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APPLICATION OF GENETIC ALGORITHM AND FOURIER COEFFICIENTS (GA-FC) IN MECHANISM SYNTHESIS

Roman Starosta

Poznan University of Technology, Institute of Applied Mechanics, Poznań, Poland
e-mail: roman.starosta@put.poznan.pl

The paper concerns synthesis of a four-bar linkage as a curve generator. Fourier coefficients of the curvature are applied to represent a closed curve. A genetic algorithm (GA) was adapted to solve the problem. The proposed method was successfully verified by many examples.

Key words: evolutionary algorithms, mechanism synthesis

1. Introduction

Rapid increase of computer calculation power enables application and development of new computation techniques. It affects the development of computer methods in the mechanism synthesis.

The subject of mechanism synthesis is not a new problem. The problem investigated in the presented paper consists in generation of a given curve by a chosen point of a mechanism. In the paper, it is a coupler point of a four-bar linkage (Fig. 1).

In general, the coupler draws a curve that is not identical to the given one. Owing to this, great attention is focused on the problem of approximation and numerical costs.

The genetic algorithm (GA) used in the calculation is numbered among deterministic-probabilistic methods (Goldberg, 1989, 2003; Michalewicz, 1992). GAs are used and effectively applied to various optimization tasks, among others to synthesis of mechanism (Fang, 1994; Kunjur and Krishnamurty, 1997; Laribi *et al.*, 2004; Białas-Heltowski *et al.*, 2004). Laribi *et al.* (2004) showed that in the case of small dimension of the work space, a genetic algorithm significantly improves accuracy of the solution. For a large scope of possible parameters, deterministic methods are more efficient than GAs.

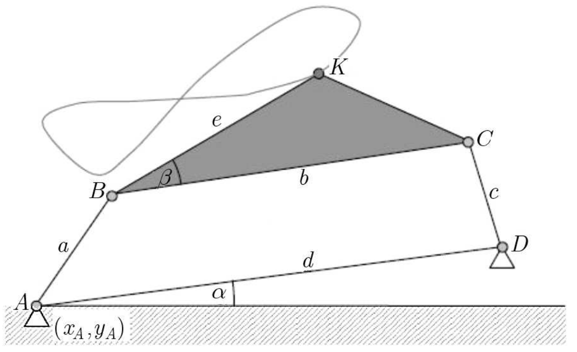

Fig. 1. Parameters defining geometry and position of a four-bar linkage

An approach using neural networks to synthesize a curve is also developed in the last years (Vasiliu and Yannou, 2001; Walczak, 2006; Starosta, 2006).

There are 9 unknown quantities in the synthesis of a four-bar linkage: lengths of links a, b, c, d, e, angles α, β and co-ordinates of the point A, i.e. x_A and y_A. In the present paper, the number of unknowns was reduced to 5.

The way of defining the goal function plays the major role in the synthesis task. In the paper, a new method to evaluate the distance between the given and the generated curve is proposed. The goal function is constructed in such a way so not to take into consideration the location, orientation in space, and scaling of the mechanism dimension in the process of synthesis. It effectively shortens the time of calculation.

2. Aim of the work

One of the methods of curve description consists in its representation by means of Fourier Coefficients FCs. So far, the curvature of a curve was not used in the mechanism synthesis. The possibility of using the curvature and algorithm of normalization of Fourier expansion coefficients was presented in Buśkiewicz (2006). The objective of the paper is to apply such a description for construction the goal function in the mechanism synthesis solved by the GA. The curve description minimizes the number of sought parameters of the mechanism drawing a given curve and, therefore, the additional algorithm of determination of its dimensions, location, and orientation on the plane is elaborated.

The paper introduces a method of optimal determination of link lengths of a four-bar linkage. The mechanism is arranged in order to do a given kinematic job.

3. Curve description

The essential part of the paper is a method of description of both the given and synthesized curves (Buśkiewicz, 2006), which is presented in brief below.

A curve of the length s is given by the set of m points A_i of coordinates (x_i, y_i). $sb_i = \sum_{l=2}^{i} |A_{l-1}A_l|$ is the arc length measured from the beginning point to the point i. $s_i^* = sb_i/s$ is the natural parameter taking values from the interval $[0, 1)$. $s^* = 1$ is the normalized length of the curve, α_i is the angle between the tangent to the curve at the point i and the horizontal line. The curvature function is defined as follows

$$f_i(s^*) = \Big(\frac{d\alpha}{ds^*}\Big)_i \approx \frac{\alpha_{i+1}(s_{i+1}^*) - \alpha_i(s_i^*)}{\varDelta s_i^*} \tag{3.1}$$

The curvature is approximated by the Fourier series

$$f(s^*) = a_0 + \sum_{n=1}^{k} [a_n \cos(2n\pi s^*) + b_n \sin(2n\pi s^*)] \tag{3.2}$$

with the coefficients (FC)

$$\begin{aligned}
a_0 &= \sum_{i=1}^{m} f(s_i^*) \varDelta s_i^* \\
a_n &= 2\sum_{i=1}^{m} f(s_i^*) \cos(2n\pi s_i^*) \varDelta s_i^* \\
b_n &= 2\sum_{i=1}^{m} f(s_i^*) \sin(2n\pi s_i^*) \varDelta s_i^*
\end{aligned} \tag{3.3}$$

Rotation by an angle, translation by a vector and scaling do not change the curvature function. The normalization of the FCs provides their invariance with respect to the change of the beginning point, the direction reversal and the mirror reflection of the curve (Fig. 2).

The description minimizes the number of parameters of a curve and enables one to effectively use numerical algorithms based on the GA in the mechanism synthesis.

The goal function is the distance norm between the FC of the given and generated curves

$$E = \sum_{i=1}^{n} [(a'_{1i} - a'_{2i})^2 + (b'_{1i} - b'_{2i})^2] \tag{3.4}$$

where (a'_{1i}, b'_{1i}), (a'_{2i}, b'_{2i}) are normalized FCs (NFC) of the curves, respectively.

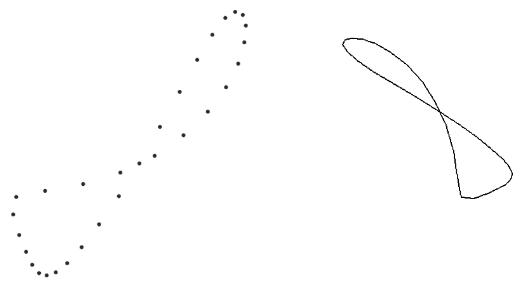

Fig. 2. Examples of curves of the same FC

4. Discussion on the genetic algorithm (GA)

The method presented in the paper belongs to the evolutionary programming class. The algorithm searches through the space of alternative solutions using the rules of natural selection and inheritance. The method combines the evolutionary principle of survival of the best fitted individual with systematic and partially random swap of information. There are many papers containing tests of optimization problems using the evolutionary idea (Michalewicz, 1992; Michalewicz *et al.*, 1994, 1996). Many authors apply GAs to optimization of mechanical structures. Among others, optimization of shape and topology of two dimensional structures was investigated by Sokołowski and Żochowski (1999) also in combination with other methods (Burczyński and Kokot, 1998, 2003). A comparison of four kinds of GA applied to mechanisms synthesis was discused in Białas-Heltowski *et al.* (2004).

The idea of GA is adopted from rules governing biological systems that makes them more resistant than their deterministic equivalents. This is the reason of great interest in evolutionary algorithms for solving a wide spectrum of problems. A maximized quantity in a GA is the so-called "fitting". This is a property or a set of properties favourable with respect to the desired goal function. In the investigated synthesis problem, an individual (alternative solution, mechanism) has the better fitting when the generated curve is more "similar" to the desired one.

In calculations, the fitting is determined for each alternative individual as the inverse to distance norm (3.4) between the generated and desired curves.

A block diagram of the synthesis algorithm of a four-bar linkage is presented in Fig. 3. The vector containing lengths of the mechanism links and the angle β is the individual.

The basic operations of the GA are: natural selection, crossover and mutation. The selection of the individuals is carried out using a roulette method, i.e. by drawing of an individual with the probability proportional to its fitting. The crossover and mutation are made on an encoded form. The binary representation is applied, i.e. each real number being a dimension or an angle is converted into a binary chain. The crossover consists in replacing a fragment of the binary chain between two randomly chosen individuals. One point crossing was admitted in the paper. In each generation, the whole population is randomly divided into two parts, and every individual of the first one is crossed with another from the second part.

The mutation occurs with some probability and consists in replacing a bit 0 with 1 or vice versa at a randomly chosen position in the binary representation.

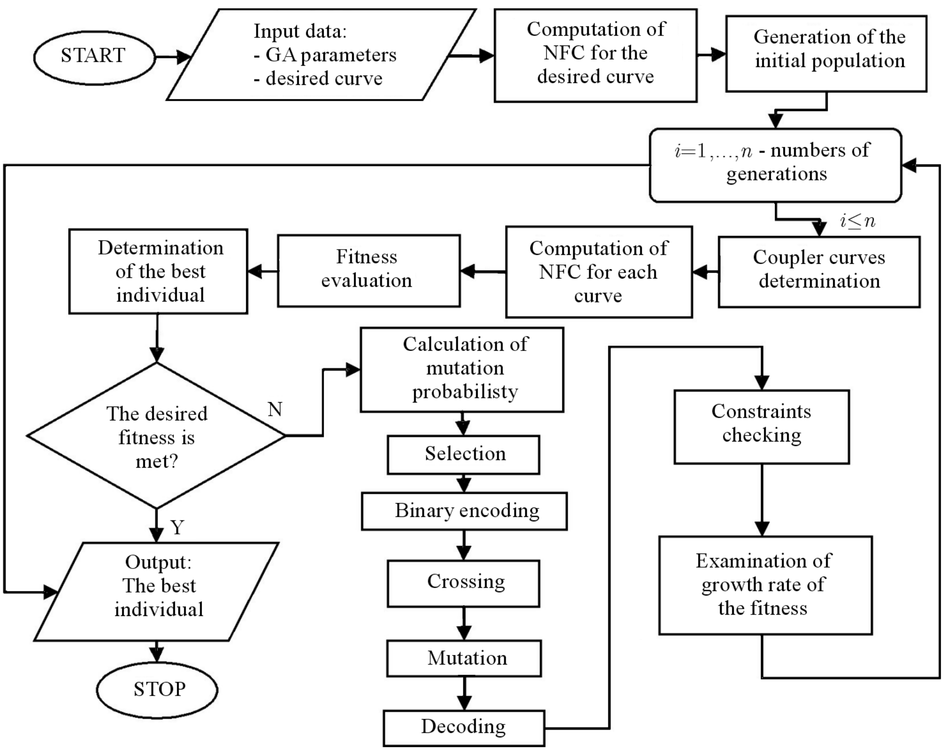

Fig. 3. Computational algorithm

The mechanism synthesis is carried out in accordance with the algorithm presented in Fig. 3 and leads to inheritance and improving advantageous properties of individuals in the next generations. Some additional new improvements are implemented in order to increase the efficiency of GA. Not all of them are placed in the block diagram in order not to blur its legibility.

The new enhancements are the following:

- The probability of mutation is adaptively controlled, it is inversely proportional to the fitting of the best individual.
- If the growth in fitting does not occur in the established number of generations the whole population is randomly disturbed (the point significantly improves convergence to the optimum solution). The magnitude of disturbance is inversely proportional to the fitting of the best individual.
- The best individual is always stored up, i.e. passes to the next generation which guarantees that the best properties are not lost.

Each solution is constrained by the Grashof conditions. When a solution does not meet these conditions it is rejected and one of its parents remains. Due to this, the number of populations is preserved.

5. Determination of the links lengths and mechanism location

The GA searches for the optimum solutions among the mechanisms (Fig. 1) attached at the origin $(A = (0, 0))$ of the crank length $a = 1$ and with $\alpha = 0$. The found mechanism generates a curve similar to the desired one. Proper dimensions of the mechanism, its location and orientation on the plane have to be determined so as the coupler curve fits best the desired one. To perform these transformations, the following geometric quantities are used: the curve length, centroid of a curve, second moments of the curve and position of the principal axis.

The desired and generated curves are defined by the coordinates of points. The geometrical properties are determined by means of the following formulae (5.1)-(5.4).

The centroids of the curve

$$x_C = \frac{1}{2s}\sum_{i=1}^{n} s_i(x_{i+1} + x_i) \qquad y_C = \frac{1}{2s}\sum_{i=1}^{n} s_i(y_{i+1} + y_i) \tag{5.1}$$

where $s_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$.

The central second moments

$$
\begin{aligned}
I_{xC} &= \sum_{i=1}^{n} s_i \Big(\frac{(y_{i+1}-y_i)^2}{12} + \Big(y_C - \frac{y_{i+1}+y_i}{2}\Big)^2 \Big) \\
I_{yC} &= \sum_{i=1}^{n} s_i \Big(\frac{(x_{i+1}-x_i)^2}{12} + \Big(x_C - \frac{x_{i+1}+x_i}{2}\Big)^2 \Big) \\
I_{xyC} &= \sum_{i=1}^{n} s_i \Big(\frac{(y_{i+1}-y_i)(x_{i+1}-x_i)}{12} + \Big(y_C - \frac{y_{i+1}+y_i}{2}\Big)\Big(x_C - \frac{x_{i+1}+x_i}{2}\Big) \Big)
\end{aligned}
\tag{5.2}
$$

In the foregoing equations, $x_{n+1} = x_1$.

The principal second moments about the centroidal axes are

$$
I_{1,2} = \frac{1}{2}(I_{xC} + I_{yC})^2 \pm \sqrt{\frac{1}{4}(I_{xC} - I_{yC})^2 + I_{xyC}^2} \tag{5.3}
$$

and the principal angle

$$
\alpha = \frac{1}{2} \arctan \frac{-2I_{xyC}}{I_{xC} - I_{yC}} \tag{5.4}
$$

An algorithm for computing real geometrical parameters of the mechanism is presented in Fig. 4. The subscripts d and g mean that the quantity corresponds to the desired and generated curves, respectively. The following denotations are introduced: $\boldsymbol{S}_k$ – scaling with respect to the centroid with the scaling coefficient equal to k in the x and y directions, $\boldsymbol{R}_\alpha$ – rotation by α about the centroid, $\boldsymbol{T}_{\boldsymbol{t}}$ – translation by a vector $\boldsymbol{t}$.

Figure 5 illustrates geometrical properties of the curve used to find the mechanism dimensions, its position and orientation on the plane.

6. Numerical solutions by GA

Below are presented five examples which illustrate correctness of the method. All calculations were made on a PC class computer, Intel Core 2 Duo 2.66 GHz, 1 GB RAM. The algorithms presented in Figs. 3 and 4 were implemented in FORTRAN 95.

In all examples, the length of the binary chain was assumed 32.

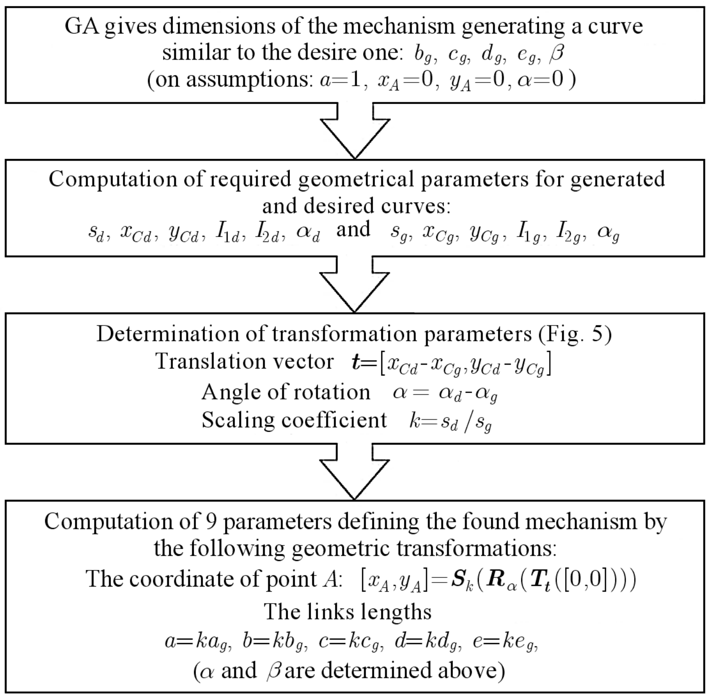

Fig. 4. A block diagram for determination of the real dimensions as well as location and orientation of the mechanism

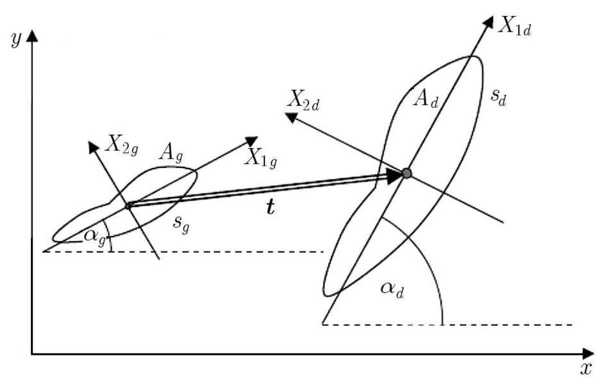

Fig. 5. Translation vector and orientation of the centroidal axes for both the desired and generated curves (indexed with subscripts d and g, respectively)

Example I

The goal of the example is to compare the results obtained by the proposed (GA-FC) method with those obtained by other authors. The example was presented by Kunjur and Krishnnamuty (1997) – KK, Cabrera *et al.* (2002) – CSP, and Laribi *et al.* (2004) – LMRZ. In the mentioned papers, the synthesis was made using some variants of genetic algorithms and a genetic algorithm coupled with a fuzzy logic controller . Results obtained by Laribi *et al.* (2004) are compared to those obtained by means of other classical methods (central difference and gradient), and superiority of the genetic algorithm is proved.

The desired curve is defined by a set of points. Their coordinates are given in Table 1.

Table 1. Coordinates of points defining the desired curve

Point	1	2	3	4	5	6	7	8	9
x	0.5	0.4	0.3	0.2	0.1	0.005	0.02	0.0	0.0
y	1.1	1.1	1.1	1.0	0.9	0.75	0.6	0.5	0.4
Point	10	11	12	13	14	15	16	17	18
x	0.03	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.6
y	0.3	0.25	0.2	0.3	0.4	0.5	0.7	0.9	1.0

Figure 6 shows evolution of norm (3.4) with respect to the number of generations.

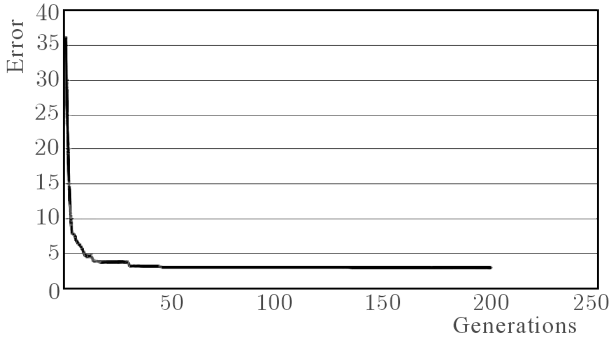

Fig. 6. Evolution of the error considered as the value of norm (3.4)

The measure of closeness and similarity of the desired and generated curves proposed in Laribi *et al.* (2004) is the conformity of chosen geometrical parameters. The same procedure was applied in order to compare the results obtained by using GA-FC with results taken from the references.

The vector $\boldsymbol{G}$ characterizes a given curve

$$\boldsymbol{G} = [w, h, s, A, x_C, y_C, I_{xC}, I_{yC}, I_{xyC}] \tag{6.1}$$

where

$$w = \max(x_i) - \min(x_i) \qquad \text{for} \qquad i = 1, \dots, n$$

and

$$h = \max(y_i) - \min(y_i) \qquad \text{for} \qquad i = 1, \dots, n$$

The remaining elements of $\boldsymbol{G}$ are expressed by Eqs. (5.1)-(5.4) and then they are normalized to be useful as a measure of the curves similarity

$$\boldsymbol{G}_n = \Big[\frac{h}{w}, \frac{s}{w}, \frac{A}{w^2}, \frac{x_C}{w}, \frac{y_C}{w}, \frac{I_{xC}}{w^3}, \frac{I_{yC}}{w^3}, \frac{I_{xyC}}{w^3}\Big] \tag{6.2}$$

The error is computed as follows

$$e = |\boldsymbol{G}_{nd} - \boldsymbol{G}_{ng}| \tag{6.3}$$

The subscripts d and g mean that given quantity corresponds to the desired and generated curves, respectively. The results obtained by Kunjur and Krishnamurty (1997), Laribi *et al.* (2004) and Cabrera *et al.* (2002) are collected in Table 2 and compared with those obtained by the proposed (GA-FC) method.

Figure 7 shows curves generated by the computer program executing the GA-FC method in search of the desired curve.

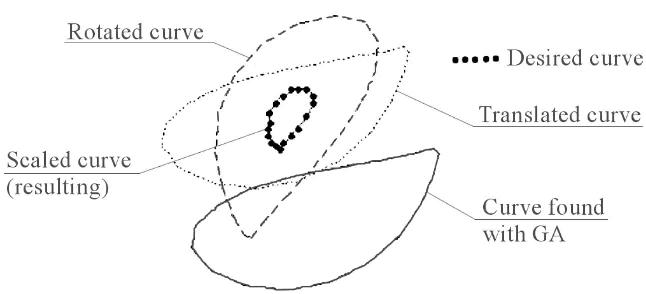

Fig. 7. Curve generated by the coupler (found with GA) and curves transformed to fit it to the desired one

The dimensions of optimal mechanisms found by the GA-FC and those described above mentioned papers are collected in Table 3.

The results presented in Table 2 show significant advantage of the presented method over the approaches used by the mentioned authors. For the tested example, the error after applying the GA-FC is about 5 times less than the

Table 2. Geometrical parameters of curves obtained with various methods

	Desired curve	KK	CSP	LMRZ	**GA-FC**
w	0.6	0.55	0.56	0.59	**0.6029**
h/w	1.5	1.63	1.47	1.50	**1.5285**
A/w^2	0.9187 1.8*	 2.25*	 1.82*	 1.96*	**0.9113**
L/w	3.9292	4.14	3.85	3.93	**3.9097**
x_C/w	0.4458	0.42	0.48	0.47	**0.4436**
x_C/w	1.1494	1.24	1.19	1.17	**1.1437**
I_{xC}/w^3	0.8963	1.16	0.88	0.93	**0.9032**
I_{yC}/w^3	0.4652	0.52	0.44	0.49	**0.4605**
I_{xyC}/w^3	0.2880 2.05*	 1.97*	 2.33*	 2.15*	**0.2959**
e	–	0.62	0.29	0.20	0.03775

* The values A and I_{xy} presented in Laribi *et al.* (2004) are incorrect. Most likely, these values for the desired and generated curves were computed from the same wrong equation.

Table 3. Parameters defining the mechanism and its location obtained by various authors

	a	b	c	d	e	x_A	y_A	β	α
KK	0.42	2.32	3.36	4.07	3.90	−3.06	−1.3	−15.6°	−9.1°
CSP	0.27	1.18	2.13	1.87	0.91	1.13	0.66	204°	249°
LMRZ	0.23	5.58	2.05	3.05	2	1.77	−0.64	67.7°	57°
AG-FC	**0.28**	**0.36**	**0.98**	**1.01**	**0.36**	**0.074**	**0.191**	**247°**	**−139.8°**

error of the results cited in the papers by Kunjur and Krishnamurty (1997), Cabrera *et al.* (2002) and Laribi *et al.* (2004). The calculation time is estimated at 0.8 s for the assumed parameters of GA: the number of specimens in each generation $N_s = 100$, the initial probability of mutation $M_p = 0.01$, the number of generations $G_n = 200$. In the last two cited papers, for the same mechanism the authors worked for 3.25 s on a Pentium processor 800 MHz and 1.32 s on the Athlon 1800 MHz, respectively.

Shorter times of calculation ensure from the computer power. Better fitting of the generated and given curves is probably due to the effective way of curve encoding as a FC and a lower number of design variables in GA. Moreover, an improvement of the convergence is caused by some amendments to the GA described in Section 4.

Example II

In example II, the desired curve is generated by a four bar linkage. It is done certain that there exists a synthesized mechanism the coupler of which draws a desired curve.

The parameters of the mechanism generating the desired curve are: $a = 1.0$, $b = 2.4$, $c = 1.9$, $d = 3.12$, $e = 3.1$, $\alpha = \pi/6$, $x_A = 0.0$, $y_A = 0.0$, $\beta = 0.0$.

The desired curve is then reflected with respect to the axis Oy, translated by the vector $[2.0, 0.0]$ and rotated by the angle $\pi/4$ with respect to the origin of the coordinate system.

The parameters of the AG assumed in the calculation are the same as in example I ($N_s = 100$, $M_p = 0.01$, $G_n = 200$). The number of points defining the desired curve equals 18.

The result found by the GA is presented in Fig. 8. The transformations on the generated curve are done according to the scheme given in Fig. 5. Figure 8 shows that the resultant curve very accurately render the original shape.

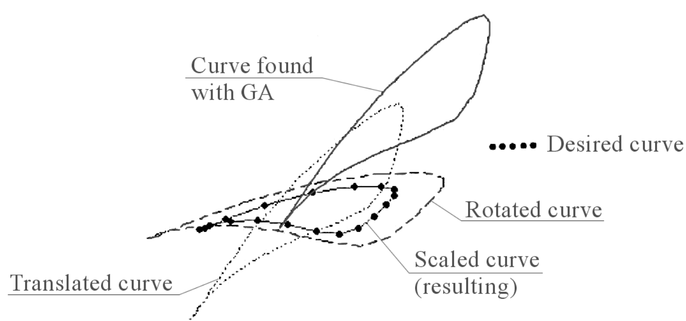

Fig. 8. Resulting coupler curves and their transformations aimed at the best fitting to the desired curve

To generalize the concept of the error, a new measure of the solution correctness is proposed. The measure of congruence of the desired and generated curves is a relative error expressed by their principal second moments of inertia about the centroidal axis normalized with respect to the curve length

$$e_1 = \frac{|I^*_{1g} - I^*_{1d}|}{I^*_{1d}} \qquad e_2 = \frac{|I^*_{2g} - I^*_{2d}|}{I^*_{2d}} \tag{6.4}$$

where

$$I^*_{1g} = \frac{I_{1g}}{s_g^3} \qquad I^*_{2g} = \frac{I_{2g}}{s_g^3} \qquad I^*_{1d} = \frac{I_{1d}}{s_d^3} \qquad I^*_{2d} = \frac{I_{2d}}{s_d^3}$$

The foregoing normalized quantities enable computation of the error of the AG results without necessity of performing the geometrical transformations described in Fig. 4.

The geometrical properties are calculated for:

— desired curves

$$s_d = 4.87737 \qquad I^*_{1d} = 1.9498 \cdot 10^{-2} \qquad I^*_{2d} = 1.13210 \cdot 10^{-3}$$

— curves generated by the GA

$$s_g = 4.87738 \qquad I^*_{1g} = 1.94983 \cdot 10^{-2} \qquad I^*_{2g} = 1.13107 \cdot 10^{-3}$$

The errors computed from Eqs. (6.4) are

$$e_1 = 4.420 \cdot 10^{-5} \qquad e_2 = 7.616 \cdot 10^{-4}$$

Table 4 contains parameters defining the link lengths, position and orientation of the transformed four-bar linkage on the plane, the coupler point which traces the curves presented in Fig. 8.

Table 4. Parameters defining the mechanism and its location

a	b	c	d	e	x_A	y_A	β	α
0.66	1.25	1.58	2.05	2.46	−0.95	1.56	0.87	−0.61

The time of numerical evaluations is below 1 s. The comparison of I_1 and I_2 allows one to state that the synthesis problem is executed satisfactorily.

Example III

An arc of an ellipse is taken as the desired curve. The method of curve description presented in this paper is related to closed curves. It is assumed that the coupler point goes through the given curve and returns by the same trajectory. Parametric equations of the arc are

$$x = 2 + 2\cos\phi \qquad y = 3 + \sin\phi$$

where $\phi \in \langle \pi/6, 2\pi/3 \rangle$.

The number of points defining the path is 30. The other parameters of the GA are the same as in Examples I and II ($N_s = 100$, $M_p = 0.01$, $G_n = 200$). The results of synthesis are shown in Fig. 9.

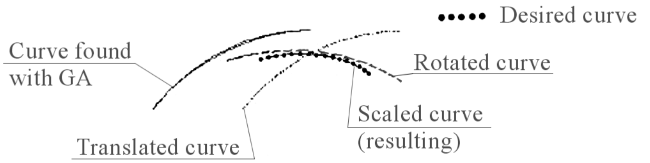

Fig. 9. Results of the ellipse arc synthesis and the curve drawn by the coupler point K

The geometrical properties are calculated for:
— desired curves

$$s_d = 5.697 \qquad I^*_{1d} = 2.010 \cdot 10^{-2} \qquad I^*_{2d} = 2.372 \cdot 10^{-4}$$

— curves generated by the AG

$$s_g = 9.136 \qquad I^*_{1g} = 2.003 \cdot 10^{-2} \qquad I^*_{2g} = 2.581 \cdot 10^{-4}$$

The errors computed from Eqs. (6.4) are

$$e_1 = 3.48 \cdot 10^{-3} \qquad e_2 = 8.82 \cdot 10^{-2}$$

Table 5. Dimensions of the four-bar linkage which generates the arc

a	b	c	d	e	x_A	y_A	β	α
0.623	1.77	1.95	1.29	2.57	2.27	−0.013	0.22	−0.59

Time of numerical evaluations is estimated at 0.7 s.

Example IV

The problem of synthesis of a straight-line mechanism is now considered – the coupler draws a straight line.

Table 6. Points of the straight line

Point	1	2	3	4	5	6	7	8	9	10
x	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
y	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0

Figure 10 presents the final solution.

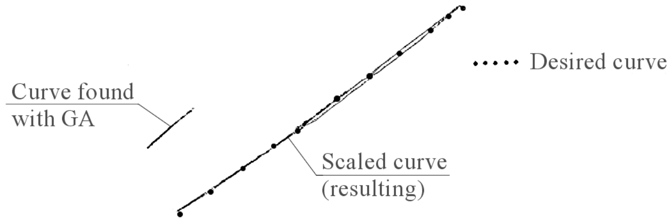

Fig. 10. Results of the straight line synthesis

The geometrical properties calculated for the line segment are:
— for desired line segment

$$s_d = 25.46 \qquad I^*_{1d} = 2.083 \cdot 10^{-2} \qquad I^*_{2d} = 0.0$$

— segment generated by the AG

$$s_g = 4.34 \qquad I^*_{1g} = 2.082 \cdot 10^{-2} \qquad I^*_{2g} = 5.145 \cdot 10^{-6}$$

The error computed from Eqs. $(6.4)_1$ is:

$$e_1 = 4.8 \cdot 10^{-4}$$

The dimensions of the four-bar linkage generating the straight line approximation are collected in Table 7.

Table 7. Dimensions of the four-bar linkage which generates the straight line

a	b	c	d	e	x_A	y_A	β	α
5.87	19.38	31.77	36.73	21.46	4.99	1.90	0.409	−3.37

Parameters of the AG assumed in the calculation are: $N_s = 100$, $M_p = 0.01$, $G_n = 600$. Time of calculations: 1.56 s.

7. Conclusions

The synthesis carried out by the GA-FC method gives satisfactory results in comparison with other methods (example I). The reduction in the number of coefficients describing a curve effectively improves the convergence of the GA. The relevant geometric transformations are performed on the mechanism after completing a particular synthesis. It enables one to determine all 9 parameters defining the position and orientation of the four-bar linkage.

In every run of the GA, the results are slightly different. It is known that evolutionary programs usually do not give an absolutely optimal solution. The presented method may be used as a convenient way to obtain a good approximation to start a classical deterministic method.

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Zastosowanie algorytmu genetycznego i współczynników Fouriera (GA-FC) w syntezie mechanizmów

Streszczenie

Rozważanym zagadnieniem jest synteza czworoboku przegubowego jako generatora krzywej. Zastosowano nowy sposób reprezentowania krzywej zamkniętej za pomocą współczynników Fouriera. Do rozwiązania zadania został zaadaptowany algorytm genetyczny. Proponowana metoda została z sukcesem przetestowana na przykładach.

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