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WORM-LIKE LOCOMOTION AS A PROBLEM OF NONLINEAR DYNAMICS

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Non-pedal forms of locomotion show their advantages in inspection techniques or applications to the medical technology for diagnostic systems and minimally invasive surgery. Observing the locomotion of worms, one recognizes conversion of (mostly periodic) internal and internally driven motions into a change in the external position (undulatory locomotion). This paper presents some theoretical and practical investigations of worm-like motion systems that have the earthworm as a live prototype. The locomotion of worms can be described by introduction of nonlinear non-symmetric frictional forces. In the first part of the paper these systems are modelled in form of straight chains of interconnected mass points. The ground contact can be described by non-symmetric dry friction. The investigations are concentrated on motion in a tube or channel, and on motion on a horizontal plane as well. In both cases, the body is modeled as a viscous Newtonian fluid. The effect of the mass flow through a cross section on the disturbance and material parameters (viscosity, dimensions) is discussed. The paper presents first prototypes of technically implemented artificial worms.

Key words: worm-like motion, modelling, oscillations, non–symmetric friction, peristaltic motion

1. Introduction

Biologically inspired locomotion systems are currently dominated by walking machines, i.e., systems performing *pedal* locomotion. *Non-pedal* forms of locomotion show their advantages in inspection techniques or applications to the medical technology for diagnostic systems and minimally invasive surgery. Observing the locomotion of worms, one recognizes conversion of (mostly periodic) internal and internally driven motions into a change in the external position (undulatory locomotion). For realization of this type of locomotion, non-symmetry in external friction forces acting on the system is needed.

The motion of a chain of mass points placed on a rough straight line and connected consecutively by equal linear viscoelastic elements under action of a non-symmetric Coulomb dry frictional force and related by unilateral differential constraints was described by Steigenberger *et al.* (2003), Zimmermann *et al.* (2001, 2002), Steigenberger (1999). In the case of three masses, an expression for velocity of "slow" motion is obtained (Zimmermann *et al.*, 2002). A limiting case of non-symmetric friction, when motion is possible only in one direction (realized by means of scales that prevent backward displacement of points of the ground contact) was proposed by Miller (1998) in connection with a realistic computer animation of worms. A thorough discussion of such systems, where the point masses can also be equipped with massless steerable runners described via knife-edge conditions, was given in Steigenberger (1999).

2. Motion under action of nonlinear friction and periodic external forces

We consider the motion of $n = 2k + 1$ mass points in a common straight line. We suppose that all of them except the middle one are equipped with scales contacting the ground. The middle mass is subjected to a harmonic external force $-M\Phi_0 \sin \Omega t$ (Fig. 1). The equations of motion are

$$\begin{aligned}
&\ddot{x}_1 + \omega^2(x_1 - x_2) + F(\dot{x}_1) = 0 \\
&\ldots \\
&\ddot{x}_k + \omega^2(2x_k - x_{k-1} - x_{k+1}) + F(\dot{x}_k) = 0 \\
&\ddot{x}_{k+1} + \omega_0^2(2x_{k+1} - x_k - x_{k+2}) + \Phi_0 \sin \Omega t = 0 \\
&\ddot{x}_{k+2} + \omega^2(2x_{k+2} - x_{k+1} - x_{k+3}) + F(\dot{x}_{k+2}) = 0 \\
&\ldots \\
&\ddot{x}_{2k+1} + \omega^2(x_{2k+1} - x_{2k}) + F(\dot{x}_{2k+1}) = 0
\end{aligned} \tag{2.1}$$

where

$$\omega^2 = \frac{c}{m} \qquad \omega_0^2 = \frac{c}{M} \qquad \Phi_0 > 0$$

The function $F(\dot{x}_s)$ describes non-symmetric dry friction, i.e., the frictional force is taken to be different in magnitude depending on the direction of motion of the body. $F(\dot{x}_s)$ may be specified as follows

$$F(\dot{x}) = \begin{cases} F_+ & \text{for} \quad \dot{x} > 0 \\ F_0 & \text{for} \quad \dot{x} = 0 \\ -F_- & \text{for} \quad \dot{x} < 0 \end{cases} \tag{2.2}$$

where $F_- > F_+ \geqslant 0$ are fixed, whereas F_0 may assume any value in the interval $(-F_-, F_+)$.

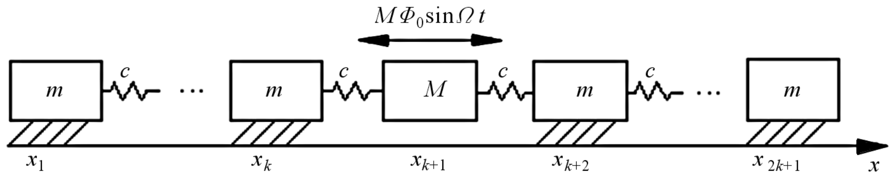

Fig. 1.

Following the method of direct separation of motion (Zimmermann *et al.*, 2001), we seek for a solution x_i to system (2.1) in the form $(i = 1, \dots, 2k+1)$

$$x_i(t) = X_i(t) + \Psi_i(t, \Omega t) \tag{2.3}$$

Here X_i denotes the "slow" component of motion x_i, whereas Ψ_i is the "fast" one. Ψ_i is assumed to be a smooth periodic function of the "fast" time $\tau = \Omega t$ $(\Omega \gg \omega)$ having the average value zero

$$\langle \Psi_i(t, \tau) \rangle = \frac{1}{2\pi} \int_0^{2\pi} \Psi_i(t, \tau) \, d\tau = 0$$

We assume that the amplitude of the external force is $\Phi_0 \gg \max(F_-, F_+)$, then the equations for "slow" motion take the final form

$$\begin{aligned}
&\ddot{X}_1 + \omega^2(X_1 - X_2) + V_1^*(\dot{X}_1) + \frac{1}{2}(F_+ - F_-) = 0 \\
&\dots \\
&\ddot{X}_k + \omega^2(2X_k - X_{k-1} - X_{k+1}) + V_k^*(\dot{X}_k) + \frac{1}{2}(F_+ - F_-) = 0 \\
&\ddot{X}_{k+1} + \omega_0^2(2X_{k+1} - X_k - X_{k+2}) = 0 \\
&\ddot{X}_{k+2} + \omega^2(2X_{k+2} - X_{k+1} - X_{k+3}) + V_k^*(\dot{X}_{k+2}) + \frac{1}{2}(F_+ - F_-) = 0 \\
&\dots \\
&\ddot{X}_{2k+1} + \omega^2(X_{2k+1} - X_{2k}) + V_1^*(\dot{X}_{2k+1}) + \frac{1}{2}(F_+ - F_-) = 0
\end{aligned} \tag{2.4}$$

where

$$V_i^*(\dot{X}) = \begin{cases} \frac{1}{2}(F_- + F_+) & \text{for} \quad \dot{X} > \Theta_i \Omega \\ \frac{1}{\pi}(F_- + F_+) \arcsin \frac{\dot{X}}{\Theta_i \Omega} & \text{for} \quad |\dot{X}| \leqslant \Theta_i \Omega \\ -\frac{1}{2}(F_- + F_+) & \text{for} \quad \dot{X} < -\Theta_i \Omega \end{cases} \tag{2.5}$$

and $\Theta_i = \Theta_i(\Phi_0, \Omega, \omega, \omega_0)$, $i = 1, \dots, k$.

There is a steady motion with constant velocities $\dot{X}_1^* = \ldots = \dot{X}_{2k+1}^* = \dot{X}^*$, where $\dot{X}^*$ is determined from the equation

$$\sum_{i=1}^{k} V_i^*(\dot{X}^*) = k(F_- - F_+) \tag{2.6}$$

By means of (2.5), it yields an equation for the velocity of the steady "slow" motion.

Equation (2.6) should be solved numerically and, what is different from the case with three masses where two of which have contact with the ground (Zimmermann *et al.*, 2002), the steady motion does not always exist. Thus in the considered case the "slow" motion of the chain as a single whole is not always possible.

If $F_+ = F_-$, then $\dot{X}^* = 0$ which follows from (2.6). That means that in the case of symmetrical friction the system "in average" does not move at all. On the other hand, if $F_- > F_+$, there can be a solution $\dot{X}^* > 0$, which entails a unidirectional average motion under non-symmetric friction.

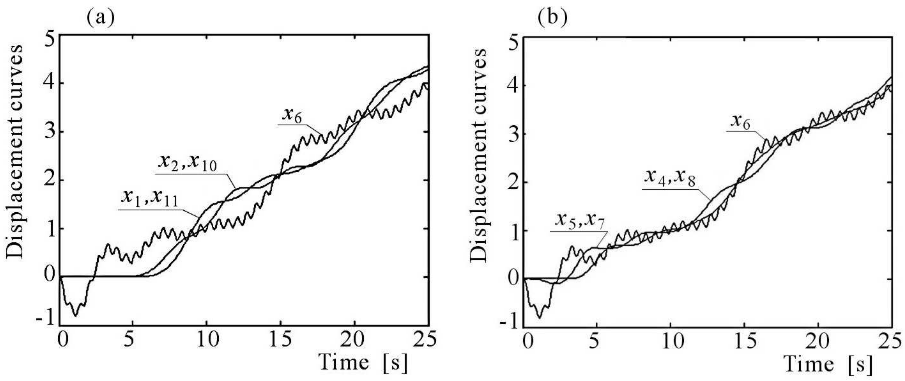

Fig. 2.

For the case of eleven masses, displacement curves vs. time for masses 1(11), 2(10), 6, are shown in Fig. 2a, and for masses 4(8), 5(7), 6, in Fig. 2b (mass 3(9) is not shown). The result follows from the numerical solution of the exact equations of motion (2.1). The following values of parameters have been taken

$$\omega = \omega_0 = 1 \qquad \Omega = 10 \qquad \Phi = 10$$
$$F_+ = 0 \qquad F_- = \frac{1}{2}$$

The numerical solution to equation (2.6) for eleven masses gives $\dot{X}^* = 0.22$.

3. Peristaltic motion

We consider the motion of a deformable continuum (index 0, "worm") within a second medium (index 1), where the extension of the latter is constrained by straight walls. The deformable body whose motion is to be described is bounded by an impermeable skin. Both media are modeled as incompressible linear-viscous media with different viscosities. The transport of the matter (medium with index 0) is based on a wave-like disturbance of the skin (Fig. 3). This peristaltic motion is the subject of present investigation. The amplitude of the disturbed surface is assumed to be small in comparison to the transversal size of the body. The equations of motion for both media and the constraints at the wave-like boundary were given in Zeidis and Zimmermann (2000). For motion within a channel (plane deformation) or within a tube (rotationally symmetric case), Cartesian coordinates (x, y) or cylindrical coordinates (x, r) are used, respectively. The Reynolds numbers of both media and the frequency of wave-like disturbances are assumed to be small. This means that the inertial forces are small compared to the viscous forces. Following these considerations, the flow of an incompressible viscous fluid is described by the equations

$$\operatorname{grad} p = \mu \Delta \boldsymbol{v} \qquad \operatorname{div} \boldsymbol{v} = 0 \tag{3.1}$$

where $\boldsymbol{v}$ is the velocity vector, p is the pressure and μ – the shear viscosity.

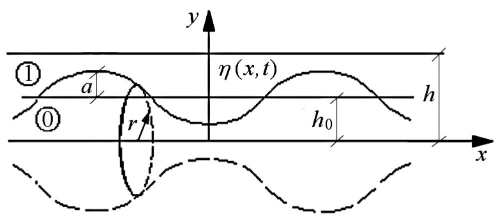

Fig. 3.

For both rotational symmetry (motion in a tube) and plane deformation (motion in a channel), a two-dimensional problem occurs, and from continuity equation (3.1) the existence of a stream function Ψ follows.

In the case of rotational symmetry, the stream function $\Psi(x,r,t)$ satisfies a differential equation

$$\Big(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\Big)\Big(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\Big)\Psi^s = 0 \qquad s = 0,1 \tag{3.2}$$

whereas in the case of plane deformation the stream function $\Psi(x,y,t)$ must be determined from

$$\frac{\partial^4\Psi^s}{\partial y^4} + 2\frac{\partial^4\Psi^s}{\partial y^2\partial x^2} + \frac{\partial^4\Psi^s}{\partial x^4} = 0 \qquad s = 0,1 \tag{3.3}$$

Let us introduce dimensionless variables (quantities with stars have dimensions)

$$\begin{array}{lll} m = \dfrac{\mu_1}{\mu_0} & \delta = \dfrac{h_0}{h} & t = t^*\dfrac{U}{h} \\ \Psi = \dfrac{\Psi^*}{Uh} & (u,v) = \dfrac{(u^*,v^*)}{U} & (x,y,r) = \dfrac{(x^*,y^*,r^*)}{h} \end{array}$$

with U as a characteristic velocity for motion of the boundary surface.

The following boundary conditions can be formulated. At the fixed walls of the tube (channel) there holds: $u^1 = v^1 = 0$. On the boundary surface described by the equation $\eta(x,t) = \delta + \varepsilon\cos[\omega(x-ct)]$, a kinematic condition and continuity of normal tensions and of the velocities are required. The conditions of symmetry relative to the axis OX must be added as well.

Applying perturbation theory, the interesting functions are represented as power series with respect to the small parameter ε, e.g., the stream function Ψ

$$\Psi^s = \Psi_0^s + \varepsilon\Psi_1^s + \varepsilon^2\Psi_2^s + \ldots = \sum_{k=0}^{\infty}\varepsilon^k\Psi_k^s \qquad s = 0,1 \tag{3.4}$$

To see the efficiency of peristaltic motion, the flow q is determined, i.e., the amount of the matter which flows through a given cross-section per time unit (Zeidis and Zimmermann, 2000)

$$q(x,t) = \int\limits_0^{\eta(x,t)} u^0(y)\;dy$$

For a flow Q within a period $\lambda = 2\pi/(\omega c)$, there holds

$$Q = \int\limits_0^{\lambda} q(x,t)\;dt$$

For calculation of Q, terms up to the second order in ε are taken into account

$$Q = Q_0 + \varepsilon Q_1 + \varepsilon^2 Q_2 \tag{3.5}$$

Here $Q_0 = Q_1 = 0$. How Q depends on ε (which is the amplitude of the disturbed surface divided by the extension of surrounding medium 1) is shown in Fig. 4 for different values of m and δ and the parameters $\omega = 0.5$, $c = 1$.

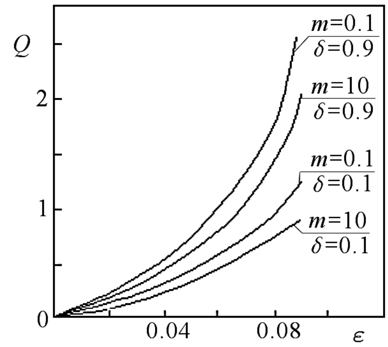

Fig. 4.

4. Conclusions

The locomotion of worms can be described by introduction of nonlinear non-symmetric frictional forces or by imposing unilateral differential constrains which provide motion only in one direction. The ”slow” motion of the system acted upon by ”small” non-symmetric dry friction and the ”fast” external force are equivalent to motion under viscous friction and a constant external force.

The mass flow in the case of a viscous fluid is proportional to the square of amplitude of the surface disturbance.

Based on the model-aided analysis of biological motion systems, technical systems with functionally defined rigid and compliant structures are developed. Some worm prototypes applying the principles outlined above have been constructed and proved positive.

Future considerations should focus on possibilities of realizing wave-like deformations with a given amplitude and frequency of a surface using ferrofluids (Zimmermann *et al.*, 2004).

References

1. Blekhmann I.I., 2000, *Vibrational Mechanics: Nonlinear Dynamic Effects, General Approach*, Applications, World Scientific
2. Miller G., 1998, The motion dynamics of snakes and worms, *Computer Graphics*, **22**, 4, 169-173
3. Steigenberger J., 1999, *On a Class of Biomorphic Motion Systems*, Faculty of Mathematics and Natural Sciences, Ilmenau Technical University, Preprint M12/99
4. Steigenberger J., Zeidis I., Zimmermann K., 2003, Model of worm-like motion as a basis of creation of new microrobots, *International Advanced Robotics Program. Proceeding of Workshop on Micro Robots, Micro Machines and Micro Systems*, Moscow, 89-86
5. Zeidis I., Zimmermann K., 2000, Ein mathematisches Modell für die peristaltische Bewegung als Grundlage für das Design wurmartiger Mikroroboter, *Technische Mechanik*, **20**, 1, 73-80
6. Zimmermann K., Zeidis I., Naletova V.A., Turkov V.A., 2004, Waves on the surface of a magnetic fluid layer in a travelling magnetic field, *Journal of Magnetism and Magnetic Materials*, **268**, 227-231
7. Zimmermann K., Zeidis I., Steigenberger J., 2002, Mathematical model of worm-like motion systems with finite and infinite degree of freedom, *Romansy 14. Theory and Practice of Robots and Manipulators*, 507-515
8. Zimmermann K., Zeidis I., Steigenberger J., Huang Jianjun, 2001, An approach to the modelling of worm-like motion systems with finite degree of freedom – first steps in technical realization, *Proceeding of the 4th International Conference of Climbing and Walking Robots*, Karlsruhe, 561-568

Ruch robaczkowy jako zagadnienie dynamiki nieliniowej

Streszczenie

Znana w naturze forma transportu i przemieszczania się z miejsca na miejsce bez użycia odnóży, tzw. forma niekrocząca, kryje w sobie ogromny potencjał aplikacyjny w technikach badawczych oraz medycynie, zwłaszcza w diagnostyce i nieinwazyjnej chirurgii. W obserwacji pełzania robaków stwierdza się konwersję ruchu wewnętrznego, lub wewnętrznie indukowanego (głównie periodycznego) w mierzalne przemieszczenie zewnętrzne (tzw. ruch falujący). W pracy przedstawiono badania teoretyczne i uwagi praktyczne na temat sztucznych układów przemieszczających się ruchem robaczkowym, których żywym prototypem może być zwykła dżdżownica. Taki rodzaj transportu opisano poprzez wprowadzenie do modelu matematycznego nieliniowej i niesymetrycznej charakterystyki tarcia. W pierwszej części pracy układy zamodelowano

prostymi łańcuchami punktów materialnych. Kontakt z podłożem odwzorowano niesymetryczną siłą tarcia suchego. Analizę skoncentrowano na pełzaniu w rurze i kanale oraz ruchu po poziomej powierzchni. We wszystkich przypadkach poruszające się ciało zamodelowano jako ciecz Newtonowska. Przedyskutowano efekt przepływu masy przez przekrój poprzeczny na zakłócenia w ruchu i zmianę parametrów ciała (lepkość, wymiary). Zaprezentowano również pierwsze prototypy sztucznych robaków przeznaczonych do zadań technicznych.

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