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FORECASTING THE GLOBAL AND PARTIAL SYSTEM CONDITION BY MEANS OF MULTIDIMENSIONAL CONDITION MONITORING METHODS

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Machines have many faults which evolve during their operation. If one observes some number of symptoms during the machine operation, it is possible to capture fault oriented information. One of the methods to extract fault information from such a symptom observation matrix is to apply the Singular Value Decomposition (SVD), obtaining in this way the generalized fault symptoms. The problem of this paper is to find if the total damage symptom, being a sum of all generalized symptoms is the best way to infer on machine condition or is it better to use the first generalized symptom for the same purposes. There were some new software created for this purpose, and two cases of machine condition monitoring considered, but so far it is impossible to state that one of the inference methods is better. Moreover, it seems to the author that both inference methods are complimentary for each other, and should be used together to increase the reliability of diagnostic decision.

Key words: condition monitoring, multidimensional observation, singular value decomposition, generalized fault symptoms, grey models, forecasting

1. Introduction

The contemporary advancement in measurement technology allows us to measure almost any component of the phenomenal field inside or outside the working machine. The only condition for such diagnostic is some kind of proportionality to gradual worsening of the machine condition which takes place during it operation. If it is so, we can name the measured component of the machine phenomenal field as the *symptom* of condition. In this way, we measure a dozen of would be symptoms, and our condition monitoring is multidimensional from the beginning. Due to this situation, the application of multidimensional machine condition observation is now a well established fact, see Cempel (1999), Korbicz *et al.* (2004), Tumer and Huff (2002), Jasiński (2004) – for example. And there exist some differences in application and processing of the multidimensional signals and/ or symptom observation matrix. For signals and symptoms one can also apply the so called data fusion (Hall and Llinas, 1997; Roemer *et al.*, 2001; Korbicz *et al.*, 2004), and similar techniques developed lately. In the case of multi symptom observation, one can apply Principal Component Analysis (PCA), or Singular Value Decomposition (SVD), looking for principal or singular components, which may have some diagnostic meaning. For the case of Singular Value Distribution (SVD) method, there exists the body of experimental evidence (Cempel, 2004; Cempel and Tabaszewski, 2007a,b)that singular components and the quantities created from them can be treated as *generalized* fault symptoms.

All that transformation and symptom processing starts from the data base called the Symptom Observation Matrix (SOM). Let us explain now how the SOM is structured and obtained.

During the machine life θ we can observe its condition by means of several symptoms $S_m(\theta)$ measured at some moments of life θ_n, $n = 0, 1, \ldots, p > r$, $\theta_p < \theta_b$, (θ_b – *anticipated breakdown time*). This creates sequentially the Symptom Observation Matrix (SOM), the only source of information on the condition evolution of a machine in its lifetime $0 < \theta < \theta_b$. We assume additionally that the condition degradation is also multidimensional and is described by semi-independent faults $F_t(\theta)$, $t = 1, \ldots, u < r$, which are evolving in the machine body, as the expression of gradual degradation of the overall machine condition. This degradation proceeds from the *not faulty condition*[1] up to its near breakdown state. Generalizing, one can say now that we have m-dimensional symptom space for condition observation, and r-dimensional fault space $(m > r)$, which we are trying to extract from the observation space by using SVD or PCA.

Moreover, some of *would be* symptoms are redundant; it means not carrying enough information on the evolving faults during the machine life. But of course there is not a unique criterion of the redundancy. During the course of our research, several measures of redundancy have been applied, the volume of observation space (Vol_1), pseudo Frobenius norm $(Frob_1)$ of SOM (Cempel and Tabaszewski, 2007a,b), and others. But they seem to be not good enough with respect of the quality of the final diagnostic decision. This means addi-

[1] We assume machine is new, or after the overhaul and repair process.

tionally, when optimizing the observation space, we should take into account the adequate assessment of the *current* and the *future machine condition*, in the form of condition forecast with a possibly small error. The paper considers this problem, and it is done on the level of previous SVD works of the author. As the forecasting technique with minimal error, the *grey system model* with rolling window (Yao and Chi, 2004) was adopted for diagnostic purposes, and has been applied here (Cempel and Tabaszewski, 2007a). But having the multidimensional problem of fault assessment and the observation, it is important now what type of generalized symptom we use for the forecasting. Do we use the overall degradation symptom of the machine or some specified generalized symptom proportional to one fault only.

The results of such a new approach to multidimensional diagnosis presented here were verified on the real data of machine vibration condition monitoring. Concerning the software, some modification of the last program for the data processing was needed as well. As a result it was found that this approach seems to be promising and enabling a better understanding of machine condition and also better current and future condition assessment.

2. Extraction method of partial faults of the system

As it was said in the introduction, our information on machine condition evolution is contained in $p \cdot r$ Symptom Observation Matrix (SOM), where in r columns are presented p rows of the successive readings of each symptom made at equidistant system lifetime moments θ_n, $t = 1, 2, \ldots, p$. The columns of such SOM are next centered and normalized to three point average of the three initial readings of every symptom. This is in order to make the SOM dimensionless, to diminish starting disturbances of symptoms, and to present the evolution range of every symptom from zero up to few times of the initial symptom value S_{0n}, measured in the vicinity of lifetime $\theta_1 = 0$.

After such preprocessing, we will obtain the dimensionless Symptom Observation Matrix ($\mathsf{\mathbf{SOM}}$) in the form

$$\mathsf{\mathbf{SOM}} \equiv \mathsf{\mathbf{O}}_{pr} = [S_{nm}] \qquad\qquad S_{nm} = \frac{\widetilde{S}_{nm}}{S_{0m}} - 1 \tag{2.1}$$

where $\widetilde{S}_{nm}$ letters indicate primary measured and averaged dimensional symptoms.

As was said in the introduction, we apply now to the dimensionless $\mathsf{\mathbf{SOM}}$ (2.1) the Singular Value Decomposition (SVD) (Golub, 1983; Will, 2005), to obtain singular components (vectors) and singular values (numbers) of **SOM** in the form

$$\mathbf{O}_{pr} = \mathbf{U}_{pp}\mathbf{\Sigma}_{pr}\mathbf{V}_{rr}^{\top} \qquad (\top - \text{ matrix transposition}) \tag{2.2}$$

where $\mathbf{U}_{pp}$ is a p-dimensional orthonormal matrix of the left-hand side singular vectors, $\mathbf{V}_{rr}$ is a r-dimensional orthonormal matrix of the right-hand side singular vectors, and the diagonal matrix of singular values $\mathbf{\Sigma}_{pr}$ is defined as below

$$\mathbf{\Sigma}_{pr} = \text{diag}\,(\sigma_1, \dots, \sigma_l) \tag{2.3}$$

with nonzero singular vectors

$$\sigma_1 > \sigma 2 > \dots > \sigma_u > 0$$

and zero singular values

$$\sigma_{u+1} = \dots = \sigma_l = 0 \qquad \begin{array}{l} l = \max(p, r) \\ u \leqslant \min(p, r) \\ u < r < p \end{array}$$

In terms of machine condition monitoring, above (2.3) means that from the r primarily measured symptoms (dimension of observation space) we can extract only $u \leqslant r$ nonzero independent sources of diagnostic information, describing the evolving generalized faults $F_t(\theta)$, $t = 1, \dots, u$, and creating in this way the less dimensional *fault space*. But only a few faults developing in a machine are making essential contribution to total fault information (are enough developed). The rest of potential generalized faults, symbolized here by small σ_u, are usually below the standard 10% level of noise. What is important here, that such SVD decomposition can be made currently, after each new observation (reading) of the symptom vector $\boldsymbol{S}_m$; $n = 1, \dots, p$, and in this way we can trace the faults evolution, and their advancement in any operating mechanical system.

3. Diagnostic interpretation of SVD

From the current research and implementation of this idea (Cempel, 2003), one can say that the most important fault oriented indices obtained from SVD is the generalized fault symptom $\boldsymbol{SD}_t$ $t = 1, 2, \dots$, and also the sum of all generalized fault symptoms $\boldsymbol{SumSD}_i$, as some equivalent symptom of total (cumulated) machine damage. In another way, the generalized fault symptom $\boldsymbol{SD}_t$ can be named also as discriminant, or the generalized symptom of the fault order t, and one can obtain this as the SOM product and singular vector v_t, or in general in matrix notation as below

$$\mathbf{SD} = \mathbf{O}_{pr}\mathbf{V} = \mathbf{U}\boldsymbol{\Sigma} \tag{3.1}$$

and in particular

$$\boldsymbol{SD}_t = \mathbf{O}_{pr}\boldsymbol{v}_t = \sigma_t \boldsymbol{u}_t \qquad t = 1, \dots, u < r$$

We know from SVD theory (Golub, 1983; Will, 2005) that all singular vectors $\boldsymbol{v}_t$ and $\boldsymbol{u}_t$, as the components of singular matrices, are normalized to one, so the energy norm of this new discriminant (generalized fault symptom) gives simply the respective singular value σ_t

$$\text{Norm}\,(\boldsymbol{SD}_t) \equiv \|\boldsymbol{SD}_t\| = \sigma_t \qquad t = 1, \dots, u \tag{3.2}$$

The above defined discriminant $\boldsymbol{SD}_t(\theta)$ can also be named as the lifetime fault profile, and the respective singular value $\sigma_t(\theta)$ as the function of the lifetime seems to be its life advancement (energy norm) and the same the measure of importance of the fault. That is the main reason why we use dimensional or dimensionless singular values for the ordering of importance of generalized symptoms (faults).

The similar fault inference can be postulated to the meaning, and the evolution of summation quantities, the total damage profile $\boldsymbol{SumSD}_i(\theta)$ as below

$$\begin{aligned} &\boldsymbol{SD}_t(\theta) \propto \boldsymbol{F}_t(\theta) \qquad t = 1, 2, \dots \\ &\boldsymbol{SumSD}_i(\theta) = \sum_{i=1}^{z} \boldsymbol{SD}_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta)\boldsymbol{u}_i(\theta) \propto \boldsymbol{F}(\theta) \end{aligned} \tag{3.3}$$

Currently it seems that the condition inference based on the first summation damage measure; $\boldsymbol{SumSD}_i$, (total damage measure) may stand as the first approach to multidimensional condition inference, as it was lately shown in the previous papers (see for example Cempel, 2004, 2005, 2006).

Going back to SVD itself, it is worthwhile to show that every perpendicular matrix has such decomposition, and it may be interpreted also as the product of three matrices (Will, 2005), namely

$$\mathbf{O}_{pr} = (\boldsymbol{Hanger}) \times (\boldsymbol{Stretcher}) \times (\boldsymbol{Aligner})^{\top} \tag{3.4}$$

This is very metaphorical description of SVD transformation, but it seems to be a useful analogy for the inference and decision making in our case. The diagnostic interpretation of formulae (3.4) one can obtain very easily. Namely, using its left hand side part, we are stretching our SOM over the life (observations) dimension, obtaining the matrix of generalized symptoms as the columns of the matrix $\mathbf{SD}$ (see below). And using its right hand side part of (3.4), we are stretching SOM over the observed symptoms dimension obtaining the assessment of contribution of every primary measured symptoms in the matrix $\mathbf{AL}$, assessing in this way the contribution of each primary symptom to the generalized fault symptom $\boldsymbol{SD}_i$

$$\mathbf{SD} = \mathbf{O}_{pr}\mathbf{V}_{rr} = \mathbf{U}_{pp}\boldsymbol{\Sigma}_{rr} \qquad \text{and} \qquad \mathbf{AL} = \mathbf{U}_{pp}^{\top}\mathbf{O}_{pr} = \boldsymbol{\Sigma}_{rr}\mathbf{V}_{rr}^{\top} \tag{3.5}$$

This means that $\mathbf{SD}$ matrix is stretched along the life coordinate giving us the life evolution of the weighted $(\boldsymbol{\sigma}_i)$ singular vectors. And $\mathbf{AL}$ matrix is aligned along the symptom dimension with the same way of weighting by $\boldsymbol{\sigma}_i$, giving the assessment of information contribution of each primary symptom.

We will calculate numerically the above matrices and use them for better interpretation of monitoring results ($\mathbf{SD}$), and optimization of dimension of the observation space ($\mathbf{AL}$).

4. SOM information measure and optimization

Having in mind the redundancy of some primary symptoms, i.e. the primary observation space, some additional considerations should be made concerning the SOM information assessment. In terms of previous findings this can be done by calculating the Frobenius norm (Frob) of this matrix, and the volume (Vol) created by u-dimensional generalized fault space identified by application of SVD. One can calculate easily both information indices as the sum and the product of singular values in the following way (Golub, 1983; Kiełbasiński, 1992)

$$\mathrm{Frob}\,(\mathbf{SOM}) \equiv \sqrt{\sum_{i=1}^{u}\sigma_i^2} \qquad \text{and} \qquad \mathrm{Vol}\,(\mathbf{SOM}) \equiv \prod_{i=1}^{u}\sigma_i$$

But squaring the small singular values of σ_i (less than one) make them much smaller, giving seemingly smaller contributions to the matrix information asset, and to the volume of the observation space. Due to this it was proposed by Cempel and Tabaszewski (2007a,b) to use not the exact Frobenius norm but its modification as below

$$\mathrm{Frob}\,1 = \sum_{i=1}^{u} \sigma_i \qquad \text{and} \qquad \mathrm{Vol}\,1 = \prod_{i=1}^{u} \sigma_i \tag{4.1}$$

This will give us possibility to look for small, just evolving faults, not omitting them when we try to reduce the redundancy of the observation vector. Consequently, one can get less redundancy of a new optimized SOM with a less number of columns, but also keeping in observation the small just evolved fault information (σ_i).

The use of Frobenius measure for a matrix has also mathematical validation. In general, one can understand this as the problem of approximation of matrix $\mathbf{B}$, by the so called k-rank approximation. Following the paper Berry *et al.* (1999), we can make the quantitative assessment of such a k-rank approximation of the matrix $\mathbf{B}$ as the difference below

$$\|\mathbf{B} - \mathbf{B}_k\|_F = \sqrt{\sigma_{k+1}^2 + \ldots + \sigma_u^2} \tag{4.2}$$

where the subscript u stands for the maximal dimension of nonzero singular value, i.e. the rank of our primary SOM.

This also means that instead of (3.5) we write a simplified measure of approximation of SOM in the form of deviation from primary SOM rank, as below

$$\Delta_k\,\mathrm{Frob}\,1 \equiv \mathrm{Frob}\,1_o - \mathrm{Frob}\,1_k = \{\sigma_{k+1} + \ldots + \sigma_u\} \tag{4.3}$$

Using this quality index of matrix approximation measure, we can form an additional objective measure of the SOM redundancy. And minimization of the SOM rank may be carried out by excluding some primary measured symptoms $\boldsymbol{S}_m$ with low information contribution, which produces mainly small (less than one) singular vales σ_u.

Such criteria of redundancy minimization we have used quite recently. But following the last papers Cempel and Tabaszewski (2007a,b), one may notice that after some symptom rejection, it gives an expected increase in the volume of information space (Vol 1). Also the rank approximation of SOM gives only a slight drop in Frob 1 measure, but the result of prognosis is not good enough, *giving erroneous future values*, even less than the previous one. How to avoid such errors in forecasting? There seem to be one possibility more to make the symptom rejection more objective and anticipating the goodness of the condition forecast. We have to consider the contribution of primary measured symptoms to the creation of first generalized symptoms $\boldsymbol{SD}_1$, and also the creation of the total damage generalized symptom $\boldsymbol{SumSD}_i$. The first overall contribution measure can be calculated separately to each primary symptom from the correlation matrix of our SOM (with appended lifetime in the first column) as the centered and normalized sum of column elements. The second measure can be obtained if we append additionally to the previous matrix the vector $\boldsymbol{SumSD}_i$ as the first column. When calculating the covariance matrix from these and in the first row, we will have needed information. After needed normalization to the first element of this row, this will give us the contribution of every primary symptom to the total damage symptom $\boldsymbol{SumSD}_i$.

5. The global and partial fault inference

We have gathered above all necessary analytical and inference knowledge concerning the processing of the symptom observation matrix, the extraction of fault information and optimization of the SOM rank. So, there is a right moment to validate these findings and proposal by some experimental data taken from real situations of vibration condition monitoring. In order to do this, the last Matlab® program svdopt1gs.m presented in Cempel and Tabaszewski (2007a) has been modified to svdopt2gs.m. The inference basis for the first program is the total damage generalized symptom $\boldsymbol{SumSD}_i$, while in the modified program such on inference basis is the first generalized symptom $\boldsymbol{SD}_1$. Just to catch the inference and the followed diagnostic decision difference, we will take some uneasy case of a heavy fan (power of 3MW) working in unstable and load uncontrolled regime (random supply of the air to the mine shaft), serving as the source of fresh air for ventilation at a deep copper mine. The main troubles with this fan were the unbalance and nonalignment between the fan and the driving electric motor. Due to that, it was constantly monitored.

Figure 1 presents below six pictures as a result of fan data processing by specially prepared program svdopt1gs.m[2] made in the Matlab® environment, and here is adopted the inference according to the total damage symptom $\boldsymbol{SumSD}_i$. The first top left picture shows results of 30-week measurements of symptom life curves of vibration velocity at five points located on the fan aggregate structure. One may notice here the great instability of symptom readings, symptom No. 4 in particular. This is better seen at the picture middle left when the data are centered and normalized to the average value of the

[2]Author is indebted to Dr M. Tabaszewski for the help in making the program and for the vibration condition monitoring data.

three initial symptom readings. The picture bottom left presents the generalized symptoms as the result of SVD processing, indicating also the symptom limit value calculated for the generalized symptom of the total damage $\boldsymbol{SumSD}_i$. We may notice here that the initial instability of primary symptoms is even enlarged, giving a situation where the main generalized symptoms are falling down at the end of the fan life, giving erroneous assessment of the fan condition.

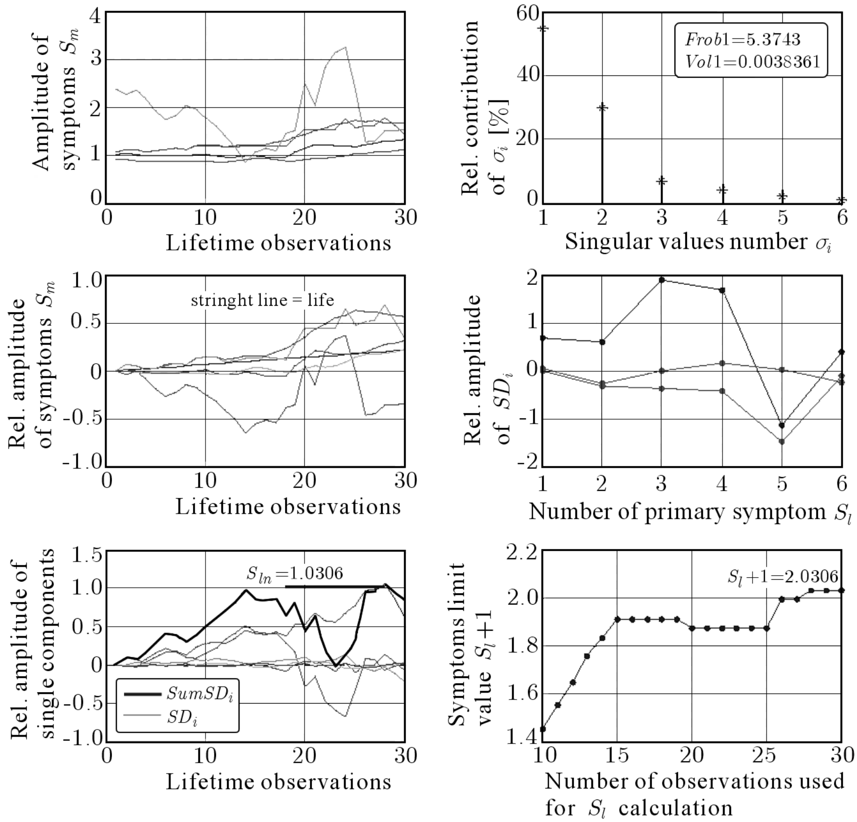

Fig. 1. Results of SVD processing of vibration data of a huge fan pumping air into the copper mine shaft

From the diagnostic decision point of view, it is a very critical situation to observe such instability of primary and generalized symptoms. The next picture top right at this figure (dimensionless singular values) give us the indication how much independent sources of diagnostic information can be observed by these five primary symptoms with appended lifetime θ. One can see here that in reality we can count on two independent sources of information, it may mean two independent faults, like shaft unbalance and machine aggregate misalignment. The picture middle right gives us the contribution of each primary symptom (the first = lifetime) to creation of the first three generalized fault symptoms taken directly from SVD as the product of **SOM** and **U** matrices. We may notice that primary symptom No. 5 gives small contribution to all three generalized symptoms, and may serve as one of the candidates to rejection in the course of further processing. The last picture, the bottom right one, gives us the result of current calculation of the symptom limit value S_l by our concept of symptom reliability (Cempel *et al.*, 2000). This program calculates S_l with data of the total damage generalized symptom $\boldsymbol{SumSD}_i$ (the picture bottom left). One may notice that the limit value S_l is steadily evolving up to the value $S_l = 1.0306$. But the application of this limit value to the total damage symptom $\boldsymbol{SumSD}_i$ is impossible as this symptom is falling down. Having this in mind, we should decide now which symptom we should reject here in order to obtain stable course of the generalized fault symptom of the total damage $\boldsymbol{SumSD}_i$ in order to make a reliable forecast and diagnostic inference.

Let us now change the inference basis and calculate the symptom limit value S_l not from the total damage generalized symptom $\boldsymbol{SumSD}_i$ but from the first generalized symptom $\boldsymbol{SD}_1$ only. So, let the same SOM of the fan will be treated by modified program svdopt2gs.m, as below.

With this modified program and our primary data, only the last row of pictures in Fig. 2 has been changed, namely the course of the symptom limit value calculation and its value (as it is seen at the bottom right picture). But due to this change, the assessed value of S_l is lower and can serve us for further elaboration of diagnostic decision, as one can see from the bottom left picture. Let us now check if we can correct the situation by reducing the SOM redundancy. As one can see from Fig. 2, there is a big redundancy of the observation space (SOM), which is reflected here by a very small volume of the fault space $\mathrm{Vol}\,1 = 0.00383$. This means that some primary symptoms carry minimal fault information, which is reflected by a very small σ_I, much smaller than one.

Trying to diminish this redundancy, let us now use the already mentioned correlation calculation of information contribution, i.e. contribution of each primary symptom to the overall information resource of our data and to the generalized symptom of total damage $\boldsymbol{SumSD}_i$. The calculation of these was described in the previous Section. Figure 3 gives the result of such a contribution assessment, and one can notice that the primary symptom number 4 is the first candidate (not symptom No. 5) as it influences negatively (decreasing)

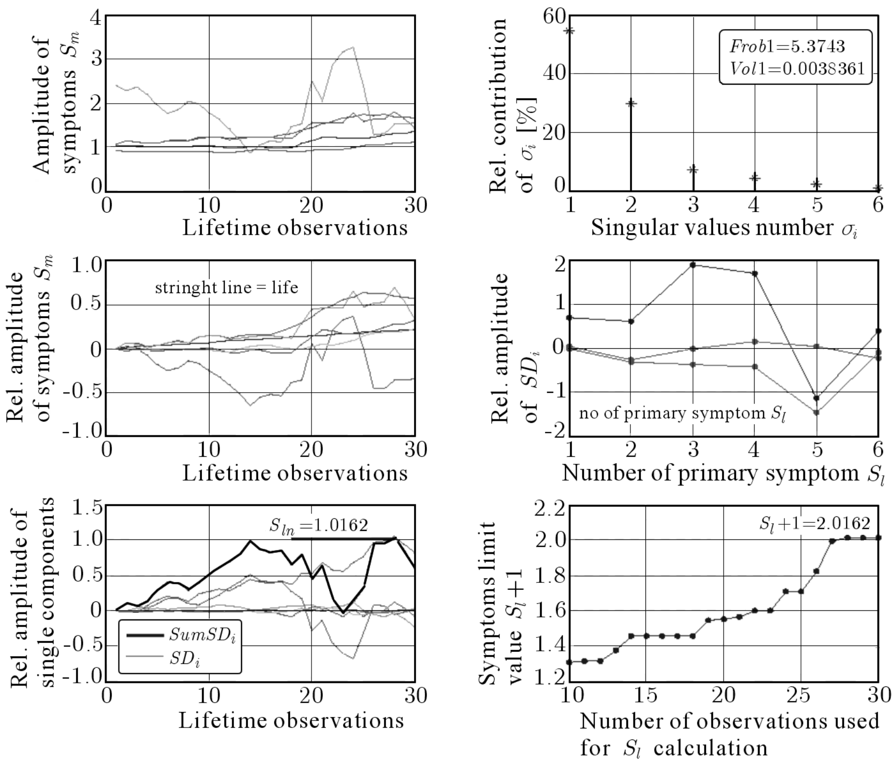

Fig. 2. The vibration symptom observation matrix of a huge fan (see Fig. 1) processed by the modified program, where S_l value is calculated on the basis of the first generalized symptom $\boldsymbol{SD}_1$

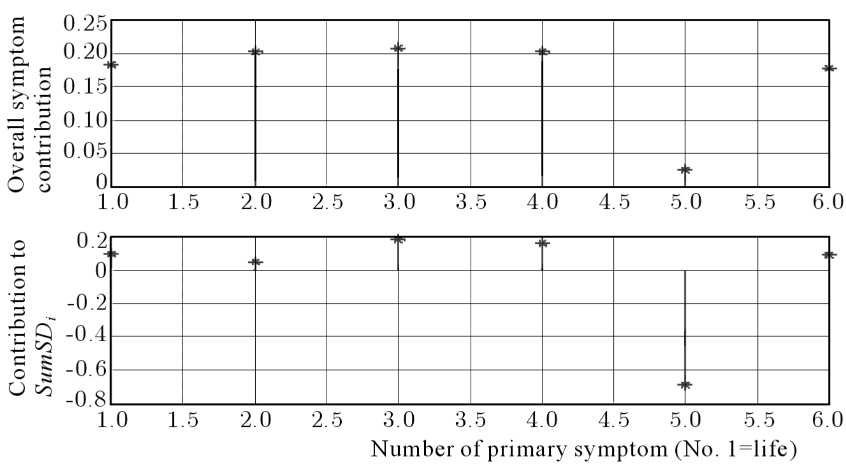

Fig. 3. The assessment of overall contributions of primary symptoms (top picture) and the contribution to the total damage generalized symptom

the total damage symptom (bottom picture), and also makes the smallest contribution to the overall information resource (top picture of Fig. 3).

Having such strong indication which a symptom to reject as the first (No. 4), we have performed this by the same program, and the results can be seen in Fig. 4 presented in the same manner as it was explained in detail for Fig. 1 and Fig. 2. Now one can see from Fig. 4 that the rejected symptom No. 4 was the symptom with the greatest amplitude and instability. One can also notice that the course of all generalized symptoms (bottom left picture) and the limit value (bottom right picture) evolves smoothly, giving a strong basis for the future diagnostic decision. It is also good to notice here a drop of the Frobenius measure after symptom rejection; $\Delta\,\mathrm{Frob}\,1 = 2.4073$, and more than a double increase of the volume of the observation space $Vol1 = 0.0082$.

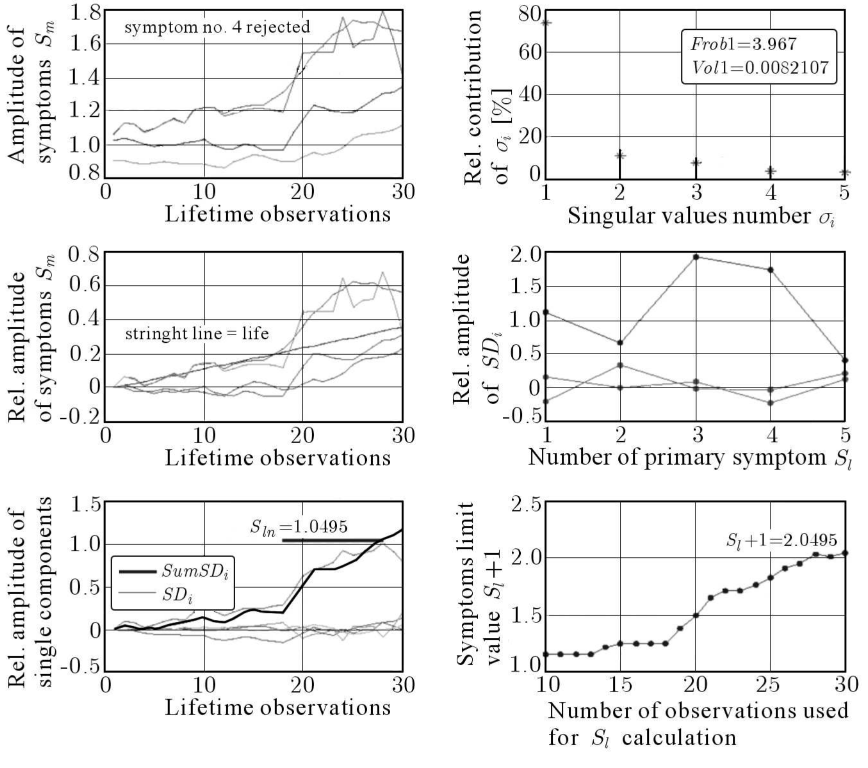

Fig. 4. Condition inference of a huge fan by the same program svdopt1gs, but with rejection of primary symptom No. 4, and total damage symptom $\boldsymbol{SumSD}_i$

It will be very interesting what result can be achieved in calculation of the symptom limit value rejecting the same symptom No. 4, but using modified the program svdopt2gs.m, which calculates the symptom limit value S_l on the basis of the first generalized symptom $\boldsymbol{SD}_1$. Figure 5 shows the result of such calculations, presenting the detailed results in the same manner as it was done before (Fig. 1 and Fig. 2).

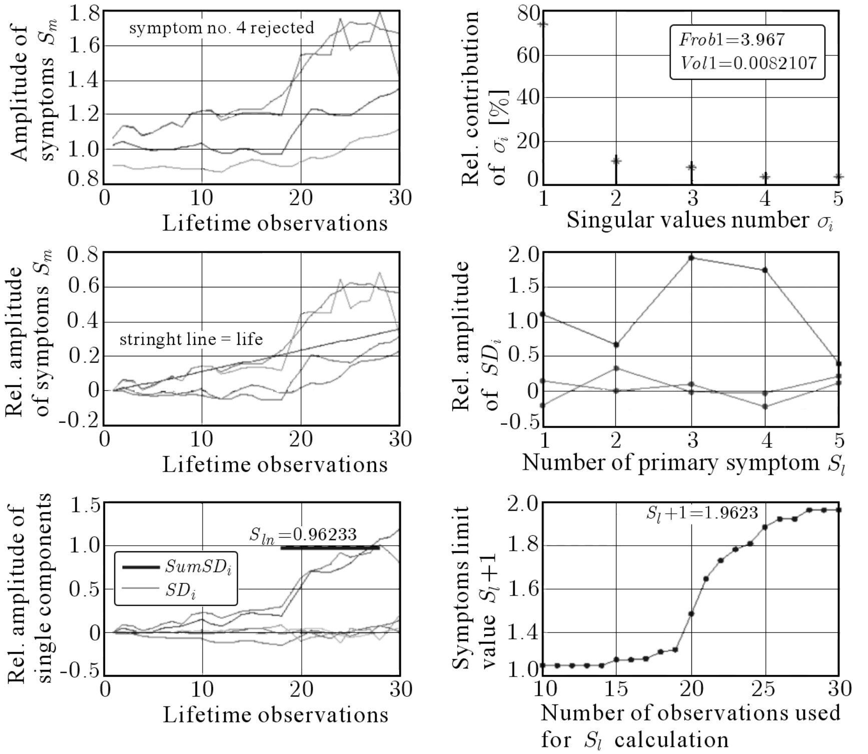

Fig. 5. The huge fan monitoring data with a change of inference basis to the first generalized symptom $\boldsymbol{SD}_1$

Analyzing now the last two figures (Fig. 1 and Fig. 2), one can find that the only change is noted at the bottom pictures where the symptom limit value S_l has been calculated and presented against the generalized symptoms (bottom left pictures). One can notice here that the calculation of the limit value using the first generalized symptom $\boldsymbol{SD}_1$ gives us a lower value, and this can give us a more safe assessment of the lifetime moment for machine shut down, and renewal. From the point of view of reliability of diagnostic decision, this seems to be important to have two sources of the symptom limit vale assessment, and to confront these values with the associated knowledge.

6. Forecasting of global system damage and partial faults advancement

The final quality of diagnostic decision one may judge by making the forecast of the future condition in terms of the total damage symptom $\boldsymbol{SumSD}_i$ and the first generalized fault symptom $\boldsymbol{SD}_1$. It was said in the introduction that the forecast will be made by the Grey System Theory (GST) (Deng, 1989), together with the rolling window method using the first order grey model $GM(1,1)$ (Yao and Chi, 2004).

In general, GST assumes that our incomplete and uncertain observation can be the output of some dynamic multi-input system of high order, described by a grey differential or difference model (Wen and Chang, 2005). In the condition monitoring, we may assume that it is enough to take the first order system described by the grey differential equation, and one forcing or control input. This simplest case in GST is denoted as $GM(1,1)$, which means the grey model of order 1 with one input only. The output of the system is the series of discrete observations (our symptom readings) denoted here as

$$\boldsymbol{x}^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} \tag{6.1}$$

where $n \geqslant 4$ is the number of observation made on the system (machine).

We will not present GST theory here, just using the final formulae for the forecasting, and the rolling window concept is implemented into the forecasting software only.

The application of GST to the above symptom readings gives the possibility to forecast the future symptom value, starting from a very small number observation, and using the formula

$$\widehat{x}^{(0)}(k+1) = \Big[x^{(0)}(1) - \frac{u}{a}\Big](\mathrm{e}^{-ak} - \mathrm{e}^{-a(k-1)}) \qquad k = 2, 3, \dots, n \tag{6.2}$$

where u and a are parameters to be estimated by a special least square matrix procedure using the observed data (6.1), and the hat $\widehat{(\cdot)}$ in (6.2) means the future value of the forecasted quantity.

This concept was adjusted to the purposes of vibration condition monitoring in one of the earlier papers Cempel and Tabaszewski (2007a,b). One can notice now from the top left picture of Fig. 6 that the total damage generalized symptom $\boldsymbol{SumSD}_i$ after rejection of the primary symptom No. 4 is well evolved, enabling a good forecast even without the rolling widow. But of course, as usually in the case of grey system modelling, the rolling window forecast gives the smallest error. This error can be even smaller if we diminish the span of window (w) as it is clearly seen from the picture bottom right in Fig. 6.

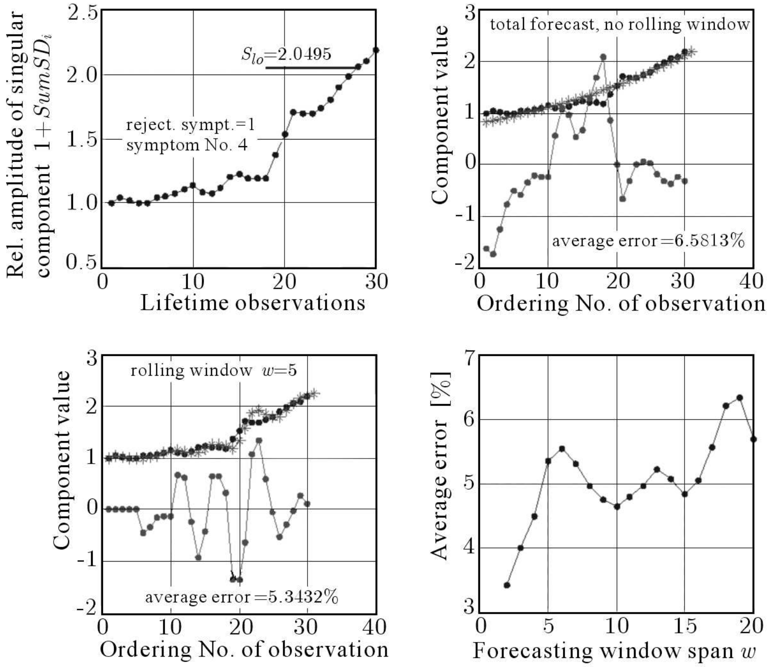

Fig. 6. Grey rolling forecast of the total damage generalized symptom $\boldsymbol{SumSD}_i$ for the huge fan sier1

It is also worthwhile to focus on other pictures of this figure. The picture top left clearly presents that the rejection of No. 4 symptom was a good idea allowing us to determine the symptom limit value S_l and having this information act properly to shut down the fan ahead of the breakdown time. The picture top right presents the total forecast of the total damage symptom $\boldsymbol{SumSD}_i$ with the model $GM(1,1)$. It seems to be a good forecast with a small average error, but the picture bottom left with the rolling window forecast have a smaller error and the actual forecast adapts smoothly to the course of $\boldsymbol{SumSD}_i$.

Knowing this, let us take into consideration the first generalized symptom $\boldsymbol{SD}_1$ and the way and property of inference obtained by this new approach. Next figure (Fig. 7) presents the results obtained according to this new way of thinking by the modified program svdopt2gs.m, where the basic quantity for the S_l and forecast calculation is the first generalized symptom $\boldsymbol{SD}_1$.

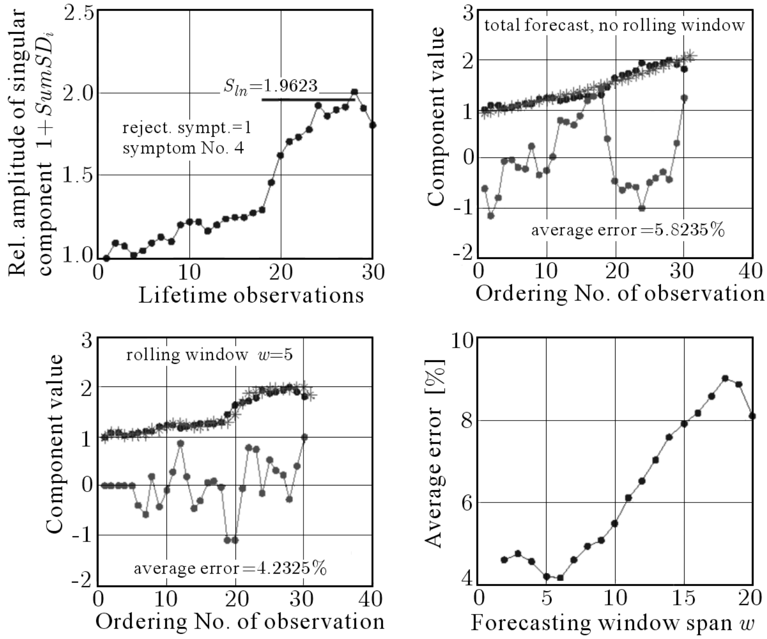

Fig. 7. Grey rolling forecast of the fan condition using the first generalized symptom $\boldsymbol{SD}_1$

It is seen in the left top picture of Fig. 7 that the course of $\boldsymbol{SD}_1$ symptom is decreasing at the end of the fan life, but the assessed symptom limit value S_l warns enough in advance to undertake shut down decision on time. However, comparatively to Fig. 6, this is some drawback in the clarity of diagnostic decision. On the contrary, the forecasting error is smaller here, having some minimum around the chosen window span $w = 5$.

As one can see from the above, it is hard to decide in advance if we should use for inference the total damage symptom $\boldsymbol{SumSD}_1$, or the first generalized symptom $\boldsymbol{SD}_1$. We need more experimental validation of this important issue. Let us take now quite another object, a small ball bearing of the type 6305, which was tested at the durability test-stand, where 7 symptoms were measured, each hour of the test. This particular bearing ($krak3$) broke down after 40 hours, and the results of observation were processed by these two mentioned programs. Figures 8 and 9, as the analogy to the Figs. 6 and 7, present the last stage of calculation of the forecast for the bearing $krak3$. One can conclude from these data that, again, the forecasting error is smaller if we use the first generalized symptom $\boldsymbol{SD}_1$ (Fig. 9 bottom left), but the transparency of diagnostic decision is better if one takes the total damage symptom

$\boldsymbol{SumSD}_i$ (Fig. 8 top left). Hence, it is impossible to decide in advance which quantity we should use for diagnostic inference – the total damage generalized symptom $\boldsymbol{SumSD}_i$ or the first generalized symptom $\boldsymbol{SD}_1$. We should decide on this important issue each time separately if we know the machine and the data.

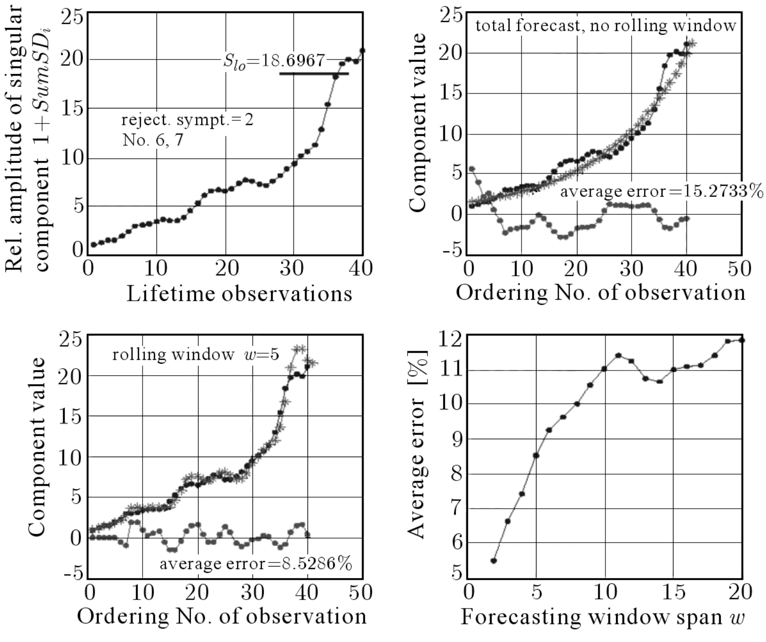

Fig. 8. The diagnostic decision and forecast premise for ball bearing $krak3$ on the basis of the total damage symptom (see next figure)

7. Conclusions

The premise to write this paper was supposition that a separate inference based on the first generalized machine symptom may be better than an inference on the basis of the total damage generalized symptom of the machine condition. As tipical in the multidimensional condition monitoring, we used the singular value decomposition to extract the fault information from the symptom observation matrix. Having calculated the just mentioned generalized symptoms, the symptom reliability and the symptom limit value S_l were assessed on that basis for the total damage symptom $\boldsymbol{SumSD}_i$ and for the dominating generalized symptom $\boldsymbol{SD}_1$. The last stage of inference was the forecast of the

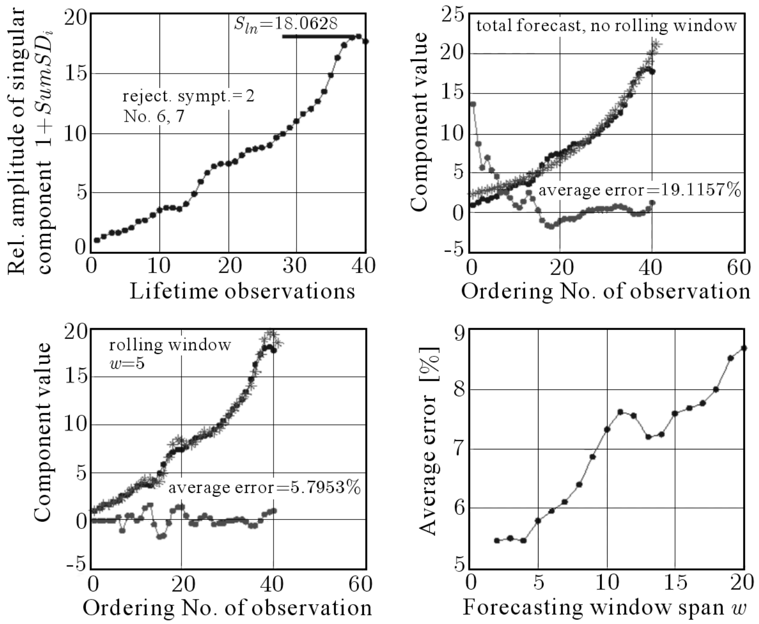

Fig. 9. The diagnostic decision and forecast premise for the same situation (as in Fig. 8), but on the basis of the first generalized symptom

future value of both symptoms made by the grey system theory and $GM(1,1)$ model. However, taking into consideration two cases of system monitoring, it was impossible to validate which one of these two approaches was better. Moreover, it seems that they are complementary, and both calculation should be made and the decision on machine condition carefully undertaken on a such basis.

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Prognozowanie globalnego i cząstkowego stanu za pomocą metod wielowymiarowej diagnostyki maszyn

Streszczenie

Maszyny mają wiele uszkodzeń, które ewoluują podczas ich pracy (życia). Jeśli obserwujemy pewną liczbę symptomów stanu podczas pracy maszyny, to jesteśmy w stanie uchwycić informację uszkodzeniową zorientowaną, za pomocą tzw. symptomowej macierzy obserwacji (SOM). Jedną z metod dalszej ekstrakcji tej informacji diagnostycznej jest zastosowanie rozkładu według wartości szczególnych (SVD) do SOM.

Problem postawiony w tej pracy polega na rozstrzygnięciu kwestii, czy w diagnostyce stanu używać uogólnionego symptomu całkowitego uszkodzenia maszyny, czy też posłużyć się tylko uogólnionym symptomem dominującego uszkodzenia. W tym celu stworzono dodatkowe oprogramowanie, dzięki któremu pokazano, że takie dychotomiczne postawienie kwestii nie jest niewłaściwe. Najlepiej używać obydwa symptomy uogólnione, wtedy nasza wiedza o stanie maszyny jest pełniejsza.

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