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FLOW PATTERNS GENERATED BY A STRONG MAGNETIC FIELD

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The influence of a strong magnetic field on a paramagnetic fluid in a cylindrical enclosure of thermosyphon-like geometry was studied. A variety of spoke patterns was experimentally obtained and recorded. The analysis led to some data about the temperature field and, indirectly, about the flow structure. The results related to the heat transfer rate confirmed the conclusions coming from the flow visualization.

Key words: spoke pattern, thermosyphon, magnetic field

1. Introduction

In nature, a variety of different kinds of patterns can be found. The beauty of nature can also be recognized in industrial processes, for example in solidification (dendrites) or spoke patterns during growth of a single crystal. The spoke pattern is also observed in a thermosyphon-like enclosure in which natural convection appears.

Pattern formation for natural convection has been studied since the beginning of the 20th century. Benard (1901) observed appearance of hexagonal cells as a result of instabilities in a shallow layer of a fluid heated from below. Rayleigh (1916) presented the theory related to Benard's experiment. He proposed a non-dimensional parameter called the Rayleigh number to characterize the convection. The works done by Jeffreys (1928, 1930), Pellew and Southwell (1940) or Chandrasekhar (1961) should also be mentioned. Recently, Hof *et al.* (1999) reported experimental results of natural convection investigations in the vertical cylinder heated from below. They studied the stability of flow and found that at the same Rayleigh number various patterns can exist. The onset of convective instabilities in cylindrical cavities heated from below was described by Touihri *et al.* (1999). They numerically analyzed this phenomenon with the attention paid to the nonlinear evaluation of convection. The multiple steady states for the same Rayleigh number were obtained by Boronska and Tuckerman (2004) who simulated numerically Hof's experiment (Hof *et al.*, 1999).

Numerous studies on the dynamic patterns of convection in a melt have been carried out, and many researchers studying crystal growth have reported the occurrence of "spoke patterns" on the melt surface. The flow loses its stability and the axisymmetric structures break its symmetry and become non-axisymmetrical. It is observed that flow is divided into several identical rolls. Adjacent roll structures (sectors) have an opposite azimuthal velocity component. Moreover, computed results show that the azimuthal velocity of the structure modifies the temperature profile from axisymmetric to non-axisymmetric. Numerical computation applied to the analysis of buoyancy driven flow in a vertical cylinder shows a non-axisymmetric character of the flow even for axisymmetric boundary conditions. Kowalewski *et al.* (1998) and Gelfgat *et al.* (1999) observed the same spoke structures and axisymmetry breaking instability with a large azimuthal number.

The pattern formation in a cylindrical enclosure of thermosyphon-like geometry was described, for example, by Japikse *et al.* (1971), Mallinson *et al.* (1981) or Ishihara *et al.* (2002a,b). They observed appearance of alternate streams of a hot and cold fluid in various numbers depending on the fluid properties, geometrical and thermal conditions.

All mentioned examples were related to the natural convection phenomenon. However, due to the development of superconducting magnets, a new phenomenon called "magnetic convection" could be investigated. Braithwaite *et al.* (1991) as the first researcher reported the influence of magnetic field on the natural convection of a paramagnetic fluid. Their work related to the magnetic field was continued, among others, by Ikezoe *et al.* (1998), Wakayama (1993), Wakayama and Wakayama (2000), Kaneda *et al.* (2002), who studied the influence of magnetic field on many other phenomena in various research areas. Wide knowledge of the magnetic field effect on engineering processes is described in a book by Ozoe (2005).

The present paper gives an insight into the magnetically induced flow in the thermosyphon-like cylinder with the attention paid to the pattern formation. It should be added that in such a configuration of the cylinder, the natural convection is very limited and no pattern can be recognized.

2. Basics

Ordinary substances (like water or air) show some magnetic effects, although very small. This small magnetism is of two kinds. Some materials are attracted toward the magnetic field, others are repelled (Feynman *et al.*, 1977). The substances, which are repelled, are called diamagnetic, whereas the substances which are attracted are called paramagnetic.

The paramagnetic materials have positive magnetic susceptibility which is usually a function of absolute temperature in contradiction to diamagnetic materials whose magnetic susceptibility is negative and independent of temperature. The equation taking into account Curie's law and describing the magnetic force can be written in a form obtained by Tagawa and Ozoe (2002)

$$\boldsymbol{F}_m = \frac{-\rho_0\chi_0\beta(\theta-\theta_0)}{\mu_m}\boldsymbol{\nabla} b^2 \tag{2.1}$$

where ρ_0 denotes reference density at the reference temperature θ_0, χ_0 – reference magnetic susceptibility, β – thermal expansion coefficient, θ – temperature, θ_0 – reference temperature, μ_m – magnetic permeability, b – magnetic induction.

The driving force for convection is usually the density difference between hot and cold regions of the fluid. If the fluid has magnetic susceptibility that varies with temperature, the magnetic forces, rather than buoyancy, can drive convective motion (Braithwaite *et al.*, 1991). The magnetic buoyancy force which drives convective motion is proportional to the gradient of square of magnetic induction.

3. Experimental enclosure

The experimental enclosure is presented in Fig. 1. The cylinder diameter was 0.04 m and the heated and cooled side walls were made of copper of 0.028 m height separated by thin Plexiglas cylindrical plate of 0.004 m thickness. Four holes were drilled from the side wall of this plate to place four Light Emitting Diodes (LEDs) for visualization of the temperature field in the chosen cross-section. The top and bottom end plates of the cylinder were made of Plexiglas of 0.005 m thickness. The outside surface of the enclosure side wall was heated with a rubber-coated nichrome wire. The wire was connected to a DC power supply whose current and voltage were constantly controlled by multimeters (Fig. 2). The side wall of the second half of the cylinder was kept at a constant temperature with water from a constant temperature bath. The temperature difference between the heated and cooled parts of the enclosure was measured by Copper-Constantan thermocouples. Two were located on the outer side of the heated wall, while the other two were attached to the coolant outlet and inlet. The thermocouples were connected to a data acquisition system. The measured temperature was recorded and stored for further analysis.

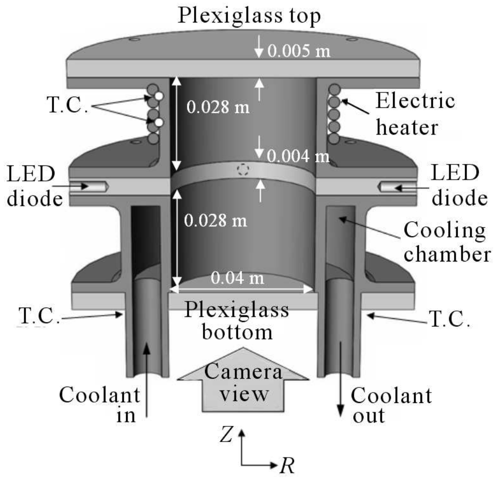

Fig. 1. Experimental enclosure or apparatus

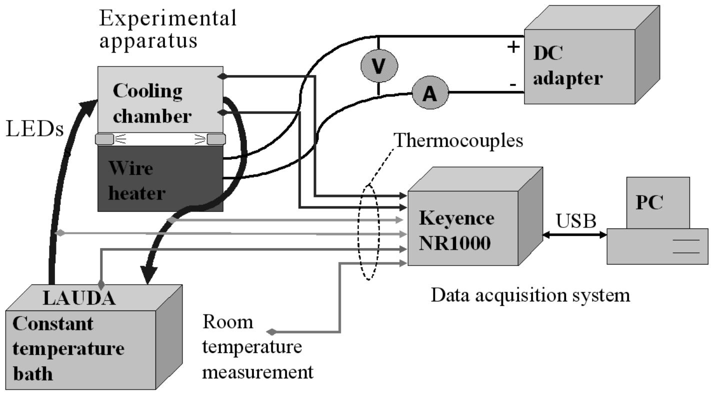

Fig. 2. Scheme of the experimental set-up

4. Experimental fluid

A 50% volume glycerol aqueous solution was chosen as the working fluid, because glycerol itself is hard to handle. This mixture is diamagnetic however, the aim was to study a paramagnetic fluid. Therefore, crystals of gadolinium nitrate hexahydrate ($Gd(NO_3)_3 \times 6H_2O$) were added to the fluid increasing its magnetic susceptibility up to $\chi_{exp} = 13.926 \cdot 10^{-8}\,\mathrm{m^3/kg}$.

The molar concentration of gadolinium nitrate hexahydrate was 0.5 mol/kg. Before starting the main part of the experiment, the density, magnetic susceptibility and viscosity of the fluid were measured. Other properties were estimated from [26] for the 50% volume glycerol aqueous solution. The properties of the working fluid are listed in Table 1. The liquid crystal slurry was mixed with the working fluid to visualize the temperature field in the middle-height cross-section.

Table 1. Properties of 50% aqueous solution of glycerol at $\theta_0 = 298\,\mathrm{K}$ (the properties marked by asterisk were estimated from [26])

Property	Value	Unit
α^*	$1.1415 \cdot 10^{-7}$	$\mathrm{m^2/s}$
β^*	$0.445 \cdot 10^{-3}$	$\mathrm{K^{-1}}$
λ^*	0.422	$\mathrm{W/(m{\cdot}K)}$
μ_{exp}	$6.145 \cdot 10^{-} \pm 0.064 \cdot 10^{-3}$	$\mathrm{Pa{\cdot}s}$
ν_{exp}	$4.80 \cdot 10^{-6} \pm 0.05 \cdot 10^{-6}$	$\mathrm{m^2/s}$
ρ_{exp}	1281 ± 1	$\mathrm{kg/m^3}$
χ_{exp}	$13.926 \cdot 10^{-8} \pm 0.329 \cdot 10^{-8}$	$\mathrm{m^3/kg}$

5. Parameters

The experiment was carried out for various Rayleigh numbers defined as

$$\mathrm{Ra} = \frac{g\beta(\theta_{hot} - \theta_{cold})r_0^3}{\alpha\nu} \tag{5.1}$$

where g is the gravitational acceleration, β – thermal expansion coefficient, θ_{hot} – temperature of the heated part of the side wall, θ_{cold} – temperature of the cooled part of the side wall, α – thermal diffusivity of the fluid, ν – kinematic viscosity, r_0 – inner radius of the cylinder.

Thermal measurements were carried out to investigate the influence of the magnetic field on the heat transfer rate. The Nusselt number was defined as follows

$$\mathrm{Nu} = \frac{Q_{net\_conv}}{Q_{net\_cond}} \tag{5.2}$$

The net convection (Q_{net_conv}) and net conduction (Q_{net_cond}) heat fluxes were estimated by the method invented by Ozoe and Churchill (1973) and applied, e.g., in Maki *et al.* (2002)

$$\begin{aligned} Q_{net\_cond} &= Q_{cond} - Q_{loss} \\ Q_{net\_conv} &= Q_{conv} - Q_{loss} \end{aligned} \tag{5.3}$$

The net convection heat flux (Q_{net_conv}), Eq. $(5.3)_1$ was estimated as the difference between the total heat supply during the convection experiment and the heat loss. The net conduction heat flux (Q_{net_cond}), Eq. $(5.3)_2$ was estimated as the difference between the total heat supply during the conduction experiment and the heat loss. The ratio of these net heat fluxes gives the Nusselt number. The convection heat flux (Q_{conv}) was calculated from the electrical current and voltage values, while the conduction heat flux (Q_{cond}) from an additional conduction experiment described in detail in Fornalik *et al.* (2005). The heat loss was assumed to depend on the heater temperature itself and not on the mode of heat transfer inside the enclosure.

6. Procedure

The experimental enclosure filled with the experimental fluid and insulated with a vinyl foil was placed into the bore of a superconducting magnet. The middle height cross-section of the cylinder was placed at 0.01 m from the opening level of the bore. That position guaranteed the minimum radial component of the magnetic buoyancy force in the tested volume.

The experiment was carried out for magnetic induction from 0 to 5 T with a step of 1 T. The visualization of fluid temperature was done by the encapsulated liquid crystal slurry. Depending on the temperature of the fluid, the liquid crystals showed different colors: the blue color represented hot fluid, while the red color indicated the cold fluid. The temperature indication range for color visualization was 291-294 K. The color images of flow modes were taken by a digital camera working in a long exposure time of 4 seconds. The thermal measurements by thermocouples were carried out to investigate the influence of the magnetic field on the heat transfer rate. The procedure and definitions were reported in detail in Fornalik *et al.* (2005).

The experimental apparatus was maintained at a constant environmental temperature. At the beginning, the cooling water bath and the heater power supply were switched on. The temperature level and the value of supplied power were set. The temperatures on the heated and cooled side walls were monitored. When the system reached the steady state after 90-120 minutes (it depended on the temperature difference), temperatures were recorded and, at the same time, the picture of fluid temperature field was taken. The steady state was assumed when the temperature stayed at the same level for 30 minutes. Then, the magnetic field was applied to the system. It should be emphasized that after each step (for example after changing the magnetic induction from 1 T to 2 T), the system was kept in a constant condition to obtain another steady state. When it was reached (after about 40-60 minutes – depending on the magnetic induction), the temperature values and the fluid flow structure were recorded.

7. Results

Figure 3 shows the visualized isotherms for selected Rayleigh numbers and various strengths of magnetic induction. The pictures in Fig. 3 at 0 T were taken for various Rayleigh numbers without the magnetic field. The uniform color of the pictures indicates the presence of an isothermal layer of the fluid in the middle height cross-section. The color represents the mean fluid temperature. The spoke pattern was not observed, which suggests that convective motion was not present.

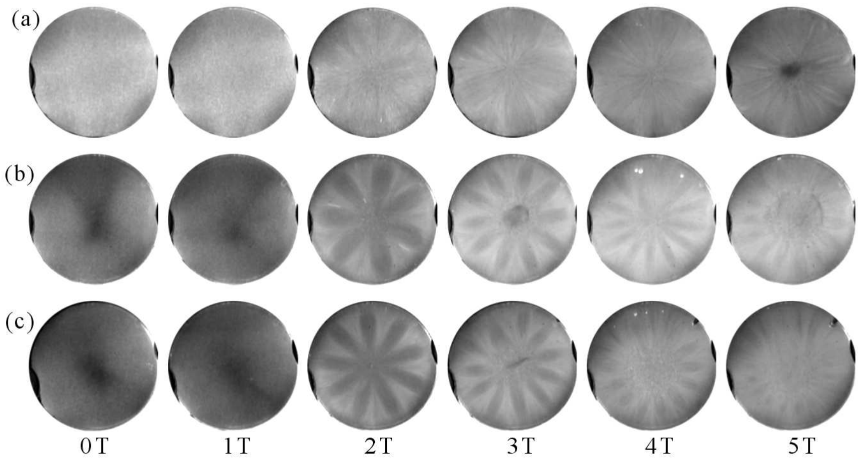

Fig. 3. Isotherms for various strengths of the magnetic field in the stable configuration, (a) $\mathrm{Ra} = 0.14 \cdot 10^5$, (b) $\mathrm{Ra} = 2.79 \cdot 10^5$, (c) $\mathrm{Ra} = 5.22 \cdot 10^5$

When the level of 2 T was achieved, the spokes appeared suddenly in the visualized cross-section. In Fig. 3a at $\mathrm{Ra} = 0.14 \cdot 10^5$ and 2 T, three spokes suddenly appeared, while in Fig. 3b at $\mathrm{Ra} = 2.79 \cdot 10^5$ seven spokes and in Fig. 3(c) at $\mathrm{Ra} = 5.22 \cdot 10^5$ nine spokes. The number of spokes increased in Fig. 3a from 0 to 8, In Fig. 3b from 0 to 14 and in Fig. 3c from 0 to 17 with an increase in the magnetic induction from 0 T to 5 T. The appearance of spoke patterns is a sign of induced convection in the system. Increasing with the magnetic induction, the number of spokes is related to enhanced convection.

Figure 4 presents calculated isotherms at the middle-height cross-section with the three-dimensional demonstration of long time streak lines for better understanding of spokes origins.

The complete numerical analysis of the phenomena is presented in Filar *et al.* (2006). The fluid moves upwards near the lower heated side wall, then proceeds toward the enclosure center at the middle-height, and near the center ascends toward the top of cylinder. Finally, the fluid descends along the upper

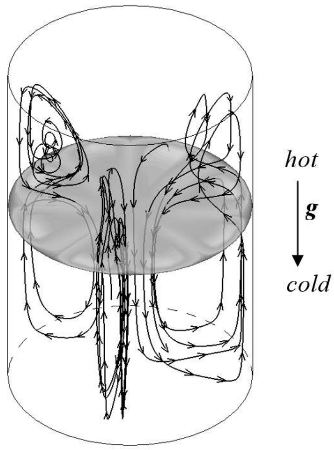

Fig. 4. Computed isotherms at the cylinder mid-height and three-dimensional demonstration of long time streak lines for the stable system at $\mathrm{Ra} = 1.89 \cdot 10^5$ and $b = 2\,\mathrm{T}$

cooled side wall. This trajectory is typical for thermosyphon systems and was reported by Japikse *et al.* (1971), Ishihara *et al.* (2002a,b).

In Fig. 5, the configuration of gravitational buoyancy (F_g) and magnetic buoyancy (F_m) forces in the experimental enclosure are presented. This schematic drawing can help one to understand the fluid behaviour.

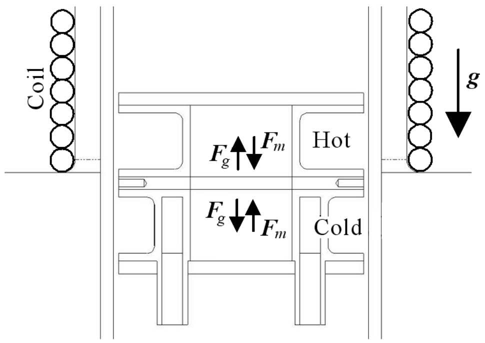

Fig. 5. Configurations of the system $(F_g$ – gravitational buoyancy force, F_m – magnetic buoyancy force)

The relatively cold fluid inside the lower cooled region having a higher density tended to stay in the lower part due to the gravitational buoyancy force. However the magnetic buoyancy force acted upward on the colder fluid with a larger value of the magnetic susceptibility. On the other hand, the hot fluid was kept in the upper part of the cylinder by the gravitational buoyancy force, but it was also repelled by the magnetic buoyancy force due to the smaller value of the magnetic susceptibility. Therefore, without the magnetic field, the system was almost thermally stable showing no pattern. It was thermally stable until the magnetic buoyancy force was smaller or equal to the gravitational buoyancy force. When the magnetic buoyancy force acting in the opposite direction exceeded the gravitational buoyancy force, the magnetic convection was induced in the system. It could be deduced from the appearance of spoke patterns characteristic for natural convection in a thermosyphon-like enclosure with the upper side wall cooled and lower one heated. The increasing of the magnetic induction caused an increase in the number of spokes suggesting enhanced convective motion.

Figure 6 shows the Nusselt number versus the maximum magnetic induction. The measurements were done for various Rayleigh numbers. For the maximum magnetic induction of 0 T, the Nusselt number was close to one for all cases except $\mathrm{Ra} = 5.22 \cdot 10^5$. It was mentioned above that without the magnetic field the system is "almost" thermally stable, however slow convective motion is present in the vicinity of side walls. As it can be seen, for high Rayleighs numbers, the convection close to the wall is "slower" because the heat transfer rate is close to 6. For the magnetic induction of 2 T, the suppression of convective motion can be suggested by the decreasing Nusselt number. The magnetic buoyancy force compensates the gravitational buoyancy force. Further growth of the magnetic induction causes growth of the Nusselt number, which means intensified heat transfer by enhanced magnetic convection.

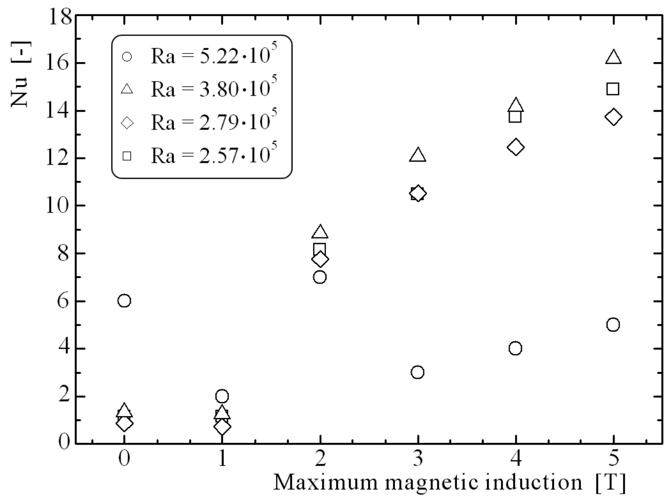

Fig. 6. The Nusselt number versus the maximum magnetic induction for various Rayleigh numbers

8. Conclusions

Experimental visualizations of the flow and heat transfer rates were presented for a single-phase thermosyphon under various thermal and magnetic conditions. Without a magnetic field, the convective flow was not observed and the Rayleigh number had no visible influence on the flow structure. After applying a magnetic field, the convective flow was obtained. It was found that the magnetic buoyancy force of about 1 T appeared to be equal to the gravitational buoyancy force. Magnetic fields greater than 1 T induced the convective flow. In the discussed experiment, the induction of convective flow resulted in the appearance of a spoke pattern in the middle height cross-section and also in an increase of the Nusselt number.

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Wpływ silnego pola magnetycznego na strukturę przepływu

Streszczenie

Przedstawione zostały badania dotyczące struktury przepływu w geometrii cylindrycznej typu odwrócony termosyfon (tzn. górna część cylindra była grzana, a dolna chłodzona). Naczynie eksperymentalne napełnione cieczą paramagnetyczną zostało umieszczone w przestrzeni badawczej magnesu nadprzewodzącego, generującego silne pole magnetyczne (maksymalna indukcja magnetyczna 5 Tesli). Badano wpływ pola magnetycznego na strukturę przepływu, dokonano wizualizacji pola temperatury przy pomocy ciekłych kryształów oraz pomiaru temperatury za pomocą termopar. Bez pola magnetycznego oraz dla indukcji magnetycznej poniżej 1 Tesli w naczyniu nie obserwowano ruchu konwekcyjnego, a co za tym idzie również żadnej struktury przepływu. Dla indukcji magnetycznej powyżej 1 Tesli w naczyniu eksperymentalnym została zapoczątkowana konwekcja magnetyczna, co objawiło się pojawieniem struktury szprychowej. Struktura szprychowa, typowa dla konwekcji naturalnej w termosyfonie (tzn. górna część cylindra chłodzona, a dolna grzana) uległa zmianie – wraz ze wzrostem indukcji magnetycznej wzrastała liczba płatków, co sugerowało intensyfikację konwekcji. Wyniki wizualizacji zostały potwierdzone przez pomiary termiczne oraz wyznaczoną liczbę Nusselta.

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