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EVALUATION OF VIBRATION DAMPING IN THE MODELLING OF DYNAMICS OF A FLEXIBLE ROTOR

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Selection of a method for evaluation of vibration damping is relevant in the analysis of flexible rotors operating between the first two critical speeds, their dynamics as well as in the modelling of dynamical states. The correspondence between results of theoretical analysis and experimental data depends on this choice. Three cases of evaluation of vibration damping in rotary systems are discussed in the paper. The presented modelling dynamical states of rotors comprises three methods for the assesment of vibration damping. Results of the theoretical analysis are compared with experimental results and discussed.

Key words: vibrations, damping modelling, vibrodiagnostics

1. Introduction

In the study of rotor system dynamics, the Method of Finite Elements (FEM) is extensively applied. This method is an important tool in the modelling of various dynamical situations that take place in such system, enabling determination of effect of likely defects and the efficiency of vibration damping due to introduced agents, etc. When developing a dynamic model of a rotor system, the intention is to make it adequately consistent with the real system. Here, the evaluation of vibration damping is of appreciable importance. Vibration damping in rotor systems is dependent on a number of factors: rotor material and is structure, its bearings and their design, complexity of the rotor system assembly, etc. When setting up a rotor system dynamical model, the evaluation of all these factors is a significant consideration with a variable degree of uncertainty. The methods applied for damping vibration modelling are prone to be approximate (Predin, 1995; Wettergren and Olsson, 1996; Genta and Tonali, 1997; Ziliukas and Barauskas, 1997).

This paper covers the study of the effect of three most popular methods used for evaluation of vibration damping in the modelling of a low power (50 MW) steam turbine. The dynamical model of the steam turbine has been developed by FEM and theoretically and experimentally analysed.

2. Dynamical model

The steam turbine rotor and its two supports are divided into 23 finite elements (Junusas and Juzenas, 2000). The rotor wheels are modelled as disc type elements, each of them having four degrees of freedom (Fig. 1).

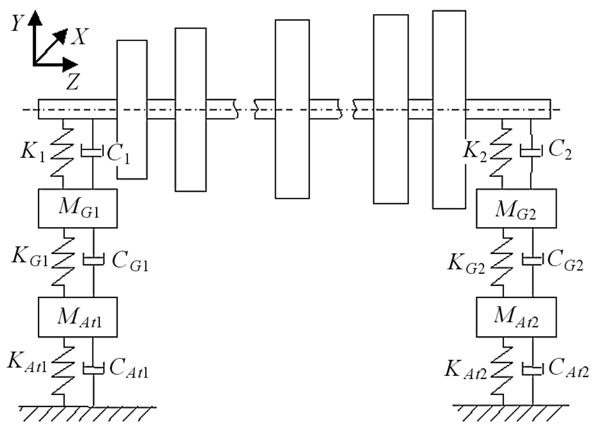

Fig. 1. Dynamical model of a steam turbine rotor: $K_{1,2}$and $C_{1,2}$ are coefficients of lubricant film stiffness and damping in the radial direction, respectively, $M_{G1,G2}$ – masses of bearings, $M_{At1,At2}$ – masses of rotor supports, $C_{G1,G2}$, $C_{At1,At2}$ – coefficients of journal bearings and supports damping, $K_{G1,G2}$, $K_{At1,At2}$ – coefficients of journal bearings and supports stiffness, respectively

The equation describing forced vibrations of the rotor are

$$(\mathbf{M} + \mathbf{M}')\ddot{\mathbf{U}} + (\omega\mathbf{G} + \mathbf{C})\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} + \mathbf{P}_k + \mathbf{P}_c \tag{2.1}$$

here $\mathbf{M}$ is the matrix of rotor masses; $\mathbf{M}'$ – matrix defining rotation of rotor cross-sections around the coordinate axes ($\mathbf{M}$ matrix describes masses of beam-shape elements, while $\mathbf{M}'$ allows one to evaluate the rotation of cross sections (Ziliukas and Barauskas, 1997; Junusas and Juzenas, 2000)); $\mathbf{G}$ – gyroscopic matrix; $\mathbf{C}$ – damping matrix; $\mathbf{K}$ – stiffness matrix; $\mathbf{U}$ – matrix of displacements of rotor elements; $\mathbf{F}$ – matrix of centrifugal forces acting on the rotor; $\mathbf{P}_k$, $\mathbf{P}_c$ – matrices of hydrodynamic forces; ω – angular speed of the rotor. The expressions of matrix elements are cumbersome, therefore they are not given here.

Hydrodynamic forces acting the journal bearings in the radial direction are (Panofko, 1960)

$$\begin{aligned} P_x &= -K_{xx}x - K_{xy}y - C_{xx}\dot{x} - C_{xy}\dot{y} \\ P_y &= -K_{yy}y - K_{yx}x - C_{yy}\dot{y} - C_{yx}\dot{x} \end{aligned} \tag{2.2}$$

here $K_{xx,xy,yx,yy} = (\mu\omega l/\psi^3)I_{1,2,3,4}$ and $C_{xx,xy,yx,yy} = (\mu l/\psi^3)I_{5,6,7,8}$ denote dynamic characteristics of bearings; l – length of the bearing; μ – dynamic viscosity of the lubricant; $\psi = \varDelta/R$ – relative gap; R – radius of the journal; $\varDelta$ – absolute gap; $I_1, \dots, I_8$ – dynamic coefficients of the bearings.

Having evaluated the dynamic characteristics the journal bearings, the matrices defining hydrodynamic forces can be set up: values of coefficients K_{qq} are inserted into the matrix $\mathbf{P}_K$, while values of coefficients C_{qq} – into the matrix $\mathbf{P}_C$ $(q = x, y)$.

3. Evaluation of vibration damping

The following three methods of evaluation of vibration damping are most frequently used.

In the first method, vibration damping is evaluated by coefficients defining internal and external friction. The effect of external friction forces (ambient effect) is evaluated as follows

$$P_{ext}(u) = c_{ext}\dot{u} \tag{3.1}$$

where $P_{ext}(u)$ is the resisting force of the external environment; $\dot{u}$ – speed of vibrations; c_{ext} – coefficient of vibration damping.

Vibration damping due to internal friction in the rotor (damping in the rotor material) is defined by introducing an additional damping coefficient whose variation depends on the vibration amplitude. It has been assumed that vibration damping due to internal friction is not very significant and does not dependend on the vibration frequency. Then, the mean damping coefficient defining the dissipated energy amount due to the hysteresis resulting in the rotor substance can be computed according to the following expression [7]

$$c_{int,mid} = \frac{\delta}{\pi}\sqrt{mk} \tag{3.2}$$

where δ denotes the logarithmic decrement of vibrations; m – mass of the rotor; k – stiffness of the rotor.

The logarithmic decrement depends on physical properties of the rotor material and internal stresses in it (on the vibration amplitude) (Pysarenko *et al.*, 1971)

$$\delta = \frac{2^{n+1}\upsilon(n-1)A^{n-1}}{n(n+1)} \tag{3.3}$$

where n and υ are coefficients defining the hysteresis loop of the material, which dependend on material physical properties; A is the amplitude of vibrations. The coefficients n and υ are derived as follows

$$n = 1 + \frac{\ln\frac{\delta_1}{\delta_2}}{\ln\frac{\sigma_{01}}{\sigma_{02}}} \qquad \upsilon = \frac{\delta_1(n+1)nE^{n-1}}{2^{n+1}(n-1)\sigma_{01}^{n-1}} \tag{3.4}$$

where δ_1, δ_2 denote decrements of vibration damping corresponding to the internal stresses at the beginning and end of the material hysteresis loop σ_{01} and σ_{02}; E is the elasticity modulus of the material. Evaluation of (3.3) and (3.4) results in the following damping matrix

$$\mathbf{C} = \frac{2^{n+1}\upsilon(n-1)A_0^{n-1}}{\pi n(n+1)}\boldsymbol{\Lambda} + \mathbf{C}_{ext} \tag{3.5}$$

where A_0^{n-1} is the amplitude of vibrations obtained during the previous computation; $\boldsymbol{\Lambda}$ – matrix describing damping properties of the rotor material. The elements of this matrix are $\sqrt{m_{ij}k_{ij}}$ ($m_{ij} > 0$, $k_{ij} > 0$ – mass and stiffness of finite elements, respectively). $\mathbf{C}_{ext}$ is the matrix of damping due to external friction.

It is obvious that damping is dependent on the amplitude of vibrations. It is of great importance for the case of flexible rotors when vibration amplitudes of different elements can markedly differ.

The second method involves the evaluation of damping by applying the so-called proportional damping matrix (Ziliukas and Barauskas, 1997), whose coefficients are computed according to thwe experimentally determined coefficient

$$Q' = \frac{\omega_r}{\Delta\omega} \tag{3.6}$$

where $\Delta\omega = (\omega_r - \omega_1) + (\omega_r - \omega_2)$; ω_r is the resonant frequency; ω_1 – frequency prior the resonance at which the vibration amplitude is equal to 0.71 of the corresponding to value the resonance amplitude; ω_2 – frequency post the resonance at which the vibration amplitude drops to the same level.

In this case, the damping matrix can be expressed as follows

$$\mathbf{C} = \frac{\omega_r}{Q'}\mathbf{M} \tag{3.7}$$

In the third method, damping is evaluated on the basis of a proportional damping matrix obtained while separately analysing free vibrations of the rotor (characteristics of free vibrations are found experimentally) (Pysarenko *et al.*, 1971) and completing the damping elements affected with both external and hydrodynamic forces

$$Q'' = \frac{\omega_0 \ln \frac{A_i}{A_{i+1}}}{2\pi} \tag{3.8}$$

where ω_0 is the frequency of rotor free vibrations. Then, the proportional matrix obtained analogically as in the second method is used (Timoshenko *et al.*, 1985)

$$\mathbf{C} = \frac{1}{Q''}\mathbf{M} \tag{3.9}$$

Defining damping by this method, it is assumed that it is linear and independent of the vibration amplitude and frequency. The developed damping matrix is additionally supplied with elements evaluating damping affected by hydrodynamic forces.

4. Experimental analysis

The experiments have been carried out at "ACHEMA", Ltd. Dynamics of the centrifugal compressor K-1290-121-1 with the steam turbine K-15-41-1 has been analysed (Fig. 2). Vibrations have been measured with the apparatus SYSTEM 2 (Pruftechnik AG).

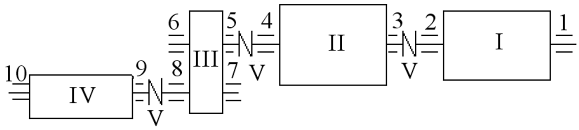

Fig. 2. Compressor K-1290-121-1 connected to steam turbine ST-K-15-41-1: I – steam turbine; II – low pressure frame; III – reducer; IV – high pressure frame; V – clutches between rotors; $1, 2, \ldots, 10$ – journal bearings

The experiments were made in two stages. At the first stage, the coefficients of the rotor vibration damping were set. To that end, vibration measuring sensors were installed on the supports and in the rotor interior. They recorded vibration damping curves when an impact pulse excited the rotor free vibrations (Fig. 3).

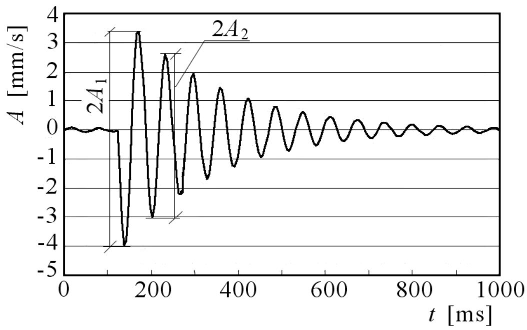

Fig. 3. Time history of free vibrations: A_1 and A_2 are amplitudes of adjacent free vibrations

The ratio of adjacent free oscillations have been evaluated, which enabled determination of the damping decrement. The vibration damping resulting from not only rotor material properties but from the rotor structural features as well (e.g. the effect of tightly assembled wheels) has been evaluated in that way.

At the second stage, amplitude-frequency characteristics of the rotor have been established. Vibrations were measured in standard operational conditions of the rotor as well as in transient regimes, i.e. during starting off and breaking.

The experimental research data have been used in computation of dynamical coefficients and in setting-up the vibration damping matrix.

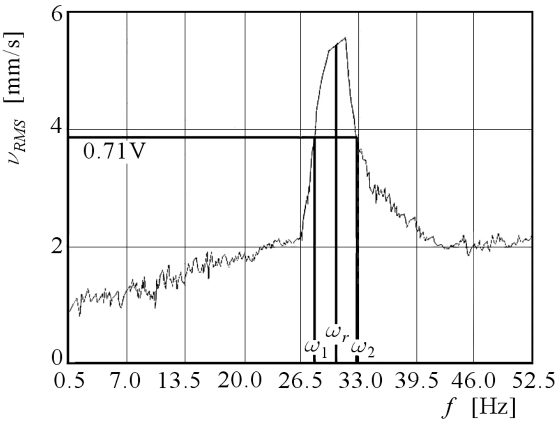

Fig. 4. Amplitude-frequency characteristic of vertical vibrations of the rotor first support

5. Modelling of rotor vibrations with different methods used for damping evaluation

To simulate the vibration level, the above given dynamic model has been used (2.1). Vibrations of the steam turbine supports were modelled in the vertical and horizontal directions by applying all the three above given methods for evaluation of damping.

To simplify the solution of the dynamic model, it has been assumed that rotor vibrations were excited by centrifugal forces whose action frequency coincided with the rotor rotation frequency, being consistent with the forces resulting from the rotor residual unbalance and its deflection. In computations, it has been assumed that the rotor residual unbalance was at the highest allowable level, while the deflection was dependent on the rotor rotation frequency. The modelling has been realised within the range of rotation frequencies from 500 rev/min up to 6500 rev/min, i.e. rotor vibrations have been simulated while passing the first critical speed and in the case of significantly exceeded rotation speeds (3100-3300rev/min). The obtained typical curves for vertical vibrations of the rotor first support are given in Fig. 5.

The application of such a dynamic model makes it possible to obtain vibration characteristics of any rotor element. They may help in the evaluation of the rotor dynamical state when modelling the effect of potential rotor defects on the vibration level.

The comparison of the curves in Fig. 5 among themselves and with the curve obtained experimentally (Fig. 4) indicates that different method of dam-

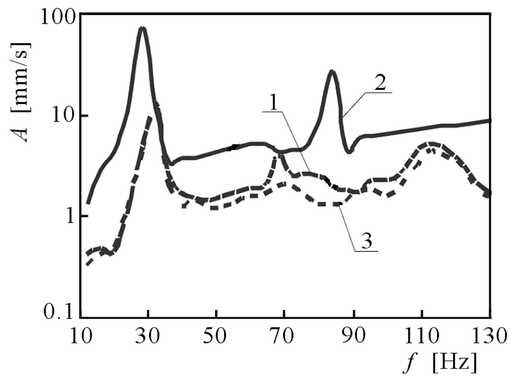

Fig. 5. Amplitude-frequency characteristics of vibrations of the first support of the steam turbine obtained by application of three different methods used for damping evaluation

ping evaluation yield different results even under the same excitation and the same remaining conditions. The results nearest to the experimental curve are obtained through application of the first and third method, whereas, the evaluation according to the second method provides a significantly higher vibration level of rotor supports than the level found experimentally. The greatest difference is noticed when the vibration level passes through the first critical speed and beyond it. The explanation may be double. Firstly, by applying this method, the vibration measurement errors exert great influence on shape of amplitude-frequency characteristics during turbine start-off and breaking, because to prevent the turbine damage resulting from high vibration levels, the critical rotation speed is passed over very fast. Moreover, when this method is applied, the damping does not dependend either on vibration amplitude or rotation speed, i.e. the damping is constant in the whole range of modelling. Whereas, when applying the first and third evaluation approach, the obtained results do not depend on the experimental errors, or they can be eliminated in the case of the third approach, many more experiments can be made.

6. Conclusions

- While modelling the rotor whose speed is between the first and second critical speed and its simulating dynamics, the preference is to be given to the first and third evaluation methods for vibration damping. In these two approaches, the amplitude-frequency characteristics of the rotor system determined theoretically and experimentally are very close.

- Applying the third method, the obtained dynamical model is simpler. It allows one to avoid the errors related to the rotor design, because when setting up the damping characteristics it is aprequisite to evaluate the effect of the operation of tighty assembled rotor wheels on the shaft, the presence of cavities in the rotor, etc.

- The comparison of experimental and theoretical research results in application of different methods for damping evaluation indicates that the dynamical model of the rotor system is consistent with the real system.

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Szacowanie poziomu tłumienia drgań w modelowaniu dynamiki podatnego wirnika

Streszczenie

Wybór metody szacowania poziomu tłumienia drgań jest ważną kwestią w analizie dynamiki podatnych wirników pracujących w obszarze pomiędzy pierwszą, a drugą krytyczną Prędkością wirowania oraz w modelowaniu stanu dynamicznego w ogóle. Zgodność wyników badań teoretycznych i eksperymentalnych zależy od tego wyboru. W pracy zaprezentowano trzy metody szacowania poziomu tłumienia drgań w układach wirnikowych. W przedstawionym zagadnieniu modelowania dynamiki wirników zastosowano każdą z nich. Otrzymane rezultaty rozważań teoretycznych porównano z wynikami badań doświadczalnych.

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