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SENSITIVITY ANALYSIS OF PLANAR MECHANISMS

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When designing a new mechanism, the designer has to know its kinematic parameters: position, velocity, acceleration, the number of design options, specific positions, motion ranges of particular links, effects of manufacturing tolerances of links and clearances in kinematic pairs on functioning of the mechanism. The latter parameter is associated with sensitivity of the mechanism, understood as its ability to respond to even minimal variations of the driving link position. The source of information about these parameters becomes the objective function obtained through the application of the modification method. This study explores the effects of clearances in kinematic pairs and structure of the mechanism on its sensitivity.

Key words: kinematics, planar mechanisms, sensitivity

1. Introduction

Sensitivity (kinematic efficiency) of a mechanism is understood as its ability to respond to even minimal variations of the driving link position. Sensitivity thus defined is affected by several factors (Młynarski and Romaniak, 2002).

There are certain aspects of sensitivity to be considered:

- structural sensitivity-determining factors that include structure of a mechanism, coordinates of external kinematic pairs, links lengths,
- technical (constructional, technological) sensitivity affected by rigidity or flexibility of links, clearances in kinematic pairs and manufacturing tolerances of links,
- operational sensitivity, depending on service conditions, load types and wearing of the kinematic pairs.

This study investigates the structural sensitivity of mechanisms and certain aspects of constructional sensitivity, focusing on the effects of structure and clearances in kinematic pairs. The modification method is presented (Młynarski and Romaniak, 2001), which finally yields the objective function underlying the sensitivity analysis of planar mechanisms.

2. Modification method

The modification method is applied to kinematic analysis of high class mechanisms, enabling us to determine positions, velocity and accelerations of particular links and kinematic pairs. The method uses transformation of the kinematic chain such that the high class unit should become a mechanism of the II class. That is achieved by disconnecting the unit at one or more points, which involves one or more decision variables. Further analysis is performed to check whether the links can be then reconnected. This relationship for the mechanism disconnected by the link l_k is given by the objective function of the form

$$f_c = \varDelta l_k = l_k - \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \to 0 \tag{2.1}$$

where (x_i, y_i), (x_{i+1}, y_{i+1}) are coordinates of kinematic pairs of the link l_k. Zeroing of the objective function implies such a value of the decision variable for which the mechanism can be reconnected.

Modification of a high class mechanism yields a mechanism of the II class, incorporating k groups of the II class. Depending on the number of k, k iterative steps are required to obtain objective functions, the number of which is 2^k.

Figure 1a shows a mechanism of the III class analysed using the modification method. A kinematic unit of the III class, series 3 is extracted and the modification procedure is applied accordingly. When all external pairs are connected to the base, the angle α_2 is chosen as the decision variable (the driving link) and the mechanism is disconnected by excluding link 5 (Fig. 1b).

For the thus obtained mechanism of the II class, comprising a single kinematic unit of the II class, we get the coordinates of kinematic pairs C, D, F and angles α_3, α_4 in accordance with the formula

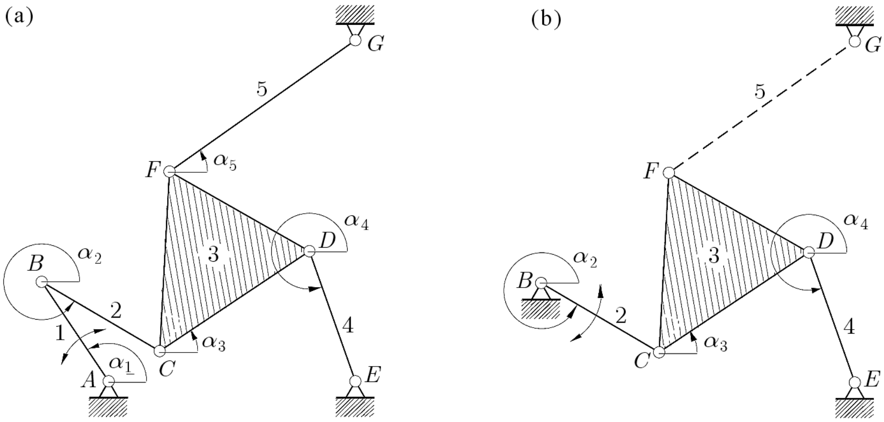

Fig. 1. Modified kinematic unit of the III class, series 3 (b), derived from the mechanism of the III class (a)

$$
\begin{aligned}
&x_C = x_B + l_2 \cos\alpha_2 \qquad\qquad y_C = y_B + l_2 \sin\alpha_2 \\
&x_C + l_{CD} \cos\alpha_3 + l_4 \cos\alpha_4 - x_E = 0 \\
&y_C + l_{CD} \sin\alpha_3 + l_4 \sin\alpha_4 - y_E = 0 \\
&x_F = x_C + l_{CF} \cos(\alpha_3 + \kappa_{3C}) \\
&y_F = y_C + l_{CF} \sin(\alpha_3 + \kappa_{3C})
\end{aligned}
\tag{2.2}
$$

Two objective functions are obtained, in accordance with the formula

$$
f_{5c} = l_5 - \sqrt{(x_G - x_F)^2 + (y_G - y_F)^2} \tag{2.3}
$$

The following data: $x_A = 40$, $y_A = 50$, $x_E = 170$, $y_E = 50$, $x_G = 170$, $y_G = 230$, $l_1 = 63.25$, $l_2 = 72.11$, $l_{CD} = 94.87$, $l_{CF} = 94.87$, $l_4 = 72.8$, $l_5 = 120$, $\kappa_{3C} = 0.927\,\mathrm{rd}$, yield a plot of the objective function graphed in Fig. 2.

Zeros of the objective functions imply those values of the decision variable α_2 for which the mechanism can be reconnected. Two design options are obtained for $\alpha_2 = -0.5317\,\mathrm{rd}$ and $\alpha_2 = 0.54583\,\mathrm{rd}$ shown in Fig. 3.

3. Sensitivity of planar mechanisms

The first motion of the driving links eliminates the clearances in kinematic pairs. During this movement the position of the driving link does not lead to any

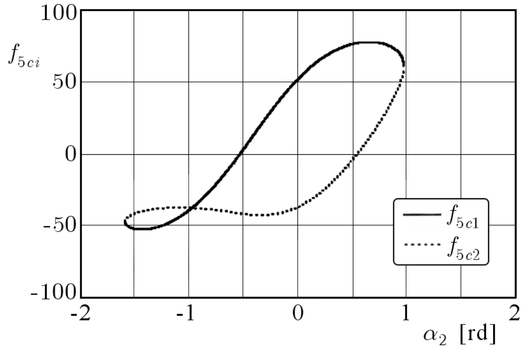

Fig. 2. Plot of an objective function

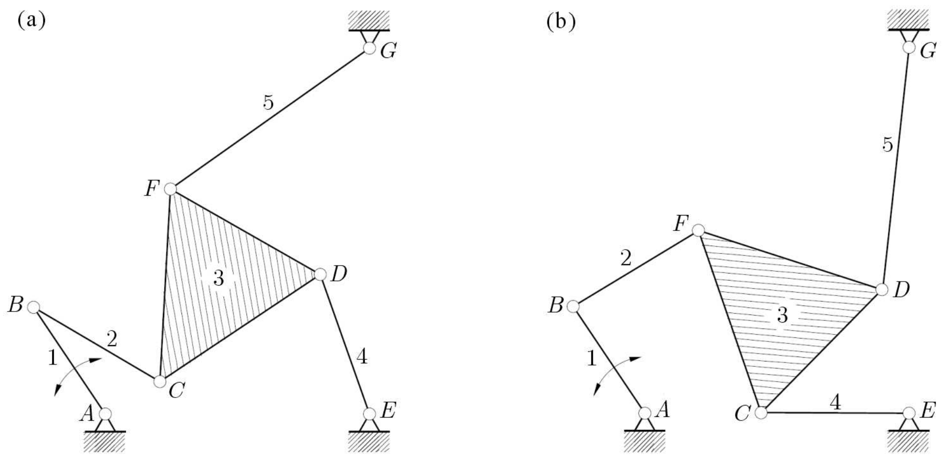

Fig. 3. Third class mechanisms obtained for $\alpha_2 = -0.5317\,\mathrm{rd}$ (a) and $\alpha_2 = 0.54583\,\mathrm{rd}$ (b)

variations of the driven link position. Sensitivity analysis uses the sensitivity factor expressed as (Romaniak, 1998)

$$\mu = 1 - \frac{\Delta\alpha_i}{\Delta} = 1 - \frac{b}{\Delta}\frac{\sqrt{1+\Big(\frac{d}{d\alpha_i}f_c(\alpha_i)\Big)^2}}{\frac{d}{d\alpha_i}f_c(\alpha_i)} \tag{3.1}$$

where $\Delta\alpha_i$ is the distance between the corresponding zeros of the extreme objective functions, Δ is the operating range of the driven link in a given design option, b is the distance between the extreme objective functions and $f_c(\alpha_i)$ is the objective function corresponding to nominal dimensions.

The following denotation is used

$$\beta_k = \frac{b}{\varDelta} \qquad\qquad \beta_s = \frac{\sqrt{1+(y')^2}}{y'} \tag{3.2}$$

where $y' = df_c(\alpha_i)/d\alpha_i$.

There are several determinants of β_k, including the design, manufacturing technology, mode of operation, wearing of kinematic pairs. This study investigates only the prognosticated constructional clearances. The factor β_k also depends on the structure of the mechanism. Furthermore, the condition is imposed

$$0 \leqslant \beta_k\beta_s < 1 \tag{3.3}$$

to preclude situations when the objective function is tangent to the abscissa axis.

Because of the clearance between the shaft and the bearing bushing, the shaft might occupy various positions in the opening. The extreme shaft positions correspond to the objective function forming a band. Actually, the shaft assumes the middle position and the real objective function is contained inside the band. The plot of the function is ambiguous, mainly because of the acting forces and the presence of friction. If initially the shaft occupied the minimum position, and motion was performed such that the shaft should assume another extreme position due to eliminated clearances, then the insensitivity range should stretch from one to another extreme function. That is the greatest insensitivity level to be achieved by the given mechanism. The quotient present in the formula becomes the measure of maximal insensitivity of the mechanism, depending on the type of fit applied in kinematic pairs. Actually, it expresses the minimal sensitivity of the mechanism, which in fact is greater.

The objective function was analysed for two design options of the third class mechanism shown in Fig. 3, taking into account the clearances in kinematic pairs (8H8/f9).

In the case of design option I (Fig. 3a), the motion range of the driven link 5 is $\varDelta = 0.6529\,\mathrm{rd}$ and for option II (Fig. 3b) is $\varDelta = 0.4914\,\mathrm{rd}$. For the angle α_1 varying by $0.2\,\mathrm{rd}$, the following parameters were computed: angle α_5, derivatives of the objective function at its zeros, the width of the band of the objective function $\varDelta\alpha_5$, sensitivity factor μ. Results are summarised in Table 1 (the value $f'_{c1}(\alpha_5)$ is denoted by y' in equation (3.2)).

Tests were performed for those values of the angle α_1 for which all the objective functions have zeros. Taking into account the clearances in kinematic pairs, the motion range will be reduced. If one regarded those values of the

Table 1. Angles α_1, α_5, derivative of the objective function, width of the band of the objective function $\Delta\alpha_5$, sensitivity factor μ

Design option I					
No.	α_1	α_5	$f'_{c1}(\alpha_5)$	$\Delta\alpha_5$	μ
1	5.23622	3.7992	−12.517	0.01986	0.98318
2	5.4	3.9745	−80.7428	0.00065	0.99945
3	5.6	4.113	−95.9276	0.00013	0.99989
4	5.8	4.2249	−94.4389	0.00065	0.99945
5	6	4.3015	−89.1443	0.0011	0.99907
6	6.2	4.3274	−92.3389	0.0013	0.9989
7	0	4.3226	−97.1505	0.00128	0.99891
8	0.2	4.2833	−112.822	0.00113	0.99904
9	0.4	4.2182	−127.76	0.00095	0.99919
10	0.6	4.1391	−138.774	0.00082	0.9993
11	0.8	4.0540	−147.212	0.00075	0.99937
12	1	3.9709	−156.376	0.00073	0.99938
13	1.2	3.8964	−169.766	0.00077	0.99935
14	1.4	3.8348	−190.257	0.00084	0.99929
15	1.6	3.7871	−219.938	0.00095	0.9992
16	1.8	3.7515	−260.472	0.00107	0.9991
17	2	3.7255	−314.031	0.00119	0.99899
18	2.2	3.70663	−385.245	0.00133	0.99888
19	2.4	3.693	−486.257	0.00147	0.99875
Design option II					
No.	α_1	α_5	$f'_{c1}(\alpha_5)$	$\Delta\alpha_5$	μ
1	−0.6185	4.9647	10.14966	0.0269	0.94522
2	−0.6	4.924	29.3017	0.0063	0.9872
3	−0.4	4.81262	70.9574	0.00089	0.9982
4	−0.2	4.79756	74.6982	0.00007	0.9999
5	0	4.844	76.9359	0.00011	0.9998
6	0.2	4.9393	74.6412	0.00012	0.9998
7	0.4	5.0822	55.84301	0.00072	0.9985
8	0.53685	5.2867	5.65659	0.0221	0.9551

angle α_1 for which the objective function expressing the minimal dimension had zeros, the mechanism might assume the singular position much earlier. The tangent point of the objective function to the abscissa axis determines those values of the decision variable for which the mechanism should assume the singular position. Figure 4 shows the band of the objective function for $\alpha_1 = 0.53797$ rd (design option II), when one of the extremal objective functions does not vanish anywhere.

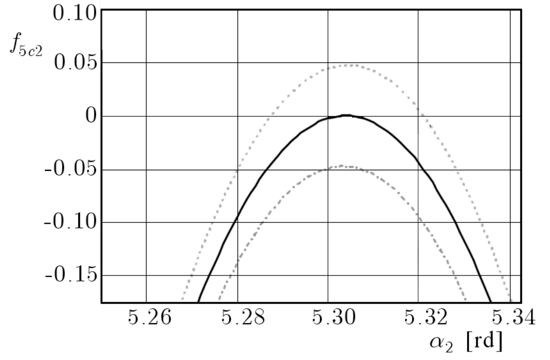

Fig. 4. Band of the objective function for $\alpha_1 = 0.53797$ rd (design option II)

Tests were performed to check how the types of fit in kinematic pairs should affect the width of the function band and sensitivity of the mechanism. For comparative purposes, according to the fixed hole principle, the basic movable fit 8H7/g6 and movable loose fit 8H11/a11 were assumed. Figures 5 and 6 show bands of the objective function for the considered fit types used in the mechanism in Fig. 3b. The band width is changed and so is the motion range of the driving link and, hence, the sensitivity factor.

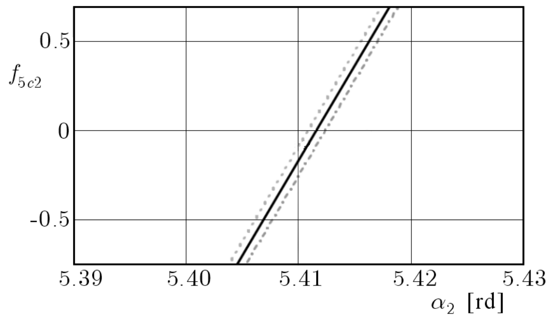

Fig. 5. Objective function band for $\alpha_1 = 0.5604$ rd (8H7/g6)

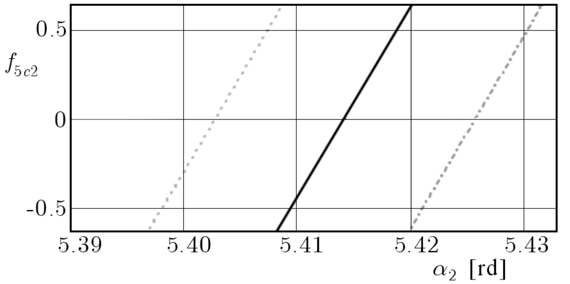

Fig. 6. Objective function band for $\alpha_1 = 0.5676\,\mathrm{rd}$ (8H11/a11)

4. The effects of mechanism structure on its sensitivity

The structure of the investigated mechanism is modified, assuming that only external kinematic pairs A, G, J, driving link 1 and driven 7 should not change (Fig. 7).

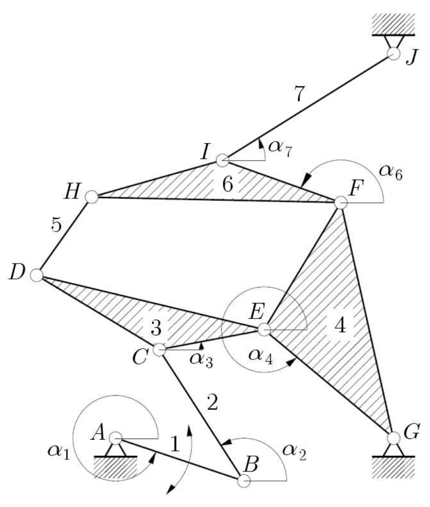

Fig. 7. Mechanism of the IV class

The motion range of driven link 7 for one of the design options is $0.511\,\mathrm{rd}$. For α_1 varying stepwise by $0.2\,\mathrm{rd}$, the following parameters were obtained: angle α_7, derivatives of the objective function at its zeros, distance $\Delta\alpha_7$, sensitivity factor μ. Results are summarised in Table 2.

Table 2. Derivatives of the objective function for various angles α_1

Design option I					
No.	α_1	α_7	$f'_{c1}(\alpha_7)$	$\varDelta\alpha_7$	μ
1	-0.4647	4.4706	17.5807	0.02556	0.0759
2	-0.4	4.447	64.7012	0.0051	0.81536
3	-0.2	4.4464	117.6593	0.0031	0.88878
4	0	4.4565	139.2007	0.00266	0.90396
5	0.2	4.4593	143.9309	0.00253	0.90867
6	0.4	4.4182	124.0576	0.00286	0.89655
7	0.585	4.4727	17.0135	0.0267	0.03349

5. Final remarks

The analysis of sensitivity data for various design options reveals that sensitivity is affected by the fit type and, hence, the clearances in kinematic pairs, mechanism structure and the length of particular links. That is associated with the width and inclination of the objective function band. The motion range of the driven link appears to be the major determinant of the sensitivity factor. The larger the rotation angle of the driven link is, the weaker are the effects of clearances in kinematic pairs. When the motion range of the driven link is rather small, the type of fit in kinematic pairs will strongly affect the mechanism sensitivity. The motion range of the driven link is also associated with lengths of particular links.

It appears that the width of the band of the objective function, which changes when the driving link is in motion, depends on the inclination of the objective function, and hence is closely associated with the derivative of the objective function. If the inclination angle of the tangent line to the plot of the objective function at its zero is close to 0 (i.e., the derivative of the objective function at this point is near zero), then the distance $\varDelta\alpha_1$ shall be the largest whilst sensitivity of the mechanism–the lowest. Besides, the width of the objective function band depends on the type of fit assumed for kinematic pairs.

Modification of the mechanism structure leads to variations of the motion ranges of particular links and, hence, to changes in the sensitivity of the mechanism. It also affects the inclination angle of the objective function, which in turn causes changes of the objective function band width and, finally, the mechanism sensitivity.

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Badania wrażliwości mechanizmów płaskich

Streszczenie

Konstruktor już na etapie projektowania mechanizmu chce wiedzieć, jakie będą jego właściwości kinematyczne. Interesują go położenia, prędkości, przyspieszenia, liczba opcji montażowych, położenia osobliwe, zakresy ruchu poszczególnych ogniw, wpływ tolerancji wykonania ogniw oraz luzów w parach kinematycznych na działanie mechanizmu. Ostatnia z wymienionych właściwości łączy się z pojęciem wrażliwości mechanizmu, rozumianej jako jego zdolności do reagowania na najmniejszą zmianę położenia ogniwa napędzającego. Źródłem informacji na temat wszystkich wymienionych właściwości jest funkcja celu otrzymana w wyniku stosowania metody modyfikacji. W niniejszym opracowaniu przedstawiono wpływ luzów w parach kinematycznych i struktury mechanizmu na jego wrażliwość.

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