,z) &= A_i\Delta S_i\Big(-\frac{\omega^2}{2\pi c}\frac{1}{r_i}+\mathrm{i}\frac{\omega}{2\pi}\frac{1}{r_i^2}\Big)\frac{z-z_i}{r_i}\mathrm{e}^{\mathrm{i}(\omega t-kr_i)} \end{aligned} \tag{2.6}$$

The next problem is the idea of superposition of the components of pressures and partial valocities comming from the set of sub-areas in the resultant form in the analysed point P. The following relationship, formulated previously by Mann *et al.* (1987) for the set of N-point acoustic sources, is applied futher

$$\boldsymbol{I} = \frac{1}{2}\Big(\sum_{i=1}^{N}p_i\Big)\Big(\sum_{j=1}^{N}\boldsymbol{v}_j^*\Big) \tag{2.7}$$

After suitable manipulations, based on relationships $(2.5)_1$, (2.6) and (2.7), are obtained formulas for the components of the real and imaginary parts of the resultant complex acoustic intensity vector in the chosen control point

$$\begin{aligned}
\mathrm{Re}\,(I_x) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} x_j c_j A_j\Big) + \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} x_j d_j A_j\Big)\Big]\\
\mathrm{Re}\,(I_y) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} y_j c_j A_j\Big) + \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} y_j d_j A_j\Big)\Big]\\
\mathrm{Re}\,(I_z) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} z_j c_j A_j\Big) + \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} z_j d_j A_j\Big)\Big]
\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}
\mathrm{Im}\,(I_x) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} x_j d_j A_j\Big) - \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} x_j c_j A_j\Big)\Big]\\
\mathrm{Im}\,(I_y) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} y_j d_j A_j\Big) - \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} y_j c_j A_j\Big)\Big]\\
\mathrm{Im}\,(I_z) &= K\Big[\Big(\sum_{i=1}^{n} a_i A_i\Big)\Big(\sum_{j=1}^{n} z_j d_j A_j\Big) - \Big(\sum_{i=1}^{n} b_i A_i\Big)\Big(\sum_{j=1}^{n} z_j c_j A_j\Big)\Big]
\end{aligned} \tag{2.9}$$

The parameters K, a_i, b_i, c_i and d_i, $i = 1, \ldots, N$ standing in the above formulas are defined as

$$\begin{aligned}
K &= \frac{1}{8\pi^2}\rho_0\omega^3 \qquad a_i = \frac{\varDelta S_i}{r_i}\cos(kr_i) \qquad b_i = \frac{\varDelta S_i}{r_i}\sin(kr_i)\\
c_i &= \frac{\varDelta S_i}{r_i^2}\Big[k\cos(kr_i) - \frac{1}{r_i}\sin(kr_i)\Big]\\
d_i &= \frac{\varDelta S_i}{r_i^2}\Big[k\sin(kr_i) + \frac{1}{r_i}\cos(kr_i)\Big]
\end{aligned} \tag{2.10}$$

Basing on the knowledge of the amplitudes of vibrations for each subarea, it is possible to obtain the resultant complex acoustic intensity vector for a given frequency. Moreover, the ratio between values of the real and imaginary parts of the complex vector gives the information of the type of acoustic field in the chosen point (nearfield, farfield). The method in the presented form does not take into account the effects of absorption or reflection of acoustic waves from any surfaces.

The knowledge of the acoustic intensity vector is usually enough to make acoustic analysis, particularly in energy forms. But if an values of the acoustic pressure in the chosen point are the most important, it can be obtained in an approximate way based on the assumption of the plane acoustic wave in the control point area, in the form

$$I = \frac{p^2}{\rho_0 c} \tag{2.11}$$

The other way of estimation of the acoustic pressure is application of the formulas given in ISO 11205 (ISO, 2003) in the form

$$L_p = 10 \log \sqrt{\left(10^{\frac{L_{I_x}}{10}}\right)^2 + \left(10^{\frac{L_{I_y}}{10}}\right)^2 + \left(10^{\frac{L_{I_z}}{10}}\right)^2} \tag{2.12}$$

where L_{I_x}, L_{I_y} and L_{I_z} are levels $[dB]$ of the components of the acoustic intensity vector in the x, y and z directions, respectively.

3. Hybrid method for random amplitudes of vibrations

3.1. General formulation

Let us assume that the amplitude of vibrations for the i-th sub-area A_i is a random process, usually with the zero middle value. Random processes are defined by the probability density function $f_i(z_i)$. Moreover, let us reduce the analysis down to the narrow frequency band, so that the description of propagation of the acoustic wave with a given central frequency band is valid.

Hence, in equations $(2.5)_2$ and (2.6) instead of only deterministic amplitudes A_i, the probability density functions $f_i(z_i)$ are the input. Then the relations are put into formula (2.7) which is multiplied with the same one and integrated over the whole appropriate probability spaces. As a result, dispersion of components of the acoustic intensity vector is obtained $(3.1)_1$ as function of the statistical moment of the fourth order between random variables (amplitudes of the transverse displacement) A_i, A_j, A_k and A_l with the probability density functions f_i, f_j, f_k and f_l – $\mathrm{m}[A_i, A_j, A_k, A_l]$ $(3.1)_2$

$$\begin{aligned}
\boldsymbol{\sigma^2}_{\boldsymbol{I}} &= \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \frac{1}{2}\Big[\sum_{i=1}^{N} p_i(z_i)\Big]\Big[\sum_{j=1}^{N} \boldsymbol{v}_j^*(z_j)\Big]\frac{1}{2}\Big[\sum_{k=1}^{N} p_k(z_k)\Big]\Big[\sum_{l=1}^{N} \boldsymbol{v}_l^*(z_l)\Big] \cdot \\
&\quad \cdot dz_i dz_j dz_k dz_l \\
\mathrm{m}[A_i, A_j, A_k, A_l] &= \\
&= \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} z_i z_j z_k z_l f_i(z_i, A_i) f_j(z_j, A_j) f_k(z_k, A_k) f_l(z_l, A_l)\, dz_i dz_j dz_k dz_l
\end{aligned} \tag{3.1}$$

The final explicite formulas are relatively complicated, and the level of complication is nonlinearly growing with the increasing number of sub-areas. For example, for two sub-areas, the formulas for determination of the x-component of the real and imaginary parts of the complex acoustic vector have the forms

$$
\begin{aligned}
\sigma^2_{\mathrm{Re}\,(I_x)}(x,y,z) = K^2\Big\{&\mathrm{m}[A_1,A_1,A_1,A_1](a_1^2c_1^2\widetilde{x}_1^2 + b_1^2d_1^2\widetilde{x}_1^2 - a_1^2d_1^2\widetilde{x}_1^2 - b_1^2c_1^2\widetilde{x}_1^2 + \\
&+4a_1b_1c_1d_1\widetilde{x}_1^2) + \mathrm{m}[A_1,A_1,A_1,A_2](2a_1^2c_1c_2\widetilde{x}_1\widetilde{x}_2 + 2a_1a_2c_1^2\widetilde{x}_1^2 + \\
&+4a_1b_1c_1d_2\widetilde{x}_1\widetilde{x}_2 + 4a_1b_2c_1d_1\widetilde{x}_1^2 + 4a_1b_1c_2d_1\widetilde{x}_1\widetilde{x}_2 + 4a_2b_1c_1d_1\widetilde{x}_1^2 + \\
&+2b_1^2d_1d_2\widetilde{x}_1\widetilde{x}_2 + 2b_1b_2d_1^2\widetilde{x}_1^2 - 2a_1^2d_1d_2\widetilde{x}_1\widetilde{x}_2 - 2a_1a_2d_1^2\widetilde{x}_1^2 - 2b_1^2c_1c_2\widetilde{x}_1\widetilde{x}_2 + \\
&-2b_1b_2c_1^2\widetilde{x}_1^2) + \mathrm{m}[A_1,A_1,A_2,A_2](4a_1a_2c_1c_2\widetilde{x}_1\widetilde{x}_2 + 4a_1b_2c_1d_2\widetilde{x}_1\widetilde{x}_2 + \\
&+a_1^2c_2^2d_2\widetilde{x}_2^2 + 4a_1b_1c_2d_2\widetilde{x}_2^2 + 4a_1b_2c_2d_1\widetilde{x}_1\widetilde{x}_2 + a_2^2c_1^2\widetilde{x}_1^2 + 4a_2b_1c_1d_2\widetilde{x}_1\widetilde{x}_2 + \\
&+4a_2b_2c_1d_1\widetilde{x}_1^2 + 4a_2b_1c_2d_1\widetilde{x}_1\widetilde{x}_2 + 4b_1b_2d_1d_2\widetilde{x}_1\widetilde{x}_2 + b_1^2d_2^2\widetilde{x}_2^2 + b_2^2d_1^2\widetilde{x}_1^2 \\
&-4a_1a_2d_1d_2\widetilde{x}_1\widetilde{x}_2 - a_1^2d_2^2\widetilde{x}_2^2 - a_2^2d_1^2\widetilde{x}_1^2 - 4b_1c_1c_2b_2\widetilde{x}_1\widetilde{x}_2 - b_1^2c_2^2\widetilde{x}_2^2 - b_2^2c_1^2\widetilde{x}_1^2) + \\
&+\mathrm{m}[A_1,A_2,A_2,A_2](2a_1a_2c_2^2\widetilde{x}_2^2 + 4a_1b_2c_2d_2\widetilde{x}_2^2 + 2a_2^2c_1c_2\widetilde{x}_1\widetilde{x}_2 + \\
&+4a_2b_2c_1d_2\widetilde{x}_1\widetilde{x}_2 + 4a_2b_1c_2d_2\widetilde{x}_2^2 + 4a_2b_2c_2d_1\widetilde{x}_1\widetilde{x}_2 + 2b_1b_2d_2^2\widetilde{x}_2^2 + \\
&+2b_2^2d_1d_2\widetilde{x}_1\widetilde{x}_2 - 2a_1a_2d_2^2\widetilde{x}_2^2 - 2a_2^2d_1d_2\widetilde{x}_1\widetilde{x}_2 - 2b_1b_2c_2^2\widetilde{x}_2^2 - 2b_2^2c_1c_2\widetilde{x}_1\widetilde{x}_2) + \\
&+\mathrm{m}[A_2,A_2,A_2,A_2](a_2^2c_2^2\widetilde{x}_2^2 + b_2^2d_2^2\widetilde{x}_2^2 - a_2^2d_2^2\widetilde{x}_2^2 - b_2^2c_2^2\widetilde{x}_2^2 + 4a_2b_2c_2d_2\widetilde{x}_2^2)\Big\}
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
\sigma^2_{\mathrm{Im}\,(I_x)}(x,y,z) = K^2\Big\{&\mathrm{m}[A_1,A_1,A_1,A_1](-2a_1b_1c_1^2\widetilde{x}_1^2 + 2a_1^2c_1d_1\widetilde{x}_1^2 + \\
&+2a_1b_1d_1^2\widetilde{x}_1^2 - 2b_1^2c_1d_1\widetilde{x}_1^2) + \mathrm{m}[A_1,A_1,A_1,A_2](2a_1^2c_1d_2\widetilde{x}_1\widetilde{x}_2 + \\
&+4a_1c_1d_1a_2\widetilde{x}_1^2 + 4a_1b_1d_1d_2\widetilde{x}_1\widetilde{x}_2 + 2b_1d_1^2a_2\widetilde{x}_1^2 + 2a_1^2d_1c_2\widetilde{x}_1\widetilde{x}_2 + \\
&-4a_1b_1c_1c_2\widetilde{x}_1\widetilde{x}_2 - 2b_1c_1^2a_2\widetilde{x}_1^2 - 2a_1c_1^2b_2\widetilde{x}_1^2 - 2b_1^2d_1c_2\widetilde{x}_1\widetilde{x}_2 - 4b_1c_1d_1b_2\widetilde{x}_1^2 + \\
&-2b_1^2c_1d_2\widetilde{x}_1\widetilde{x}_2 + 2a_1d_1^2b_2\widetilde{x}_1^2) + \mathrm{m}[A_1,A_1,A_2,A_2](4a_1c_1a_2d_2\widetilde{x}_1\widetilde{x}_2 + \\
&+4a_1d_1a_2c_2\widetilde{x}_1\widetilde{x}_2 - 4b_1c_1a_2c_2\widetilde{x}_1\widetilde{x}_2 + 4b_1d_1a_2d_2\widetilde{x}_1\widetilde{x}_2 + 4a_1d_1b_2d_2\widetilde{x}_1\widetilde{x}_2 + \\
&-4b_1c_1b_2d_2\widetilde{x}_1\widetilde{x}_2 - 4a_1c_1b_2c_2\widetilde{x}_1\widetilde{x}_2 - 4b_1d_1b_2c_2\widetilde{x}_1\widetilde{x}_2 + 2a_1^2c_2d_2\widetilde{x}_2^2 + \\
&+2c_1d_1a_2^2\widetilde{x}_1^2 - 2a_1b_1c_2^2\widetilde{x}_2^2 - 2c_1^2a_2b_2\widetilde{x}_1^2 + 2a_1b_1d_2^2\widetilde{x}_2^2 + 2d_1^2a_2b_2\widetilde{x}_1^2 + \\
&-2b_1^2c_2d_2\widetilde{x}_2^2 - 2c_1d_1b_2^2\widetilde{x}_1^2 - 2d_1b_2^2c_2\widetilde{x}_1\widetilde{x}_2) + \mathrm{m}[A_1,A_2,A_2,A_2](4a_1a_2c_2d_2\widetilde{x}_2^2 + \\
&+2d_1a_2^2c_2\widetilde{x}_1\widetilde{x}_2 + 2a_1b_2d_2^2\widetilde{x}_2^2 + 4d_1a_2b_2d_2\widetilde{x}_1\widetilde{x}_2 + 2c_1a_2^2d_2\widetilde{x}_1\widetilde{x}_2 - 2b_1a_2c_2^2\widetilde{x}_2^2 + \\
&-4c_1a_2b_2c_2\widetilde{x}_1\widetilde{x}_2 - 4b_1b_2c_2d_2\widetilde{x}_2^2 - 2c_1b_2^2d_2\widetilde{x}_1\widetilde{x}_2 - 2a_1b_2c_2^2\widetilde{x}_2^2 + 2b_1a_2d_2^2\widetilde{x}_2^2) + \\
&+\mathrm{m}[A_2,A_2,A_2,A_2](a_2^2c_2d_2\widetilde{x}_2^2 + 2a_2b_2d_2^2\widetilde{x}_2^2 - 2a_2b_2c_2^2\widetilde{x}_2^2 - 2b_2^2c_2d_2\widetilde{x}_2^2)\Big\}
\end{aligned}
$$

where $\widetilde{x}_1 = x - x_1$, $\widetilde{x}_2 = x - x_2$.

The probability density function for realistic cases can be obtained by the assumption of a random process or as a result of analysis of random vibrations of structures, e.g. by the finite element method.

3.2. Formulation by probability density functions

In this attempt, probability density functions of a random process which describes amplitudes of vibrations of the sub-areas are assumed. For example, if the process is a normal (Gaussian) one with the zero middle value and dispersion $\sigma_i^2 = A_i^2$, the probbility density functions have the form

$$f_i(z_i) = \frac{1}{A_i\sqrt{2\pi}} \mathrm{e}^{-\frac{z_i^2}{2A_i^2}} \tag{3.3}$$

Then the whole formulas should be integrated over the probability spaces.

3.3. FEM analysis of random vibrations

For realistic cases, the finite element method is often applied to analysis of vibrations of a randomly excited structure. As a result, some probability functions, such as variance or covariance of amplitudes for each finite surface element are obtained. Then, based on these functions, the analysis of radiation is performed having in mind the discusssed general formulation.

4. Application of the method for FEM analysis of random vibrations

Let us consider randomly excited vibrations of a square steel plate with thickness of 2 mm and edge length of 1 m. The excitation are distributed external surface loadings of a random type with a constant power spectral density of 0.1 N/Hz. The analysis is preformed for the narrow frequency band arround the basic natural frequency 9.86 Hz. The analysis of random vibrations is done by finite element package *Ansys*. The resultant random functions are a base for further analysis in the above described way. In the analysis, the dispersion of real and imaginary parts of the acoustic intensity vector component perpendicular to the plate in chosen control points is calculed.

Based on these values, levels of dispersion of real and imaginary parts of the acoustic pressure are estimated on the assumpion of the plane acoustic

Table 1. Values of dispersion levels of the acoustic pressure (real and imaginary parts)

Distance h [m]	$L_{\sigma^2_{\mathrm{Re}\,(p)}}$ [dB]	$L_{\sigma^2_{\mathrm{Im}\,(p)}}$ [dB]
1.0	128.0	132.7
2.0	107.8	108.7
5.0	80.5	68.9
10.0	59.7	57.4

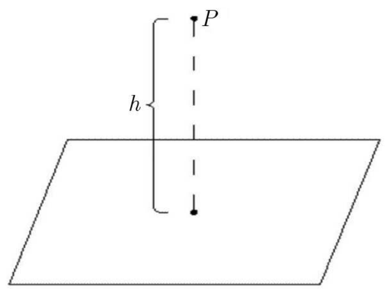

Fig. 2. Position of the control point P

wave (2.11). These values are shown in Table 1 for a few control points whose positions are schematically shown in Fig. 2

The obtained values are realistic and give good interpretation of the acoustic near- and far-field too. Unfortunately, the applied numerical procedures of the hybrid method in random formulation are rather complicated and not easy algorithmised.

5. Conclusions

The presented analysis reveals the theoretical background and some numerical simulations of the generalisation of the previously formulated hybrid method for the case of random vibrations of plates. The results show the possibility of application of the method, but the obtained formulas are complicated and they are not easily alghorithmised. The main idea of the hybrid method for deterministic cases is the possibility to easily estimate the acoustic intensity or pressure with no need to model the acoustic medium. Unfortunately, this idea does not hold in the presented random formulation.

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Wektor natężenia akustycznego generowany przez układ małych płaskich elementów drgających z losowymi amplitudami

Streszczenie

W artykule omówiono rozszerzenie metody hybrydowej oszacowania dźwięku promieniowanego przez drgające powierzchnie, sformułowanej pierwotnie dla przypadku drgań deterministycznych, na przypadek drgań losowych. Rozważono przypadek drgań z losowo zmienną amplitudą w wąskim paśmie częstotliwości. Rezultaty analiz pokazują złożoność uzyskanych formuł w stosunku do zagadnień deterministycznych. Dlatego też wydaje się, że metoda ta w prezentowanym podejściu nie jest tak użyteczna w zastosowaniach inżynierskich, jak to ma miejsce w sformułowaniu deterministycznym.

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