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AN EXPERIMENTAL STUDY ON THE SHEAR STRENGTH OF SFRC BEAMS WITHOUT STIRRUPS

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The use of steel fibers in reinforced concrete (RC) structural members aims at the improvement of mechanical properties of such members. This study focuses on shear strength characteristics of steel fiber reinforced concrete (SFRC) beams without stirrups. Test specimens consisting of three RC and ten SFRC beams without stirrups have been tested under three-point loading in order to investigate the effects of fiber content and shear span-to-effective depth ratio on the shear strength. Furthermore, an equation developed previously for predicting the ultimate shear strength of SFRC beams without stirrups is proposed to predict the diagonal cracking strength of SFRC beams without stirrups.

Keywords: steel fiber, reinforced concrete, shear strength, beam

1. Introduction

Concrete is a brittle material that has a relatively low tensile strength. This makes reinforced concrete (RC) beams without any shear reinforcement vulnerable to shear failure. Such brittle materials have been reinforced by using various types of fibers since ancient times. A substantial amount of research has been carried out to investigate the use of steel fibers for enhancing mechanical properties of concrete over the last half century (ACI, 1996). The focus of this study is on shear strength characteristics of low- and normal-strength steel fiber reinforced concrete (SFRC) beams without stirrups. It has been shown through experimental studies that the use of steel fibers improves those characteristics significantly (Batson *et al.*, 1972; Kadir and Saeed, 1986; Mansur *et al.*, 1986; Uomoto *et al.*, 1986; Lim *et al.*, 1987; Narayanan and Darwish, 1987; Li *et al.*, 1992; Swamy *et al.*, 1993; Noghabai, 2000; Kwak *et al.*, 2002; Rosenbusch and Teutsch, 2002; Dupont and Vandewalle, 2003; Cucchiara *et al.*, 2004; Parra-Montesinos, 2006; Parra-Montesinos *et al.*, 2006; Dinh *et al.*, 2010; Ding *et al.*, 2011; Aoude *et al.*, 2012; Minelli and Plizzari, 2013; Minelli *et al.*, 2014; Sahoo and Sharma, 2014; Shoaib *et al.*, 2014; Singh and Jain, 2014; Sahoo *et al.*, 2015). Besides, various studies have been conducted to develop an accurate model for predicting the shear strength of SFRC beams without stirrups (Sharma, 1986; Narayanan and Darwish, 1987; Ashour *et al.*, 1992, Swamy *et al.*, 1993; Imam *et al.*, 1997, Khuntia *et al.* 1999; Kwak *et al.*, 2002; RILEM, 2003; Yakoub, 2011; Gandomi *et al.*, 2011; Dinh *et al.*, 2011; Arslan, 2014). Despite these studies, SFRC has not yet achieved a widespread structural use. It is essential to increase the experimental database for both verifying the current models and developing more accurate ones.

The objective of this study is to investigate shear strength characteristics of low- and normal-strength SFRC beams without stirrups experimentally. A total of thirteen specimens, three of which being RC and the others SFRC beams, have been tested under three-point loading in order to examine the effects of volume fraction of steel fibers V_f and shear span-to-effective depth ratio a/d on the shear strength of SFRC beams without stirrups. Furthermore, the ultimate shear strengths and diagonal cracking strengths of test specimens have been predicted by using a number of equations available in the literature, and a comparison of those predictions is presented. A modified version of the equation proposed by Arslan (2014) for predicting the ultimate shear strength of SFRC beams without stirrups is suggested for predicting the diagonal cracking strength of SFRC beams without stirrups.

2. Experimental program

Test specimens consisting of three RC and ten SFRC beams have been divided into three groups as A2.5, A3.5 and A4.5 series based on the shear span-to-effective depth ratio. The beams of A2.5 series have a shear span-to-effective depth ratio of 2.5, which is usually defined as the lower limit for slender RC beams without stirrups. The shear span-to-effective depth ratios of the beams of A3.5 and A4.5 series have been chosen as 3.5 and 4.5, respectively, in order to observe shear failure resulting from diagonal tension. The geometrical properties of test specimens are shown in Fig. 1. All beams have the same cross-section of 150 mm by 230 mm with an effective depth of 200 mm. The beams of A2.5 series are 1400 mm long, whereas the beams of A3.5 and A4.5 series are 2200 mm long.

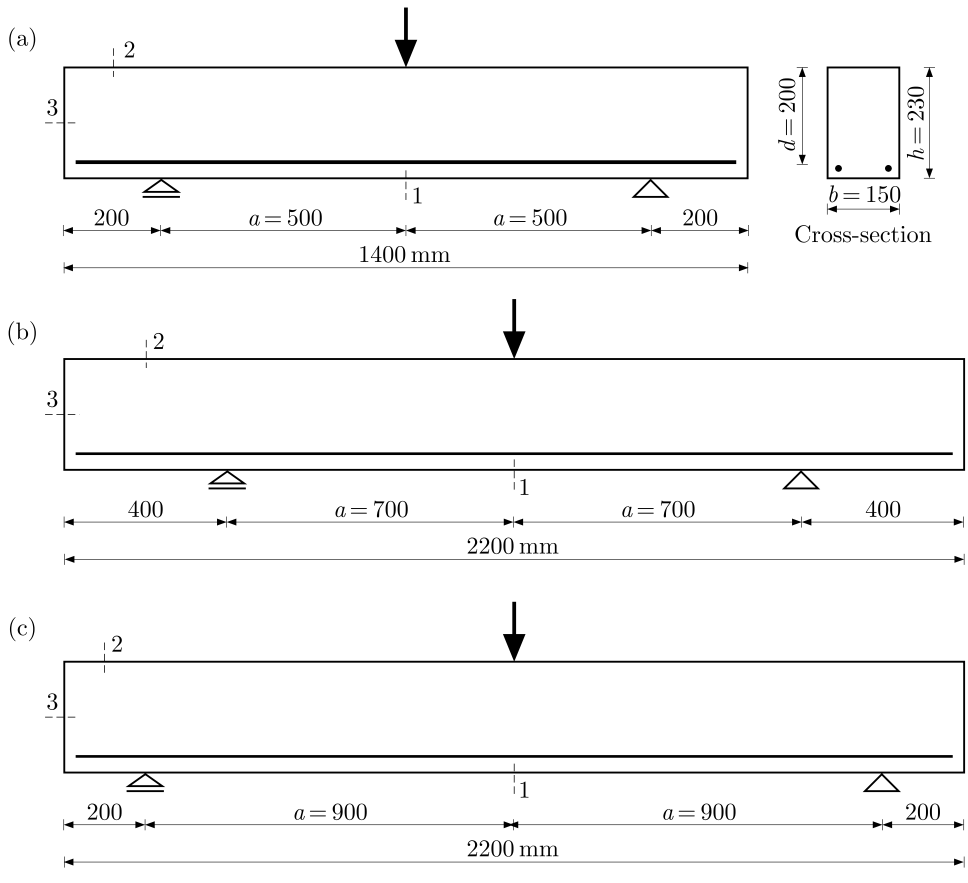

Fig. 1. Configuration and geometry of test specimens; (a) A2.5 series, (b) A3.5 series, (c) A4.5 series

A longitudinal reinforcement ratio ρ of 1.34% has been chosen to avoid any premature flexural failure. Two deformed (S420 grade) steel bars of 16 mm diameter were used as the longitudinal reinforcement for all beams. Hooked-end steel fibers with a length L_f of 30 mm and a nominal diameter D_f of 0.55 mm, resulting in an aspect ratio L_f/D_f of 54.5, were used as the only shear reinforcement. Each group of test specimens includes beams with the volume fraction of steel fibers ranging from 0.0% to 3.0%.

A specimen label consists of a combination of letters and numbers. Each label starts with an "A" followed by 2.5, 3.5 or 4.5 to designate the shear span-to-effective depth ratio and continues with an "F" to indicate the volume fraction of steel fibers followed by 1.0, 2.0 or 3.0 to designate the volume fraction of steel fibers. For example, a beam having a shear span-to--effective depth ratio of 3.5 with a volume fraction of steel fibers equal to 1.0% is labeled as A3.5F1.0. The specimens labeled as A2.5R, A3.5R and A4.5R are the reference beams that do not contain any steel fibers.

Table 1. Properties of test specimens

Test specimen	f_c [MPa]	V_f [%]	a/d	l [mm]
A2.5R	39.00	0.0	2.5	1400
A2.5F1.0a	33.68	1.0		
A2.5F1.0b	24.53	1.0		
A2.5F2.0	21.43	2.0		
A2.5F3.0	9.77	3.0		
A3.5R	31.52	0.0	3.5	2200
A3.5F1.0	20.21	1.0		
A3.5F2.0	21.43	2.0		
A3.5F3.0	27.91	3.0		
A4.5R	41.82	0.0	4.5	2200
A4.5F1.0	24.53	1.0		
A4.5F2.0	21.43	2.0		
A4.5F3.0	27.91	3.0		

The properties of test specimens are summarized in Table 1, where f_c is the concrete compressive cylinder strength and l is the length of test specimen. All specimens have been cast with the concrete mix given in Table 2. The concrete compressive strength of each specimen has been determined by using either 150×150×150 mm cube or 100×100 mm cylinder samples. The concrete compressive strength of A2.5F3.0 is notably low compared to the others. This might have been occurred due to poor mixing and/or compacting of concrete leading to a relatively low concrete compressive strength.

Table 2. Concrete mix

Materials	A2.5R, A2.5F3.0, A3.5R, A3.5F3.0, A4.5R, A4.5F3.0	A2.5F1.0a, A2.5F1.0b, A2.5F2.0, A3.5F1.0, A3.5F2.0, A4.5F1.0, A4.5F2.0
	Mixture proportions [kg/m^3]	
0-5 mm crushed sand	1180	1150
5-12 mm crushed stone	721	310
12-22 mm crushed stone	–	470
Fly ash (40% of binder)	80	90
Cement CEMI 42.5R	240	220
Water/binder	0.55	0.55
Superplasticizer	3.20	3.10

A load-controlled test procedure has been followed such that all specimens were incrementally loaded up to the failure. After each load increment, the deflections were measured by means of linear variable differential transducers (LVDTs) placed at locations 1, 2 and 3 shown in Fig. 1, and the crack pattern was monitored visually throughout the test.

3. Experimental results

At the early stages of loading, fine vertical cracks were observed in the vicinity of mid-span of each beam. With the increasing load, new flexural cracks formed away the mid-span. With a further increase in the load, the flexural cracks started then to propagate diagonally towards the applied load and additional diagonal cracks began to form farther away the mid-span. The failure mechanisms of test specimens except for A4.5F2.0 and A4.5F3.0 were characterized by a wide diagonal crack. It was observed that (i) the failure mechanisms were controlled by an increased shear strength and the dowel effect, and reduced crack spacing and crack width resulting from the crack-bridging ability of steel fibers, and (ii) the specimens exhibited large deflections at failure. The crack patterns of test specimens at failure are shown in Fig. 2.

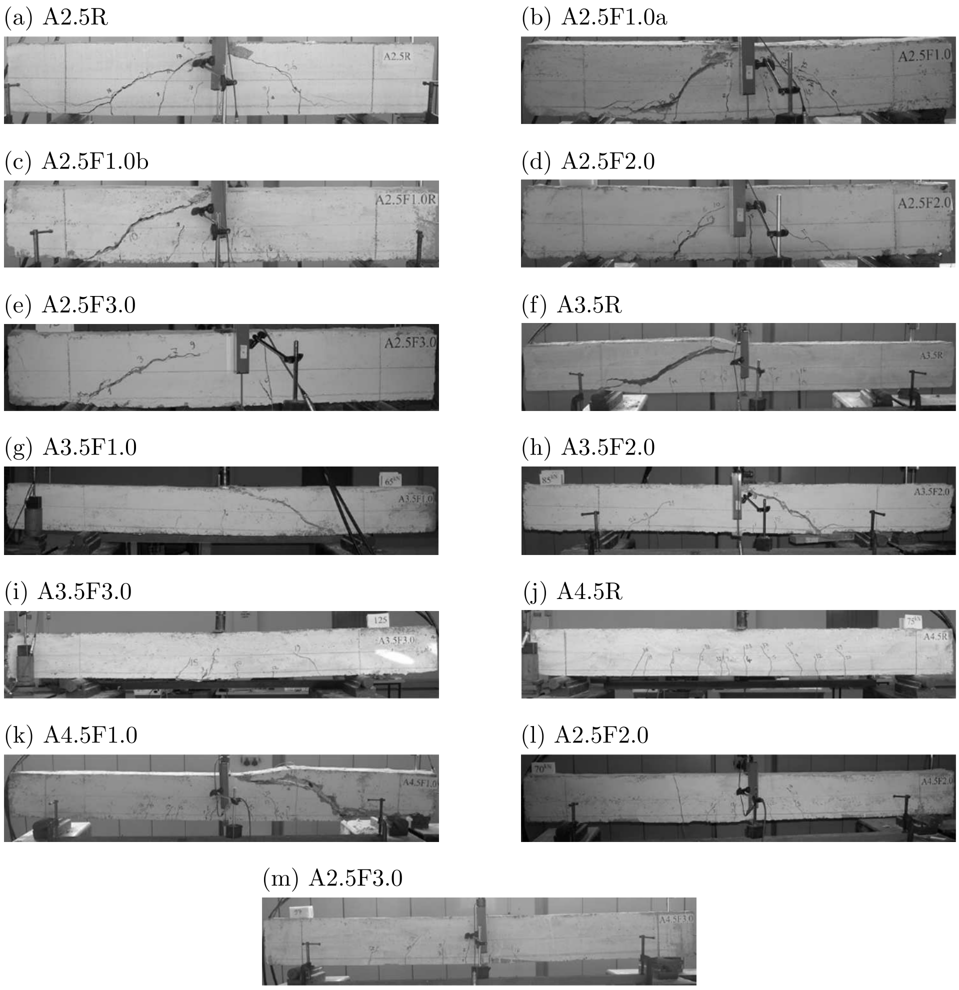

Fig. 2. Crack patterns of test specimens

It is usually assumed that the diagonal tension failure of an RC beam without stirrups initiates when the principal tensile stresses within the shear span exceed the tensile strength of concrete and a diagonal crack propagates through the beam web. In the studies of Arslan (2008, 2012) and Arslan and Polat (2013), a diagonal crack is defined as a major inclined crack extending from the level of the longitudinal reinforcement towards the applied load, and the load at the growth of the first inclined crack is termed as the diagonal cracking load. The diagonal cracking load P_{cr}, the mid-span deflection at diagonal cracking δ_{cr}, the ultimate load P_u, the ultimate mid-span deflection δ_u, the diagonal cracking strength v_{cr} and the ultimate shear strength v_u for each test specimen are given in Table 3, where the diagonal cracking strength and the ultimate shear strength are the diagonal cracking load and the ultimate load divided by the product of beam thickness b_w and effective depth, respectively. The load-deflection curves of all specimens are plotted in Fig. 3.

Table 3. Experimental results

Test specimen	P_{cr} [kN]	δ_{cr} [mm]	P_u [kN]	δ_u [mm]	v_{cr} [MPa]	v_u [MPa]	δ_u/δ_{cr}	v_u/v_{cr}
A2.5R	65	1.23	81	1.89	1.08	1.35	1.54	1.25
A2.5F1.0a	85	4.31	130	6.69	1.42	2.17	1.55	1.53
A2.5F1.0b	70	4.41	88	5.71	1.17	1.47	1.29	1.26
A2.5F2.0	70	2.75	100	5.09	1.17	1.67	1.85	1.43
A2.5F3.0	60	4.90	78	7.79	1.00	1.30	1.59	1.30
A3.5R	60	2.81	62	2.83	1.00	1.03	1.01	1.03
A3.5F1.0	55	2.80	65	4.26	0.92	1.08	1.52	1.17
A3.5F2.0	60	3.08	85	6.36	1.00	1.42	2.06	1.42
A3.5F3.0	95	5.09	117	6.83	1.58	1.95	1.34	1.23
A4.5R	60	5.51	76	7.37	1.00	1.27	1.34	1.27
A4.5F1.0	60	4.60	85	8.70	1.00	1.42	1.89	1.42
A4.5F2.0*	45	8.54	70	15.35	0.75	1.17	1.80	1.56
A4.5F3.0**	–	–	97	18.63	–	1.62	–	–

* Failed in shear-flexure; ** failed in flexure

3.1. Influence of volume fraction of steel fibers

Experimental results given in Table 3 and Fig. 3 clearly show that the use of steel fibers improved the shear strength and deformation capacity considerably. In the case of A2.5 series, the use of steel fibers in the amounts of 1.0% and 2.0% by volume increased the ultimate shear strength by 9% and 23%, respectively, and increased the deflection capacity 3.02 and 2.69 times, respectively. It is to be noted that increasing the volume fraction of steel fibers from 1.0% to 2.0% for approximately the same concrete compressive strength (A2.5F1.0b and A2.5F2.0) increased the ultimate shear strength by 14% but did not increase the deflection capacity. A2.5F3.0 cannot be compared directly since its concrete compressive strength is significantly low. In the case of A3.5 series, an increase in the ultimate shear strength due to the use of steel fibers in the amounts of 1.0%, 2.0% and 3.0% is 5%, 37% and 89%, respectively, and the deflection capacity is increased 1.51, 2.25 and 2.41 times, respectively.

For a better understanding of the effect of fiber content on the shear strength, the normalized maximum shear stress against the volume fraction of steel fibers for each beam is plotted in Fig. 4. It is clearly observed that the normalized maximum shear stress increases with the fiber content in the case of A2.5 and A3.5 series. A similar trend cannot be observed in the case of A4.5 series since A4.5F2.0 and A4.5F3.0 failed in flexure.

Even though the use of steel fibers enhanced the shear strengths and deformation capacities of the beams considerably, it was still not able to change the failure mechanisms of the beams of A2.5 and A3.5 series. On the other hand, the use of steel fibers in the amount of 3.0% in the case of A4.5 series both increased the shear strength and the deformation capacity by 28% and 152%, respectively, and changed the failure mode from shear to flexure. It is to be noted that a high volume fraction of steel fibers, i.e. 3.0%, was required to prevent the shear failure and use the flexural capacity. However, it may not be practical to work with a concrete mix having such a high volume fraction of steel fibers. Instead, steel fibers can be used together with a limited amount of stirrups to modify the failure mode.

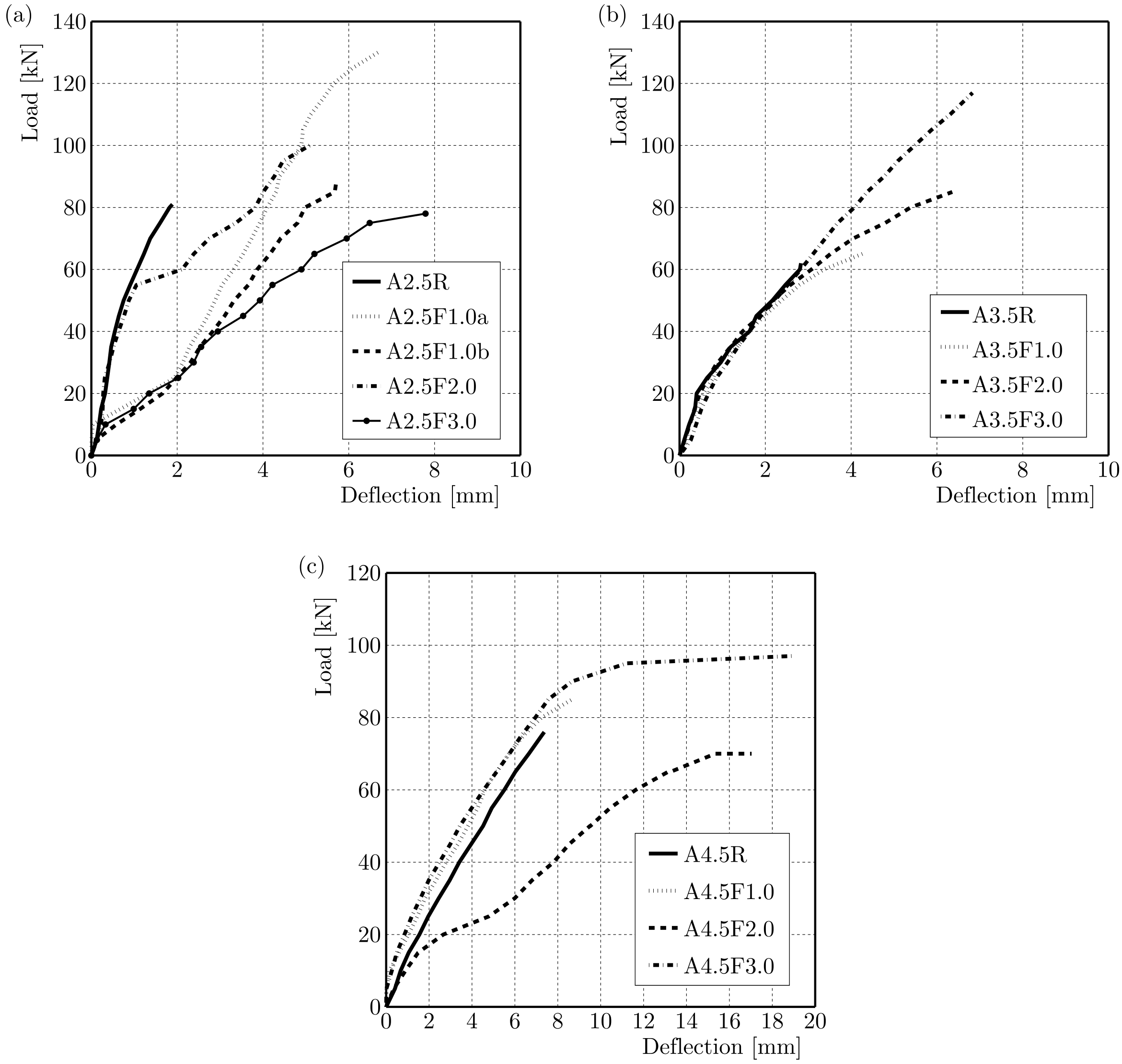

Fig. 3. Load-deflection curves; (a) A2.5 series, (b) A3.5 series, (c) A4.5 series

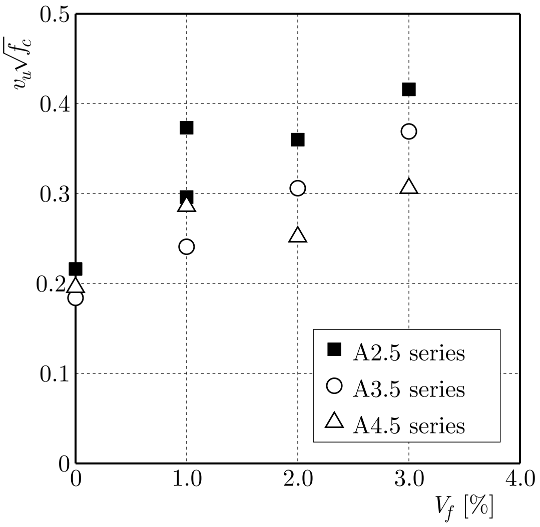

Fig. 4. Normalized maximum shear stress vs. fiber content

It is observed in Table 3 that the ratio of the ultimate shear strength to the diagonal cracking strength increases with the volume fraction of steel fibers up to 2.0% for the beams of all three series, but decreases with an increase in the volume fraction of steel fibers from 2.0% to 3.0% for the beams of A2.5 and A3.5 series. A4.5F3.0 exhibited flexural failure and the diagonal cracking was not observed in this specimen.

3.2. Influence of shear span-to-effective depth ratio

It can be seen in Table 3 that the diagonal cracking strength decreases with the increasing shear span-to-effective depth ratio as a result of the increased flexural moment and the associated principal stresses and the diminishing arching effect. The shear span-to-effective depth ratio eventually affects the ultimate shear strength. As expected for the reference beams, the ultimate shear strength of A2.5R, which has the smallest shear span-to-effective depth ratio, is greater than that of the other reference beams; however it has a smaller deflection capacity. A similar relationship between the beams with the volume fraction of steel fibers of 2.0% is observed. A2.5F2.0 has a greater load carrying capacity and a smaller deflection capacity than A3.5F2.0 and A4.5F2.0 do.

It is observed that increase in the deflection capacity decreases with the increasing shear span-to-effective depth ratio for a given fiber content. The deflection capacities of A2.5F1.0b, A3.5F1.0 and A4.5F1.0 are 3.02, 1.51 and 1.18 times those of A2.5R, A3.5R and A4.5R, respectively. The use of steel fibers in the amount of 2.0% by volume resulted in deflection capacities 2.69, 2.25 and 2.08 times those of the reference specimens for A2.5F2.0, A3.5F2.0 and A4.5F2.0, respectively. Experimental results manifest that it is essential to consider the effect of shear span-to-effective depth ratio in predicting the shear strength of SFRC beams, as done by Sharma (1986), Narayanan and Darwish (1987), Ashour *et al.* (1992), Imam *et al.* (1997), Kwak *et al.* (2002), Gandomi *et al.* (2011) and Arslan (2014).

4. Predicting the shear strengths of test specimens

A number of equations proposed for predicting the ultimate shear strength and diagonal cracking strength of SFRC beams without stirrups are considered. The statistical evaluations of the equations considered within the scope of this study are available in the work of Arslan (2014). In this study, the equations are used only for predicting the ultimate shear strength and diagonal cracking strength of test specimens.

4.1. The equations for ultimate shear strength

Sharma (1986) proposed a simple empirical equation for predicting the ultimate shear strength of SFRC beams without stirrups. The equation, which is recommended by ACI (1988) (v_u in MPA), is

$$v_u = kf_{ct}\Big(\frac{d}{a}\Big)^{0.25} \tag{4.1}$$

where f_{ct} is the concrete tensile strength, $k = 1$ if f_{ct} is obtained by a direct tension test, $k = 2/3$ if f_{ct} is obtained by an indirect tension test and $k = 4/9$ if f_{ct} is obtained by using the modulus of rupture or $f_{ct} = 0.79\sqrt{f_c}$.

Narayanan and Darwish (1987) proposed an empirical equation as

$$v_u = e\Big(0.24f_{sp} + 80\rho\frac{d}{a}\Big) + v_b \tag{4.2}$$

where $e = 1.0$ for $a/d > 2.8$ and $e = 2.8d/a$ for $a/d \leqslant 2.8$, $f_{sp} = f_{cuf}/(20-\sqrt{F}) + 0.7 + \sqrt{F}$ is the computed value of split-cylinder strength of fiber concrete, f_{cuf} is the cube strength of fiber reinforced concrete, $F = (L_f/D_f)V_f d_f$ is the fiber factor (d_f is the fiber bond factor that is 0.5 for round, 0.75 for crimped and 1.0 for indented fibers), $v_b = 0.41\tau F$ is the pull-out strength of fibers along the inclined crack and τ is the average fiber matrix interfacial bond stress equal to 4.15 MPa.

Ashour *et al.* (1992) revised the equations given by the ACI 318 code (ACI, 2014) and Zsutty (1971) for predicting the ultimate shear strength of RC beams without stirrups in order to propose two empirical equations for SFRC beams with $a/d \geqslant 2.5$ as

$$v_u = (0.7\sqrt{f_c} + F)\frac{d}{a} + 17.2\rho\frac{d}{a} \qquad v_u = (2.11\sqrt[3]{f_c} + 7F)\sqrt[3]{\rho\frac{d}{a}} \tag{4.3}$$

respectively.

Swamy *et al.* (1993) proposed an equation based on the truss model as

$$v_u = 0.37\tau V_f\frac{L_f}{D_f} + 0.167\sqrt{f_c} \tag{4.4}$$

where τ is assumed to be 4.15 MPa as suggested by Narayanan and Darwish (1987).

Imam *et al.* (1997) modified the equation that Bazant and Sun (1987) had developed to predict the ultimate shear strength of RC beams without stirrups to propose the relationship

$$v_u = 0.6\frac{1+\sqrt{5.08/d_a}}{\sqrt{1+d/25d_a}}\sqrt[3]{\rho(1+4F)}\left(f_c^{0.44} + 275\sqrt{\frac{\rho(1+4F)}{(a/d)^5}}\right) \tag{4.5}$$

where d_a is the maximum aggregate size and d_f is 0.5 for smooth, 0.9 for deformed and 1.0 for hooked fibers.

Khuntia *et al.* (1999) proposed the following equation

$$v_u = (0.167 + 0.25F)\sqrt{f_c} \tag{4.6}$$

where d_f is 2/3 for plain and round, 1.0 for hooked or crimped fibers.

Kwak *et al.* (2002) developed an equation by using the form of the equation proposed by Zsutty (1971) combined with an additional term accounting for the contribution of steel fibers and proposed two versions of the equation with different constants as

$$v_u = 2.1ef_{sp}^{0.7}\left(\rho\frac{d}{a}\right)^{0.22} + 0.8v_b^{0.97} \tag{4.7}$$

where $e = 1.0$ for $a/d > 3.5$ and $e = 3.5d/a$ for $a/d \leqslant 3.5$, and

$$v_u = 3.7e\sqrt[3]{f_{sp}^2}\sqrt[3]{\rho\frac{d}{a}} + 0.8v_b \tag{4.8}$$

where $e = 1.0$ for $a/d > 3.4$ and $e = 3.4d/a$ for $a/d \leqslant 3.4$.

According to RILEM (2003), the ultimate shear strength of SFRC beams without stirrups is calculated as

$$v_{Rd,3} = 0.12k\sqrt[3]{100\rho f_c} + 0.7k_f k_1\tau_{fd} \tag{4.9}$$

where $k = 1 + \sqrt{200/d} \leqslant 2$ (d is in mm), $\rho \leqslant 0.02$, k_f is a factor considering the contribution of flanges in a T-section and is equal to 1 for rectangular sections, $k_1 = 1 + \sqrt{200/d} \leqslant 2$ (d is in mm), $\tau_{fd} = 0.12 f_{Rk,4}$ is the design value of increase in shear strength due to steel fibers and $f_{Rk,4}$ is the characteristic residual strength for the ultimate limit state.

Yakoub (2011) used an expression developed for predicting the contribution of steel fibers to the shear strength of SFRC beams to modify the equations given by Bazant and Kim (1984) and CSA A23.3-04 (CSA, 2004). The resulting equations for $a/d \geqslant 2.5$ are

$$\begin{aligned} v_u &= 0.83\xi \sqrt[3]{\rho}\left(\sqrt{f_c} + 249.28\sqrt{\frac{\rho}{(a/d)^5}}\right) + 0.162F\sqrt{f_c} \\ v_u &= \beta\sqrt{f_c}(1 + 0.70F) \end{aligned} \tag{4.10}$$

respectively, where $\xi = 1/\sqrt{1 + d/(25da)}$ is the aggregate size effect factor, $\beta = [0.4/(1 + 1500\varepsilon_x)][1300/(1000 + s_{xe})]$ (s_{xe} is in mm), $\varepsilon_x = (M/d_v + V)/(2E_sA_s)$ is the longitudinal strain at the mid-depth of the beam web, M and V are the external failure moment and shear acting on the section, respectively, d_v is the flexural lever arm equal to $0.9d$ or $0.72h$ (h is the beam height), whichever is greater, $s_{xe} = 35s_x/(16 + d_a) \geqslant 0.85s_x$ is the equivalent crack spacing factor that accounts for the maximum aggregate size effects on the shear strength, s_x is the crack spacing parameter that accounts for the crack spacing at the mid-depth of the beam and d_f is 0.79 for sheared, 0.83 for crimped, 0.89 for duoform, 0.91 for rounded, 0.92 for indented cut wire and 1.00 for hooked fibers.

Gandomi *et al.* (2011) developed a nonlinear model by means of linear genetic programming as

$$v_u = 2\frac{d}{a}(\rho f_c + v_b) + 2\frac{d}{a}\frac{\rho}{(288\rho - 11)^4} + 2 \tag{4.11}$$

Dinh *et al.* (2011) proposed an equation as the summation of the shear stress carried across the compression zone and the vertical component of the diagonal tension resistance provided by steel fibers, such that

$$v_u = 0.13\rho f_y + 1.2\sqrt[4]{\frac{V_f}{0.0075}}\Big(1 - \frac{c}{d}\Big) \tag{4.12}$$

where f_y is the yield strength of flexural reinforcement and c is depth of the compression zone, which can be simply taken as $0.1h$.

Arslan (2014) proposed an equation by considering the influences of the shear span-to--effective depth ratio, dowel strength of tensile reinforcement and contribution of steel fibers to the shear strength as

$$v_u = \Big(0.2\sqrt[3]{f_c^2}\frac{c}{d} + \sqrt{\rho(1 + 4F)f_c}\Big)\sqrt[3]{\frac{3}{a/d}} \tag{4.13}$$

where $(c/d)^2 + (600\rho/f_c)(c/d) - 600\rho/f_c = 0$.

4.2. The predictions for ultimate shear strength

The ultimate shear strengths of test specimens excluding the reference beams (A2.5R, A3.5R and A4.5R), A4.5F2.0 and A4.5F3.0 – since they failed in shear-flexure and flexure, respectively – have been predicted by using Eqs. (4.1)-(4.13) and the predictions were compared with the experimental values. The mean value (MV), standard deviation (SD) and coefficient of variation (COV) of the ratios of the experimental values to the corresponding predictions are given in Table 4.

Table 4. Statistics of the ratios of the experimental values to the predictions

The model	MV	SD	COV
Sharma (1986)	1.241	0.186	0.150
Narayanan and Darwish (1987)	0.593	0.183	0.309
Ashour *et al.* (1992), Eq. $(4.3)_1$	0.504	0.186	0.369
Ashour *et al.* (1992), Eq. $(4.3)_2$	0.783	0.214	0.274
Swamy *et al.* (1993)	0.734	0.239	0.326
Imam *et al.* (1997)	0.633	0.201	0.317
Khuntia *et al.* (1999)	0.853	0.192	0.225
Kwak *et al.* (2002), Eq. (4.7)	0.543	0.128	0.235
Kwak *et al.* (2002), Eq. (4.8)	0.574	0.145	0.253
RILEM (2003)	1.361	0.227	0.167
Yakoub (2011), Eq. $(4.10)_1$	0.858	0.126	0.147
Yakoub (2011), Eq. $(4.10)_2$	1.342	0.329	0.246
Gandomi *et al.* (2011)	0.480	0.116	0.242
Dinh *et al.* (2011)	0.725	0.172	0.238
Arslan (2014)	0.833	0.098	0.118

The equations proposed by Sharma (1986), RILEM (2003) and Yakoub (2011) (Eq. $(4.10)_2$) underestimate the ultimate shear strengths of test specimens involved in this study, whereas the ones proposed by Narayanan and Darwish (1987), Ashour *et al.* (1992) (Eq. $(4.3)_1$), Imam *et al.* (1997), Kwak *et al.* (2002) and Gandomi *et al.* (2011) largely overestimate the experimental values. It is observed from the statistics given in Table 4 that the equation proposed by Yakoub (2011) (Eq. $(4.10)_2$) and Arslan (2014) provide the most accurate predictions for the specimens involved in this study, where as the predictions of the equation of Arslan (2014) are slightly better. The ratios of the experimental values to the corresponding predictions of the equation of Arslan (2014) have a mean value of 0.833 with the lowest coefficient of variation equal to 0.118.

4.3. The equations for diagonal cracking strength

Narayanan and Darwish (1987) proposed an empirical equation for predicting the diagonal cracking strength as

$$v_{cr} = 0.24 f_{sp} + 20\rho\frac{d}{a} + 0.5F \tag{4.14}$$

Kwak *et al.* (2002) proposed an equation by following a procedure similar to the one followed for developing Eqs. (4.7) and (4.8) as

$$v_{cr} = 3\sqrt[3]{f_{sp}^2}\sqrt[3]{\rho\frac{d}{a}} \tag{4.15}$$

The equation proposed by Arslan (2014) and given in Eq. (4.13) has been modified by introducing a strength reduction factor of 0.6, which was obtained through a regression analysis undertaken to identify the strength reduction factor in calculating the diagonal cracking strength of SFRC slender beams without stirrups by using the results of existing experimental data. The resulting equation is

$$v_u = 0.6\Big(0.2\sqrt[3]{f_c^2}3\frac{c}{d} + \sqrt{\rho(1+4F)f_c}\Big)\sqrt[3]{\frac{3}{a/d}} \tag{4.16}$$

4.4. The predictions for diagonal cracking strength

The diagonal cracking strengths of test specimens containing steel fibers excluding A4.5F3.0 that failed in flexure have been predicted by using Eqs. (4.14)-(4.16). The mean value, standard deviation and coefficient of variation of the ratios of the experimental values to the corresponding predictions are given in Table 5. It is observed from Table 5 that Eq. (4.16) performed better in predicting the diagonal cracking strengths of the considered test specimens than the other equations do. The ratios of the experimental values to the corresponding predictions obtained from Eq. (4.16) have a mean value of 1.032 with the lowest coefficient of variation equal to 0.079.

Table 5. Statistics of the ratios of the experimental values to the predictions

The model	MV	SD	COV
Eq. (4.16)	1.032	0.082	0.079
Narayanan and Darwish (1987)	0.899	0.166	0.185
Kwak *et al.* (2002)	1.102	0.131	0.119

5. Conclusion

An experimental study has been conducted to investigate shear strength characteristics of low- and normal-strength SFRC slender beams without stirrups. The fact that the use of steel fibers improves the ultimate shear strength, diagonal cracking strength and ductility significantly is justified based on the following observations.

- The use of steel fibers with volume fractions of 1.0% and 2.0% increased the ultimate shear strength by 9% and 23%, respectively, in the case of a shear span-to-effective depth ratio of 2.5 and by 5% and 37%, respectively, in the case of a shear span-to-effective depth ratio of 3.5.
- The use of steel fibers in an amount of 3.0% was not able to change the failure mode of test specimens with a shear span-to-effective depth ratio of either 2.5 or 3.5, but it made the beam with a shear span-to-effective depth ratio of 4.5 fail in flexure instead of shear. However, it is to be noted that 3.0% may not be a practical volume fraction in the context of workability of a concrete mix. The use of steel fibers with a limited amount of stirrups can be a more practical way to modify the failure mode.
- The ratio of the ultimate shear strength to the diagonal cracking strength increased with the volume fraction of steel fibers up to 2.0% for all series of beams.
- The diagonal cracking strength decreased with the increasing shear span-to-effective depth ratio, which eventually affected the ultimate shear strength. This implies that it is essential to consider the effect of shear span-to-effective depth ratio in predicting the shear strength of SFRC beams.
- The use of steel fibers increased the deflection capacities significantly in all cases.

Besides the experimental study, the ultimate shear strengths and diagonal cracking strengths of SFRC beams involved in the experimental study were predicted by various equations available in the literature. Among the fifteen equations considered for predicting the ultimate shear strength of SFRC beams without stirrups, the equation proposed by Arslan (2014) had the best performance. The equation of Arslan (2014) was modified to predict the diagonal cracking strengths of SFRC beams involved in this study by introducing a strength reduction factor equal to 0.6. The modified equation had a better performance than those of the other two considered equations. Since the number of test specimens was limited, the modified version of the equation of Arslan (2014) should be verified with more data.

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