# STEADY PERIODIC REGIME OF ROTARY MOTION OF ROLLERS IN VIBRATING CLASSIFIERS

Victor A. Ostapenko

*Dnepropetrovsk National University, Ukraine*
*e-mail: duo-mmf@ff.dsu.dp.ua*

Vladimir P. Naduty
Vladimir F. Yagnyukov

*Institute of Geotechnical Mechanics, NAS, Ukraine*
*e-mail: astasdn@rambler.ru*

The problem of dynamics of vibrating classifiers with rollers intended for sorting of loose materials is considered in the paper. The main goal of research consists in substantiation of a choice of such parameters of rollers and classifiers which provide a steady periodic mode of rollers rotation. Differential equations of rollers rotation under action of inertial forces are essentially nonlinear. These equations are transformed to equations concerning the value of delay of moving rollers concerning rotation of axles on which these rollers are freely suspended. As values of such delays is small, the equations of motion can be linearized with a sufficient degree of accuracy. The linearized equations represent the inhomogeneous Hill equations, which can be under certain conditions transformed into the Mathieu equations. Periodic solutions to these equations are obtained, and also stability of these solutions is investigated.

*Key words:* classifier, roller, rotation

## 1. Introduction

Classifiers with rollers are widely applied in the mountain, metallurgical and building industries, providing high efficiency of classification and reliability ([1]; Caughey, 1960; [4]; [8]). In total, in the world, in manufacture and operation of roll screens and classifiers are engaged more than fifty firms in the

United States, the Great Britain, France, Germany, Finland, Italy, Spain, Japan, Sweden and Russia. Germany is one of the largest manufacturers and consumers of roll screens ([1]; Caughey, 1960). Firm Zemag have been making roll screens since 1992 [8]. Numerous sources specify high operational qualities of such machines ([1]; Caughey, 1960; 4]; [8]).

At the same time, the necessity of transfering of rotary movement to each of numerous rollers essentially complicates the design, making it bulky and metal-consuming. For instance, every roller of screens made by Zemag has its own individual drive gear with an electric motor. Only in small classifiers every two rollers have a separate drive gear [8]. Therefore classifiers with rollers demand further constructive improvement. One of perspective directions is the application of a vibrating drive and use of vibrating movements of working body for rotation of rollers. In this case, in general, there is no necessity of creation of drives for rollers. In a suggested design illustrated in Fig. 1, the rigid frame of the classifier is oscillatory driven with the help of vibrators. Along the frame, in equal distances the axles rigidly connected to it are placed and located in the horizontal plane. On each of axles, the roller it is freely put, representing a ring in its cross section.
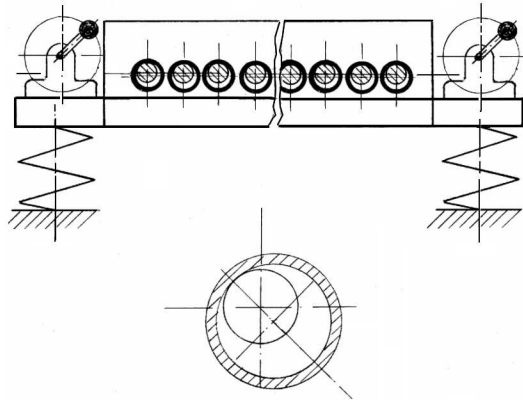


Fig. 1. Scheme of a classifier

Under the action of vibrators, the frame, and together with it the axles of rollers, move on an elliptic or, in particular, a circular trajectory in a vertical plane. Besides, under the action of inertial forces, also the rollers come into movement. From the point of view of qualitative work of the classifier, it is important to obtain conditions of periodic, synchronous and in phase rotation of rollers. Besides, it is important to investigate conditions of stability of rollers motion and to provide their steady rotation independent of initial conditions

as they have random enough character. The problem of formulation of steady periodic solutions to the equations motion of rollers is considered in the paper.

## 2. Linearization of equation of motion

A thorough research of literature shows that there is only one attempt creating a mathematical model for a more or less similar device. We mean the hula-hoop, rotation of hoop around of man's body (Caughey, 1960). In this work, an essentially simplified model of motion of a hoop is considered. The system: human torso-hoop is considered as a certain coulisse mechanism making given movements. The analysis of forces causing this movement is absent. The choice of a certain direction of influence of the human torso on the hoop, in our opinion, obviously does not correspond to real interactions in the system.

This circumstance induces the authors to create a mathematical model of vibrating classifiers with rollers (Ostapenko *et al.*, 2005). The equation of relative rotation of the rollers in such a model looks like (Ostapenko *et al.*, 2005)

$$\frac{d^2\alpha}{dt^2} = -\frac{R_2 m}{J_{kz}}\left[-(R_2 - R_1 + R\cos\alpha)\frac{d\omega}{dt} + R\omega^2\sin\alpha - g\sin(\phi - \alpha)\right] \quad (2.1)$$

where $\alpha$ is the angle of delay of the roller in relation to the angle $\phi$, $\phi = \omega t$, $t$ – time, $\omega$ – angular velocity of rotation, $R$ – radius of rotation of the center of the axis, $m$ – mass of roller, $J_{kz}$ – moment of inertia of the roller with respect to the point of its contact with the axle, and

$$J_{kz} = \frac{m}{2}(R_2^2 + R_3^2) + mR_2^2 \quad (2.2)$$

Here $R_2$ and $R_3$ are the internal and external radii of the roller, respectively, and $R_1$ is the radius of the axle.

Equation (2.1) is essentially nonlinear and cannot be integrated in quadratures. However, at small values of the angle $\alpha$ compared with its first and second derivatives, equation (2.1), with a sufficient degree of accuracy, can be linearized. We have for small $\alpha$ ($\sin\alpha \approx \alpha$, $\cos\alpha \approx 1$)

$$\sin(\phi - \alpha) = \sin\phi\cos\alpha - \cos\phi\sin\alpha \approx \sin\phi - \alpha\cos\phi \quad (2.3)$$

and in the stationary mode, that is when the equality

$$(R_2 - R_1 + R\cos\alpha)\frac{d\omega}{dt} = 0 \quad (2.4)$$

holds, we obtain that linearized equation (2.1) becomes

$$\frac{d^2\alpha}{dt^2} + (a_1 + b\cos\omega t)\alpha = b\sin\omega t \tag{2.5}$$

where

$$a_1 = \frac{R_2 R m \omega^2}{J_{kz}} \qquad b = \frac{R_2 m g}{J_{kz}} \tag{2.6}$$

Now it is necessary to investigate equation (2.5) from the point of view of the existence of its steady periodic solutions. That permits one to choose parameters of the researched system which provide realization of such operating modes. It is most preferable to obtain periodic solutions with the period equal to $\omega$ to make one turnover of the roller correspond to one turnover of its axle. These requires constant clearances between the rollers and, therefore, higher quality of classification of a processable material.

Equation (2.5) can be considered as a special case of the inhomogeneous Hill equation, or, in particular, as an inhomogeneous Mathieu equation.

## 3. Hill's equation

A homogeneous equation

$$y''(x) + [\Phi(x) + \lambda]y(x) = 0 \tag{3.1}$$

with a periodic function $\Phi(x)$ and a constant $\lambda$ is the Hill equation. The basic interest in the investigation of equations (2.5) and (3.1) connected to it represents, as it was mentioned above, a question on the existence and stability $\omega$-periodic solutions to these equations as only at such a periodicity the constant backlash between the rollers can be provided, and hence, qualitative work of the classifier. To this purpose, we shall consider the Hill equation in a more general view (Smirnoff, 1969)

$$y''(x) + p(x)y'(x) + q(x)y = 0 \tag{3.2}$$

with $\omega$-periodic factors $p(x)$ and $q(x)$.

If $y_1(z)$, $y_2(z)$ is the fundamental system of solutions to equation (3.2), its general solution can be written as

$$y(x) = C_1 y_1(x) + C_2 y_2(x) \tag{3.3}$$

According to the Floquet theorem (McLachlan, 1947), linear homogeneous equation (3.2) with periodic factors has the solution in the form $y(x) = \exp(\mu x)U(x)$, where $U(x)$ is a $\omega$-periodic function. To obtain this solution, a condition is entered

$$y(x + \omega) = \sigma[C_1 y_1(x) + C_2 y_2(x)] = \sigma y(x) \tag{3.4}$$

with some constant $\sigma$. Then assume $\sigma = \exp(-\omega\mu)$ and enter function

$$U(x) = e^{-\omega\mu} y(x) \tag{3.5}$$

Then under condition (3.4), the function $U(x)$ becomes $\omega$-periodic. Numbers $\sigma$ in equality (3.4) are determined as roots of the equation

$$\sigma^2 - 2A\sigma + 1 = 0 \tag{3.6}$$

The number A is called the characteristic Lyapunov constant. For real $q(x)$, $A$ is real as well. If the roots of equation (3.6) $\sigma_1$ and $\sigma_2$ are simple there are two linearly independent solutions to equation (3.2) of kind (3.4)

$$\eta_1(x + \omega) = \sigma_1 \eta_1(x) \qquad \eta_2(x + \omega) = \sigma_2 \eta_2(x) \tag{3.7}$$

Then, according to the Floquet theorem, in the case of various roots of equation (3.6), the functions $\eta_1(x)$ and $\eta_2(x)$ can be represented as

$$\eta_1(x) = e^{x\mu_1} U_1(x) \qquad \eta_2(x) = e^{x\mu_2} U_2(x) \tag{3.8}$$

where $\sigma_i = \exp(\omega\mu_i)$, that is $\mu_i = (\ln \sigma_i)/\omega$, $U_i(x)$ are $\omega$-periodic functions, $i = 1, 2$. If equation (3.6) has a multiple root $\sigma$, then only one function of $\eta_1(x)$ of kind (3.8) exists. Any other solution to the Hill equation, linearly independent of $\eta_1(x)$, cannot look like (3.8). From equation (3.6), it follows that this equation has a multiple root only for $|A| = 1$. In this case, two linearly independent solutions to equation (3.2) can be represented as

$$\eta_1(x) = e^{\mu x} U(x) \qquad \eta_2(x) = e^{\mu x}\left(\frac{a_{21}}{\sigma\omega} x U(x) + U_3(x)\right) \tag{3.9}$$

where the functions $U(x)$ and $U_3(x)$ are $\omega$-periodic, and the constant $a_{21} \neq 0$. So, the general solution to homogeneous Hill equation (3.2)

$$y(x) = C_1 \eta_1(x) + C_2 \eta_2(x) \tag{3.10}$$

will be not periodic. Moreover, if the multiple root of equation (3.6) $\sigma = 1$, we obtain $\mu = 0$, and therefore solutions (3.9) become

$$\eta_1(x) = U(x) \qquad \eta_2(x) = \frac{a_{21}}{\omega} x U(x) + U_3(x) \tag{3.11}$$

From (3.11) it follows that at $x \to \infty$ the function $\eta_2(x)$ unbounded modulo grows. Therefore, in the case $|A| = 1$, the general solution to equation (3.2) is not periodic and stable. That is why this case does not represent practical interest.

In general, the characteristic Lyapunov constant $A$ essentially influence the solutions to the Hill equation. If $|A| > 1$, the functions $\eta_1(x)$ and $\eta_2(x)$ can be represented as (3.8). As the functions $U_1(x)$ and $U_2(x)$ are periodic and continuous, they will be bounded at any $x$. In this case, roots of equation (3.6) are different and real. Accordingly to Viet's theorem, we have $\sigma_1 \sigma_2 = 1$, with one of the roots greater and another less than 1. So, the values $\exp(x\mu_i) = \exp(x\omega^{-1} \ln \sigma_i)$ $(i = 1, 2)$ at $x \to \infty$ behave differently. Let, for definiteness, $|\sigma_1| > |\sigma_2|$. Then $|\sigma_1| > 1$, and $|\sigma_2| < 1$. The real part of $\ln \sigma$ is equal to $\ln |\sigma|$. Therefore, we have $\ln |\sigma_1| > 0$ and $\ln |\sigma_2| < 0$. That means

$$\lim_{x \to \infty} e^{\frac{x}{\omega} \ln \sigma_1} = \infty \qquad \lim_{x \to 0} e^{\frac{x}{\omega} \ln \sigma_2} = 0 \qquad (3.12)$$

Consequently, the first summand in the right part of equality (3.10) unbounded modulo grows at $x \to \infty$, which makes the general solution to the Hill equation (if $C_1 \neq 0$) unstable.

Hence, at $|A| \geqslant 1$ there are no two linearly independent solutions to the Hill equation, simultaneously periodic and steady. It means that such a case, from the point of view of the considered problem, does not represent practical interest. Therefore, it is necessary to only consider the case $|A| < 1$. At $|A| < 1$, the roots $\sigma_1$ and $\sigma_2$ are determined by the equality

$$\sigma_{1,2} = A \pm i\sqrt{1 - A^2} \qquad (3.13)$$

where $\sigma_1$ and $\sigma_2$ are complex conjugate and $|\sigma_1| = |\sigma_2| = 1$. For complex $\sigma = \gamma + i\nu$, the determination of $\mu$ with the help of the equality $\sigma = \exp(\omega\mu)$ leads to the correlation $\mu = (\ln \sigma)/\omega$ with the function ln of a complex argument. Such a function is determined by the equality

$$\ln \sigma = \ln(\gamma + i\nu) = \ln |\sigma| + i \arg \sigma \qquad (3.14)$$

where

$$\arg \sigma = \begin{cases} \arctan \dfrac{\nu}{\gamma} & \text{for} \quad \gamma > 0 \\[2mm] \arctan \dfrac{\nu}{\gamma} + \pi & \text{for} \quad \nu \geqslant 0, \quad \gamma < 0 \\[2mm] \arctan \dfrac{\nu}{\gamma} - \pi & \text{for} \quad \nu < 0, \quad \gamma < 0 \end{cases} \qquad (3.15)$$

Owing to the equality of modules of $\sigma_1$ and $\sigma_2$ to 1, in equality (3.14) we obtain $\ln|\sigma_1| = \ln|\sigma_2| = 0$ for these values and therefore

$$\mu_1 = \mathrm{i}\frac{\arg\sigma_1}{\omega} \qquad\qquad \mu_2 = \mathrm{i}\frac{\arg\sigma_2}{\omega} \qquad\qquad (3.16)$$

Having denoted $\gamma = A$, $\nu = \sqrt{1 - A^2}$, we obtain that $\sigma_1 = \gamma + \mathrm{i}\nu$; $\sigma_2 = \gamma - \mathrm{i}\nu$. As the values $\sigma_1$ and $\sigma_2$ are complex conjugate, the next equality is valid

$$\arg\sigma_2 = -\arg\sigma_1 \qquad\qquad (3.17)$$

Therefore $\mu_2 = -\mu_1$ and, hence, $\exp(\omega\mu_2) = \exp(-\omega\mu_1)$. Thus, as in the case of $|A| \neq 1$, two linearly independent solutions to the Hill equation are represented as in (3.8), and in the considered case these solutions will become

$$\eta_1(x) = \mathrm{e}^{\mathrm{i}\frac{\beta}{\omega}x}U_1(x) \qquad\qquad \eta_2(x) = \mathrm{e}^{-\mathrm{i}\frac{\beta}{\omega}x}U_2(x) \qquad\qquad (3.18)$$

where $\beta = \arg\sigma_1$. In equalities (3.16), the functions $U_1(x)$ and $U_2(x)$ are $\omega$-periodic. Therefore, in the case when $2\pi/\beta$ is a rational number, that is can be expressed as $2\pi/\beta = m/n$, where $m$ and $n$ are integers, the functions $\eta_1(x)$ and $\eta_2(x)$ in formulas (3.12) will have the period $T = m\omega$. That is why the general solution to Hill equation (3.10) will be also an $m\omega$-periodic function.

If the number $2\pi/\beta$ is irrational, the functions $\exp(\pm\beta x/\omega)$ on the one hand and the function $U_1(x)$ and $U_2(x)$ on the other hand will have not a multiple period, therefore the function $\eta_1(x)$ and $\eta_2(x)$ in (3.12) and also general solution (3.10) will be oscillation, but not periodic. Owing to the periodicity of all functions making general solution (3.10), this solution will be limited at $x \to \infty$, and therefore steady.

Thus, it appears that the Hill equation can have a steady periodic solution only for $|A| < 1$ and the rational ratio $2\pi/\beta$. This case represents practical interest and should be duly addressed.

It is important here to note that the solution to the Hill equation can be represented only in the form of Fourier series. It is well known that Fourier series converges very slowly, and to approximate the solution with an essential accuracy, it is necessary to keep a large number of terms of such a series. Therefore, taking into account the fact that we are able to select parameters of the system in a broad region and we need only to provide a steady periodic mode of its operation, to obtain a compact explicit expression for solution to equation (2.5) we will consider this one as a Mathieu equation.

## 4. Mathieu's equation

In this particular case, Hill's equation (2.5) by means of transformation

$$z = \frac{\omega t}{2} \tag{4.1}$$

can be represented as the Mathieu equation in one of the standard forms (McLachlan, 1947)

$$\frac{d^2\alpha}{dz^2} + (a + 16q\cos 2z)\alpha = 16q\sin 2z \tag{4.2}$$

where

$$a = \frac{4a_1}{\omega^2} = 4\frac{R_2 R m}{J_{kz}} \qquad\qquad q = \frac{b}{4\omega^2} = \frac{R_2 mg}{4\omega^2 J_{kz}} \tag{4.3}$$

It is necessary to note that if $J_{kz}$ is proportional to $m$, then the values of $a$ and $q$ do not depend on the weight of rollers, and are determined only by geometry of the system. The parameter $q$, in addition, depends on the acceleration of gravity $g$. Owing to presence of the factor $\omega^2$ in the denominator of the formula for $q$, the parameter $q$ is small in comparison with the parameter $a$. It is much less than unit and strongly decreases with growing $\omega$.

Let us consider the homogeneous equation of Mathieu corresponding to (4.2)

$$\frac{d^2\alpha}{dz^2} + (a + 16q\cos 2z)\alpha = 0 \tag{4.4}$$

As it has already been marked, from the point of view of stable work of the classifier, it is necessary to obtain a periodic solution to equation (2.5) with the period equal to $2\pi/\omega$. It means that concerning the variable $z$, the period of this solution should be equal to $\pi$. If we find periodic linearly independent solutions to equation (4.4) in form of (3.18), it is necessary to put $b/\omega = m\pi$. Then the period of the function $\exp(\mathrm{i}\beta x/\omega) = \exp(\mathrm{i}m\pi x)$ will be equal to $m\pi$.

Therefore, in view of such a transformation of the period, functions (3.18) for equation (4.4) become

$$\eta_1(z) = \mathrm{e}^{\mathrm{i}m\pi z} U_1(z) \qquad\qquad \eta_2(z) = \mathrm{e}^{-\mathrm{i}m\pi z} U_2(z) \tag{4.5}$$

In such a representation, odd $\mathrm{se}_i(z,q)$ and even $\mathrm{ce}_i(z,q)$ Mathieu functions of the first kind are constructed. The greatest interest is put to the cases, when the index $i$ is odd as the solution to Mathieu's equation has the period $2\pi/\omega$. For cases of the even index $i$ the period of the solution is equal to $4\pi/\omega$,

which means that the solution will be repeated not through one but through two turnovers of the roller.

It is necessary to note that each of the mentioned Mathieu functions will be periodic solutions to equation (4.4) not for any value of the parameters $a$ and $q$, but only in the case when the parameter $a$ is a certain function of the parameter $q$. Such functions $a(q)$, are called the own values of Mathieu functions and are denoted by $a_i$ for functions $ce_i(z, q)$ and by $b_i$ for the function $se_i(z, q)$. Functions $a(q)$ are analytical, and the initial terms of their expansion in Taylor series are given in reference books. Thus, for small enough $q$ (in comparison with unit) the series converge quickly.

On the basis of the above stated, there is a following technique for selection of parameters of the system assuming periodic movements of rollers. It is necessary to take one of Mathieu functions to calculate $a_m$ or $b_m$ and to select such parameters of the system that equality (4.3) is valid. On the other hand, after defining $a_m$ and $b_m$ in view of (4.3) and (2.6), the parameters should be selected so that the equality

$$4\frac{R_2 R m}{J_{kz}} = c_m \tag{4.6}$$

holds, where $c_m = a_m$ or $c_m = b_m$ depending on the choice of a given Mathieu function.

## 5.  Inhomogeneous Mathieu's equation

Let us now consider inhomogeneous equation (4.2). For brevity, we shall denote its right hand side by $F(z)$. If $\eta_1(z)$ and $\eta_2(z)$ are a fundamental system of solutions to equation (4.4), the particular solution to equation (4.2) is usually obtained by a method of variation of arbitrary constants. This means that the particular solution is found in the form of (3.10) considering $C_1$ and $C_2$ as functions of $z$. Applying the method of variation of arbitrary constants, we obtain such a particular solution to be

$$\alpha(z) = C_1(0)\eta_1(z) + C_2(0)\eta_2(z) + \int_0^z \frac{F(\zeta)}{\Delta(\zeta)}[\eta_1(\zeta)\eta_2(z) - \eta_1(z)\eta_2(\zeta)]\,d\zeta \tag{5.1}$$

where

$$\Delta(z) = \eta_1(z)\eta_2'(z) - \eta_1'(z)\eta_2(z) \tag{5.2}$$

The function $\alpha(z)$ will be the general solution to equation (4.2) as well. However, the solutions to Mathieu's equation are represented as

$$\eta_1(z) = e^{i\beta z}U_1(z) \qquad\qquad \eta_2(z) = e^{-i\beta z}U_1(-z) \qquad\qquad (5.3)$$

Functions (5.3) have the period equal to $n\pi$ for integer $\beta$. Nevertheless, for integer $\beta$, functions (5.3) are dependent in the Jacobi sense and, consequently, cannot create a fundamental system of solutions to Mathieu's equation. In the case when $\beta$ is not integer, functions (5.3) do not have the required period.

Therefore, the obtaining of solutions to Mathieu's equation in view of (5.1) does not contain the cases representing practical interest. In connection with the above stated, other method for obtaining the particular solution to inhomogeneous equation (4.2), using not the fundamental system of solutions to equation (4.4), but only on one solution to this equation is here applied. It is clear that such a unique solution is necessary for the Mathieu function to be of integer order. The mentioned method consists in the following.

Let $\psi(z)$ be a solution to the equation (4.4). Having substituted in equation (4.2) the following transformation

$$\alpha(z) = U(z)\psi(z) \qquad\qquad (5.4)$$

we obtain

$$U''(z)\psi(z) + 2U'(z)\psi'(z) = F(z) \qquad\qquad (5.5)$$

In the domain, where $\psi(z)$ is not equal to zero, this equation, by replacing $V(z) = U'(z)$, results in a linear equation of the first order

$$V'(z) + \frac{2\psi'(z)}{\psi(z)}V(z) = \frac{F(z)}{\psi(z)} \qquad\qquad (5.6)$$

whose general solution is the function

$$V(z) = \exp\left(-\int \frac{2\psi'(z)}{\psi(z)}\,dz\right)\left[C_1 + \int \frac{F(z)}{\psi(z)}\exp\left(\int \frac{2\psi'(z)}{\psi(z)}\,dz\right)dz\right]$$

Therefore

$$U(z) = C_1 \int \exp\left(-\int \frac{2\psi'(z)}{\psi(z)}\,dz\right) +$$

$$+ \int\left[\exp\left(-\int \frac{2\psi'(z)}{\psi(z)}\,dz\right)\int \frac{F(z)}{\psi(z)}\exp\left(\int \frac{2\psi'(z)}{\psi(z)}\,dz\right)\right]dz + C_2$$

Taking into account that

$$\int \frac{\psi'(z)}{\psi(z)}\,dz = \ln\psi(z)$$

we obtain

$$\exp\!\Big(2\int \frac{\psi'(z)}{\psi(z)}\,dz\Big) = \exp\ln\psi^2(z) = \psi^2(z)$$

therefore

$$U(z) = C_1\int \frac{1}{\psi^2(z)}\,dz + \int\Big(\frac{1}{\psi^2(z)}\int F(z)\psi(z)\,dz\Big)\,dz + C_2$$

Hence, the general solution to equation (4.2) is

$$\alpha(z) = C_2\psi(z) + C_1\psi(z)\int \frac{dz}{\psi^2(z)} + \psi(z)\int\Big(\frac{1}{\psi^2(z)}\int F(z)\psi(z)\,dz\Big)\,dz \quad (5.7)$$

The integral of the periodic function under certain conditions is a periodic function of the same period as well. So, while $\psi(z)$ and $F(z)$ are periodic functions of the same period and satisfy such conditions, solution (5.7) will be a periodic function of the same period for any values of $C_1$ and $C_2$.

In the considered case, the state of matter is following. If the function $\psi(z)$ is assumed to be the Mathieu function of an integer order, the period of the function $\psi(z)$ will be equal to $\pi$ or $2\pi$. The right hand side of equation (4.2) $F(z) = 16q\sin 2z$ has the period equal to $\pi$, therefore solution (5.7) will have a period which would be equal to the period of the Mathieu function.

Concerning solution (5.7), it is necessary to make one essential remark. This solution contains in the denominators some terms of the Mathieu functions. As the Mathieu functions have zero, therefore in neighborhoods of zero of the corresponding Mathieu functions, solution (5.7) loses sense, and in these neighborhoods there will be essential features of the solution.

In these cases, it is possible to offer the following procedure. As in the neighborhood of zero the solution to equation (4.2) linearly depends on $\alpha$ and becomes small, then in such circumstances, with a high degree of accuracy, it is possible to limit considerations to the equation

$$\frac{d^2\alpha}{dz^2} = 16q\sin 2z \quad (5.8)$$

whose general solution will be a function

$$\alpha(z) = C_1 z + C_2 - 4q\sin 2z$$

Function (5.8) at $C_1 = 0$ has the period equal to $\pi$. At $C_1 \neq 0$, $\alpha(z)$ quickly grows, $\alpha(z)$ no longer stays close to zero, which allow us to come back to solution (5.7).

## 6.   Conclusions

As a result of the carried out analysis, it was possible to show that the line-
arised inhomogeneous equation of motion of rollers yields periodic solutions of
periods equal to $2\pi/\omega$ and $4\pi/\omega$. These solutions are obtained in an explicit
form. It was shown that periodic solutions are steady, but not asymptotic. On
the basis of these solutions, a technique of selection of parameters for design
purposes of classifiers with rollers is offered.

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**Obszar ustalonego okresowego ruchu obrotowego rolek drgającego
sortownika**

### Streszczenie

    W pracy zaprezentowano problem dynamiki drgającego sortownika przeznaczo-
nego do materiałów sypkich. Głównym celem analizy jest znalezienie i uzasadnienie
takiego wyboru parametrów konstrukcyjnych sortownika, które zapewnią ustalony
i okresowy stan pracy rolek. Równania ruchu są nieliniowe. W pracy dokonano prze-
kształcenia równań tak, aby opisywały opóźnienie ruchu rolek w stosunku do rotacji

względem osi, na których te rolki zamocowano. Przy niewielkich wartościach tego opóźnienia, równania ruchu mogą być zlinearyzowane z utrzymaniem wystarczającej dokładności. Linearyzacja prowadzi do równań Hilla, które przy spełnieniu pewnych warunków mogą zostać przekształcone do postaci Mathieu. W artykule przedstawiono rozwiązania tych równań oraz zbadano ich stabilność.