JOURNAL OF THEORETICAL
and APPLIED MECHANICS
45, 2, pp. 203-223, Warsaw 2007

ADIABATIC HEAT EVALUATION FOR DYNAMIC PLASTIC LOCALIZATION

Patrice Longère

Laboratoire de Génie Mécanique et Matériaux, Université de Bretagne Sud, Lorient Cedex, France
e-mail: patrice.longere@univ-ubs.fr

André Dragon

Laboratoire de Mécanique et de Physique des Matériaux, CNRS-ENSMA, Futuroscope-Chasseneuil Cedex, France

In most cases, adiabatic plastic heating is evaluated using the inelastic heat fraction coefficient, also known as Taylor-Quinney coefficient, which is usually assumed to be constant. From the thermodynamic viewpoint, this method consists in neglecting (or including some part of) various thermomechanical couplings in the heat equation. Nevertheless this coarse method can lead to over-estimation of the temperature rise. In this paper, adiabatic shear banding incipience is considered in the context of thermal instability. It is shown that the accuracy in the prediction of favourable conditions for the onset of this plastic localization is strongly dependent on the technique retained for evaluating the plasticity induced heating. This paper aims at showing the influence of various levels of simplification of the thermal equation on the critical conditions at localization onset, the latter being obtained from a criterion based on the linear perturbation method.

Key words: adiabatic shear banding, inelastic heat fraction, non-linear mechanics, viscoplasticity

1. Introduction

Dynamic plastic localization in the form of Adiabatic Shear Banding (ASB) is known to occur at high strain rates under quasi adiabatic conditions as thermal softening prevails against strain and strain rate hardening. Depending on the application considered, this precursor of structural failure is sought – it is typically the case for metal cutting (see Rhim and Oh, 2006), or apprehended – it is the case in penetrator-armour steel interaction (see Odeshi *et al.*, 2006).

The knowledge of favourable conditions for ASB localization onset and further post-critical behaviour is thus of major interest, particularly in numerical simulations of the processes mentioned above.

In parallel, most of dynamic engineering finite element calculation codes take into account thermomechanical coupling only through the inelastic heat fraction which characterizes the fraction of plastic work converted into heat which is usually assumed to be constant. However, the inelastic heat fraction can be strongly dependent on strain, strain rate and temperature as observed experimentally by Mason *et al.* (1994), Kapoor and Nemat-Nasser (1998) for example, and shown theoretically by Aravas *et al.* (1990), Longère and Dragon (2006). For a better evaluation of plastic deformation-induced temperature rise and related thermal softening, a specific 'user material' routine implemented in the FE engineering code is needed including other aspects appearing in the formulation of the heat equation (see Longère *et al.* (2005) for computations involving LS-DYNA® code).

According to ASB-devoted experimental studies, the band orientation is mainly controlled by the maximum shear stress (see Bai and Dodd, 1992; Pęcherski, 1998), and a weak localization of deformation, as a consequence of thermal instability, is primarily observed followed by the genuine strong localization of deformation leading to the ASB process (see Marchand and Duffy, 1988; Liao and Duffy, 1998). Starting from these results, this paper aims at studying the consequences of simplifications assumed to evaluate dissipative mechanisms-induced heating on the determination of dynamic plastic localization occurrence. The latter is obtained herein from the theory of linear perturbation (see Bai, 1982; Clifton *et al.*, 1984) in the form of a criterion relating the resolved shear stress to strain hardening, thermal softening and viscosity (see Batra and Chen, 2001; Longère *et al.*, 2003) in the context of ASB induced localization. The purpose of the present study is not to discuss or extend the theory but to apply the method in order to obtain a practical criterion reproducing experimental observation – the reader can refer to Fressengeas and Molinari (1987) and Anand *et al.* (1987) for extensive analysis. For this purpose, various levels of simplification of the heat equation formulation are considered, yielding various criteria for the ASB onset.

In Section 2, thermodynamic fundamentals are recalled leading to the expression of the heat equation in the case of thermo/elastic-viscoplastic modelling. The context considered in this work concerns loading at strain rates high enough to allow for the assumption of adiabatic conditions while making it possible to assume the (visco)plastic material behaviour to be pressure insensitive. The strain rates range is thus typically comprised between 10^2 and $10^4\,\mathrm{s}^{-1}$, and maximum value of temperature remains lower than the melting point. Three levels of simplification are studied: first, due to its weak contribution to temperature changes, the thermo/elastic coupling is neglected;

secondly thermodissipative couplings are neglected as well; finally, the temperature rise is supposed to be directly linked to the plastic work rate via the inelastic heat fraction assumed to have a constant value.

In Section 3, the consequences of simplifications mentioned above are examined in the context of thermal instability induced dynamic plastic localization. With this aim in view, the linear perturbation method is applied to the case of simple shearing of a volume element consisting of a thermo/viscoplastic material. Note that in the case of the ASB phenomenon, the extension of the 1D loading to a complex 3D loading can be performed today (see Longère *et al.* (2003)). Simplifications are assumed in order to reproduce qualitatively different stages of deformation localization (weak and strong) as observed notably by Marchand and Duffy (1988). Further parametric analysis shows how the value of the critical shear strain at localization incipience is strongly dependent on the temperature rise evaluation method.

2. Evaluation of temperature rise under adiabatic conditions

Irreversible thermodynamics framework is used here to describe the thermo/elastic-inelastic response of a material (see Perzyna, 1966; Bataille and Kestin, 1975; Raniecki and Nguyen, 1984; Petryk, 2000; Mróz and Oliferuk, 2002). At first, the internal variable procedure is summarized leading to expressions of dissipated energy and thermal equation (see Perzyna (1998) for computational issues for the evolution problem). Different levels of simplification of the latter are then introduced in connection with the further study of heat evaluation at the dynamic localization onset.

2.1. Thermodynamic context and constitutive framework

The state of the material is supposed to be described by a thermodynamic potential, namely Helmholtz free energy per unit mass $\psi(T,z)$, where T represents absolute temperature and z a set of normal variables. Gibbs relation takes the following form

$$\rho_0\dot{\psi} = -\rho_0 s\dot{T} + Z\dot{z} \qquad s = -\frac{\partial\psi}{\partial T}\Big|_z \qquad Z = \rho_0\frac{\partial\psi}{\partial z}\Big|_T \tag{2.1}$$

where ρ_0 represents the mass density in the initial configuration, s – the entropy and Z – the set of thermodynamic (conjugate) forces associated with the state variables z.

According to the second law of thermodynamics, the mechanical part of dissipated energy is written as

$$D = \boldsymbol{\tau} : \mathbf{d} - Z\dot{z} \geqslant 0 \tag{2.2}$$

where $\mathbf{d}$ represents the rate of deformation tensor and $\boldsymbol{\tau}$ is the Kirchhoff stress tensor defined as the Cauchy stress tensor $\boldsymbol{\sigma}$ multiplied by the Jacobian determinant J of the deformation gradient $\mathbf{F}$ ($J = \det \mathbf{F} = \rho_0/\rho$), i.e. $\boldsymbol{\tau} = J\boldsymbol{\sigma}$.

A combination of the first law of thermodynamics and the Gibbs relation gives a local form of the heat equation

$$\rho_0 c_z \dot{T} + \operatorname{div} \boldsymbol{q} + r = \boldsymbol{\tau} : \mathbf{d} - \Big(Z - T\frac{\partial Z}{\partial T}\Big)\dot{z} \tag{2.3}$$

where c_z represents the specific heat, $\boldsymbol{q}$ – the heat flux vector per unit area and r – the heat supply per unit volume.

Equation (2.3) relies on thermal terms on the left hand-side including the temperature rate $\dot{T}$, heat conduction $\operatorname{div} \boldsymbol{q}$, and heat supply r, and mechanical terms on the right hand-side including the mechanical work rate $\boldsymbol{\tau} : \mathbf{d}$ and other work rates $Z\dot{z}$, while the last term $T\dot{z}\partial Z/\partial T$ is referred explicitly to thermomechanical couplings.

The study is now reduced to thermo-elastic/viscoplastic behaviour in the case of isotropic strain hardening. The set of state variables is assumed in the form $z \equiv (\boldsymbol{\varepsilon}^e, p)$, where $\boldsymbol{\varepsilon}^e$ represents a measure of finite elastic strain ($\boldsymbol{\varepsilon}^e = \ln \mathbf{V}^e$; $\mathbf{F}^e = \mathbf{V}^e\mathbf{R}^e$) and p is the isotropic strain hardening variable. The set of conjugate forces is thus given in the form $Z \equiv (\boldsymbol{\tau}, R)$, where R represents the isotropic hardening force (affinity).

The rate of deformation (symmetric part of the velocity gradient) $\mathbf{d} = [\partial \boldsymbol{v}/\partial \boldsymbol{x}]^S$ is furthermore decomposed into a reversible ('elastic') part $\mathbf{d}^e = \overset{\nabla}{\boldsymbol{\varepsilon}}{}^e$ (where ∇ designates the objective Jaumann derivative of a 2nd order tensor) and an irreversible part $\mathbf{d}^p$ such that $\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p$ (see Sidoroff and Dogui, 2001).

Mechanical dissipation (2.2) becomes

$$D = \boldsymbol{\tau} : \mathbf{d}^p - R\dot{p} \geqslant 0 \tag{2.4}$$

The constitutive laws are thus expressed as

$$s = -\frac{\partial \psi}{\partial T}\Big|_{\boldsymbol{\varepsilon}^e,p} \qquad \boldsymbol{\tau} = \rho_0 \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e}\Big|_{T,p} \qquad R = \rho_0 \frac{\partial \psi}{\partial p}\Big|_{T,\boldsymbol{\varepsilon}^e} \tag{2.5}$$

The free energy density $\psi(T, \boldsymbol{\varepsilon}^e; p)$ is expressed in the following form

$$\psi(T, \boldsymbol{\varepsilon}^e; p) = \psi^e(T, \boldsymbol{\varepsilon}^e) + \psi^{th}(T) + \psi^b(T; p) \tag{2.6}$$

where $\psi^e(T, \boldsymbol{\varepsilon}^e)$ represents the recoverable energy which includes isotropic linear thermo-elasticity, $\psi^{th}(T)$ – purely thermal energy, and $\psi^b(T; p)$ – stored energy reflecting competition in the material between isotropic strain hardening and thermal softening. These contributions are written as follows

$$\begin{aligned}
&\rho_0\psi^e(T,\boldsymbol{\varepsilon}^e) = \frac{\lambda}{2}(Tr\boldsymbol{\varepsilon}^e)^2 + \mu\boldsymbol{\varepsilon}^e:\boldsymbol{\varepsilon}^e - \alpha K Tr\boldsymbol{\varepsilon}^e\vartheta\\
&\rho_0\psi^{th}(T) = -\rho_0 c_0\Big[T\ln\Big(\frac{T}{T_0}\Big) - \vartheta\Big] - h(0)f(T) \qquad (2.7)\\
&\rho_0\psi^b(T,p) = h(p)f(T) - h(0)f(0)
\end{aligned}$$

where λ and μ represent the Lamé elasticity constants, $K = \lambda + 2\mu/3$ – bulk modulus, α – thermal dilatation coefficient, c_0 – constant heat capacity, $\vartheta = T - T_0$ – temperature rise, $h(p)$ – stored energy of cold work and $f(T)$ – thermal softening function. For simplification, the thermo-elasticity coefficients are supposed to be temperature independent.

From now on, the elastic deformations will be considered as small (infinitesimal).

After partial derivation of (2.6) with respect to $z \equiv (T,\boldsymbol{\varepsilon}^e,p)$, the thermodynamic forces $Z \equiv (-s,\boldsymbol{\tau},R)$ are written as

$$\begin{aligned}
&-\rho_0 s = -\alpha K Tr\boldsymbol{\varepsilon}^e - \rho_0 c_0\ln\Big(\frac{T}{T_0}\Big) - [h(0) - h(p)]f'(T)\\
&\boldsymbol{\tau} = (\lambda Tr\boldsymbol{\varepsilon}^e - \alpha K\vartheta)\boldsymbol{\delta} + 2\mu\boldsymbol{\varepsilon}^e \qquad (2.8)\\
&R = h'(p)f(T)
\end{aligned}$$

By rewriting mechanical dissipation (2.4) in the form $D = \overline{Z}\dot{\overline{z}} \geqslant 0$, $\overline{Z} = (\boldsymbol{\tau},R)$ represents a set of thermodynamic forces associated to the set of fluxes $\dot{\overline{z}} = (\mathbf{d}^p,-\dot{p})$. Considering now the time-dependent plasticity and applying the normality rule with respect to the dual $\varPhi(\overline{Z})$ of the dissipation potential, depending on $\overline{Z}$ via the loading (yield) function $F(\overline{Z})$ (i.e. standard rule for $\dot{\overline{z}}$), the internal state variable evolution laws are deduced from

$$\dot{\overline{z}} = \varLambda\frac{\partial F(\overline{Z})}{\partial\overline{Z}} \qquad \varLambda = \frac{\partial\varPhi(F)}{\partial F} \geqslant 0 \qquad (2.9)$$

where $\varLambda$ represents the viscous multiplier governing the dissipative mechanisms of plasticity.

Applying the normality rule to the present case yields

$$\mathbf{d}^p = \varLambda\frac{\partial F}{\partial\boldsymbol{\tau}} \qquad -\dot{p} = \varLambda\frac{\partial F}{\partial R} \qquad (2.10)$$

The yield function $F(\overline{Z})$ in (2.9) is assumed in the form of Huber-von Mises criterion

$$F(\boldsymbol{\tau},R;T) = J_2(\boldsymbol{\tau}) - g(R,T) \qquad (2.11)$$

where

$$J_2(\boldsymbol{\tau}) = \sqrt{\frac{3}{2}\mathbf{s} : \mathbf{s}}$$

and $\mathbf{s}$ represents the stress deviator tensor

$$\mathbf{s} = \boldsymbol{\tau} - \frac{Tr\boldsymbol{\tau}}{3}\boldsymbol{\delta}$$

The strain hardening function $g(R,T)$ in (2.11) representing the Huber-von Mises surface radius is expressed by

$$g(R,T) = R_0 f(T) + R(T,p) \tag{2.12}$$

The evolution laws in (2.10) are thus detailed as follows

$$\mathbf{d}^p = \frac{3}{2}\Lambda\frac{\mathbf{s}}{J_2} \qquad\qquad \dot{p} = \Lambda \tag{2.13}$$

The model is completed by the expression of the force potential in (2.9)

$$\varPhi(F) = \frac{Y}{m+1}\Big\langle\frac{F}{Y}\Big\rangle^{m+1} \qquad\qquad \Lambda = \Big\langle\frac{F}{Y}\Big\rangle^{m} \tag{2.14}$$

Furthermore, Eq. (2.3) becomes

$$\rho_0 c_0 \dot{T} + \operatorname{div}\boldsymbol{q} + r = \boldsymbol{\tau} : \mathbf{d}^p + T\frac{\partial\boldsymbol{\tau}}{\partial T} : \mathbf{d}^e - \Big(R - T\frac{\partial R}{\partial T}\Big)\dot{p} \tag{2.15}$$

The context considered here concerns the loading at high strain rate excluding heat supply, and for which conditions can be assumed as adiabatic (dynamic loading). Relation (2.15) is thus reduced to

$$\rho_0 c_0 \dot{T} = \boldsymbol{\tau} : \mathbf{d}^p + T\frac{\partial\boldsymbol{\tau}}{\partial T} : \mathbf{d}^e - \big(R - T\frac{\partial R}{\partial T}\big)\dot{p} \tag{2.16}$$

In (2.16), $\boldsymbol{\tau} : \mathbf{d}^p$ represents the plastic part of the mechanical work rate (plastic work rate), $R\dot{p}$ – the stored energy rate, the difference $\boldsymbol{\tau} : \mathbf{d}^p - R\dot{p}$ represents the unrecoverable energy rate dissipated by heating, see (2.4), $(T\partial\boldsymbol{\tau}/\partial T) : \mathbf{d}^e$ – thermo-elastic coupling contribution which describes cooling during the tensile loading and heating during compression, and $T\dot{p}\partial R/\partial T$ – the thermo-dissipative coupling contribution which expresses the stored energy release rate during the temperature rise (see Clayton (2005) for a similar approach).

2.2. Levels of simplification of heat equation

As mentioned above, heat equation (2.16) can be decomposed into various contributions. By denoting

$$\dot{Q} = \rho_0 c_0 \dot{T} \qquad\qquad D = \boldsymbol{\tau} : \mathbf{d}^P - R\dot{p} \geqslant 0$$

$$\dot{W}^{\boldsymbol{\tau}} = T\frac{\partial \boldsymbol{\tau}}{\partial T} : \mathbf{d}^e \qquad\qquad \dot{W}^R = T\frac{\partial R}{\partial T}\dot{p}$$

Eq. (2.16) can be rewritten as

$$\dot{Q} = \dot{W} \qquad \text{with} \qquad \dot{W} = D + \dot{W}^{\boldsymbol{\tau}} + \dot{W}^R \tag{2.17}$$

Three cases are now distinguished depending on the terms retained on the right side of $(2.17)_{1,2}$. In the first case, called 'quasi complete heat evaluation', thermo-elastic coupling effects are neglected due to their weak contribution to the temperature change: $\dot{W}^{\boldsymbol{\tau}} = 0$. Relation $(2.17)_2$ is thus reduced to

$$\dot{W} = D + \dot{W}^R = \Big[J_2 - \Big(R - T\frac{\partial R}{\partial T}\Big)\Big]\dot{p} \tag{2.18}$$

In the second case, heat equation (2.16) is further simplified by neglecting both thermo-elastic and thermo-dissipative couplings contributions (see Voyiadjis and Abed (2006) for similar simplification). The resulting equation is called 'simplified heat evaluation'. This means that $\dot{W}^{\boldsymbol{\tau}} = \dot{W}^R = 0$, and $(2.17)_2$ becomes

$$\dot{W} = D = (J_2 - R)\dot{p} \tag{2.19}$$

The last case is equivalent to the representation involving the constant inelastic heat fraction β – also known as Taylor-Quinney coefficient (see Mason *et al.*, 1994) – as frequently done in engineering problems. The resulting equation is called 'basic heat evaluation'. The expression for the work rate $\dot{W}$ in $(2.17)_2$ is then reduced to

$$\dot{W} = \beta J_2 \dot{p} \tag{2.20}$$

3. Condition for dynamic localization occurrence

The condition for localization (in form of adiabatic shear banding) onset is obtained here using the linear perturbation method. In order to analyse the consequences of simplifications assumed to evaluate the temperature rise, the loading path considered is simple shearing.

3.1. Linear perturbation method

The linear perturbation method is, in general, applied to the case of simple shear under constant velocity boundary conditions. Assuming negligible elastic effects, laminar viscoplastic flow and adiabatic conditions, the problem can be reduced to a one-dimensional formulation, see Bai (1982), Clifton *et al.* (1984), Molinari (1985), Batra and Wei (2006), and also Anand *et al.* (1987) when three-dimensional generalization is presented. Admitting analytical solutions, the linear perturbation method provides in this case a criterion of instability onset, which is interpreted as the incipience of the adiabatic shear banding process, providing in fact the necessary condition for the onset of formation of bands (possibility of the shear band type instability).

Starting from the governing equations (momentum balance, energy balance, kinematics and constitutive law) in the case of simple shear in (1,2) plane under adiabatic conditions and neglecting elasticity, one obtains the following system

$$\begin{aligned} & s_{12,2} = \rho_0 \dot{v}_1 && \rho_0 c_0 \dot{T} = \dot{W} \\ & \dot{p} = \mathit{\Lambda}(s_{12}, p, T) && d_{12} = \frac{1}{2}(v_{1,2} + v_{2,1}) \approx d^p_{12} = \frac{3}{2}\dot{p}\frac{s_{12}}{J_2} \end{aligned} \tag{3.1}$$

With $J_2 = \sqrt{3}s_{12},\ v_1 = \dot{\mathit{\Gamma}} x_2,\ v_2 = 0$ and $\mathit{\Gamma} = \dot{\mathit{\Gamma}} t$, system (3.1) is reduced to

$$\begin{aligned} & s_{12,2} - \rho_0 \dot{v}_1 = 0 && \rho_0 c_0 \dot{T} - \dot{W} = 0 \\ & v_{1,2} - \sqrt{3}\dot{p} = 0 && \dot{p} - \mathit{\Lambda}(s_{12}, p, T) = 0 \end{aligned} \tag{3.2}$$

A small perturbation $\delta U = (\delta v_1, \delta s_{12}, \delta p, \delta T)$ is now superimposed on the set of homogeneous solutions $U = (v_1, s_{12}, p, T)$

$$U \Rightarrow U + \delta U \qquad \text{with} \qquad \delta U \ll U$$

The perturbed system is linearized from (3.2) as follows

$$\begin{aligned} & \delta s_{12,2} - \rho_0 \delta \dot{v}_1 = 0 && \rho_0 c_0 \delta \dot{T} - \delta \dot{W} = 0 \\ & \delta v_{1,2} - \sqrt{3}\delta \dot{p} = 0 && \delta \dot{p} - \delta \mathit{\Lambda}(s_{12}, p, T) = 0 \end{aligned} \tag{3.3}$$

With the notations in Appendix – see in particular (A.2) and (A.3), the above system can be equivalently written as follows

$$\begin{aligned} & \delta s_{12,2} - \rho_0 \delta \dot{v}_1 = 0 && \rho_0 c_0 \delta \dot{T} - (w_s \delta s_{12} + w_p \delta p + w_T \delta T + w_{\dot{p}} \delta \dot{p}) = 0 \\ & \delta v_{1,2} - \sqrt{3}\delta \dot{p} = 0 && \delta \dot{p} - 2P_{12}\delta s_{12} - B\delta p - E\delta T = 0 \end{aligned} \tag{3.4}$$

where the expressions for w_i are given in Table 1 depending on the level of simplification of the energy balance.

Table 1. Expressions for w_i

	Basic	Simplified	Quasi-complete
w_s	$\sqrt{3}\beta\dot{p}$	$\sqrt{3}\dot{p}$	$\sqrt{3}\dot{p}$
w_p	0	$-Q\dot{p}$	$-(Q-TY)\dot{p}$
w_T	0	$-S\dot{p}$	$TZ\dot{p}$
$w_{\dot{p}}$	$\sqrt{3}\beta s_{12}$	$\sqrt{3}s_{12}-R$	$\sqrt{3}s_{12}-(R-TS)$

Let the perturbation have a wave-like form

$$\delta U = \overline{U}\exp(\varpi t + \mathrm{i}kx_2) = \overline{U}\exp(\varpi_R t)\exp[\mathrm{i}k(ct + x_2)] \tag{3.5}$$

where $\overline{U}$ represents the perturbation magnitude, ϖ – wave pulsation, k – wave number, x_2 – wave plane normal, ϖ_R and ϖ_I – real and imaginary parts of the wave pulsation ϖ, respectively, and $c = \varpi_I/k$ – wave velocity.

According to the right-hand side of (3.5), the case $\varpi_R = 0$ points the transition between the stable and unstable states:

- if $\varpi_R > 0$, the perturbation may grow with time and the instability mentioned is possible;
- if $\varpi_R < 0$, the perturbation decreases with time.

The objective consists thus in looking for the conditions of transition from the stable state to the possible unstable state by studying the sign of ϖ_R.

Injecting (3.5) into system (3.4), yields

$$\begin{aligned} &\mathrm{i}k\overline{s}_{12} - \rho_0\varpi\overline{v}_1 = 0 && (\rho_0 c_0\varpi - w_T)\overline{T} - w_s\overline{s}_{12} - (w_p + w_{\dot{p}}\varpi)\overline{p} = 0 \\ &\mathrm{i}k\overline{v}_1 - \sqrt{3}\varpi\overline{p} = 0 && (\varpi - B)\overline{p} - 2P_{12}\overline{s}_{12} - E\overline{T} = 0 \end{aligned} \tag{3.6}$$

or otherwise

$$\begin{bmatrix} -\rho_0\varpi & \mathrm{i}k & 0 & 0 \\ 0 & -w_s & -(w_p + w_{\dot{p}}\varpi) & (\rho_0 c_0\varpi - w_T) \\ \mathrm{i}k & 0 & -\sqrt{3}\varpi & 0 \\ 0 & -2P_{12} & (\varpi - B) & -E \end{bmatrix} \begin{Bmatrix} \overline{v}_1 \\ \overline{s}_{12} \\ \overline{p} \\ \overline{T} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3.7}$$

(3.7) can also be put as

$$[\mathbf{A}]\{\overline{\boldsymbol{U}}\} = \{\mathbf{0}\} \tag{3.8}$$

The determinant of the matrix $[\mathbf{A}]$ in (3.8), whose components are denoted a_{ij}, is simply

$$\det[\mathbf{A}] = a_{11}a_{22}a_{33}a_{44} - a_{11}a_{42}a_{33}a_{24} + a_{12}a_{31}(a_{23}a_{44} - a_{43}a_{24}) \tag{3.9}$$

The spectral equation deduced from (3.8) and (3.9) is a 3rd degree polynomial in ϖ

$$\det[\mathbf{A}] = a_3\varpi^3 + a_2\varpi^2 + a_1\varpi^1 + a_0 \tag{3.10}$$

with

$$\begin{array}{ll} a_3 = \sqrt{3}\rho_0^2 c_0 2P_{12} & a_2 = \rho_0[k^2 c_0 + \sqrt{3}(Ew_s - 2P_{12}w_T)] \\ a_1 = -k^2(Ew_{\dot{p}} + w_T + \rho_0 c_0 B) & a_0 = k^2(Bw_T - Ew_p) \end{array} \tag{3.11}$$

3.2. Localization vs. instability

As observed by Marchand and Duffy (1988) and demonstrated by Molinari (1985), the instability does not imply rigorously localization. This means that the use of the linear perturbation method provides the necessary condition only, which means that it represents a 'lower' bound for the effective localization incipience. The idea is here to 'delay' the instability onset, i.e. push it towards the 'upper' bound in the sense of approaching the strong localization incipience. In other words, we distinguish indeed the 'instability point' characterizing locally the equilibrium between strain hardening and thermal softening (maximum of the shear stress-shear strain curve in most cases) and the 'localization point' beyond which the shear stress drops strongly. The linear perturbation method provides the 'instability point', and we look, via a pragmatic engineering evaluation approach, for the 'localization point' which eventually succeeds the 'instability point'.

The adiabatic shear banding occurs as thermal softening overcomes the strain hardening. Before the 'instability point', strain hardening is predominant and the material is necessarily stable, while past the 'instability point', the thermal softening becomes predominant and the material may become unstable. In the linear perturbation method, attenuating thermal softening allows consequently for pushing forward the 'instability point' and approaching the 'localization point'.

Let now examine the sources of thermal softening. In the yield function presented previously, see (2.12), the thermal softening source is double – in the function $R_0 f(T)$ and in the isotropic hardening force $R(T,p)$, see $(2.8)_3$. Neglecting the contribution of $R_0 f(T)$ – by prescribing that $R_0 f(T) = 0$ in the linear perturbation method solely while maintaining this thermal contribution in the constitutive law – constitutes a manner to attenuate the thermal softening and pushes forward the instability onset (without altering the maximum of the shear stress-shear strain curve).

This simplification is studied here by considering the basic evaluation of temperature rise. In the first case, the problem is solved with both sources of thermal softening (assuming that localization and instability are concomitant), while in the second case, the first source of thermal softening is neglected (the localization onset is delayed regarding instability onset).

Employing Table 1, when considering the basic evaluation, the coefficients a_0 to a_3 in (3.11) become

$$\begin{aligned} a_3 &= \sqrt{3}\rho_0^2 c_0 2P_{12} \qquad & a_2 &= \rho_0(k^2 c_0 + 3E\beta\dot{p}) \\ a_1 &= -k^2(E\sqrt{3}\beta s_{12} + \rho_0 c_0 B) \qquad & a_0 &= 0 \end{aligned} \tag{3.12}$$

Spectral equation is, in this case, indeed reduced to a 2nd degree polynomial in ϖ and $\varpi > 0$ if and only if $a_1 a_3 < 0$. The condition for possible perturbation growth (instability occurrence) is accordingly given by

$$\sqrt{3}s_{12} > \frac{\rho_0 c_0}{\beta}\frac{B}{-E} \tag{3.13}$$

The instability incipience in the case of ”two sources of thermal softening” is given by injecting $(A.4)_2$ and $(A.4)_3$ (see Appendix) into (3.13)

$$\sqrt{3}s_{12} > \frac{\rho_0 c_0}{\beta}\frac{Q}{-(R_2 f' + S)} = \frac{\rho c_y}{\beta}\frac{h''(p)f(T)}{-[R_2 + h'(p)]f'(T)} \tag{3.14}$$

In order to delay the instability onset (i.e. to approach the localization one), we are now considering a ”single source of thermal softening” ($f(T) = 0$ in $(A.4)_3$). In this case, the localization is supposed to occur as soon as

$$\sqrt{3}s_{12} > \frac{\rho_0 c_0}{\beta}\frac{Q}{-S} = \frac{\rho c_y}{\beta}\frac{h''(p)f(T)}{-h'(p)f'(T)} \tag{3.15}$$

Relations (3.14) and (3.15) are available for any material behaviour. In the following application, we are interested in a class of metallic materials exhibiting strain hardening with saturation stress and linear thermal softening. Stored energy of cold work $h(p)$ and thermal softening function $f(T)$ in $(2.7)_3$ are given by

$$h(p) = R_\infty\Big[p + \frac{1}{k}\exp(-kp)\Big] \qquad\qquad f(T) = 1 - AT \tag{3.16}$$

Model constants have been reported in Table 2 below.

Table 2. Material constants for the numerical modelling

E [GPa]	ν	ρ [kg/m^3]	c [J/(kg K)]	α [K^{-1}]
200	0.33	7800	420	10^{-6}

R_∞ [MPa]	k	A [K^{-1}]	R_0 [MPa]	Y [MPa s$^{1/m}$]	m
1000	20	10^{-3}	1000	100	6

The response of the above material to shear loading is given for various initial temperatures and various strain rates in Fig. 1 and Fig. 2, respectively. The critical plastic strain rate for adiabatic conditions is supposed to be equal to about $100\,\mathrm{s}^{-1}$.

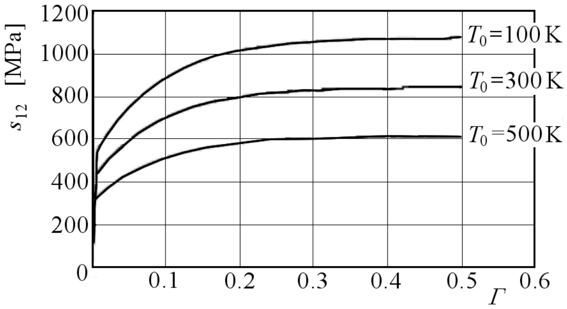

Fig. 1. Shear stress vs. shear strain. Influence of initial temperature. Instability criterion deactivated. $\dot{\Gamma} = 10^{-1}\,\mathrm{s}^{-1}$

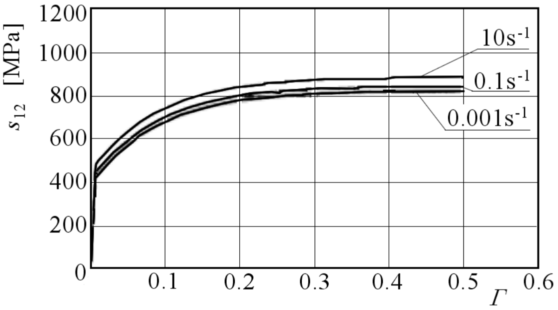

Fig. 2. Shear stress vs. shear strain. Influence of strain rate. Instability criterion deactivated. $T_0 = 20°\mathrm{C}$

Detection of the plastic localization onset is essential to activate the process of softening behaviour (see Longère *et al.*, 2003, 2005) for three-dimensional modelling of a thermoelastic/viscoplastic material incorporating adiabatic shear banding formation and growth). The description of the post–localization behaviour is not the purpose of the present paper. But in order to visualize the drop in stress induced by adiabatic shear banding, the ASB deterioration model developed by these authors has been used. Details regarding numerical calculations and a corresponding algorithmic procedure can be found in Longère *et al.* (2005). It is notable that instability criteria (3.14) and (3.15) can be applied to any constitutive equations based on irreversible thermodynamics (see nevertheless the recommendations in Longère and Dragon (2006)). In Fig. 1 and Fig. 2, the criteria have been deactivated (no deterioration), while in Fig. 3 and Fig. 4 the corresponding criteria are accounted for and considered as a necessary and sufficient condition for the localization onset (plastic localization induced ASB deterioration is thus producing strong softening behaviour according to the model by Longère *et al.* (2005)).

The simplification leading to the delay of the instability point is illustrated in Fig. 3 and Fig. 4 for given shear strain rates and two values for β.

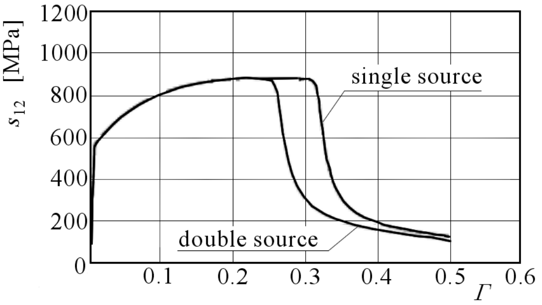

Fig. 3. Shear stress vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$; $\beta = 1$

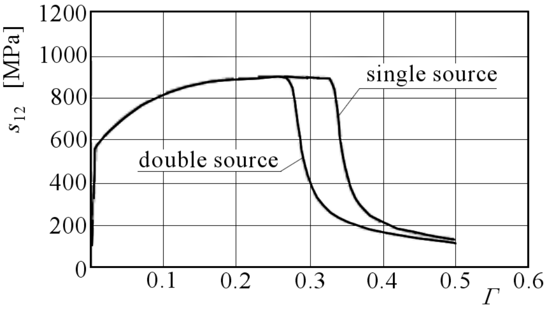

Fig. 4. Shear stress vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$; $\beta = 0.8$

For $\beta = 1$ (see Fig. 3), the value of localization deformation (drop in stress) is close to 25% for the double source of thermal softening, while it is close to 30% for the single source of thermal softening. For $\beta = 0.8$ (see Fig. 4), the value of localization deformation is close to 27% for the double source of thermal softening, while it is close to 32% for the single source of thermal softening.

With the double source of thermal softening, Figures 3 and 4 clearly show that the instability appears at the maximum of the shear stress-shear strain curve. This is in opposition with the experimental results obtained by Marchand and Duffy (1988) who distinguished two stages in the deformation localization (a weak one preceding the genuine strong one in form of adiabatic shear bands). In this sense, the single source of thermal softening seems to be more realistic, i.e. reproduces the delay between the thermal instability and genuine localization onset. The reader can also refer to works by Estrin *et al.* (1997) on full non-linear analysis of adiabatic shear band formation in the case of a shear sample containing a geometric defect.

3.3. Condition for delayed dynamic localization onset

As mentioned in the previous section, delaying the strong localization onset with respect to the supposed instability onset is favoured by setting $f(T) = 0$ in the linear perturbation method (concept of 'upper' bound approximation). This approach is consequently used in the following.

According to Table 1, the coefficients a_0 to a_3 in (3.11) become in the "simplified evaluation"

$$\begin{aligned} a_2 &= \rho_0\Big[k^2c_0 + 3\dot{p}\Big(E + \frac{2}{\sqrt{3}}P_{12}S\Big)\Big] & a_3 &= \sqrt{3}\rho_0^2c_0 2P_{12} \\ a_1 &= -k^2[E(\sqrt{3}s_{12} - R) - S\dot{p} + \rho_0c_0B] & a_0 &= k^2\dot{p}(EQ - BS) \end{aligned} \tag{3.17}$$

with $E + 2P_{12}S/\sqrt{3} = 0$ and $EQ - BS = 0$.

The corresponding constants in (3.17) are thus expressed by

$$\begin{aligned} a_3 &= \sqrt{3}\rho_0^2c_0 2P_{12} & a_2 &= \rho_0k^2c_0 \\ a_1 &= -k^2[E(\sqrt{3}s_{12} - R) - S\dot{p} + \rho_0c_0B] & a_0 &= 0 \end{aligned} \tag{3.18}$$

The spectral equation is once more reduced to a 2nd degree polynomial in ϖ. and the condition for perturbation growth is given by

$$\sqrt{3}s_{12} > R - \frac{\dot{p}}{\alpha} + \rho_0c_0\frac{\frac{\partial R}{\partial p}}{-\frac{\partial R}{\partial T}} \tag{3.19}$$

Following the previous procedure for quasi-complete evaluation, gives

$$\begin{aligned} a_3 &= \sqrt{3}\rho_0^2c_0 2P_{12} \\ a_2 &= \rho_0\Big[k^2c_0 + 3\dot{p}\Big(E - \frac{2}{\sqrt{3}}P_{12}TZ\Big)\Big] \\ a_1 &= -k^2\{E[\sqrt{3}s_{12} - (R - TS)] + TZ\dot{p} + \rho_0c_0B\} \\ a_0 &= k^2\dot{p}[EQ - T(EY - BZ)] \end{aligned} \tag{3.20}$$

The spectral equation remains a 3rd degree polynomial in ϖ for which the condition of instability onset is not trivial. In order to obtain a practical criterion in the sense of Bai (1982), the condition for instability onset is again deduced from the sign of the product a_1a_3.

The condition for perturbation growth is, in this case, given by

$$\sqrt{3}s_{12} > R - T\left(\frac{\partial R}{\partial T} - \frac{\frac{\partial^2 R}{\partial T^2}}{\frac{\partial R}{\partial T}}\frac{\dot{p}}{\alpha}\right) + \rho_0 c_0 \frac{\frac{\partial R}{\partial p}}{-\frac{\partial R}{\partial T}} \tag{3.21}$$

Figures 5 and 6 give the material response to shearing and temperature evolution for three cases of heat evaluation (two values for β are given), respectively.

According to Fig. 5, the value of shear strain at localization (critical shear strain) onset is close to 30% for basic evaluation with $\beta = 1.0$, to 32% for basic evaluation with $\beta = 0.8$, to 35% for simplified evaluation, and to 38% for complete evaluation. In parallel, the influence of heat evaluation is shown in Fig. 6, which gives the evolution of temperature. According to the "basic evaluation", temperature increases until about 380 K, while according to the "quasi-complete evaluation", the maximum of temperature remains under 350 K. This difference of 30 K is enough to provoke a difference of 8% for the estimation of the critical shear strain in the conditions prescribed here.

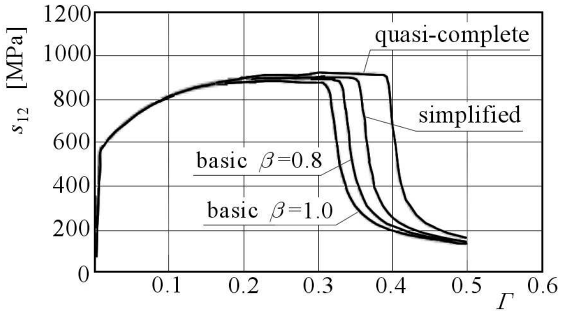

Fig. 5. Shear stress vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$

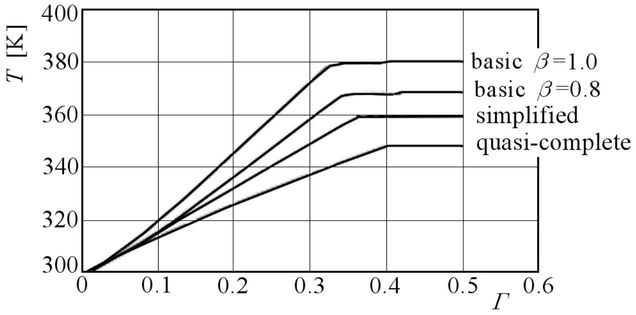

Fig. 6. Temperature vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$

In this example, the shear stress-shear strain curves diverge from each other from the shear strain close to 10%, see Fig. 5. As a consequence, the value of shear stress at the maximum of each curve is different (the highest is obtained for the quasi-complete evaluation, while the lowest is obtained for the basic evaluation for $\beta = 1$). So, we consider next here shear stress-shear strain curves with the same value of the shear stress maximum value (see Table 3 for new material constants). Numerical results including the instability criterion are given in Fig. 7 and Fig. 8.

Table 3. Material constants

Evaluation type	β	R_0 [MPa]	A [K^{-1}]
Basic	1.0	1000	$1 \cdot 10^{-3}$
	0.8	1015	$1.05 \cdot 10^{-3}$
Simplified	X	1050	$1.1 \cdot 10^{-3}$
Quasi-complete	X	1080	$1.15 \cdot 10^{-3}$

The values of critical shear strain are lower than the previous ones (see Fig. 5), but the tendency is preserved: the critical shear strain is greater for the quasi complete evaluation.

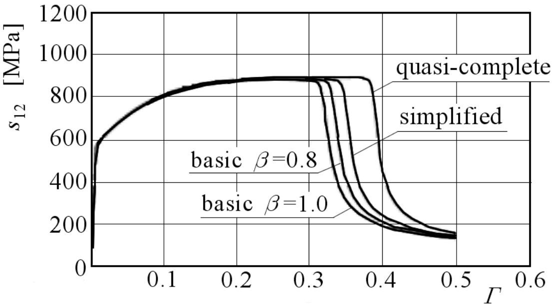

Fig. 7. Shear stress vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$

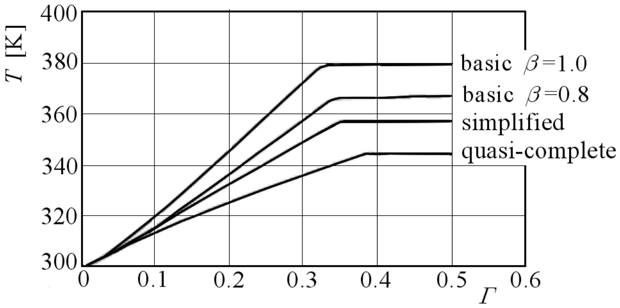

Fig. 8. Temperature vs. shear strain. Instability criterion activated. $T_0 = 300\,\mathrm{K}$; $\dot{\Gamma} = 10^3\,\mathrm{s}^{-1}$

In Figs. 7 and 8, the maximum shear stress is the same for all simulations.

4. Concluding remarks

In many boundary value problems involving high strain rates under the hypothesis of adiabaticity and employing numerical simulations via engineering finite element calculation codes, thermomechanical couplings are usually accounted for by using the approximation of the linear dependency of the temperature growth on the plastic work rate. In practice, a constant inelastic heat fraction coefficient, also known as Taylor-Quinney coefficient, is used. This method consists in neglecting genuine thermomechanical couplings explicitly occuring in the heat equation. In most cases, this simplification appears to be sufficient and conservative from the engineering viewpoint and regarding the calculation time. However, if the thermal instability induced plastic localization is concerned – it is the case of adiabatic shear banding as typically encountered in speed machining and impact ballistics for example – the evaluation of temperature growth and its consequences in terms of the related material behaviour softening must be more accurate. We have applied here the linear perturbation method to obtain a criterion for adiabatic shear banding initiation taking into account the strain hardening, thermal softening and viscous effects. This criterion is suitable for a wide range of thermo/viscoplastic materials. It has been used to study the influence of terms retained in the plasticity-induced heating and related adiabatic shear banding incipience. The analysis shows clearly that the use of the coarse method based on the constant inelastic heat fraction (varying in the usual range between 80% and 100%) leads to over-estimation of the temperature rate and produces an early onset of adiabatic shear banding instability as compared to a more rigorous thermodynamic evaluation. If the use of a constant inelastic heat fraction appears conservative for impact problems (from the armour viewpoint, earlier detection via numerical simulation can be interpreted as a safety-oriented approach), it is not when applied to numerical simulations relevant to metal cutting (in this case, later detection is rather researched for a more realistic definition of cutting conditions guaranteeing serrated chip formation under adiabatic shear banding).

A. Appendix

The perturbed system is linearized from (3.2) as follows

$$\begin{aligned} &\delta s_{12,2} - \rho\delta\dot{v}_1 = 0 && \rho c_y \delta\dot{T} - \delta\dot{W} = 0 \\ &\delta v_{1,2} - \sqrt{3}\delta\dot{p} = 0 && \delta\dot{p} - \delta\Lambda(s_{12}, p, T) = 0 \end{aligned} \tag{A.1}$$

Perturbed viscous multiplier in $(\text{A.1})_4$ is expressed by

$$\delta\Lambda = 2\frac{\partial\Lambda}{\partial s_{12}}\delta s_{12} + \frac{\partial\Lambda}{\partial p}\delta p + \frac{\partial\Lambda}{\partial T}\delta T = 2P_{12}\delta s_{12} + B\delta p + E\delta T \tag{A.2}$$

where

$$\begin{aligned} P_{ij} &= \frac{\partial\Lambda}{\partial s_{ij}} = \alpha\frac{\partial F}{\partial s_{ij}} \qquad & B &= \frac{\partial\Lambda}{\partial p} = \alpha\frac{\partial F}{\partial p} \\ E &= \frac{\partial\Lambda}{\partial T} = \alpha\frac{\partial F}{\partial T} \qquad & \alpha &= \frac{\partial\Lambda}{\partial F} \end{aligned} \tag{A.3}$$

Furthermore, the coefficients in (A.3) become

$$P_{ij} = \frac{3}{2}\alpha\frac{s_{ij}}{J_2} \qquad B = -\alpha Q \qquad E = -\alpha[R_0 f'(T) + S] \tag{A.4}$$

where

$$Q = \frac{\partial R}{\partial p} = h''(p)f(T) \qquad S = \frac{\partial R}{\partial T} = h'(p)f'(T) \tag{A.5}$$

The expressions of the perturbed work rate depend on the heat evaluation type:

— basic evaluation

In this simple case, the perturbed work rate is given by

$$\delta\dot{W} = \beta(\delta J_2\dot{p} + J_2\delta\dot{p}) = \sqrt{3}\beta(\delta s_{12}\dot{p} + s_{12}\delta\dot{p}) \tag{A.6}$$

— simplified evaluation

The perturbed work rate is here given by

$$\delta\dot{W} = (\delta J_2 - \delta R)\dot{p} + (J_2 - R)\delta\dot{p} = (\sqrt{3}\delta s_{12} - \delta R)\dot{p} + (\sqrt{3}s_{12} - R)\delta\dot{p} \tag{A.7}$$

According to (A.4), one obtains

$$\delta R = \frac{\partial R}{\partial p}\delta p + \frac{\partial R}{\partial T}\delta T = Q\delta p + S\delta T \tag{A.8}$$

Thus, perturbed work rate (A.7) becomes

$$\delta\dot{W} = (\sqrt{3}\delta s_{12} - Q\delta p - S\delta T)\dot{p} + (\sqrt{3}s_{12} - R)\delta\dot{p} \tag{A.9}$$

— quasi complete evaluation

The perturbed work rate is here given by the following expression

$$\begin{aligned} \delta\dot{W} &= \Big[\delta J_2 - \Big(\delta R - \delta T\frac{\partial R}{\partial T} - T\delta\Big(\frac{\partial R}{\partial T}\Big)\Big)\Big]\dot{p} + \Big[J_2 - \Big(R - T\frac{\partial R}{\partial T}\Big)\Big]\delta\dot{p} = \\ &= \Big[\sqrt{3}\delta s_{12} - \Big(\delta R - \delta T\frac{\partial R}{\partial T} - T\delta\Big(\frac{\partial R}{\partial T}\Big)\Big)\Big]\dot{p} + \Big[\sqrt{3}s_{12} - \Big(R - T\frac{\partial R}{\partial T}\Big)\Big]\delta\dot{p} \end{aligned} \tag{A.10}$$

Differentiating $(\mathrm{A}.5)_2$, yields

$$\delta S = \frac{\partial S}{\partial p}\delta p + \frac{\partial S}{\partial T}\delta T = Y\delta p + Z\delta T \tag{A.11}$$

where

$$Y = \frac{\partial S}{\partial p} = \frac{\partial^2 R}{\partial p \partial T} = h''(p)f'(T) \qquad\qquad Z = \frac{\partial S}{\partial T} = \frac{\partial^2 R}{\partial T^2} = h'(p)f''(T) \tag{A.12}$$

Finally, the expression for $\delta\dot{W}$ in (A.10) becomes

$$\delta\dot{W} = [\sqrt{3}\delta s_{12} - (Q - TY)\delta p + TZ\delta T]\dot{p} + [\sqrt{3}s_{12} - (R - TS)]\delta\dot{p} \tag{A.13}$$

References

1. Anand L., Kim K.H., Shawki T.G., 1987, Onset of shear localization in viscoplastic solids, *J. Mech. Phys. Solids*, **35**, 4, 407-429
2. Aravas N., Kim K.S., Leckie F.A., 1990, On the calculations of the stored energy of cold work, *J. Eng. Mater. Technol.*, **112**, 465-470
3. Bai Y.L., 1982, Thermo-plastic instability in simple shear, 1982, *J. Mech. Phys. Solids*, **30**, 4, 195-207
4. Bai Y.L., Dodd B., 1982, *Adiabatic Shear Localization*, Pergamon Press
5. Bataille J., Kestin J., 1975, Linterprétation physique de la thermodynamique rationnelle, *J. de Mécanique*, **14**, 2, 365-384
6. Batra R.C., Chen L., 2001, Effect of viscoplastic relations on the instability strain, shear band initiation strain, the strain corresponding to the minimum shear band spacing, and the band width in a thermoviscoplastic material, *Int. J. Plasticity*, **17**, 1465-1489
7. Batra R.C., Wei Z.G., 2006, Shear band spacing in thermoviscoplastic materials, *Int. J. Impact Eng.*, **32**, 947-967
8. Clayton J.D., 2005, Dynamic plasticity and fracture in high density polycrystals: constitutive modeling and numerical simulation, *J. Mech. Phys. Solids*, **53**, 261-301
9. Clifton R.J., Duffy J., Hartley K.A., Shawki T.G., 1984, On critical conditions for shear band formation at high strain rates, *Scripta Met.*, **18**, 443-448
10. Estrin Y., Molinari A., Mercier S., The role of rate effects and of thermomechanical coupling in shear localization, *J. Eng. Mat. Tech.*, **119**, 322-330

11. Fressengeas C., Molinari A., Instability and localization of plastic flow in shear at high strain rates, *J. Mech. Phys. Solids*, **35**, 2, 185-211

12. Kapoor R., Nemat-Nasser S., 1998, Determination of temperature rise during high strain rate deformation, *Mech. Mat.*, **27**, 1-12

13. Liao S.-C., Marchand A., 1998, Adiabatic shear bands in a Ti-6Al-4V titanium alloy, *J. Mech. Phys. Solids*, **46**, 11, 2201-2231

14. Longère P., Dragon A., Trumel H., De Resseguier T., Deprince X., Petitpas E., 2003, Modelling adiabatic shear banding via damage mechanics approach, *Arch. Mech.*, **55**, 3-38

15. Longère P., Dragon A., Trumel H., Deprince X., 2005, Adiabatic shear banding induced degradation in a thermo-elastic/viscoplastic material under dynamic loading, *Int. J. Impact Eng.*, **32**, 285-320

16. Longère P., Dragon A., 2006, Plastic work induced heating under dynamic conditions: critical assessment, submitted for publication

17. Marchand A., Duffy J., 1988, An experimental study of the formation process of adiabatic shear bands in a structural steel, *J. Mech. Phys. Solids*, **36**, 3, 251-283

18. Mason J.J., Rosakis A.J., Ravichandran G., 1994, On the strain and strain rate dependence of the fraction of plastic work converted to heat: an experimental study using high speed infrared detectors and the Kolsky bar, *Mech. Mat.*, **17**, 135-145

19. Molinari A., 1985, Instabilité thermoviscoplastique en cisaillement simple, *J. Méca. Théorique et appliquée*, **4**, 659-684

20. Mróz Z., Oliferuk W., Energy balance and identification of hardening moduli in plastic deformation processes, *Int. J. Plasticity*, **18**, 379-397

21. Odeshi A.G., Al-Ammeeri, Mirfakhraei, Yazdani F., Bassim M.N., 2006, Deformation and failure mechanism in AISI 4340 steel under ballistic impact, *Th. Appl. Fract. Mech.*, **45**, 18-24

22. Pęcherski R.B., 1998, Macroscopic effects of micro-shear banding in plasticity of metals, *Acta Mech.*, **131**, 203-224

23. Perzyna P., 1966, Fundamental problems in viscoplasticity, *Adv. Appl. Mech.*, **9**, 243-377, Academic Press, New York

24. Perzyna P., 1998, Dynamic localized fracture in inelastic solids, In: *Damage Mechanics in Engineering Materials*, 183-202, G.Z. Voyiadjis, J.-W.W. Ju and J.-L. Chaboche (Edit.), Elsevier

25. Petryk H., 2000, General conditions for uniqueness in materials with multiple mechanisms of inelastic deformation, *J. Mech. Phys. Solids*, **48**, 367-396

26. Raniecki B., Nguyen H.V., 1984, Isotropic elastic-plastic solids at finite strain and arbitrary pressure, *Arch. Mech.*, **36**, 5-6, 687-704

27. RHIM S.H., OH S.I., 2006, Prediction of serrated chip formation in metal cutting process with new flow stress model for AISI 1045 steel, *J. Mat. Proc. Tech.*, **171**, 417-422

28. SIDOROFF F., DOGUI A., 2001, Some issues about anisotropic elastic-plastic models at finite strain, *Int. J. Solids Struct.*, **38**, 9569-9578

29. TAYLOR G.I., QUINNEY H., 1934, The latent energy remaining in a metal after cold working, *Proc. Roy. Soc.*, **A413**, 307-326

30. VOYIADJIS G.Z., ABED F.H., 2006, A coupled temperature and strain rate dependent yield function for dynamic deformations of bcc metals, *Int. J. Plasticity*, **22**, 1398-1431

31. WEI Q., JIAO T., RAMESH K.T., MA E., KECSKES L.J., MAGNESS L., DOWDING R., KZAYKHANOV V.U., VALIEV R.Z., 2006, Mechanical behaviour and dynamic failure of high-strength ultrafine grained tungsten under uniaxial compression, *Acta Mater.*, **54**, 77-87

Szacowanie adiabatycznego ciepła w procesie dynamicznej lokalizacji odkształceń plastycznych

Streszczenie

Adiabatyczne ogrzewanie w stanie plastycznym jest w większości przypadków opisywane za pomocą ułamkowego ciepła zwanego współczynnikiem Taylora-Quinney'a, zwykle przyjmowanego jako parametr o stałej wartości. Opis taki, z punktu widzenia termodynamiki, zasadza się na pomijaniu (lub uwzględnianiu jedynie pewnej części) sprzężeń termomechanicznych w równaniu definiującym bilans cieplny. Taki zgrubny opis może prowadzić do przeszacowania wzrostu temperatury w obliczeniach dotyczących plastycznego ogrzewania. W pracy przedstawiono proces adiabatycznego formowania pasm ścinania wynikający z termicznej niestabilności. Pokazano, że dokładność w przewidywaniu korzystnych czynników dla inicjacji procesu lokalizacji odkształceń plastycznych silnie zależy od metody szacowania ciepła powstającego wskutek uplastycznienia. Celem rozważań jest pokazanie wpływu różnego stopnia uproszczeń przyjętych w równaniu bilansu cieplnego na warunki odpowiadające za pojawienie się lokalizacji odkształceń plastycznych. Te ostatnie wyznaczono za pomocą kryterium opartego na liniowej metodzie perturbacyjnej.

Manuscript received June 1 2006; accepted for print December 1, 2006