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REGULAR AND CHAOTIC VIBRATIONS OF A VIBRATION-ISOLATED HAND GRINDER

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The paper is concerned with qualitative analysis of a non-linear model describing vibration of a vibration-isolated hand grinder. A discontinuous description of grinding forces is introduced, which accounts for the possible separation of the grinding wheel from the object during the process. Eight non-linear ordinary differential equations are obtained which describe dynamics of the system. Numerical analysis is done using methods of numerical integration and the Fast Fourier Transform. The influence of selected parameters on the character of vibration is studied and some measures are calculated which characterize the quality of the vibration isolation system.

Key words: vibrations, chaos, grinding, vibration isolation, non-linear

1. Introduction

Harmful vibrations during mechanical processing (such as grinding or milling) have to be avoided since they deteriorate the quality of the product as well as have a negative effect on the human-operator. The main sources of these vibrations are kinematic and inertial excitations (Alfares and Elsharkawy, 2002; Gradišek *et al.*, 2001; Karube *et al.*, 2002; Łuczko and Markiewicz, 1986; Suh *et al.*, 2002). In order to reduce vibration levels transmitted to the operator, vibration isolation systems are mounted between the tool body and the handle. In the case of passive vibration isolation systems their parameters are usually selected using linear models.

However, for large vibration amplitudes it is necessary to account for non-linear phenomena, such as brought about e.g. by the loss of contact of the grinding wheel with the object being worked (Łuczko *et al.*, 2003). Moreover, it is desirable to determine the influence of parameters that undergo changes during the grinding process, such as rotational speed or pressure on the tool handle, on dynamic characteristics of the system.

2. Model of the system

Figure 1 shows a schematic view of a hand grinder equipped with a vibration isolation system. The following elements have been taken into account in the model: tool body (1) (with the subassembly motor-spindle-grinding wheel), handle (2) and object being processed (base) (3). The resilient connections between the tool body and the handle represent the passive vibration isolation system, whereas the flexible elements which link the body to its surroundings are a simplified model of the operator interaction. The flexible connections consist of extension-compression springs of stiffnesses c_j $(j = 1, 2)$ and torsion springs of stiffnesses k_j, which can be considered as a result of the reduction of any resilient connections to the point at a distance l_{nj} from the respective centre of mass S_n $(n = 1, 2)$. In a similar way, one can define parameters which describe energy dissipation. Assuming that the damping matrix is proportional to the stiffness matrix, the damping of a given connector is given by a single coefficient ε_j. In order to limit the displacements of the handle relative to the body, a motion limiter (4) is introduced into the model. The properties and geometry of the motion limiter are determined by the parameters c_4, ε_4, δ, l_{14}, l_{24}.

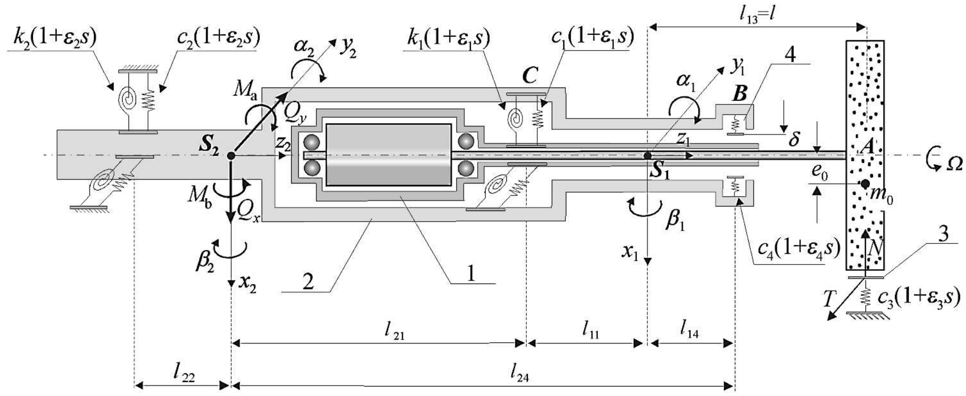

Fig. 1. Model of the system: 1 – body, 2 – handle, 3 – workpiece, 4 – limiter

It has been assumed that the tool body and the handle can undergo a general motion in space, with the exclusion of longitudinal and torsional degrees of freedom. Therefore, longitudinal and torsional vibrations are excluded from the analysis. The kinematic excitation is accounted for by defining parameters m_0 (the unbalanced mass) and e_0 (eccentricity), and assuming that the rotational speed remains constant. Introducing moving co-ordinate systems with origins at the centres of mass S_n of subassemblies: the body with the rotating elements $(n = 1)$ and the handle $(n = 2)$, motion of the system can be described by specifying the co-ordinates x_n and y_n of points S_n and the angles α_n and β_n, which for small vibrations describe rotations of the local axes with respect to the fixed ones.

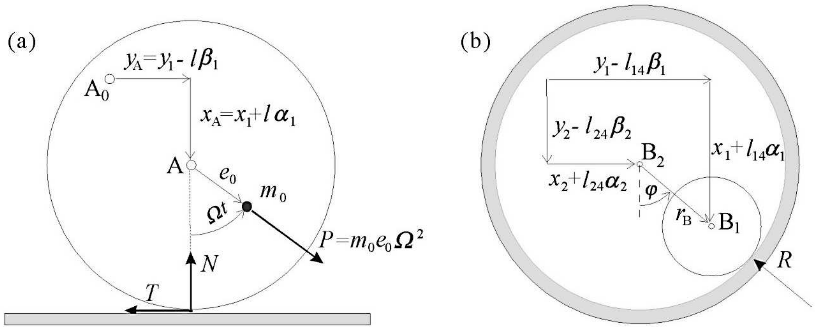

Fig. 2. Kinematic relations: (a) grinding forces, (b) reaction of the limiter

The case of processing of a flat surface is being considered. It has been assumed that when the grinding wheel remains in contact with the base (for $x_A = x_1 + l\alpha_1 > 0$), a linear relationship $T = fN$ holds between the tangential and normal components of the grinding force (Fig. 2a). Additionally, it has been assumed that the normal reaction force N is described by the Voigt-Kelvin model, but it cannot take on negative values. By introducing the notation $s = d/dt$, this reaction force is given by the following formula

$$N(x_A, sx_A) = c_3(1 + \varepsilon_3 s)x_A H(x_A)H[(1 + \varepsilon_3 s)x_A] \tag{2.1}$$

where H is the Heaviside step function. In a similar way, the normal reaction force of the limiter (Fig. 2b) is written as

$$R(r_B, sr_B) = c_4(1 + \varepsilon_4 s)r_B H(r_B - \delta)H[(1 + \varepsilon_4 s)r_B] \tag{2.2}$$

Here, it has been assumed that the impact takes place when $r_B = \sqrt{x_B^2 + y_B^2} > \delta$, where

$$x_B = (x_1 + l_{14}\alpha_1) - (x_2 + l_{24}\alpha_2) \qquad y_B = (y_1 - l_{14}\beta_1) - (y_2 - l_{24}\beta_2) \tag{2.3}$$

The Cartesian components of the limiter reactions are calculated as

$$R_x = R\frac{x_B}{r_B} \qquad R_y = R\frac{y_B}{r_B} \tag{2.4}$$

Using the laws of change of momentum and angular momentum about the centres of mass S_n, motion of the system can be described by the following set of eight second-order ordinary differential equations

$$
\begin{aligned}
&m_1s^2x_1 + c_1(1+\varepsilon_1 s)(x_1 - x_2 - l_{11}\alpha_1 + l_{21}\alpha_2) + R_x + N = m_0e_0\Omega^2\cos\Omega t\\
&m_1s^2y_1 + c_1(1+\varepsilon_1 s)(y_1 - y_2 + l_{11}\beta_1 - l_{21}\beta_2) + R_y + T = m_0e_0\Omega^2\sin\Omega t\\
&I_1s^2\alpha_1 - I_0\Omega s\beta_1 + (1+\varepsilon_1 s)[k_{11}\alpha_1 - k_{21}\alpha_2 - c_1l_{11}(x_1 - x_2)] +\\
&\qquad +R_xl_{14} + Nl = m_0e_0l\Omega^2\cos\Omega t\\
&I_1s^2\beta_1 + I_0\Omega s\alpha_1 + (1+\varepsilon_1 s)[k_{11}\beta_1 - k_{21}\beta_2 + c_1l_{11}(y_1 - y_2)] +\\
&\qquad -R_yl_{14} - Tl = -m_0e_0l\Omega^2\sin\Omega t\\
&m_2s^2x_2 - c_1(1+\varepsilon_1 s)(x_1 - x_2 - l_{11}\alpha_1 + l_{21}\alpha_2) + c_2(1+\varepsilon_2 s)(x_2 - l_{22}\alpha_2) +\\
&\qquad -R_x = Q_x \qquad\qquad (2.5)\\
&m_2s^2y_2 - c_1(1+\varepsilon_1 s)(y_1 - y_2 + l_{11}\beta_1 - l_{21}\beta_2) + c_2(1+\varepsilon_2 s)(y_2 + l_{22}\beta_2) +\\
&\qquad -R_y = Q_y\\
&I_2s^2\alpha_2 - (1+\varepsilon_1 s)[k_{21}\alpha_1 - (k_1 + c_1l_{21}^2)\alpha_2 - c_1l_{21}(x_1 - x_2)] +\\
&\qquad +(1+\varepsilon_2 s)(k_{22}\alpha_2 - c_2l_{22}x_2) - R_xl_{24} = M_\alpha\\
&I_2s^2\beta_2 - (1+\varepsilon_1 s)[k_{21}\beta_1 - (k_1 + c_1l_{21}^2)\beta_2 + c_1l_{21}(y_1 - y_2)] +\\
&\qquad +(1+\varepsilon_2 s)(k_{22}\beta_2 + c_2l_{22}y_2) + R_yl_{24} = -M_\beta
\end{aligned}
$$

Here

$$
k_{11} = k_1 + c_1l_{11}^2 \qquad k_{21} = k_1 + c_1l_{11}l_{21} \qquad k_{22} = k_2 + c_2l_{22}^2 \qquad (2.6)
$$

In equations (2.5), m_1 and m_2 are respectively the masses of subassemblies 1 and 2, I_1 and I_2 are moments of inertia of the subassemblies with respect to the x-axis (or the y-axis, thanks to axial symmetry) passing through points S_1 and S_2, and I_0 is the moment of inertia of the rotating elements about the axis of symmetry. The generalised forces Q_x, Q_y, M_α and M_β represent the operator action on the system. When the grinding wheel remains in contact with the object, one can assume that these forces remain constant. The case when the grinding wheel loses contact with the object is more complex, especially as far as the generalized forces Q_y and M_β are concerned. The human operator is an active system since he reacts to changes in the working conditions. During grinding, the operator tries to adjust the magnitudes of forces so as to equilibrate the corresponding components of the grinding forces. When the loss of contact occurs, the operator tries on one hand to bring the tool back into contact with the object (without changing the values of Q_x, M_α), and, on the other hand, counteracts sudden movements of the tool in the direction tangent to the surface being processed by a sudden change (in the model an instant change) of the force Q_y and the moment M_β. To account for this behaviour, the following simplified relations are used in the model

$$Q_y = fQ_xH(N) \qquad M_\alpha = Q_xd \qquad M_\beta = Q_yd \tag{2.7}$$

where d is the distance of the resultant operator's action from the centre of mass S_2.

The analysis is done in a dimensionless form, and non-dimensional quantities are used where the amplitudes are calculated relative to the effective amplitude of inertial excitation $e = m_0e_0/(m_1+m_2)$, angles are taken relative to e/l ($l = l_{13}$) and the non-dimensional time $\tau = \omega_0 t$ is referred to the circular frequency $\omega_0 = \sqrt{c_2/(m_1+m_2)}$ of the simplified linear model.

The equations, when written in the dimensionless form, depend on the following parameters

$$\begin{array}{llll}
\omega = \dfrac{\varOmega}{\omega_0} & \varDelta = \dfrac{\delta}{e} & q = \dfrac{Q_x}{c_2e} & \mu_n = \dfrac{m_n}{m_1+m_2} \\
\rho_n = \dfrac{I_n}{m_nl^2} & \chi = \dfrac{I_0}{I_1} & \gamma_j = \dfrac{c_j}{c_2} & \kappa_j = \dfrac{k_j}{c_2l^2} \\
\zeta_j = \dfrac{\varepsilon_j\omega_0}{2} & \lambda_{nj} = \dfrac{l_{nj}}{l} & \lambda_0 = \dfrac{l_0}{l} & \eta = \dfrac{d}{l}
\end{array} \tag{2.8}$$

and on the coefficient of dry friction f. In equations (2.8), the index n is the number of the subsystem ($n = 1,2$), and the index j corresponds to the number of the flexible element ($j = 1,2,3,4$). Moreover, the following conditions hold: $\gamma_2 = 1$, $\lambda_{21} = \lambda_0 - \lambda_{11}$, $\lambda_{24} = \lambda_0 + \lambda_{14}$ and $\mu_1 + \mu_2 = 1$. By introducing the state vector

$$\boldsymbol{u} = \Big[\frac{x_1}{e}, \frac{y_1}{e}, \frac{l\alpha_1}{e}, \frac{l\beta_1}{e}, \frac{x_2}{e}, \frac{y_2}{e}, \frac{l\alpha_2}{e}, \frac{l\beta_2}{e}\Big]^\top \tag{2.9}$$

the system of equations (2.5) can be written in a compact matrix form

$$\mathbf{M}\boldsymbol{u}\text{''} + (\mathbf{G} + 2\zeta_1\mathbf{C}_1 + 2\zeta_2\mathbf{C}_2)\boldsymbol{u}' + (\mathbf{C}_1 + \mathbf{C}_2)\boldsymbol{u} = \boldsymbol{p}(\tau) + \boldsymbol{q} + \boldsymbol{r} + \boldsymbol{s} \tag{2.10}$$

Here, the matrices $\mathbf{M}$, $\mathbf{G}$, $\mathbf{C}_1$ and $\mathbf{C}_2$ are respectively the mass-, gyroscopic- and stiffness matrices, and $\boldsymbol{p}(\tau)$ is the vector of inertial excitation

$$\boldsymbol{p}(t) = \omega^2[\cos\omega\tau, \sin\omega\tau, \cos\omega\tau, -\sin\omega\tau, 0, 0, 0, 0]^\top \tag{2.11}$$

The vectors $\boldsymbol{q}$, $\boldsymbol{r}$ and $\boldsymbol{s}$ describe non-linear terms, respectively related to the model of the operator

$$\boldsymbol{q} = [0, 0, 0, 0, q, fqH(n), \eta q, -\eta fqH(n)]^\top \tag{2.12}$$

the model of the limiter

$$\boldsymbol{r} = [-r_x, -r_y, -\lambda_{14}r_x, \lambda_{14}r_y, r_x, r_y, \lambda_{24}r_x, -\lambda_{24}r_y]^\top \tag{2.13}$$

and the model of the grinding forces

$$\boldsymbol{s} = [-n, -fn, -n, fn, 0, 0, 0, 0]^\top \tag{2.14}$$

In order to calculate n, r_x and r_y one makes use of formulae (2.1)-(2.4), where dimensionless components of the state vector (2.9) and the respective non-dimensional parameters (e.g. parameters γ_3, γ_4, $2\zeta_3$, $2\zeta_4$ and $\varDelta$ in lieu of c_3, c_4, ε_3, ε_4 and δ) are introduced. The matrix $\mathbf{M}$ is diagonal and has the following form

$$\mathbf{M} = \operatorname{diag}[\mu_1, \mu_1, \mu_1\rho_1, \mu_1\rho_1, \mu_2, \mu_2, \mu_2\rho_2, \mu_2\rho_2] \tag{2.15}$$

The only non-zero terms of the gyroscopic matrix $\mathbf{G}$ are given by $G_{43} = -G_{34} = \chi\mu_1\rho_1\omega$.

In order to define the stiffness matrix, we introduce an auxiliary matrix

$$\boldsymbol{\varGamma}_j(\lambda_{nj}, \lambda_{mj}) = \begin{bmatrix} \gamma_j & 0 & -\gamma_j\lambda_{mj} & 0 \\ 0 & \gamma_j & 0 & \gamma_j\lambda_{mj} \\ -\gamma_j\lambda_{nj} & 0 & \kappa_j + \gamma_j\lambda_{nj}\lambda_{mj} & 0 \\ 0 & \gamma_j\lambda_{nj} & 0 & \kappa_j + \gamma_j\lambda_{nj}\lambda_{mj} \end{bmatrix} \tag{2.16}$$

The matrices $\mathbf{C}_1$ and $\mathbf{C}_2$, which represent respective flexible links can be written as the following block matrices

$$\mathbf{C}_1 = \begin{bmatrix} \boldsymbol{\varGamma}_1(\lambda_{11}, \lambda_{11}) & -\boldsymbol{\varGamma}_1(\lambda_{11}, \lambda_{21}) \\ -\boldsymbol{\varGamma}_1(\lambda_{21}, \lambda_{11}) & \boldsymbol{\varGamma}_1(\lambda_{21}, \lambda_{21}) \end{bmatrix} \qquad \mathbf{C}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\varGamma}_2(\lambda_{22}, \lambda_{22}) \end{bmatrix} \tag{2.17}$$

3. Results

Results of the qualitative analysis of the vibration-isolated hand grinder will be described below, with emphasis put on the selection of some of the parameters of the vibration isolation system and on the explanation of physical phenomena brought about by the percussive nature of the grinding forces. The results have been obtained using methods of numerical integration and the Fast Fourier Transform, which have been used in studying non-linear oscillations, e.g. by Awrejcewicz and Lamarque (2003). More details about the use of spectrum analysis to determine the character of vibrations have been discussed in Ferdek and Łuczko (2003). In discussion of the results, the criterion index J_1 (or J_2) of the efficiency of the vibration isolation system will be used. It is defined as the ratio of the rms values of accelerations (respectively velocities) at the point B on the handle (front grip – see Fig. 1), calculated for the tool with- and without the vibration isolation system. By analysing the influence of the parameters γ_1, κ_1 and λ_{11} on the value of the criterion indices J_k, estimates of the optimum parameter values from the point of view of minimising vibration levels have been found. The character of vibration has also been studied, depending on the values of these parameters and the values of parameters (ω, q) which characterize the grinding process. The following set of values of parameters have been used in the numerical calculations: $\mu_1 = 0.8$, $\mu_2 = 0.2$, $\rho_1 = 1.5$, $\rho_2 = 0.5$, $\chi = 0.1$, $f = 0.5$, $\gamma_1 = 1.5$, $\gamma_2 = 1$, $\gamma_3 = 400$, $\gamma_4 = 100$, $\kappa_1 = 1.5$, $\kappa_2 = 1$, $\zeta_1 = 0.1$, $\zeta_2 = 0.5$, $\zeta_3 = 0.05$, $\zeta_4 = 0.05$, $\lambda_0 = 0.5$, $\lambda_{11} = 0.25$, $\lambda_{21} = 0.25$, $\lambda_{22} = 0$, $\lambda_{13} = 0.5$, $\lambda_{23} = 1.5$, $\lambda_{14} = 0.5$, $\lambda_{24} = 1$, $\Delta = 10$, $\eta = 0.75$, $q = 10$, $\omega = 5$.

Figure 3a shows the dependence of the criterion index J_1 on the parameters γ_1, κ_1 (for $\omega = 5$ and $q = 10$), and Fig. 3b illustrates the zones of different vibration types in the (γ_1, κ_1) plane. The minimum of J_1 (and also of J_2) is achieved in the neighbourhood of the point $\gamma_1 = 1.5$, $\kappa_1 = 1.5$. As seen in Fig. 3b, in the neighbourhood of this point, sub-harmonic vibrations of type 1:2 take place.

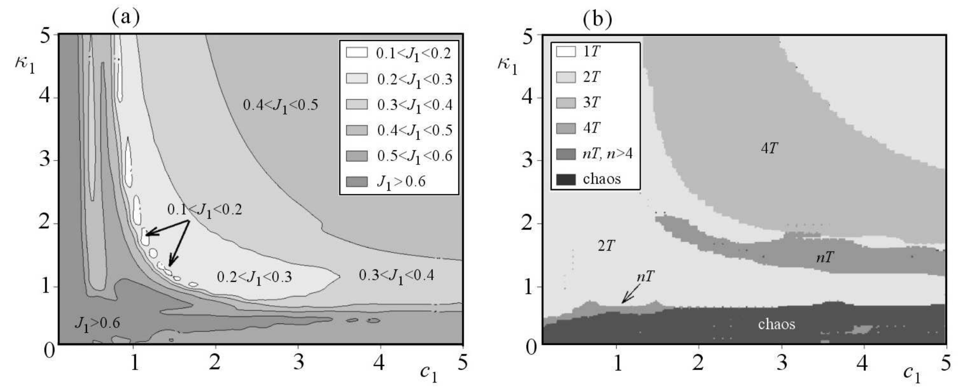

Fig. 3. Influence of parameters c_1 and κ_1 ($\omega = 5$, $q = 10$, $\lambda_{11} = 0.25$) on: (a) criterion index J_1, (b) zones of different vibration types

In a similar way, Figs. 4a and 4b illustrate the influence of the parameters ω and λ_{11}. In a relatively wide neighbourhood of $\lambda_{11} = 0.25$, the index J_2 (Fig. 4a) assumes a value close to the minimum, regardless of the rotational speed ω. This holds true, even though for high values of ω the vibrations are chaotic, as is seen in Fig. 4b.

The type of vibrations and the value of the criterion index depend also on the remaining parameters, characterizing both the model of the tool and the

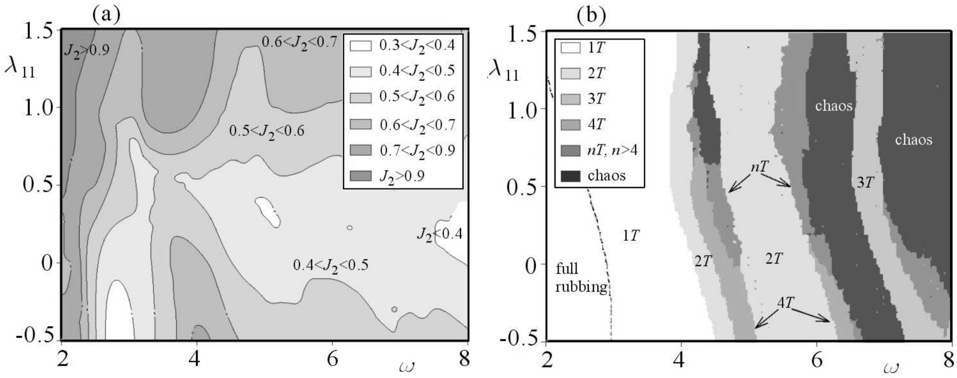

Fig. 4. Influence of ω and λ_{11} ($c1 = 1.5$, $\lambda_{11} = 1.5$, $q = 10$) on: (a) criterion index J_2, (b) zones of vibration types

grinding process. A proper choice of the vibration isolation system requires evaluation of the sensitivity of the solution to changes in these parameters. Below, the discussion is limited to the influence of the rotational speed and the operator pressure on the tool handle, since these two parameters have been found to have the biggest effect on the dynamic behaviour of the system operator-tool-base. The operator controls the grinding process mainly by changing the force q. The rotational speed ω also undergoes changes as a result of the limited power of the motor.

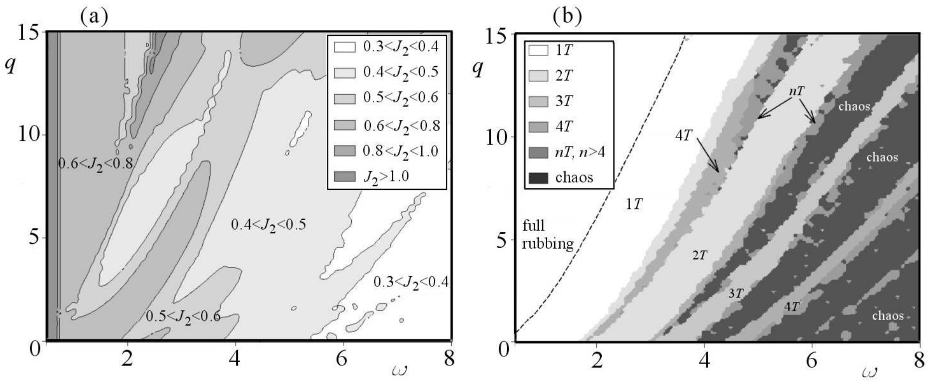

Fig. 5. Influence of ω and q ($c_1 = 1.5$, $\kappa_{11} = 1.5$, $\lambda_{11} = 0.25$) on: (a) efficiency index J_2, (b) vibration zones

Figures 5a and 5b illustrate, in the same format as in the previous figures, the influence of the parameters ω and q on the criterion index J_2 and on the character of vibrations. By studying the zones in the (ω, q) plane in Fig. 5b, one can note that only for small values of the non-dimensional rotational speed ω the grinding process takes place without separation of the grinding wheel from the motion limiter. This zone becomes a little wider as the dimensionless pressure q increases. For higher values of ω, which is the case of most practical applications, there appear zones of sub-harmonic vibrations of an order increasing with ω separated by narrow zones of sub-harmonic (mostly of type 1:4) or chaotic oscillations. The criterion index (Fig. 5a) decreases with the rotational speed, which signifies that the selected vibration isolation system is efficient also in the case of irregular vibrations.

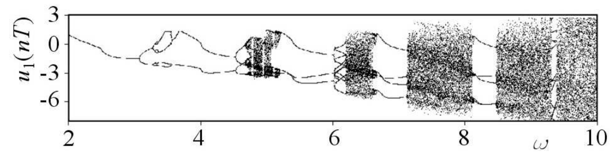

Fig. 6. Bifurcation diagram $(c_1 = 1.5,\ \kappa_{11} = 1.5,\ \lambda_{11} = 0.25,\ q = 5)$

Figure 6 shows a bifurcation diagram obtained using the stroboscopic method by taking the displacement u_1 at selected time instants (every excitation period). This diagram corresponds to the section of the (ω, q) plane shown in Fig. 5b taken for $q = 5$. In the bifurcation diagram, one can distinguish alternate regions of sub-harmonic and chaotic oscillations, where the order of the sub-harmonic vibrations increases with the rotational speed.

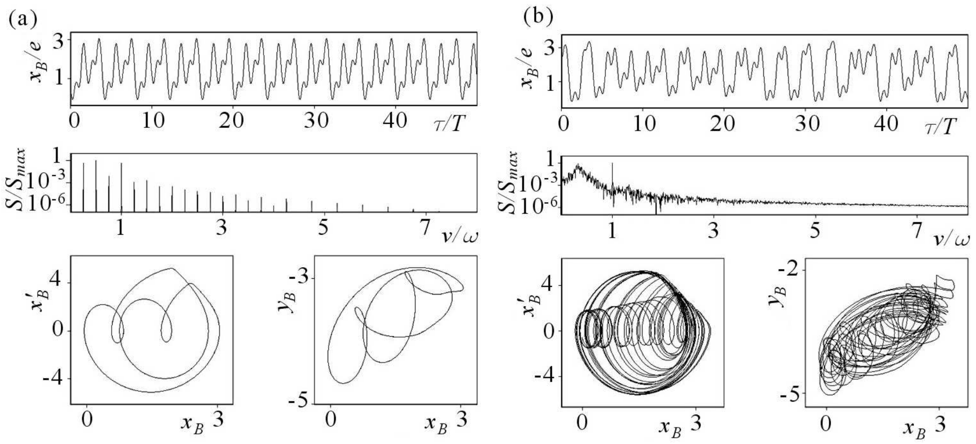

Fig. 7. Time history, spectrum, phase portrait and trajectory of the point B on the handle $(c_1 = 1.5,\ \kappa_{11} = 1.5,\ \lambda_{11} = 0.25,\ q = 5)$: (a) $4T$-periodic vibrations, $\omega = 4.6$, (b) chaotic vibrations, $\omega = 5.0$

Figure 7 shows time histories of the displacement $x_B(t)$, phase portraits (x_B, x'_B) and frequency spectra $S(\nu)$ of the signal x_B for two values of the excitation frequency ω. For the first value of ω, the period of oscillations is four times of that of the excitation (sub-harmonic oscillations of type 1:4), the spectrum has pronounced maxima at points $\nu/\omega = k/4$, where k is a natural number, and the corresponding curve in the phase plane is closed. For the second value of ω, the oscillations are chaotic, which is manifested in the irregular time history by the continuous spectrum and irregular phase portraits.

For chaotic vibrations, the phase portraits are difficult to interpret. Much more information is gained by making stroboscopic portraits or Poincaré maps. For the subsequent regions of chaotic vibrations shown in the bifurcation diagram in Fig. 6, one obtains different shapes of fractals shown in Fig. 8.

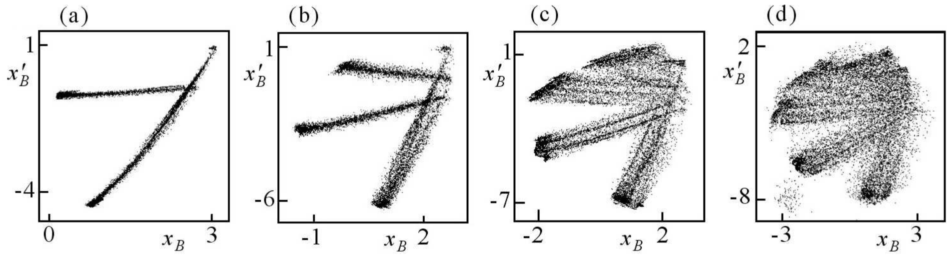

Fig. 8. Stroboscopic portraits of chaotic oscillations ($c_1 = 1.5$, $\kappa_{11} = 1.5$, $\lambda_{11} = 0.25$, $q = 5$): (a) $\omega = 5.0$, (b) $\omega = 6.5$, (c) $\omega = 7.5$, (d) $\omega = 9.0$

4. Conclusions

In the paper an approach to the modelling and to the qualitative analysis of vibrations of a non-linear system in the presence of unilateral constraints resulting both from the grinding process and the presence of the motion limiter is discussed. The adopted model of the grinding process can predict sub-harmonic and chaotic vibrations brought about by the percussive character of grinding forces. The numerical analysis has shown that the parameters ω and q have a pronounced qualitative effect on motion of the system. A little less important, but not negligible, is the effect of the parameters f, γ_3, ζ_3. The regions of different vibration types shown in Fig. 3-Fig. 5 as well as the bifurcation diagram (Fig. 6) do not undergo qualitative changes for different values of the parameters γ_3, ζ_3, f that describe the model of the grinding forces. By increasing the values of the parameters f and γ_3 or by reducing the value of ζ_3, the zones of chaotic vibrations tend to get bigger.

The inclusion of the motion limiter in the model prevents excessive static displacements and allows an efficient choice of the parameters of the passive vibration isolation system without a need of imposing additional constraints. With a badly designed vibration isolation system, the parameters of the motion limiter can have an important effect on the type of vibration in the system.

The proposed approach has proved to be very efficient for qualitative analysis of vibrations of non-linear systems. A similar approach can be used in the study of models of other mechanical machining processes.

References

1. Alfares M., Elsharkawy A., 2002, Effect of grinding forces on the vibration of grinding machine spindle system, *International Journal of Machine Tools and Manufacture*, **40**, 2003-2030
2. Awrejcewicz J., Lamarque C.-H., 2003, *Bifurcation and Chaos in Non-Smooth Mechanical Systems*, World Scientific Series of Nonlinear Science, Series A, **45**, Singapore
3. Ferdek U., Łuczko J., 2003, Qualitative analysis of vibro-impact systems [in Polish], *Czasopismo Techniczne*, **6-M**, 1-16
4. Gradišek J., Govekar E., Grabec I., 2001, Chatter onset in non-regenerative cutting: A numerical study, *Journal of Sound and Vibration*, **242**, 829-838
5. Karube S., Hoshino W., Soutome T., Sato K., 2002, The non-linear phenomena in vibration cutting system. The establishment of dynamic model, *International Journal of Non-Linear Mechanics*, **37**, 541-564
6. Łuczko J., Markiewicz M., 1986, Dynamical analysis of the system: tool-base-operator in hand grinding with a pneumatic tool [in Polish], *Zeszyty Naukowe Politechniki Rzeszowskiej*, **31**, *Mechanika*, **12**, 51-55
7. Łuczko J., Cupiał P., Ferdek U., 2003, Regular and chaotic vibration in the manual grinding process, *Proc. of the 7th Conference on Dynamical Systems – Theory and Applications*, **1**, 207-214
8. Suh C.S., Khurjekar P.P., Yang B., 2002, Characterisation and identification of dynamic instability in milling operation, *Mechanical Systems and Signal Processing*, **16**, 853-872

Drgania regularne i chaotyczne w procesach obróbki wibroizolowaną ręczną szlifierką

Streszczenie

Praca dotyczy analizy jakościowej nieliniowego modelu, opisujący drgania wibroizolowanej ręcznej szlifierki. Model opisano układem ośmiu równań różniczkowych zwyczajnych drugiego rzędu. Wprowadzono nieciągły opis sił skrawania, uwzględniający możliwość chwilowego oderwania się ściernicy od obrabianego przedmiotu.

Do analizy wykorzystano procedury matematycznego całkowania skojarzone z algorytmami szybkiej transformaty Fouriera. Zbadano wpływ parametrów na charakter drgań oraz wyznaczono pewne wskaźniki jakości działania zastosowanego układu wibroizolacji.

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