JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
47, 3, pp. 553-571, Warsaw 2009

OPTIMIZATION OF ANTI-SYMMETRICAL OPEN CROSS-SECTIONS OF COLD-FORMED THIN-WALLED BEAMS

Jerzy Lewiński

Poznan University of Technology, Institute of Applied Mechanics, Poznań, Poland

e-mail: jerzy.lewinski@put.poznan.pl

Krzysztof Magnucki

Poznan University of Technology, Institute of Applied Mechanics, Poznań, Poland

Institute of Rail Vehicles, Tabor, Poznań, Poland

e-mail: krzysztof.magnucki@put.poznan.pl

The paper deals with cold-formed thin-walled beams with the Z-, S- and Clothoid-section. A short survey of optimal designs of thin-walled beams with open cross-sections is given. Geometric properties of three cross-sections are described. Strength, local and global buckling conditions for thin-walled beams are presented. The optimal design criterion with a dimensionless objective function as a quality measure is defined. Results of numerical calculations for optimal shapes of three cross-sections are presented in tables and figures.

Key words: thin-walled beam, open cross-section, global and local buckling, optimal design

Notations

a, b, c, d	–	dimensions of cross-sections
r	–	radius of the circular arc
t	–	thickness of the beam wall
u	–	dimensionless parameter of the clothoid
A	–	area of the cross-section
H	–	depth of the beam
L	–	length of the beam
J_{S-V}	–	geometric stiffness for Saint-Venant torsion
J_y, J_z	–	inertia moments

J_ω	–	warping moment of inertia
M_0	–	loading moment
R	–	principal radius of the clothoid
α, β	–	angles of the S-section
λ	–	relative length of the beam
θ_p	–	angle to the principal axes
ω	–	warping function
Φ_j	–	objective function

1. Introduction

Shapes of open cross-sections of contemporary cold-formed thin-walled beams are rather complicated. They are usually mono-symmetrical, although sometimes anti-symmetrical too. The main constraints in designing thin-walled structures are strength and stability conditions. The beginnings of the optimal design of thin-walled structures reach back to 1959. The first paper on optimal design of a thin-walled beam with an open cross-section (I-section) in pure bending state was presented by Krishnan and Shetty (1959). A complete survey of optimal design problems of structures for the second half of the twentieth century was given by Gajewski and Życzkowski (1988) and Krużelecki (2004). A bibliography on the problems of topology and shape optimization of structures using FEM and BEM for 1999-2001 was collected by Mackerle (2003). Optimal design criteria for shapes of thin-walled beams cross-sections under strength and local and global stability constraints was presented by Cardoso (2000). Karim and Adeli (1999) presented global optimum design of cold-formed steel hat-shape beams under uniformly distributed load using a neural network model. Variational and parametric design of an open cross-section of a thin-walled beam under stability constraints was described by Magnucki and Magnucka-Blandzi (1999), Magnucki and Monczak (2000). Vinot *et al.* (2001) presented a methodology for optimizing the shape of thin-walled structures. Magnucki (2002) studied optimization of an open cross-section of a thin-walled beam with flat web and circular flange analytically and numerically. Knowledge-based global optimization of cold-formed steel columns under pure axial compression was presented by Liu *et al.* (2004). In result of the study, five anti-symmetrical open cross-sections were proposed. Theoretical and experimental study on the minimum weight of cold-formed channel thin-walled beams with and without lips were analysed by Tian and Lu (2004). Optimum design of cold-formed steel channel beams under uniformly distributed load using micro Genetic Algorithm was presented by Lee *et al.* (2005). Global optimization of cold-formed steel thin-walled beams with lipped channel sections were described by Tran and Li (2006). Optimal design of open cross-sections of cold-formed thin-walled beams with respect to the dimensionless objective function as the quality measure was presented by Magnucka-Blandzi and Magnucki (2004b), Magnucki and Ostwald (2005a,b), Magnucki *et al.* (2006a,b), Magnucki and Paczos (2008). Kasperska *et al.* (2007), Ostwald *et al.* (2007), Ostwald and Magnucki (2008), Manevich and Raksha (2007) described bicriterial optimal design of open cross-sections of cold-formed beams. Strength, global and local buckling and optimization problems of cold-formed thin-walled beams with open cross-sections were collected and described by Magnucki and Ostwald (2005a,b), Ostwald and Magnucki (2008).

The present paper provides further development of optimal shaping of anti-symmetrical open cross-sections of cold-formed thin-walled beams in pure bending state. These beams of the length L, depth H, and wall thickness t are simply supported and carry two equal moments M_0 applied to the beam ends (Fig. 1). The optimization includes three anti-symmetrical cross-sections: Z-section, S-section and clothoid-section.

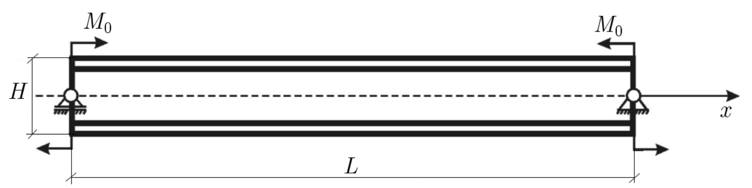

Fig. 1. A scheme of the thin-walled beam

2. Geometric properties of three cross-sections

2.1. Anti-symmetrical Z-section

A scheme of the cross-section with principal axes yz is shown in Fig. 2. The middle line of the Z-section is a broken line situated symmetrically with respect to the origin $O(0,0)$.

Geometric properties of the cross-section are defined by the following dimensionless parameters

$$x_1 = \frac{b}{a} \qquad x_2 = \frac{c}{b} \qquad x_3 = \frac{t}{b} \qquad x_4 = \frac{d}{a} \tag{2.1}$$

where: a, b, c, d – sizes of the cross-section, t – thickness of the wall.

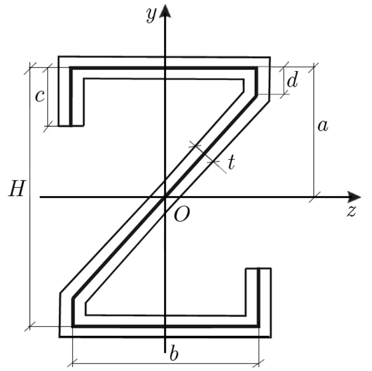

Fig. 2. A scheme of the Z-section

Depth of the beam is

$$H = 2a + t = a(2 + x_1 x_3) \tag{2.2}$$

Total area and geometric stiffness for Saint-Venant torsion of the cross-section

$$A = 2atf_0(x_i) \qquad\qquad J_{S-V} = \frac{2}{3}at^3 f_0(x_i) \tag{2.3}$$

where

$$f_0(x_i) = x_1(1 + x_2) + x_4 + \sqrt{(1 - x_4)^2 + \frac{1}{4}x_1^2}$$

The product of inertia with respect to the principal axes yz is zero

$$J_{yz} = 2a^3 t x_1[-3(x_1x_2 - x_4)(2 - x_1x_2 - x_4) + (1 - x_1)\sqrt{x_1^2 + 4(1 - x_4)^2}] = 0 \tag{2.4}$$

from which

$$x_2 = \frac{1}{x_1}(1 + \sqrt{1 - C_0}) \tag{2.5}$$

where

$$C_0 = (2 - x_4)x_4 + \frac{1}{3}(1 - x_1)\sqrt{x_1^2 + 4(1 - x_4)^2}$$

Moments of inertia of the plane area (Fig. 2) with respect to the y and z axes are

$$J_y = 2a^3 t f_2(x_i) \qquad\qquad J_z = 2a^3 t f_3(x_i) \tag{2.6}$$

where

$$f_2(x_i) = \frac{1}{4}x_1^2\Big[x_1\Big(\frac{1}{3} + x_2\Big) + x_4 + \frac{1}{6}\sqrt{x_1^2 + 4(1 - x_4)^2}\Big]$$

$$f_3(x_i) = x_1 + \frac{1}{3}[2 - (1 - x_1x_2)^3 - (1 - x_4)^3] + \frac{1}{6}(1 - x_4)^2\sqrt{x_1^2 + 4(1 - x_4)^2}$$

The warping function $\omega(s)$ for the Z-section (half section) is shown in Fig. 3.

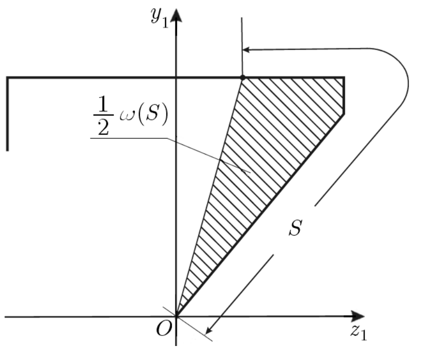

Fig. 3. Geometric interpretation of the warping function $\omega(s)$

The warping function in characteristic points of the Z-section have the following values

$$\omega_1 = 0 \qquad\qquad \omega_i = a^i\widetilde{\omega}_i \qquad\qquad i = 2,3,4 \tag{2.7}$$

where

$$\widetilde{\omega}_2 = \frac{1}{2}x_1x_4 \qquad\qquad \widetilde{\omega}_3 = \Big(1 + \frac{1}{2}x_4\Big)x_1 \qquad\qquad \widetilde{\omega}_4 = \Big[1 + \frac{1}{2}(x_1x_2 + x_4)\Big]x_1$$

The warping moment of inertia

$$J_\omega = 2a^5tf_5(x_i) \tag{2.8}$$

where

$$f_5(x_i) = \frac{1}{3}[x_4\widetilde{\omega}_2^2 + x_1(\widetilde{\omega}_2^2 + \widetilde{\omega}_2\widetilde{\omega}_3 + \widetilde{\omega}_3^2) + x_1x_2(\widetilde{\omega}_3^2 + \widetilde{\omega}_3\widetilde{\omega}_4 + \widetilde{\omega}_4^2)]$$

The centroid and the shear center of the plane area of anti-symmetrical cross-sections are located in the origin $O(0,0)$.

2.2. Anti-symmetrical S-section

A scheme of the cross-section with auxiliary axes y_1z_1 and principal axes yz is shown in Fig. 4. The middle line of the S-section is a composite curve (two circles and one line segment) situated symmetrically with respect to the origin $O(0,0)$.

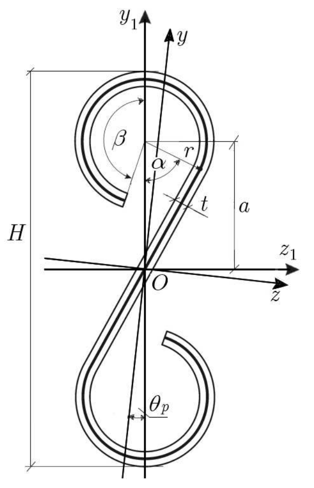

Fig. 4. A scheme of the S-section

Geometric properties of the cross-section are defined by the following dimensionless parameters

$$x_1 = \frac{r}{a} \qquad x_3 = \frac{t}{r} \qquad \text{and} \qquad \beta \tag{2.9}$$

where: a, r – sizes of the cross-section, β – angle, t – thickness of the wall.

Depth of the beam is

$$H = 2a\cos\theta_p + 2r + t = 2a\Big[\cos\theta_p + x_1\Big(1 + \frac{1}{2}x_3\Big)\Big] \tag{2.10}$$

Total area and geometric stiffness for Saint-Venant torsion of the cross-section

$$A = 2atf_0(x_i) \qquad J_{S-V} = \frac{2}{3}at^3 f_0(x_i) \tag{2.11}$$

where

$$f_0(x_i) = \sqrt{1 - x_1^2} + x_1(\pi + \beta - \alpha) \qquad \cos\alpha = x_1$$

The product of inertia with respect to the auxiliary axes $y_1 z_1$

$$J_{y_1z_1} = 2a^3tf_1(x_i) \tag{2.12}$$

where

$$f_1(x_i) = \Big\{\frac{1}{3}(1 - x_1^2)^2 + x_1\Big[\cos\beta + x_1 + \frac{1}{4}x_1(1 + \cos 2\beta - 2x_1^2)\Big]\Big\}x_1$$

Moments of inertia of the plane area (Fig. 4) with respect to the y_1 and z_1 auxiliary axes are

$$J_{y_1} = 2a^3 t f_2(x_i) \qquad\qquad J_{z_1} = 2a^3 t f_3(x_i) \tag{2.13}$$

where

$$f_2(x_i) = \Big\{\frac{1}{3}\sqrt{(1-x_1^2)^3} + \frac{1}{4}x_1[2(\pi+\beta-\alpha) + 2x_1\sqrt{1-x_1^2} - \sin 2\beta]\Big\}x_1^2$$

$$f_3(x_i) = \frac{1}{3}\sqrt{(1-x_1^2)^5} + \\ +\frac{1}{2}x_1\{(\pi+\beta-\alpha)(2+x_1^2) + x_1[\sqrt{1-x_1^2}(4-x_1^2) + (4+x_1\cos\beta)\sin\beta]\}$$

The angle θ_p defining the principal axes is

$$\tan 2\theta_p = -\frac{2J_{y_1z_1}}{J_{z_1} - J_{y_1}} \tag{2.14}$$

and, principal moments of inertia

$$\begin{aligned} J_y &= \frac{1}{2}(J_{z1} + J_{y1}) - \sqrt{\frac{1}{4}(J_{z1} + J_{y1})^2 + J_{y_1z_1}^2} \\ J_z &= \frac{1}{2}(J_{z1} + J_{y1}) + \sqrt{\frac{1}{4}(J_{z1} + J_{y1})^2 + J_{y_1z_1}^2} \end{aligned} \tag{2.15}$$

The warping function $\omega(\varphi)$ for the S-section (half section) is shown in Fig. 5.

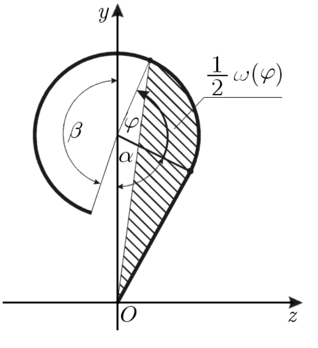

Fig. 5. Geometric interpretation of the warping function $\omega(\varphi)$

The warping function for the S-section is defined as follows

$$\omega(\varphi) = [\sin\alpha - \sin(\alpha+\varphi) + x_1\varphi]x_1a^2 \tag{2.16}$$

The warping moment of inertia

$$J_\omega = 2a^5 t f_5(x_i) \tag{2.17}$$

where

$$\begin{aligned}
f_5(x_i) &= (f_{51} - f_{52} + f_{53} + f_{54})x_1^3\\
f_{51} &= \frac{1}{2}\Big(x_1\sqrt{1-x_1^2} - \frac{1}{2}\sin 2\beta\Big)\\
f_{52} &= 2[(\pi+\beta-\alpha)x_1 + \sqrt{1-x_1^2}]\cos\beta\\
f_{53} &= [2\sin\beta - (\pi+\beta-\alpha)\sqrt{1-x_1^2}]x_1\\
f_{54} &= \frac{1}{6}(\pi+\beta-\alpha)\{9 + 2[(\pi+\beta-\alpha)^2 - 3]x_1^2\}
\end{aligned}$$

2.3. Anti-symmetrical Clothoid-section

A scheme of the cross-section with auxiliary axes y_1z_1 and principal axes yz is shown in Fig. 6. The middle line of the Clothoid-section is a curve situated symmetrically with respect to the origin $O(0,0)$.

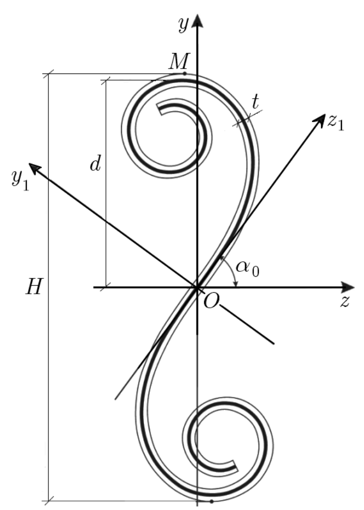

Fig. 6. A scheme of the Clothoid-section

In Cartesian auxiliary coordinates, the curve is parametrized as follows

$$y_1 = a\sqrt{\pi}\int\limits_0^{u_1} \sin\frac{\pi u^2}{2}\,du \qquad\qquad z_1 = a\sqrt{\pi}\int\limits_0^{u_1} \cos\frac{\pi u^2}{2}\,du \tag{2.18}$$

where a is the scale parameter determining the outer size of the curve, u – dimensionless parameter $(0 \leqslant u \leqslant u_1)$.

The principal curvature radius

$$R = \frac{a^2}{s} \tag{2.19}$$

where arc length $s = a\sqrt{\pi}\, u_1$.

Geometric properties of the cross-section are defined by the following dimensionless parameters

$$x_1 = u_1 \qquad\qquad x_3 = \frac{t}{a} \tag{2.20}$$

Depth of the beam is

$$H = 2d + t \tag{2.21}$$

where: u_1 is the upper integration limit, as in (2.18), deciding on the "depth" of convolutions of the curve, t – wall thickness.

The total area of the clothoid cross-section is

$$A = 2\int\limits_A dA = 2t\int\limits_{OP} ds = 2\sqrt{\pi}\, atu_1 \tag{2.22}$$

The moments of inertia of the plane area with respect to the z_1 and y_1 axes are

$$I_{z1} = \int\limits_A y_1^2\, dA = 2at\sqrt{\pi}\int\limits_0^{u_1} a^2\pi\Big(\int\limits_0^{u} \sin\frac{\pi v^2}{2}\, dv\Big)\, du = 2a^3t\sqrt{\pi^3}\int\limits_0^{u_1}[s(u)]^2\, du \tag{2.23}$$

where

$$s(u) = \int\limits_0^{u} \sin\frac{\pi u^2}{2}\, du \tag{2.24}$$

and

$$I_{y1} = \int\limits_A z_1^2\, dA = 2a^3t\sqrt{\pi^3}\int\limits_0^{u_1}[c(u)]^2\, du \tag{2.25}$$

where

$$c(u) = \int\limits_0^{u} \cos\frac{\pi u^2}{2}\, du \tag{2.26}$$

The principal axes and principal moments of inertia are defined by the same expressions as for the S-section, i.e. (2.14) and (2.15).

The warping function of the clothoid section (Fig. 7) takes the following form

$$\omega = 2\Big(\frac{1}{2}z_p y_p - \int\limits_0^{z_p} y_1\, dz_1\Big) \tag{2.27}$$

where

$$z_p = a\sqrt{\pi}\int\limits_0^{u_p} \cos\frac{\pi u^2}{2}\, du \qquad\qquad y_p = a\sqrt{\pi}\int\limits_0^{u_p} \sin\frac{\pi u^2}{2}\, du$$

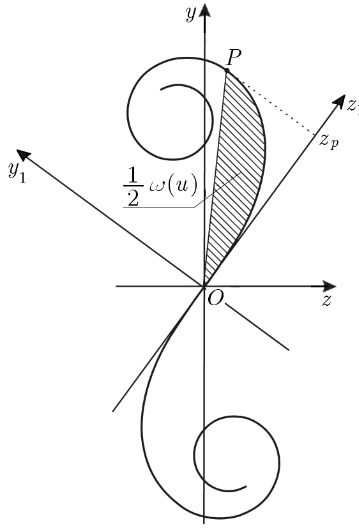

Fig. 7. Geometric interpretation of the clothoid warping function $\omega(u)$

According to earlier definitions (2.23) and (2.25)

$$z_p y_p = \pi a^2 c(u_p) s(u_p) \qquad\qquad dz_1 = a\sqrt{\pi}\cos\frac{\pi u^2}{2}\, du \tag{2.28}$$

Hence, the warping function, being a function of the parameter u_p, may be formulated as follows

$$\omega(u_p) = \pi a^2\Big[c(u_p)s(u_p) - 2\int\limits_0^{u_p} \cos\frac{\pi u^2}{2} s(u)\, du\Big] \tag{2.29}$$

Finally, the warping moment of inertia of the clothoid section is calculated as follows

$$I_\omega = \int\limits_A \omega^2 \, dA = 2t \int\limits_0^{u_p} \omega^2(u) \, ds \tag{2.30}$$

3. Formulation of the optimization problem

3.1. Optimization criterion

The minimal mass and maximal safe load are usually a basic objective in structure designing. The optimization criterion according to the papers of Magnucka-Blandzi and Magnucki (2004a,b), Magnucki *et al.* (2006a,b), has been formulated in the following form

$$\max_{x_i}\{\varPhi_1(x_i), \varPhi_2(x_i), \varPhi_3(x_i), \varPhi_4(x_i)\} = \varPhi_{max} \tag{3.1}$$

and the objective function

$$\varPhi_j(x_i) = \frac{M_j}{E\sqrt{A^3}} \tag{3.2}$$

where M_j are the allowable moments defined from the strength condition ($j = 1$), lateral buckling condition ($j = 2$), local buckling condition of the flange ($j = 3$), and local buckling condition of the web.

3.2. Constraints

Strength and buckling are main problems in thin-walled structures designing. Lateral buckling strengths of a cold-formed Z-section beam was presented by Pi *et al.* (1999). Li (2004) described lateral-torsion buckling of the cold-formed Z-beam. The effects of warping stress on the lateral torsional buckling, and local and distortional buckling of cold-formed Z-beams were described by Chu *et al.* (2004, 2006). Stasiewicz *et al.* (2004) described local buckling of a bent flange of a thin-walled beam. Analytical and numerical analysis of the stress state and global elastic buckling of a thin-walled beam with a mono-symmetrical open cross-section was presented by Magnucki *et al.* (2004). Critical stresses for open cylindrical shells with free edges were calculated by Magnucka-Blandzi and Magnucki (2004b), Magnucki and Mackiewicz (2006) and Joniak *et al.* (2008). Ventsel and Krauthammer (2001) collected and described strength and buckling problems of thin plates and shells.

The space of feasible solutions for optimal shapes of cross-sections of thin-walled beams is restrained. The strength condition has the following form

$$M_0 \leqslant M_1 \qquad\qquad M_1 = 2\frac{J_z}{H}\sigma_{all} \tag{3.3}$$

where σ_{all} is the allowable stress.

The global stability condition (lateral buckling condition) for a simply supported beam in pure bending state has the following form

$$M_0 \leqslant M_2 \qquad\qquad M_2 = \frac{M_{CR}^{(Globl)}}{c_{s1}} \tag{3.4}$$

where c_{s1} is the safety coefficient, and the lateral buckling moment for a simply supported thin-walled beam in pure bending state is (Magnucki and Ostwald, 2005a,b)

$$M_{CR}^{(Globl)} = \frac{\pi E}{L}\sqrt{\frac{J_y J_{S-V}}{2(1+\nu)}\Big[1+2(1+\nu)\frac{\pi^2}{L^2}\frac{J_\omega}{J_{S-V}}\Big]} \tag{3.5}$$

The local stability conditions for the **Z-beam** are as follows:

- for the bent flange, according to Magnucki and Ostwald (2005a,b) and Stasiewicz *et al.* (2004)

$$\sigma_{max}^{(Z-flange)} \leqslant \frac{\sigma_{CR}^{(Z-flange)}}{c_{s2}} \qquad\qquad \sigma_{CR}^{(Z-flange)} = \frac{1+x_2}{1+3x_2}x_3^2 G \tag{3.6}$$

where $\sigma_{CR}^{(Z-flange)}$ is the critical stress, $G = E/[2(1+\nu)]$ – shear modulus of elasticity, E – Young's modulus, ν – Poisson's ratio, c_{s2} – safety coefficient.

Taking into account the classical theory of plates, the local stability condition for the bent flange may be written down as

$$M_0 \leqslant M_3 \qquad\qquad M_3 = \frac{\sigma_{CR}^{(Z-flange)}}{c_{s2}}\frac{J_z}{a-e_f} = \frac{2a^2 t}{c_{s2}}G\frac{1+x_2}{1+3x_2}x_3^2\frac{f_3(x_i)}{1-\widetilde{e}_f} \tag{3.7}$$

where $\widetilde{e}_f$ is the dimensionless parameter of the centroid location of the flange

$$\widetilde{e}_f = \frac{x_2}{2(1+x_2)}f_1(x_i)$$

- for the flat web according to Ventsel and Krauthammer (2001)

$$\sigma_{max}^{(Z-web)} \leqslant \frac{\sigma_{CR}^{(Z-web)}}{c_{s2}} \qquad \sigma_{CR}^{(Z-web)} = \frac{2\pi^2}{1-\nu^2} E \frac{(x_1 x_3)^2}{x_1^2 + 4(1-x_4)^2} \tag{3.8}$$

where $\sigma_{CR}^{(Z-web)}$ is the critical stress.

Taking into account the classical theory of beams, the local stability condition for the flat web may be put down as

$$\begin{aligned} & M_0 \leqslant M_4 \\ & M_4 = \frac{\sigma_{CR}^{(Z-web)}}{c_{s2}} \frac{J_z}{a-d} = \frac{4\pi^2 a^2 t}{c_{s2}(1-\nu^2)} E \frac{(x_1 x_3)^2}{x_1^2 + 4(1-x_4)^2} \frac{f_3(x_i)}{1-x_4} \end{aligned} \tag{3.9}$$

The local stability conditions for the **S-beam** take the following forms:

- for the open circular cylindrical shell, regarding the results of Magnucka Blandzi and Magnucki (2004a), Magnucki and Mackiewicz (2006), Joniak *et al.* (2008)

$$\sigma_{max}^{(S-shell)} \leqslant \frac{\sigma_{CR}^{(S-shell)}}{C_{s2}} \qquad \sigma_{CR}^{(S-shell)} = \alpha_C \frac{E}{12.7\sqrt{3(1-\nu^2)}} x_3 \tag{3.10}$$

where $\sigma_{CR}^{(S-shell)}$ is the critical stress and α_C – coefficient

$$\alpha_C = 1 + 0.8\Big(\beta - \frac{\pi}{2}\Big)^4$$

Taking into account the classical theory of beams, the local stability condition for the circular cylindrical flange may be written down as

$$M_0 \leqslant M_3 \qquad M_3 = \frac{\sigma_{CR}^{(S-shell)}}{c_{s2}} \frac{J_z}{a} = \frac{2at^2}{12.7 c_{s2}\sqrt{3(1-\nu^2)}} E \alpha_C \frac{f_3(x_i)}{x_1} \tag{3.11}$$

- for the flat web Ventsel and Krauthammer (2001)

$$\sigma_{max}^{(S-web)} \leqslant \frac{\sigma_{CR}^{(S-web)}}{c_{s2}} \qquad \sigma_{CR}^{(S-web)} = \frac{\pi^2}{2(1-\nu^2)} E \frac{(x_1 x_3)^2}{1-x_1} \tag{3.12}$$

where $\sigma_{CR}^{(S-web)}$ is the critical stress.

Taking into account the classical theory of beams, the local stability condition for the flat web is

$$\begin{aligned}
&M_0 \leqslant M_4 \\
&M_4 = \frac{\sigma_{CR}^{(S-web)}}{c_{s2}} \frac{J_z}{a-d} = \frac{4\pi^2 a^2 t}{c_{s2}(1-\nu^2)} E \frac{(x_1 x_3)^2}{x_1^2 + 4(1-x_4)^2} \frac{f_3(x_i)}{1-x_4}
\end{aligned} \tag{3.13}$$

The local stability conditions for the **Clothoid-beam** are as follows:

- for the open cylindrical shell, according to Magnucka Blandzi and Magnucki (2004b), Magnucki and Mackiewicz (2006), Joniak *et al.* (2008)

$$\sigma_{edge}^{(Cl-shell)} \leqslant \frac{\sigma_{CR,edge}^{(Cl-shell)}}{c_{s2}} \qquad \sigma_{CR,edge}^{(Cl-shell)} = \frac{E}{12.7\sqrt{3(1-\nu^2)}} \frac{t}{R_{edge}} \tag{3.14}$$

where $\sigma_{CR,edge}^{(Cl-shell)}$ is the critical stress.

Taking into account the classical theory of beams, the local stability condition for the circular cylindrical flange assumes the form

$$\begin{aligned}
&M_0 \leqslant M_3 \\
&M_3 = \frac{\sigma_{CR,edge}^{(Cl-shell)}}{c_{s2}} \frac{J_z}{d} = \frac{2at^2}{12.7 c_{s2}\sqrt{3(1-\nu^2)}} E\alpha_C \frac{f_3(x_i)}{x_1}
\end{aligned} \tag{3.15}$$

- for the cylindrical shell

$$\sigma_{local}^{(Cl-shell)} \leqslant \frac{\sigma_{CR,local}^{(Cl-shell)}}{c_{s2}} \qquad \sigma_{CR,local}^{(Cl-shell)} = \frac{E}{\sqrt{3(1-\nu^2)}} \frac{t}{R(y)} \tag{3.16}$$

where $\sigma_{CR,local}^{(Cl-shell)}$ is the critical stress.

Taking into account the classical theory of beams, the local stability condition for the flat web is

$$M_0 \leqslant M_4 \qquad M_4 = \frac{\sigma_{CR,local}^{(Cl-web)}}{c_{s2}} \frac{J_z}{y(u)} \tag{3.17}$$

4. Numerical solution of the optimization problem

Optimization of three anti-symmetrical open cross-sections has been performed for a family of cold-formed thin-walled beams: $\sigma_{all}/E = 0.0015$, $\nu = 0.3$, $c_{s1} = 1.5$, $c_{s2} = 2.1$, with relative lengths $\lambda = L/H = 7.5$, 10.0, 12.5, 15.0, 17.5, 20.0. The results of numerical calculations for the Z-beam are specified in Table 1, for the S-beam in Table 2, and for the Clothoid-beam in Table 3.

Table 1. Optimal parameters for the Z-beam

λ	7.5	10.0	12.5	15.0	17.5	20.0
$x_{1,opt}$	0.3897	0.5262	0.6607	0.7908	0.9161	1.0359
$x_{2,opt}$	0.7259	0.4581	0.3051	0.2077	0.1400	0.0903
$x_{3,opt}$	0.1175	0.1131	0.1098	0.1072	0.1052	0.1036
$x_{4,opt}$	0.0458	0.0595	0.0725	0.0850	0.0964	0.1072
Φ_{max}	0.0020725	0.001825	0.001645	0.001507	0.0013940	0.001301

Table 2. Optimal parameters for the S-beam

λ	7.5	10.0	11.15	12.5	15.0	17.5	20.0
$x_{1,opt}$	0.3696	0.3696	0.3696	0.4145	0.4971	0.5789	0.6598
β_{opt}	π	π	π	π	π	π	π
$x_{3,opt}$	0.047951	0.047951	0.047951	0.04641	0.04383	0.04155	0.0395
Φ_{max}	0.003049	0.003049	0.003049	0.002857	0.002573	0.002356	0.002187

Table 3. Optimal parameters for the Clothoid-beam

λ	7.5	10.0	12.5	15.0	17.5	20.0
$x_{1,opt}$	2.6	2.6	2.6	2.6	2.6	2.6
$x_{3,opt}$	0.014353	0.014353	0.014353	0.014353	0.014353	0.014353
Φ_{max}	0.003553	0.003553	0.003553	0.003553	0.003553	0.003553

5. Conclusions

The criterion of effective shaping (optimal design) with dimensionless objective functions (26) enables sorting and comparing beams with arbitrary cross-sections. This criterion is a quality measure of the cross-sections of beams.

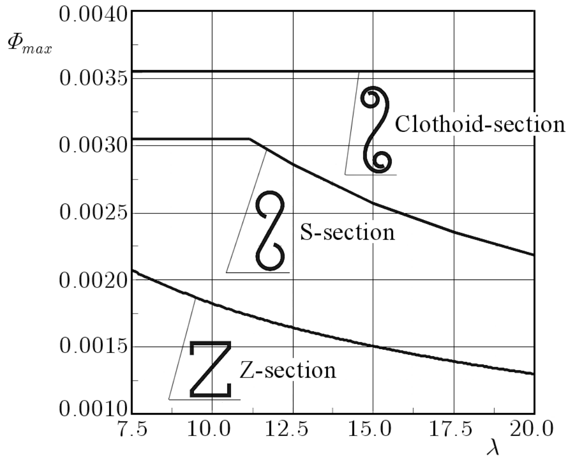

Fig. 8. Dimensionless objective function Φ_{max} for three considered types of beams

According to the plots in Fig. 8, the following conclusions may be drawn:

- In the case of the Z-Section, the lateral buckling is decisive for the beam of relative length $7.5 \leqslant \lambda$.
- For the S-Section of the relative length $\lambda \leqslant 11.15$, the lateral buckling imposes no constraint – the condition remains inactive. It is active only for $\lambda > 11.15$.
- In the case of the Clothoid-Section, the lateral buckling remains inactive within the whole considered range of the relative length λ.

The beams with the Clothoid-section are definitely better than those with Z- or S-sections.

References

1. Cardoso J.B., Imperial College Press 2000, Optimal design criteria, *Proc. of the Third Int. Conference on Coupled Instabilities in Metal Structures, CIMS'2000*, D. Camotim, D. Dubina, J. Rondal (Edit.), Lisbon, 625-634
2. Chu X., Li L., Kettle R., 2004, The effect of warping stress on the lateral-torsion buckling of cold-formed zed-purlins, *Journal of Applied Mechanics* **71**, 742-744
3. Chu X., Ye Z., Li L., Kettle R., 2006, Local and distortional buckling of cold-formed zed-section beams under uniformly distributed transverse loads, *Int. Journal of Mechanical Sciences*, **48**, 378-388

4. Gajewski A., Życzkowski M., 1988, *Optimal Structural Design under Stability Constraints*, Dordrecht, Boston, London, Kluwer Academic Publishers

5. Joniak S., Magnucki K., Szyc W., 2008, Theoretical and experimental investigations of elastic buckling of open circular cylindrical shells, *Proc. 5th Int. Conference on Coupled Instabilities in Metal Structures*, **1**, K. Rasmussen, T. Wilkinson (Edit.), The University of Sydney, Australia, CIMS, 23/25, 465-472

6. Karim A., Adeli H., 1999, Global optimum design of cold-formed steel hat-shape beams, *Thin-Walled Structures*, **35**, 275-288

7. Kasperska R., Magnucki K., Ostwald M., 2007, Bicriteria optimization of cold-formed thin-walled beams with monosymmentrical open cross-sections under pure bending, *Thin-Walled Structures*, **45**, 563-572

8. Krishnan S., Shetty K.V., 1959, On the optimum design of an I-section beam, *Journal of Aero/Space Sciences*, **26**, 9, 599-600

9. Krużelecki J., 2004, A review of optimal structural design of shells, *Proc. Third Conf. Thin-Walled Vessels*, S. Joniak et al. (Edit), Karlow, Poland, 35-72

10. Lee J., Kim S.-M., Park H.-S., Woo B.-H., 2005, Optimum design of cold-formed steel channel beams using micro Genetic Algorithm, *Engineering Structures*, **27**, 17-24

11. Li L., 2004, Lateral-torsion buckling of cold-formed zed-purlins partial-laterally restrained by metal sheeting, *Thin-Walled Structures*, **42**, 995-1011

12. Liu H., Igusa T., Schafer B.W., 2004, Knowledge-based global optimization of cold-formed steel columns, *Thin-Walled Structures*, **42**, 785-801

13. Mackerle J., 2003, Topology and shape optimization of structures using FEM and BEM. A bibliography (1999-2001), *Finite Elements in Analysis and Design*, **39**, 243-253

14. Magnucka-Blandzi E., Magnucki K., 2004a, Elastic buckling of an axially compressed open circular cylindrical shell, *Proceedings in Applied Mathematics and Mechanics, PAMM*, **4**, 546-547

15. Magnucka-Blandzi E., Magnucki K., 2004b, Optimal open cross-section of thin-walled beams, *Thin-Walled Structures. Advances in Research, Design and Manufacturing Technology. Proc. 4th Intl Conference on Thin-Walled Structures*, J. Loughlan (Edit.), Loughborough, UK, Institute of Physics Pub., Bristol and Philadelphia, 877-884

16. Magnucki K., 2002, Optimization of open cross-section of the thin-walled beam with flat web and circular flange, *Thin-Walled Structures*, **40**, 297-310

17. Magnucki K., Mackiewicz M., 2006, Elastic buckling of an axially compressed cylindrical panel with three edges simply supported and one edge free, *Thin-Walled Structures*, **44**, 387-392

18. Magnucki K., Maćkiewicz M., Lewiński J., 2006a, Optimal design of a mono-symmetrical open cross-section of a cold-formed beam with cosinusoidally corrugated flanges, *Thin-Walled Structures*, **44**, 554-562

19. Magnucki K., Magnucka-Blandzi E., 1999, Variational design of open cross-section thin-walled beam under stability constraints, *Thin-Walled Structures*, **35**, 185-191

20. Magnucki K., Monczak T., 2000, Optimum shape of open cross-section of thin-walled beam, *Engineering Optimization*, **32**, 335-351

21. Magnucki K., Ostwald M., 2005a, Optimal design of open cross-sections of cold-formed thin-walled beams, *Proc. of the Fourth Intl Conference on Advances in Steel Structures*, Z.Y. Shen *et al.* (Edit.), Shanghai, China, Elsevier, 1311-1316

22. Magnucki K., Ostwald M., 2005b, *Optimal Design of Selected Open Cross-Sections of Cold-Formed Thin-Walled Beams*, Publishing House of Poznan University of Technology, Poznań

23. Magnucki K., Paczos P., 2008, Elastic buckling and effective shaping of selected cross-sections of flanges of thin-walled channel beams, *Proc. 5th Int. Conference on Coupled Instabilities in Metal Structures*, **1**, K. Rasmussen, T. Wilkinson (Edit.), The University of Sydney, Australia, 375-382

24. Magnucki K., Rodak M., Lewiński J., 2006b, Optimization of mono- and anti-symmetrical I-section of cold-formed thin-walled beams, *Thin-Walled Structures*, **44**, 832-836

25. Magnucki K., Szyc W., Stasiewicz P., 2004, Stress state and elastic buckling of a thin-walled beam with monosymmetrical open cross-section, *Thin-Walled Structures*, **42**, 25-38

26. Manevich A.I., Raksha S.V., 2007, Two-criteria optimization of H-section bars-beams under bending and compression, *Thin-Walled Structures*, **45**, 898-901

27. Ostwald M., Magnucki K., 2008, *Optimal Design of Cold-Formed Thin-Walled Beams with Open Cross-Sections*, Poznan, Comprint [in Polish]

28. Ostwald M., Magnucki K., Rodak M., 2007, Bicriteria optimal design of open cross-sections of cold-formed beams, *Steel and Composite Structures*, **7**, 53-70

29. Ostwald M., Rodak M., 2008, Multicriteria optimization of cold-formed thin-walled beams with generalized open shape, *Proc. 5th Int. Conference on Thin-Walled Structures*, **1**, M. Mahendran (Edit.), Queensland University of Technology, Brisbane Australia, 509-516

30. Pi Y.-L., Put B.M., Trahair N.S., 1999, Lateral buckling strengths of cold-formed Z-section beams, *Thin-Walled Structures*, **34**, 65-93

31. Stasiewicz P., Magnucki K., Lewiński J., Kasprzak J., 2004, Local buckling of a bent flange of a thin-walled beam, *Proceedings in Applied Mathematics and Mechanics, PAMM*, **4**, 554-555

32. Tian Y.S., Lu T.J., 2004, Minimum weight of cold formed steel sections under compression, *Thin-Walled Structures*, **42**, 515-532

33. Tran T., Li L., 2006, Global optimization of cold-formed steel channel sections, *Thin-Walled Structures*, **44**, 399-406

34. Ventsel E., Krauthammer T., 2001, *Thin Plates and Shells. Theory, Analysis and Applications*, New York, Basel, Marcel Dekker, Inc.

35. Vinot P., Cogan S., Piranda J., 2001, Shape optimization of thin-walled beam-like structures, *Thin-Walled Structures*, **39**, 611-630

Optymalizacja otwartych antysymetrycznych przekrojów belek cienkościennych walcowanych na zimno

Streszczenie

W artykule rozważane są belki o przekrojach poprzecznych w kształcie Z-, S- oraz w kształcie klotoidy. Zamieszczono krótki przegląd zagadnień optymalnego projektowania belek cienkościennych o przekrojach otwartych. Opisano właściwości geometryczne trzech rozważanych przekrojów. Zapisano warunki wytrzymałości oraz lokalnej i ogólnej stateczności belek cienkościennych. Sformułowano kryterium optymalizacyjne z wykorzystaniem bezwymiarowej funkcji celu będącej miarą jakości przekroju. Wyniki numerycznych obliczeń optymalnych zarysów przekrojów poprzecznych przedstawiono w tablicach i na rysunkach.

Manuscript received April 3, 2009; accepted for print April 22, 2009