

THERMAL CREEP STRESS AND STRAIN ANALYSIS IN A NON-HOMOGENEOUS SPHERICAL SHELL

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The purpose of this paper is to present study of thermal creep stress and strain rates in a non-homogeneous spherical shell by using Seth's transition theory. Seth's transition theory is applied to the problem of creep stresses and strain rates in the non-homogeneous spherical shell under steady-state temperature. Neither the yield criterion nor the associated flow rule is assumed here. With the introduction of thermal effect, values of circumferential stress decrease at the external surface as well as internal surface of the spherical shell. It means that the temperature dependent materials minimize the possibility of fracture at the internal surface of the spherical shell. The model proposed in this paper is used commonly as a design of chemical and oil plants, industrial gases and steam turbines, high speed structures involving aerodynamic heating.

Keywords: stress, strain rates, thermal, spherical shell, non-homogeneous

1. Introduction

Spherical shell structures have found widespread use in modern technology such as design of chemical and oil plants, accumulator shells, pressure vessel for industrial gases or media transportation of high-pressurized fluids and piping of nuclear containment, high speed structures involving aerodynamic heating, submerged undersea structures, earth sheltered domes, and the like. These spherical systems are effective from the perspectives of both structural and architectural design. In many of these cases, the spherical shells have to operate under severe mechanical and thermal loads causing significant creep and thus reducing its service life. The collapse or damage is initiated by creep, shrinkage and thermal effects, or from their interaction with structures that both experience or do not experience environmental degradation. Consequently, demand for strengthening and upgrading the existing concrete structures, because of damage caused by long-term effects and excessive structural deformations, has been recognized. However, before the application of costly strengthening techniques, understanding of nonlinear long-term behaviour of the existing and new spherical shells is essential, and the development of suitable and reliable theoretical approaches for their analysis and safety assessment is required. Creep effects generally increase deformations of a shell structure even under room temperatures, and are usually only considered to affect behaviour at the service ability limit states. Therefore, the analysis of

long term steady state creep deformations of shells is very important in these applications (Hamed *et al.*, 2010; Kashkoli and Nejad, 2014). Due to the occurrence of these creep deformations, non-homogeneous materials are widely used in the engineering applications. Non-homogeneous materials are a specific class of composite materials known as functionally graded materials (FGM) in which constituents are graded in one or more direction with continuous variation to achieve desired properties. The smooth grading of the constituents result in better thermal properties, higher fracture toughness, improved residual stress distribution and reduced stress intensity factors. These properties allow non-homogeneous structures to withstand high pressure under elevated thermal environment. Therefore, the analysis of non-homogeneity in the spherical shell through a mathematical model by taking one and all the complexities into consideration is the major concern of researchers (Kar and Panda, 2016). Some degree of non-homogeneity is present in a wide class of materials such as hot rolled metals, magnesium and aluminum alloys. Non-homogeneity can also be introduced by a certain external field which is a thermal gradient material as the elastic moduli of the materials vary with temperature (Olszak, 1960). Penny (1967) obtained the effects of creep in spherical shells by analysis similar to the corresponding elastic ones described here. Miller (1995) evaluated solutions for stresses and displacements in a thick spherical shell subjected to internal and external pressure loads. You *et al.* (2005) presented an accurate model to carry out elastic analysis of thick-walled spherical pressure vessels subjected to internal pressure. Kellogg and King (1997) developed a finite element model of convection in a spherical axisymmetric shell that we use to simulate upwelling thermal plumes in the mantle. Thakur (2011) analyzed creep transition stresses of a thick isotropic spherical shell by infinitesimal deformation under steady state of temperature and internal pressure by using Seth's transition theory. Seth's transition theory does not acquire any assumptions like the yield condition, incompressibility condition, and thus poses and solves a more general problem from which cases pertaining to the above assumptions can be worked out. This theory utilizes the concept of a generalized strain measure and asymptotic solution at critical points or turning points of differential equations defining the deformed field and has been successfully applied to a large number of problems (Seth, 1962, 1966; Thakur, 2011, 2014; Thakur *et al.*, 2016, 2017). Seth (1962) defined the concept of generalized strain measures as

$$e_{ii} = \int_0^{e_{ii}^A} \left(1 - 2 e_{ii}^A\right)^{\frac{n}{2}-1} d e_{ii}^A = \frac{1}{n} \left[1 - \left(1 - 2 e_{ii}^A\right)^{\frac{n}{2}}\right] \quad i = 1, 2, 3 \quad (1.1)$$

where n is the measure and e_{ii}^A are the Almansi finite strain components. For $n = -2, -1, 0, 1, 2$ it gives the Cauchy, Green Hencky, Swainger and Almansi measures respectively. Non-homogeneity in a spherical shell has been taken as the compressibility of the material as

$$C = C_0 r^{-k} \quad (1.2)$$

where $a \leq r \leq b$, C_0 and k are real constants.

2. Governing equations

We consider a spherical shell whose internal and external radii are a and b , respectively, and is subjected to uniform internal pressure p_i of gradually increasing magnitude and temperature Θ_0 applied to the internal surface $r = a$ as shown in Fig. 1. The components of displacement in spherical co-ordinates are given by (Seth, 1962, 1966)

$$u = r(1 - \beta) \quad v = 0 \quad w = 0 \quad (2.1)$$

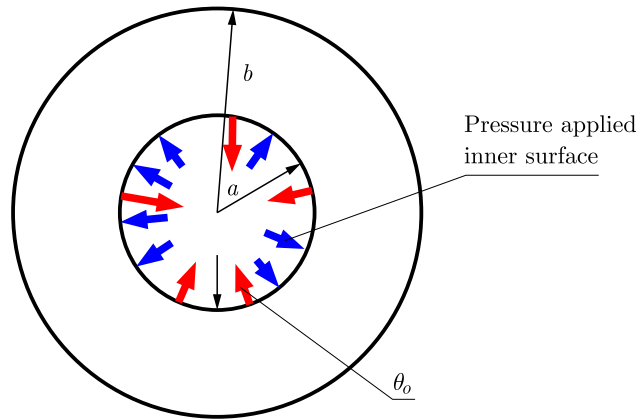


Fig. 1. Geometry of the spherical shell

where u, v, w are displacement components, β is a position function depending on r .

The generalized components of strain are given by Seth (1966)

$$e_{rr} = \frac{1}{n}[1 - (r\beta' + \beta)^n] \quad e_{\theta\theta} = \frac{1}{n}(1 - \beta^n) = e_{\varphi\varphi} \quad e_{r\theta} = e_{\theta\varphi} = e_{\varphi r} = 0 \quad (2.2)$$

where n is the measure and $\beta' = d\beta/dr$.

Stress-strain relation. The stress-strain relations for a thermo-elastic isotropic material are given by (Parkus, 1976)

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} - \xi\Theta\delta_{ij}T_{ij} \quad i, j = 1, 2, 3 \quad (2.3)$$

where T_{ij} are stress components and e_{ij} is strain component, λ and μ are Lamé's constants, $I_1 = e_{kk}$ is the first strain invariant, δ_{ij} is Kronecker's delta, $\xi = \alpha(3\lambda + 2\mu)$, α being the coefficient of thermal expansion, and Θ is temperature. Further, Θ has to satisfy

$$\nabla^2\Theta = 0 \quad (2.4)$$

Substituting the strain components from Eq. (2.2) in Eq. (2.3), the stresses are obtained as

$$\begin{aligned} T_{rr} &= \frac{2\mu}{n}[1 - (r\beta' + \beta)^n] + \frac{\lambda}{n}[3 - (r\beta' + \beta)^n - 2\beta^n] - \xi\Theta \\ T_{\theta\theta} &= T_{\varphi\varphi} = \frac{2\mu}{n}(1 - \beta^n) + \frac{\lambda}{n}[3 - (r\beta' + \beta)^n - 2\beta^n] - \xi\Theta \\ T_{r\theta} &= T_{\theta\varphi} = T_{\varphi r} = 0 \end{aligned} \quad (2.5)$$

Equation of equilibrium. The radial equilibrium of an element of the spherical shell requires

$$\frac{dT_{rr}}{dr} + \frac{2}{r}(T_{rr} - T_{\theta\theta}) = 0 \quad (2.6)$$

where T_{rr} and $T_{\theta\theta}$ are the radial and hoop stresses, respectively.

Boundary conditions. The temperature satisfying Laplace equation (2.4) with boundary conditions

$$\begin{aligned} \Theta &= \Theta_0 \quad \wedge \quad T_{rr} = -p_i \quad \text{at} \quad r = a \\ \Theta &= 0 \quad \wedge \quad T_{rr} = 0 \quad \text{at} \quad r = b \end{aligned} \quad (2.7)$$

where Θ_0 is constant, is given by (Parkus, 1976)

$$\Theta = \frac{\Theta_0 \log(r/b)}{\log(a/b)} \quad (2.8)$$

Critical points or turning points. Using Eq. (2.5) in Eq. (2.6), we get a non-linear differential equation in β as

$$nP(P+1)^{n-1}\beta^{n+1}\frac{dP}{d\beta} = \left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)\{3 - 2C - \beta^n[2(1-C) + (1+P)^n]\} - 2C'r(1-\beta^n) - n\beta^n P[2(1-C) + (1+P)^n] + 2C\beta^n[1 - (1+P)^n] - \frac{nC\bar{\Theta}_0}{2\mu\beta^n}\left(\xi + r\xi' \log \frac{r}{b}\right) \quad (2.9)$$

where $\bar{\Theta}_0 = \Theta_0/\log(a/b)$, $C = 2\mu/\lambda + 2\mu$ and $r\beta' = \beta P$ (P is a function of β and β is a function of r only). The transition or turning points of β in Eq. (2.9) are $P \rightarrow -1$ and $P \rightarrow \pm\infty$.

3. Analytical solution

For finding thermal creep stresses and strain rates, the transition function is taken through the principal stress difference (see Seth, 1962, 1966; Thakur, 2011, 2014; Thakur *et al.*, 2016, 2017) at the transition point $P \rightarrow -1$. We define the transition function ψ as

$$\psi = T_{rr} - T_{\theta\theta} = \frac{2\mu\beta^n}{n}[1 - (P+1)^n] \quad (3.1)$$

where ψ is a function of r only, and ψ is the dimension.

Taking logarithmic differentiation of Eq. (3.1) with respect to r and substituting the value of $dP/d\beta$ from Eq. (2.9), we get

$$\begin{aligned} \frac{d \log \psi}{dr} = \frac{np}{r} + \frac{\mu'}{\mu} - \frac{1}{r\beta^n[1 - (1+P)^n]} & \left[r\left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)\{3 - 2C - \beta^n[2(1-C) + (1+P)^n]\} \right. \\ & - 2rC'(1-\beta^n) - n\beta^n P[2(1-C) + (1+P)^n] \\ & \left. + 2C\beta^n[1 - (1+P)^n] - \frac{nC\bar{\Theta}_0}{2\mu}\left(\xi + r\xi' \log \frac{r}{b}\right) \right] \end{aligned} \quad (3.2)$$

Taking asymptotic value of Eq. (3.2) at $P \rightarrow -1$, we get

$$\frac{d}{dr}(\log \psi) = \frac{3\mu'}{\mu} - \frac{2C'}{C} - \frac{3n}{r} + X \quad (3.3)$$

where

$$X = \frac{2(n-1)C}{r} - \frac{2C\mu'}{\mu} + \frac{2C'}{\beta^n} - \left(\frac{\mu'}{\mu} - \frac{C'}{C}\right)\frac{3-2C}{\beta^n} + \frac{nC\bar{\Theta}_0}{2\mu r\beta^n}\left(\xi + r\xi' \log \frac{r}{b}\right)$$

Integrating equation (3.3), we get

$$\psi = A \frac{\mu^3}{C^2 r^{3n}} \exp(h) \quad (3.4)$$

where $h = \int X dr$ and A is a constant of integration, which can be determined from the boundary condition. From Eq. (3.1) and Eq. (3.4), we have

$$T_{rr} - T_{\theta\theta} = A \frac{2r\mu^3}{2C^2 r^{3n+1}} \exp(h) = \frac{ArH}{2} \quad (3.5)$$

where

$$H = \frac{2\mu^3}{C^2 r^{3n+1}} \exp(h)$$

Substituting Eq. (3.5) in Eq. (2.6) and integrating, we get

$$T_{rr} = B - A \int H dr \tag{3.6}$$

where B is a constant of integration, which can be determined from the boundary condition and asymptotic value of β as $P \rightarrow -1$ is D/r , with D being a constant.

Using boundary condition Eq. (2.7) in Eq. (3.6), we get

$$A = -\frac{p_i}{\int_a^b H dr} \quad B = -\frac{p_i}{\int_a^b H dr} \int H dr \tag{3.7}$$

where p_i is pressure at the inner surface of the spherical shell. Using the integration constants A and B in Eq. (3.6), we get

$$T_{rr} = \frac{p_i}{\int_a^b H dr} \int_a^b H dr r \tag{3.8}$$

Substituting Eq. (3.8) into Eq. (3.5), we get

$$T_{\theta\theta} = T_{\varphi\varphi} = \frac{p_i}{\int_a^b H dr} \left(\int_a^b H dr + \frac{rH}{2} \right) \tag{3.9}$$

We introduce non-homogeneity in the spherical shell due to variable compressibility as given in Eq. (1.2), then Eq. (3.8) and Eq. (3.9) become

$$T_{rr} = \frac{p_i}{\int_a^b H_1 dr} \int_a^b H_1 dr r \quad T_{\theta\theta} = T_{\varphi\varphi} = \frac{p_i}{\int_a^b H_1 dr} \left(\int_a^b H_1 dr + \frac{rH_1}{2} \right) \tag{3.10}$$

where T_{rr} , $T_{\theta\theta}$ are radial and circumferential stresses, and

$$\begin{aligned} H_1 &= \frac{2\mu^3}{C^2 r^{3n+1}} \exp(h_1) = \frac{r^{-(3n+k+1)}}{4(1 - C_0 r^{-k})^3} \exp(h_1) \\ h_1 &= -\frac{2(n-1)}{k} C_0 r^{-k} - \frac{2kC_0 r^{n-k}}{D^n(n-k)} + \frac{kC_0}{D^n} \int \frac{r^{n-k-1}(3 - 2C_0)r^{-k}}{1 - C_0(bR)^{-k}} dr + 2 \log(1 - C_0 r^{-k}) \\ &\quad + \frac{\alpha n \bar{\Theta}_0}{D^n} \int (1 - C_0 r^{-k}) \left(3 + \frac{C_0 r^{-k}}{1 - C_0 r^{-k}} - \frac{kC_0 r^{-k} \log(r/b)}{1 - C_0 r^{-k}} \right) r^{n-1} dr \\ C' &= -kC_0 r^{-k-1} \quad \frac{\mu'}{\mu} = \frac{C'}{C(1-C)} \end{aligned}$$

Equations (3.10)_{1,2} give thermal creep stresses for a spherical shell made of a non-homogeneous material under steady-state temperature. We introduce the following non-dimensional components as: $R = r/b$, $R_0 = a/b$, $\sigma_{rr} = \tau_{rr}/p_i$, $\sigma_{\theta\theta} = \tau_{\theta\theta}/p_i$ and $\alpha\Theta_0 = \Theta_1$. Equations (3.10)_{1,2} in non-dimensional form become

$$\sigma_{rr} = \frac{\int_R^1 H_2 dR}{\int_{R_0}^1 H_2 dR} \quad \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \frac{1}{\int_{R_0}^1 H_2 dR} \left(\int_R^1 H_2 dR + \frac{RH_2}{2} \right) \tag{3.11}$$

where

$$\begin{aligned}
 H_2 &= \frac{(bR)^{-(3n+k+1)}C_0\lambda^3}{4(1 - C_0b^{-k}R^{-k})^3} \exp(h_2) \\
 h_2 &= -\frac{2(n-1)}{k}C_0(bR)^{-k} - \frac{2kC_0(bR)^{n-k}}{D^n(n-k)} \\
 &\quad + \frac{kC_0b^{n-k}}{D^n} \int \frac{R^{n-k-1}(3 - 2C_0)R^{-k}}{1 - C_0(bR)^{-k}} dR + 2 \log(1 - C_0b^{-k}R^{-k}) \\
 &\quad + \frac{n\Theta_1}{D^n \ln R_0} \int (1 - C_0b^{-k}R^{-k}) \left(3 + \frac{C_0(bR)^{-k}}{1 - C_0b^{-k}R^{-k}} - \frac{kC_0(bR)^{-k} \log R}{1 - C_0b^{-k}R^{-k}} \right) R^{n-1} dR
 \end{aligned}$$

Particular case: In the absence of temperature gradient (i.e. $\Theta_1 = 0$), Eqs. (3.11), become

$$\sigma_{rr} = \frac{\int_1^R H_2^* dR}{\int_{R_0}^1 H_2^* dR} \quad \sigma_{\theta\theta} = \sigma_{\varphi\varphi} = \frac{1}{\int_{R_0}^1 H_2^* dR} \left(\int_1^R H_2^* dR + \frac{RH_2^*}{2} \right) \tag{3.12}$$

where

$$\begin{aligned}
 H_2^* &= \frac{(bR)^{-(3n+k+1)}C_0\lambda^3}{4(1 - C_0b^{-k}R^{-k})^3} \exp(h_2^*) \\
 h_2^* &= -\frac{2(n-1)}{k}C_0(bR)^{-k} - \frac{2kC_0(bR)^{n-k}}{D^n(n-k)} \\
 &\quad + \frac{kC_0b^{n-k}}{D^n} \int \frac{R^{n-k-1}(3 - 2C_0)R^{-k}}{1 - C_0(bR)^{-k}} dR + 2 \log(1 - C_0b^{-k}R^{-k})
 \end{aligned}$$

4. Estimation of creep parameters

When the creep sets in, the strains should be replaced by strain rates, then stress-strain relations are given (Sokolnikoff, 1946; Parkus, 1976)

$$e_{ij} = \frac{1 + \nu}{E} T_{ij} - \frac{\nu}{E} \delta_{ij} T + \alpha \Theta \tag{4.1}$$

where e_{ij} is the strain component and $T = T_{ii}$ is the first stress invariant and $\nu = (1 - C)/(2 - C)$ is Poisson’s ratio. Differentiating Eq. (2.2) with respect to time t , we get

$$\dot{e}_{\theta\theta} = -\beta^{n-1} \dot{\beta} \tag{4.2}$$

For Swainger measure (i.e. $n = 1$), Eq. (4.2) become

$$\dot{\epsilon}_{\theta\theta} = \dot{\beta} \tag{4.3}$$

where $\dot{\epsilon}_{\theta\theta}$ is the Swainger strain measure. From Eq. (3.1), the transition value β is given at the transition point $P \rightarrow -1$ by

$$\beta = \left(\frac{n}{2\mu} \right)^{\frac{1}{n}} (T_{rr} - T_{\theta\theta})^{\frac{1}{n}} \tag{4.4}$$

Using Eqs. (4.2)-(4.4) in Eq. (4.1), we get

$$\begin{aligned}
 \dot{\epsilon}_{rr} &= m(\sigma_{rr} - \nu\sigma_{\theta\theta} + \alpha\Theta) & \dot{\epsilon}_{\theta\theta} &= m(\sigma_{\theta\theta} - \nu\sigma_{rr} + \alpha\Theta) \\
 \dot{\epsilon}_{\varphi\varphi} &= -m[\nu(\sigma_{rr} + \sigma_{\theta\theta}) + \alpha\Theta]
 \end{aligned} \tag{4.5}$$

where $\dot{\epsilon}_{rr}$, $\dot{\epsilon}_{\theta\theta}$ and $\dot{\epsilon}_{\varphi\varphi}$ are strain rates, σ_{rr} , $\sigma_{\theta\theta}$ are stress components and $m = [n(\sigma_r - \sigma_\theta)(1 + \nu)]^{\frac{1}{n}-1}$. These are the constitutive equations used by Odquist (1974) for finding the creep stresses, provided we put $n = 1/N$ and N is the measure.

5. Numerical results and discussion

For calculating creep stresses and strain rates on the basis of the above analysis, the following values have been taken, $\nu = 0.5$ (incompressible material $C_0 = 0$), $\nu = 0.42857$ (compressible material $C_0 = 0.25$) and 0.333 (compressible material $C_0 = 0.50$), $n = 1/3, 1/5, 1/7$ (i.e. $N = 3, 5, 7$), thermal expansion coefficient $\alpha = 5.0 \cdot 10^{-5} \text{ deg F}^{-1}$ for Methyl Methacrylate (Levitsky and Shaffer, 1975) and $\Theta_1 = \alpha\Theta_0 = 0$ and 0.5 , $D = 1$. In classical theory, the measure N is equal to $1/n$. The definite integrals in Eqs. (3.11) have been evaluated by using Simpson's rule. From Figs. 2-4, curves are presented between stresses along the radii ratio $R = r/b$ in the spherical shell made of compressible as well as incompressible materials for $k = -1, 0, 1$. It can be seen from Figs. 2 and 3 that the circumferential stresses are maximum at the external surface for $n = 1/7$ and $k = -1, 0$ for a compressible material as compared to the incompressible material. From Fig. 4, the circumferential stress is maximum at the internal surface for non-homogeneity $k = 1$. The non-homogeneity increases the values of circumferential stress (i.e. $k = -1, 0$), but reverse the result for $k = 1$.

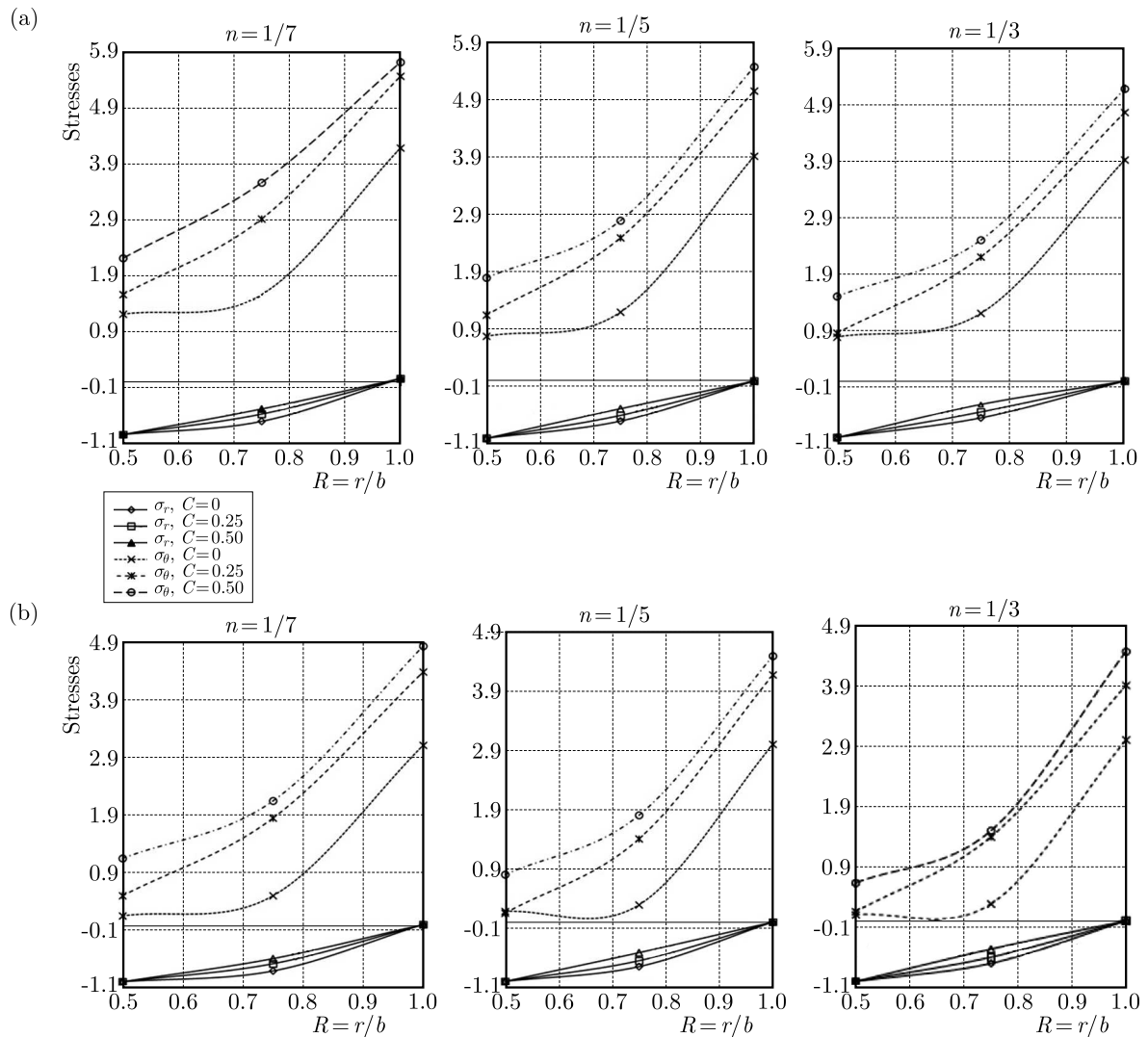


Fig. 2. Creep stresses in a non-homogeneous ($k = -1$) spherical shell along the radii ratio $R = r/b$; (a) without temperature, (b) for temperature $\Theta_1 = 0.5$

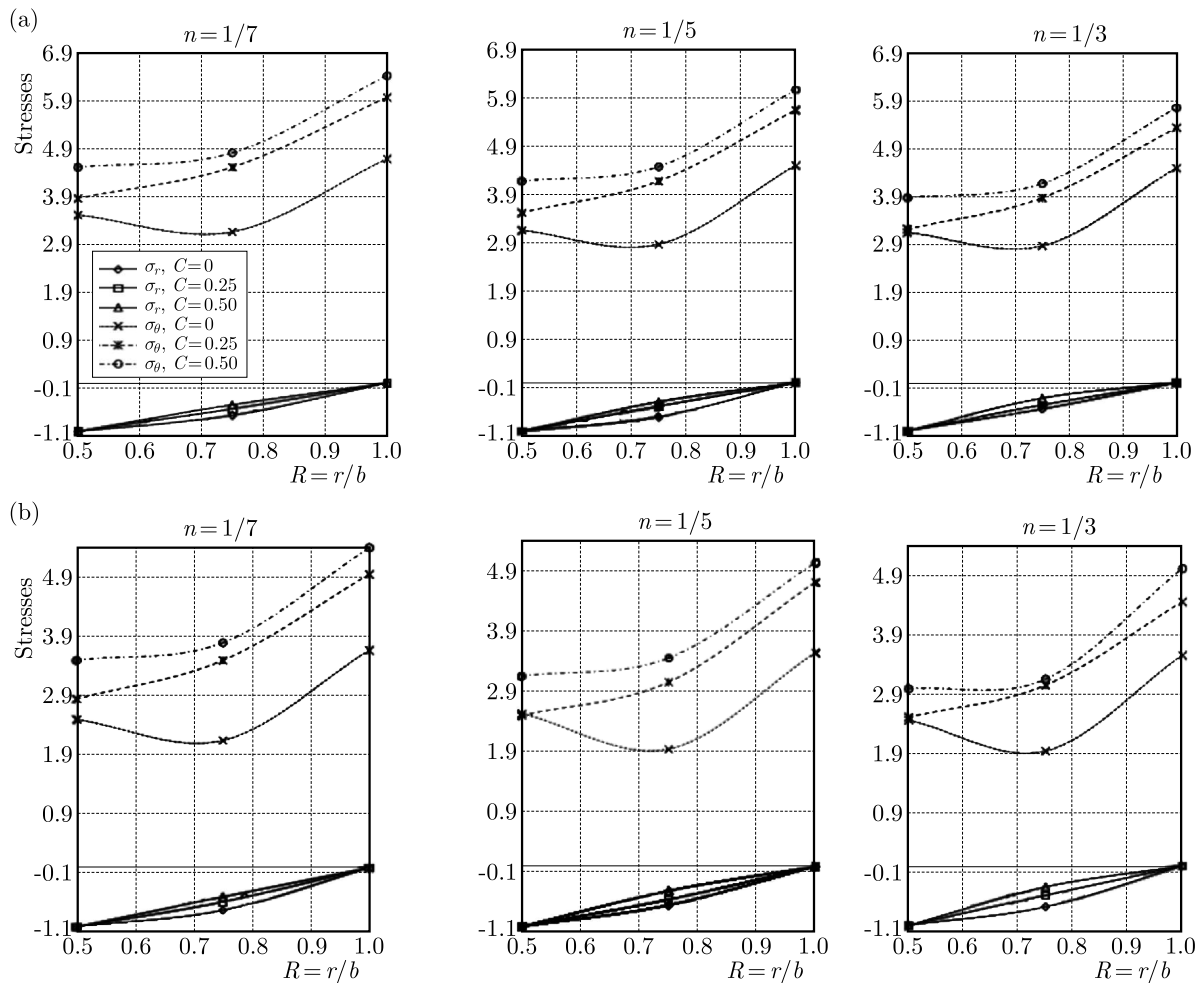


Fig. 3. Creep stresses in a non-homogeneous ($k = 0$) spherical shell along the radii ratio $R = r/b$; (a) without temperature, (b) for temperature $\Theta_1 = 0.5$

With the introduction of a temperature gradient, the values of circumferential stress are decreased at the external surface as well as internal surface of the spherical shell for different values of non-homogeneity. It means that temperature dependent materials minimize the possibility of fracture at the internal surface of the spherical shell. From (Fig. 5), curves are produced between strain rates along the radii ratio $R = r/b$ for the spherical shell made of compressible material $C = 0.25$, i.e. saturated clay for $k = -1, 0, 1$. It can be seen that the strain rates are maximum at the external surface for $k = -1, 0$ and reverse in the case $k = 1$. With the introduction of a temperature gradient, the strain rates decrease at the internal surface as well as the external surface. It means that the temperature dependent materials minimize the possibility of fracture at the internal surface of the spherical shell.

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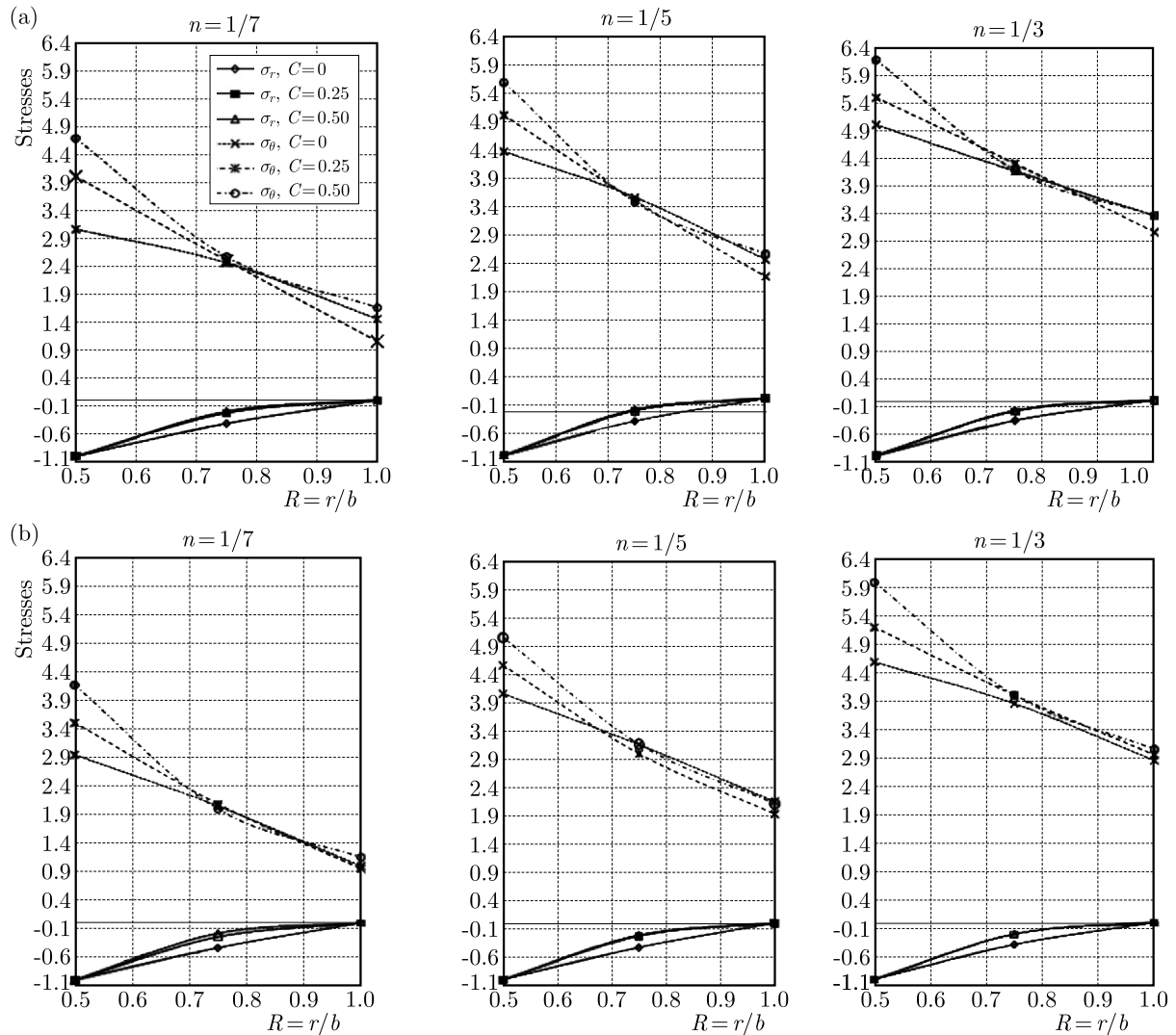


Fig. 4. Creep stresses in a non-homogeneous ($k = 1$) spherical shell along the radii ratio $R = r/b$; (a) without temperature, (b) for temperature $\Theta_1 = 0.5$

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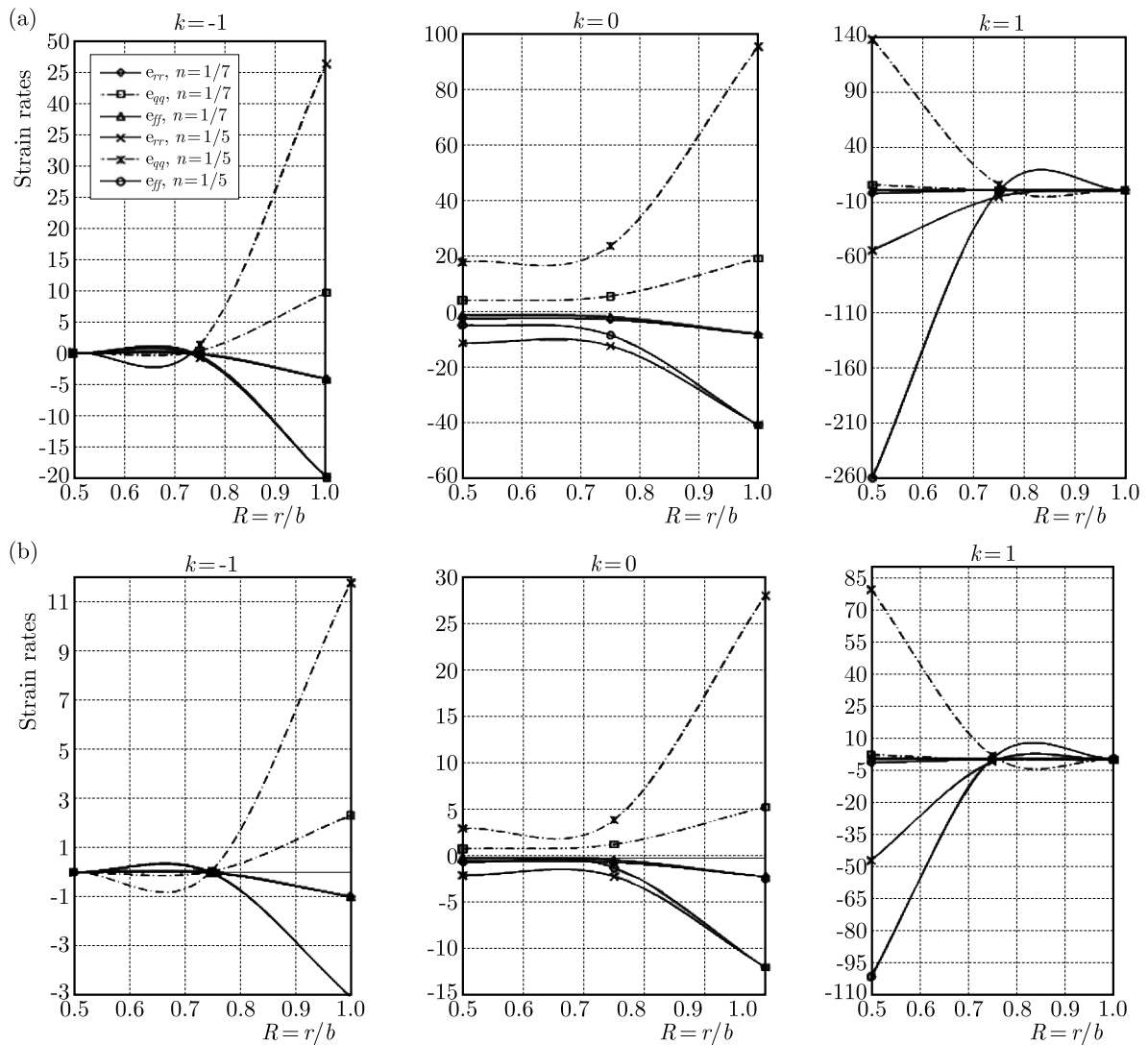


Fig. 5. Strain rates in a non-homogeneous spherical shell along the radii ratio $R = r/b$ for $C = 0.25$;
 (a) without temperature $\Theta_1 = 0$, (b) for temperature $\Theta_1 = 0.5$

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