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WEAR PATTERNS AND LAWS OF WEAR – A REVIEW

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Wear is a process of gradual removal of a material from surfaces of solids subject to contact and sliding. Damages of contact surfaces are results of wear. They can have various patterns (abrasion, fatigue, ploughing, corrugation, erosion and cavitation). The results of abrasive wear are identified as irreversible changes in body contours and as evolutions of gaps between contacting solids. The wear depth profile of a surface is a useful measure of the removed material. The definition of the gap between contacting bodies takes into account deformations of bodies and evolutions of wear profiles. The wear depth can be estimated with the aid of wear laws. Derived in this study, constitutive equations of anisotropic wear are extensions of the Archard law of wear. The equations describe abrasion of materials with microstructures. The illustrative example demonstrates calculations of the abraded mass and temperatures in pin-on-disc test rig.

Key words: contact mechanics, wear, friction, constitutive equations

1. Introduction

Everything that man makes wears out, usually, as a result of sliding between contacting and rubbing solids. Wear of materials is an every-day experience and has been observed and studied for a very long time. A large body of empirical data has been collected and some phenomenological models have been developed. The first experimental investigations of wear have been carried out by Hatchett (1803) and Rennie (1829). In our days, experimental tests of wear intensities of materials belong to standard measurements of mechanical properties of solids. Nevertheless, it is difficult to predict and to control wear of rubbing elements.

Contacting elements of structures are very common in technology. Many mechanical devices and mechanisms are constructed with the aid of component parts contacting one with another. Contact regions occur between tools and workpieces in machining processes. Loads, motions and heat are transmitted through the contacts of structures. Friction and wear accompany any sliding contact. It has been agreed that wear cannot be totally prevented. In machine technology, wear is an equally important reason of damage of materials as fracture, fatigue and corrosion.

The modelling of friction and wear is an important engineering problem. In the process of design of machine elements and tools operating in contact conditions, engineers need to know areas of contact, contact stresses, and they need to predict wear of rubbing elements. Friction, wear and contact problems are subjects of numerous experimental and theoretical studies. The very complex nature of tribological phenomena is a reason that many problems of contact mechanics are still not solved. The modelling of friction and wear can be carried out not only with the aid of laboratory tests but using also mathematical models and computer simulations. Due to computer simulation techniques, physical and mechanical phenomena in real objects can be reconstructed with a high degree of precision.

There is still a need for efficient and reliable computational procedures of contact problems taking into account complex phenomena of friction and wear. On the one hand, the accuracy of numerical procedures should be improved. On the other hand, new models of friction and wear should be included in numerical calculations. Contemporary numerical codes do not discuss how to calculate and how to predict wear. Better understanding and control of wear in materials can be done with the aid of new models of wear.

The first trials on numerical analysis of wearing out solids were given by Grib (1982), Hugnell *et al.* (1996), Strömberg (1997, 1999), Szefer (1998), Christensen *et al.* (1998), Agelet de Saracibar and Chiumenti (1999), Franklin *et al.* (2001), Ko *et al.* (2002), Shillor *et al.* (2003), McColl *et al.* (2004), Shillor *et al.* (2004), Telliskivi (2004), Kim *et al.* (2005). For instance, Szefer (1998) included in his numerical analysis the so called third body. The evolution of wear gaps in fretting problems was studied numerically by Strömberg (1997, 1999). The finite element method was applied. Numerical simulations of wear shapes due to pitting phenomena for various operating conditions have been investigated by Glodež *et al.* (1998, 1999). They made use of the fracture mechanics. In the subject literature, some authors calculate propagations of surface or near-surface fatigue cracks induced by contact stresses, see Suh (1973), Rosenfield (1980), Sin and Suh (1984), Bogdański *et al.* (1996), Dubourg *et al.*

(2003). An original analytical approach to wear is preferred by the Russian school, see Galin and Goryacheva (1980), Goryacheva and Dobychin (1988), Goryacheva (1998) and Põdra and Anderson (1999). There are various computational approaches to wear at this stage of investigations (finite elements, boundary elements, fracture mechanics, molecular dynamics).

A common way to solve nonlinear contact problems is to adopt step-by-step incremental procedures. Following a loading history, contact constraints are satisfied at any increment step with the aid of iteration techniques. Nowadays, formulations of contact problems (i.e. the mechanical problems with constraints on solution variables) are more refined by applications of the Lagrange multiplier method, penalty method and their generalizations (e.g. perturbed and augmented Lagrangian methods, mathematical programming techniques).

This study is devoted to the modelling of wear of materials. Wear patterns and laws of wear for abrasion are the main aims of the paper. Wear laws are classified as follows: empirical, phenomenological, based on failure mechanisms of materials. Archard laws of wear is extended in the study and applied in illustrative calculations.

2. Experimental observations of wear and surface damage

There are numerous advantages of contacts, e.g. transmission of loads and motions, dissipation of energy, damping of vibrations, etc. The friction process of solids operating in contact conditions always involves frictional heat generation and wear of their surfaces, e.g. wear of machine component parts (gears, bearings, brakes, clutches), wear of rubber tires, wear od shoes, wear of clothes, etc. In general, machine component parts can fail by breakage or by wear, the former being spectacular and sudden, the latter inconspicuous yet insidious. Particularly high wear can occur in mechanisms which operate in conditions of dry friction or in marginally lubricated conditions (so called mixed friction).

Wear is a process of gradual removal of a material from surfaces of solids. The detached material becomes loose wear debris. Nowadays, wear particles are the subject of intensive studies, see Godet (1984), Williams (2005) and Zmitrowicz (2005b). Two dimensional continuum mechanics-based models can describe a thin layer of the wear particles between contacting bodies, see Zmitrowicz (1987, 2004, 2005b) and Szwabowicz (1998). Next results of wear can be identified macroscopically as the generation of worn contours of the bodies (so called wear profiles) and as the increase of clearance spaces (gaps) between the contacting solids.

Wear of materials is the result of many mechanical, physical and chemical phenomena. Several types of wear have been recognized, e.g. abrasive, adhesive, fatigue, fretting, erosion, oxidation, corrosion, see Bahadur (1978), Rabinowicz (1995), Ludema (1996) and Kato (2002). Wear of solids is usually treated as the mechanical process. However, oxidation, corrosion and other chemical processes are exceptions of this rule. The abrasive wear and the contact fatigue are the most important from the technological point of view. It was estimated that the total wear of machine elements can be identified in 80-90% as abrasion and in 8% as fatigue wear. Contributions of other types of wear are small.

Most of wear observations are carried out indirectly (post factum). The rubbing process must be stopped, the wearing out elements must be disassembled, and after that the effects of the wear process can be observed. Weighting is the simplest way of detecting wear. It gives the total amount of the removed mass, but the distribution of the wear depth in the contact surface is unknown.

In order to obtain qualitative information on wear, after opening the contact, visual inspection of the worn surface and wear debris is very often used. The easiest method of the visual inspection of surface damage is to photograph the surface. Furthermore, the worn surfaces can be examined with the aid of optical microscopes and with scanning and transmission electron microscopes. The surface examination by microscope provides a two-dimensional view. To determine how much a material had been removed, surface topographic measurements must be preformed with the aid of a surface profilometer. A quantitative technique which monitors the worn surface is based on surface profiles perpendicular to the wear track. The depth of the removed material from rubbing surfaces can be obtained by surface topographic measurements, and the amount of the material worn away can be estimated. The examination of the wear profile history is a valuable indicator of the nature of the wear process. In every-day life, one can easy observe wear profiles and wear debris during the abrasion, e.g.: a pencil drawing marks on paper, a piece of chalk writing on a blackboard, a rubber eraser rubbing out pencil marks on paper, etc.

Radioactive methods and radionuclide methods are alternative direct methods of measuring wear. Sliding elements or their surfaces are made radioactive. The following quantities are measured: (a) amount of wear debris in a lubricant if the rubbing elements are operated in the presence of the circulating lubricant, (b) amount of radioactive wear debris transferred to the interface or to a non-radioactive surface. It is studied by measuring the radioactivity of the lubricant or the irradiated sliding element (Rabinowicz, 1995).

Wear, frictional heat, fracture and fatigue are main factors which govern machine life time. From the point of view of machine technology, the generation and the increase of clearance spaces in fitted exactly rubbing component parts (e.g. in sliding and rolling bearings, between teeth of gears, etc.) are important consequences of wear. Consistently, numerous disadvantageous effects follow, i.e.: increase of friction, dynamical loads, mechanical shocks, vibrations, irregular motions, noise, fracture or break of component parts in the case of great reduction of their worn cross-sections, decrease of quality of rubbing surfaces, loss of wear-resistance properties by near-surface materials, heating of elements, additional contamination, machine failure, etc. Numerous machine component parts must be taken out of service not due to failure caused by an exceed of the limit stress, but due to wear manifested in the removal of the material. Wear is an important topic from the economical point of view because it represents one of ways in which material objects lose their usefulness.

Wear cannot be eliminated completely, but it can be reduced. The simplest methods of reduction of friction and wear are as follows: lubrication, formation of sufficiently smooth surfaces, modification of near-surface materials of rubbing components, correct assembling and exploitation of fitted component parts. Friction and wear can be reduced by an optimal choice of structural, kinematical and material parameters of mechanical systems realized by: correct choice of shapes of rubbing elements, forming of loads and motions in adequate limits, correct choice of sliding materials.

Shape and size changes of machine elements are very important consequences of wear. However, in sliding electrical contacts, small wear of the machine elements can be compensated with the aid of a spring pressing the rubbing element to the counterpart. Abradable turbine seals are constructed and assembled with the so called negative clearance space. During turbine operation, the seals are submitted to the running-in process, and finally they work as leak-proof seals with a nearly zero clearance. Brake pads, for instance, can lose up to 3/4 of their original thickness and still give perfect service.

Overall wear consequences are negative, however one should remember advantages following from running-in, braking-in and from many methods of producing a surface on a manufactured object exploiting the abrasion phenomenon (e.g. finishing). Thanks to abrasion, a pencil, a crayon and a piece of chalk are useful in every-day life. A rubber eraser removes carbon particles from the pencil transferred to the paper by the wear mechanism (Rabinowicz, 1995).

3. Patterns of wear

3.1. Profiles of worn machine component parts (abrasion patterns)

In the process of wear, a material is removed from surfaces of solids, and dimensions of worn bodies are gradually reduced. The amount of the removed material can be estimated with the aid of the wear profile. The wear depth defines the removed material at the given point of the contact area. The wear profile is a function of the wear depth with respect to positions at the contact region. The wear depth and the wear profile describe irreversible changes in shapes and sizes of the worn bodies, and they are useful measures of wear.

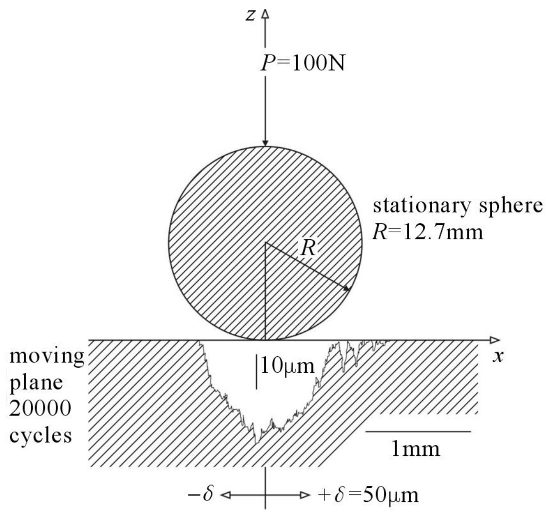

Fig. 1. The wear profile of an oscillating surface in contact with a stationary sphere after 20 000 cycles, see Fouvry *et al.* (2001). This is the wear profile in the Hertzian contact

Figure 1 shows the measured wear profile of an oscillating surface in contact with a stationary sphere after 20000 cycles (Fouvry *et al.*, 2001). This is a wear profile in Hertzian contact, i.e. contact between an elastic sphere and a plane. In the Hertzian contact, the normal pressure is semi-elipsoidally distributed in the circular contact area (Johnson, 1985). Therefore, the measured maximum wear depth is at the center of the contact zone, and it is equal to several tenth micrometers. Figure 1 shows a typical fretting process, i.e. the wear process which takes place during oscillatory sliding of small amplitudes. Notice, that motion producing fretting is very small (per cycle, 50 micrometers). In mechanical systems, oscillatory tangential displacements of small amplitudes arise from vibrations or cyclic stressing of one of the contacting bodies. Fretting contacts occur in mechanical joints, supports, screw joints, steel wires, bearings, gears, turbine blade roots, see Hills and Nowel (1994) and Arakere and Swanson (2001). Damaged surfaces in the fretting process were observed by microscopy in Vingsbo *et al.* (1990).

Weiergräber (1983) measured the wear profile of an anvil face in a half hot metal forming process after pressing 5000 elements. It is a good example of the wear profile in Nonhertzian contact, i.e. contact between a rectangular punch and a half-plane. In the Nonhertzian contact, singularities in the normal pressure distribution occur at edges of the contact zone (Johnson, 1985). The maximum wear depth, near the edges of the contact zone, was equal to several tenth micrometers. In points of the maximum wear depth, the normal pressure was high and the sliding velocity was large. Marginally lubricated conditions were in the contact area.

Measurements of the wear profiles of rails and rail wheels with respect to the distance travelled were reported in the subject literature, see Moore (1975), Seyboth (1987), Sato (1991), Olofsson and Telliskivi (2003). Sato (1991) presented consequent changes in transverse profiles of worn rail heads and worn rail wheels on curved railway tracks after wheel run 1600, 27000, 79000 and 200000 km investigated on Tokaido Shin kansen (Japan). The maximum wear depth of the worn rail was equal to several tenth millimeters. Sometimes the rail head can be designed to have such a worn profile which coincides with the shape of a new wheel (Sato, 1991).

Stresses in some points of the worn rail cross-section were measured in an experimental set-up and reported by Seyboth (1987). An increase of stresses in the head and at the foot of the worn rail cross-section was observed in comparison with the unworn rail. The increase of stresses depended on the wear depth. A great increase of the stresses was measured in the point near the contact area (i.e. in the rail head, up to 149%, when the wear depth was 28 millimeters) and a less increase in the points far from the contact (i.e. in the rail foot, up to 28%, when the wear depth was 28 millimeters). Ghonem and Kolousek (1984) presented a model describing wear of rails as a function of the angle that a wheelset forms when deviating from the radial direction in relation to the track (so called angle of attack).

Measurements of wear profiles of tools used in various machining processes have been reported in the literature. König (1985) presented the formation and development of a crater wear profile on a worn cutting tool top surface during successive stages of increasing cutting time, from 4 to 35 minutes. An average life time of the cutting edge usually vary from a few to several tens minutes depending on the cutting tool and the cutting process. Exceptionally, it is equal to several hundred minutes. The maximum wear depth was equal to 0.2 millimeters (König, 1985). Lueg (1950) showed the measured profiles of worn drawing die inserts of the wire drawing machine in dependence on the slide length (22650 m and 45000 m). Two types of the die insert were taken into account, cylindrical and non-cylindrical.

Occasionally, wear profiles of other machine component parts are presented in the literature. For instance, changes in teeth profiles of gears caused by abrasive wear were measured after some number of cycles. Extensive wear occurs in large-size open toothed gears operating in driving mechanisms applied in cement plants and in steelworks; there are emitted great amounts of dusts which have abrasive properties. Usually toothed gears are lubricated. Wear profiles of engine components have been measured, e.g. at the contact between piston rings, piston and cylinder liner. Sometimes, it becomes important to study worn surfaces of journal bearings. Changes in the journal bearing geometry due to wear strongly affect the bearing performance especially at high operating speeds, e.g. in high speed turbomachinery.

3.2. Surface fatigue

Certain forms of surface damage do not occur slowly and continuously but may suddenly cause large failure of rubbing surfaces. For instance, initiation and growth of surface and subsurface cracks can lead to the formation of surface defects by large particles of the material removed from the surface, see Suh (1973), Rosenfield (1980), Ohmae and Tsukizoe (1980), Sin and Suh (1984), Bogdański *et al.* (1996), Glodež *et al.* (1998, 1999), Dubourg *et al.* (2003). Surface fatigue wear was observed during repeated loading and unloading cycles (e.g. during repeated sliding or rolling).

3.3. Ploughing of surfaces

From the point of view of machine technology, wear is also thought as a decrease of quality of rubbing surfaces, i.e. increase of surface roughness, fogging of the surface, generation of scratches and grooves, etc. Surfaces can be ploughed by wear particles, hard particles entrapped from the environment and by hard asperities of the counterface. Irreversible plastic deformations in the surfaces are the main results of ploughing. When the surfaces are ploughed with evidence of plastic flow of a material, then scratches and grooves are generated on the surface. Surfaces ploughed (scratched) by a cone-shaped indenter and hemispherical-shaped pin were observed with the aid of microscopy by Sakamoto *et al.* (1977), Kayaba *et al.* (1986) and Magnée (1993).

3.4. Surface corrugation

Changes in shapes and sizes are observed in transverse and longitudinal profiles of worn rails. Rail corrugations are specific type of wear, since rail corrugations are variations in the longitudinal profile of a rail. They are typically periodic, characterized by long (100-400 millimeters) and short (25-80 millimeters) pitch wavelengths, see Sato *et al.* (2002), Grassie (2005), Meehan *et al.* (2005). The same phenomenon was observed for rail wheels. The rail corrugations induce vibrations in trains as they pass over them, see Knothe and Grassie (1999), Bogacz and Kowalska (2001), Meehan *et al.* (2005). Sometimes re-griding of the rail and wheel profiles may be necessary to restore the initial conditions of their co-operation.

3.5. Surface damage due to erosion and cavitation

Wear of solid surfaces as a result of bombardment by solid particles or liquid drops is an important form of surface damage. In the case of erosion, hard solid particles moving with some velocity impact the solid surface at a certain angle, then slide along the surface and finally bounce off. Moving along the surface for a small distance, these particles cut scratches and grooves, and produce wear debris. There are high-speed and low-speed processes. Therefore, surface damage and material removal may occur by the mechanism of crack initiation and crack growth. In the case of erosive wear, it is assumed that a volume of the removed material is proportional to a high power of the particle velocity (e.g. fifth, fourth, second powers).

Surface damage may also occur due to cavitation of liquids. Cavitation takes place in fluid flow systems in which negative pressure regions exist. Bubbles filled with vapor derived from the moving liquid may be formed near the solid surface. When the bubbles collapse, a jet forms, impacts the surface and damages it (Rabinowicz, 1995).

4. Wear depth predictions

In the tribology literature, measures of wear have been formulated with respect to changes of the following quantities: (a) mass of the removed material from the solid, (b) volume of the removed material, (c) reduced dimensions of the body. The measures of wear have non-zero values as long as an observable amount of the material is removed. Due to this, usually, but not always, a significant period of the sliding time (or a great number of cycles) must be taken into account.

Barber (see the paper by Kennedy and Ling; 1974) suggested that the wear rate (i.e. the depth of the material removed per time unit) is a prescribed function of the normal pressure, sliding velocity and temperature. The form of the function for a given material could be determined by laboratory wear tests. Galin and Goryacheva (1980) recognized that besides elastic deformations of the body contacting with the rigid foundation, irreversible changes of the shape of the body take place in the wear process. In the case of abrasive wear, the amount of the removed material is proportional to the work of the friction force.

Let a system of two bodies A and B be a model of rubbing and wearing out solids. It is assumed that velocities $\boldsymbol{v}_c^+$ and $\boldsymbol{v}_c^-$ describe motions of the wearing out boundaries $(\Omega_{cA}, \Omega_{cB})$ due to two reasons: deformations of the bodies $(\boldsymbol{v}_A, \boldsymbol{v}_B)$ and the wear process $(\boldsymbol{v}^+, \boldsymbol{v}^-)$. Notice, $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ are material velocities. Two quantities m_A and m_B describe rates of the mass flowing out from the bodies A and B due to the wear process

$$
\begin{aligned}
m_A &= \rho_A(\boldsymbol{v}_A - \boldsymbol{v}_c^+)\cdot\boldsymbol{n}^+ = -\rho_A\boldsymbol{v}^+\cdot\boldsymbol{n}^+ \\
m_B &= \rho_B(\boldsymbol{v}_B - \boldsymbol{v}_c^-)\cdot\boldsymbol{n}^- = -\rho_B\boldsymbol{v}^-\cdot\boldsymbol{n}^-
\end{aligned}
\tag{4.1}
$$

where, ρ_A and ρ_B are mass densities, $\boldsymbol{n}^+$ and $\boldsymbol{n}^-$ are unit vectors normal to the boundaries Ω_{cA} and Ω_{cB}. The following velocities normal to the boundaries $(\Omega_{cA}, \Omega_{cB})$ define the wear process

$$
\boldsymbol{v}^+\cdot\boldsymbol{n}^+ = v^+ \qquad\qquad \boldsymbol{v}^-\cdot\boldsymbol{n}^- = v^- \tag{4.2}
$$

and they are dependent variables of the constitutive equations of wear. Galin and Goryacheva (1980) assumed that the velocity of the wearing out boundary of the solid body is a function of the normal pressure and the sliding velocity. According to Rabinowicz (1995), v^+ defines the velocity at which the surface Ω_{cA} of the body A "travels" into other surface because of its wearing away. Kinematics of the wearing out boundary Ω_{cA} of the solid body A is shown in detail in Fig. 2. The wear profiles are drawn for successive stages of the increasing sliding time.

Kinematics of the two contacting bodies A and B can be described in Euler description by displacement functions $\boldsymbol{u}_A(\boldsymbol{x}_A, \tau)$ and $\boldsymbol{u}_B(\boldsymbol{x}_B, \tau)$, where τ is time. Velocities of displacements and the sliding velocity are given by

$$
\begin{aligned}
&\boldsymbol{v}_A = \dot{\boldsymbol{u}}_A \qquad\qquad \boldsymbol{v}_B = \dot{\boldsymbol{u}}_B \\
&\boldsymbol{V}_{AB} = (\mathbf{1} - \boldsymbol{n}^+\otimes\boldsymbol{n}^+)(\dot{\boldsymbol{u}}_A - \dot{\boldsymbol{u}}_B)
\end{aligned}
\tag{4.3}
$$

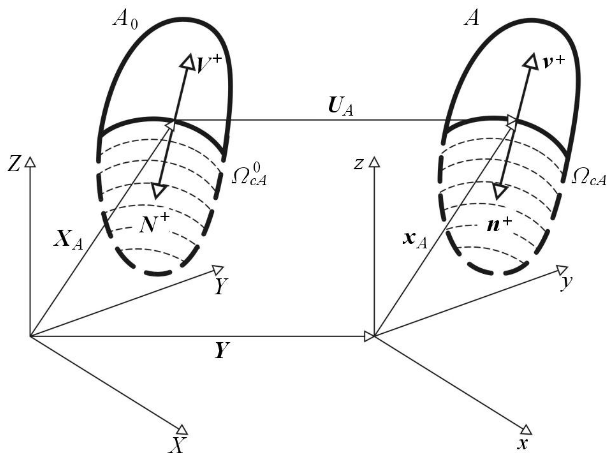

Fig. 2. Wear profiles – kinematics of a wearing out boundary of a solid body A. In Lagrange description: $\boldsymbol{X}_A$ – radius vector of the wearing out boundary of the body A_0, $\boldsymbol{N}^+$ – unit vector normal to the boundary at the contact area Ω^0_{cA}, $\boldsymbol{V}^+$ – wear velocity. In Euler description: $\boldsymbol{x}_A$ – radius vector, $\boldsymbol{n}^+$ – unit normal vector, $\boldsymbol{v}^+$ – wear velocity

In Fig. 2, kinematical quantities describing the wear profiles in Lagrange and Euler descriptions are shown schematically. Notations in Lagrange description are as follows: $\boldsymbol{X}_A$ is the radius vector of the wearing out boundary Ω^0_{cA} of the body A_0, $\boldsymbol{N}^+$ is the unit vector normal to the boundary at the contact area Ω^0_{cA}, $\boldsymbol{V}^+$ is the wear velocity. In Euler description, notations are following: $\boldsymbol{x}_A$ is the radius vector, $\boldsymbol{n}^+$ is the unit normal vector, $\boldsymbol{v}^+$ is the wear velocity.

The total clearance gap between two deformable bodies (d_n) is equal to the sum of the initial gap (g_n), the elastic deformations (u_n^A, u_n^B) and the depth worn away (u_n^+, u_n^-), i.e.

$$d_n = g_n - u_n^A + u_n^+ + u_n^B - u_n^- \qquad \text{on } \Omega_{cA} \cup \Omega_{cB} \tag{4.4}$$

where

$$\begin{aligned} \boldsymbol{u}_n^A(\boldsymbol{x}_A, \tau) &= (\boldsymbol{n}^+ \otimes \boldsymbol{n}^+)\boldsymbol{u}_A \equiv u_n^A \boldsymbol{n}^+ && \boldsymbol{x}_A \in \Omega_{cA} \\ \boldsymbol{u}_n^B(\boldsymbol{x}_B, \tau) &= (\boldsymbol{n}^- \otimes \boldsymbol{n}^-)\boldsymbol{u}_B \equiv u_n^B \boldsymbol{n}^- && \boldsymbol{x}_B \in \Omega_{cB} \end{aligned} \tag{4.5}$$

Notice, d_n is the distance between two points coming into contact. The depth of the material removed in the time interval $\langle 0, t\rangle$ is defined by the following integrals

$$u_n^+ = \int_0^t v^+(\boldsymbol{x}_A, \tau)\, d\tau \qquad \boldsymbol{x}_A \in \Omega_{cA}$$
$$u_n^- = \int_0^t v^-(\boldsymbol{x}_B, \tau)\, d\tau \qquad \boldsymbol{x}_B \in \Omega_{cB} \tag{4.6}$$

Figure 3 shows principal quantities which define the wear profile and the gap between the body A and the rigid foundation. In Table 1, definitions of the radius vector, the wear velocity and the wear depth in Euler and Lagrange descriptions are given.

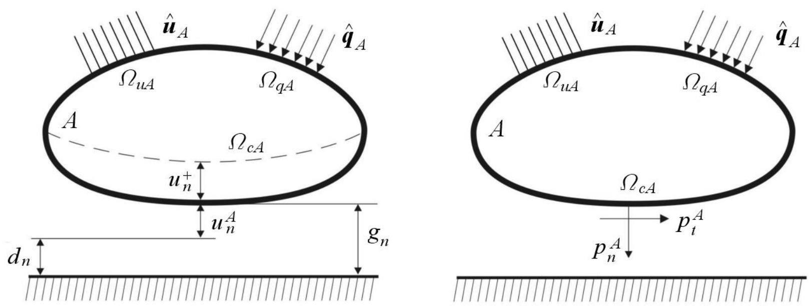

Fig. 3. Quantities which define the contact: wear depth (u_n^+), gap between contacting bodies (d_n), initial gap (g_n) and normal and tangential contact forces (p_n^A, p_t^A)

Table 1. Definitions of the radius vector, wear velocity and wear depth in Euler and Lagrange descriptions

Quantity	Euler description	Lagrange description
Radius vector of wearing out boundary	$\boldsymbol{X}^+ = \boldsymbol{X}^+(\boldsymbol{x}_A, \tau)$ $\boldsymbol{x}_A \in \Omega_{cA}$ $\tau \in \langle 0, t\rangle$	$\boldsymbol{x}^+ = \boldsymbol{x}^+(\boldsymbol{X}_A, \tau)$ $\boldsymbol{X}_A \in \Omega_{cA}^0$ $\tau \in \langle 0, t\rangle$
Wear velocity	$\boldsymbol{v}^+(\boldsymbol{x}_A, \tau) =$ $= \dfrac{\partial \boldsymbol{x}^+(\boldsymbol{X}_A, \tau)}{\partial \tau}\Big\vert_{\boldsymbol{X}_A = \boldsymbol{X}_A(\boldsymbol{x}_A, \tau)}$ $\boldsymbol{v}^+ \cdot \boldsymbol{n}^+ = v^+$	$\boldsymbol{V}^+(\boldsymbol{X}_A, \tau) =$ $= \dfrac{\partial \boldsymbol{x}^+(\boldsymbol{X}_A, \tau)}{\partial \tau}\Big\vert_{\boldsymbol{X}_A = \mathrm{const}}$ $\boldsymbol{V}^+ \cdot \boldsymbol{N}^+ = V^+$
Depth worn away	$u_n^+(\boldsymbol{x}_A, \tau) =$ $= \int_0^t v^+(\boldsymbol{x}_A, \tau)\, d\tau$	$U_n^+(\boldsymbol{X}_A, \tau) =$ $= \int_0^t V^+(\boldsymbol{X}_A, \tau)\, d\tau$

In the case of wearing out solids, we have to solve the problem in the region $(A \cup B)$ whose boundary $(\Omega_{cA} \cup \Omega_{cB})$ is partial unknown in advance. From the point of view of continuum mechanics, successive positions of the wearing out boundary Ω_{cA} of the solid A depend on the solution to the contact problem. The shape and size of the worn body A should be calculated together with the fields of stresses, strains and temperatures. It means that the evolution of the wear profile can be modelled by a material continuum with a moving boundary, see Alts and Hutter (1988, 1989). Usually, moving boundary problems (so called Stefan problems) are solved using a mapping of the continuum to a fixed domain (fixed-domain mapping) at any step from the given time interval. For this reason, the pull-back technique can be used.

In the finite element analysis of contact problems, a finite element mesh in the material continuum near the contact surface should be designed adequately with respect to the maximum wear depth. For instance, dimensions of the finite element can be equal to several micrometers. In the wear process, dimensions of the wearing out body are gradually reduced. If the calculated wear depth is equal to the dimension of the finite element, then this element is removed from the mesh. Further analysis is conducted for the mesh with the reduced number of the finite elements. This is an updating process of the contact surface geometry.

The first trials on theoretical and numerical calculations of wear profiles have been carried out by Galin and Goryacheva (1980), Grib (1982), Goryacheva and Dobychin (1988), Hugnell *et al.* (1996), Strömberg (1997, 1999), Goryacheva (1998), Christensen *et al.* (1998), Agelet de Saracibar and Chiumenti (1999), Franklin *et al.* (2001), Ko *et al.* (2002), Shillor *et al.* (2003), McColl *et al.* (2004), Shillor *et al.* (2004), Telliskivi (2004), Kim *et al.* (2005). Paczelt and Mróz (2005) discussed optimization problems with respect to the contact surface geometry generated by wear.

Analysis of the contact problem generally requires the determination of stresses and strains within contacting bodies, together with information regarding the distribution of displacements, velocities and stresses at the contact region. In most cases, two following contact constraints have to be satisfied. The contacting bodies cannot penetrate each other

$$d_n \geqslant 0 \qquad \text{on } \Omega_{cA} \tag{4.7}$$

and they are separated or pressed on each other (no tensile normal forces)

$$p_n^A \leqslant 0 \qquad \text{on } \Omega_{cA} \tag{4.8}$$

Figure 3 shows components of the contact force $(\boldsymbol{p}_A)$, i.e. the normal pressure force $(\boldsymbol{p}_n^A)$ and the tangential traction $(\boldsymbol{p}_t^A)$ given by

$$
\begin{aligned}
&\boldsymbol{p}_A = \boldsymbol{\sigma}_A \boldsymbol{n}^+ \qquad \text{on } \Omega_{cA} \\
&\boldsymbol{p}_n^A = (\boldsymbol{n}^+ \otimes \boldsymbol{n}^+)(\boldsymbol{\sigma}_A \boldsymbol{n}^+) \equiv p_n^A \boldsymbol{n}^+ \\
&\boldsymbol{p}_t^A = (\mathbf{1} - \boldsymbol{n}^+ \otimes \boldsymbol{n}^+)(\boldsymbol{\sigma}_A \boldsymbol{n}^+) \qquad \text{(stick)} \\
&|\boldsymbol{p}_t^A| \leqslant \mu_A |p_n^A| \qquad \text{(stick)} \\
&\boldsymbol{p}_t^A = -\mu_A |p_n^A| \frac{\boldsymbol{V}_{AB}}{|\boldsymbol{V}_{AB}|} \qquad \text{(slip)}
\end{aligned}
\tag{4.9}
$$

where, μ_A is the friction coefficient, $\boldsymbol{\sigma}_A$, is the Cauchy stress tensor in the body A. Material constitutive relations of the elastic body A are as follows

$$
\begin{aligned}
&\boldsymbol{\sigma}_A = \mathbf{E}_A \boldsymbol{\varepsilon}_A \\
&\boldsymbol{\varepsilon}_A(\boldsymbol{u}_A) = \frac{1}{2}(\operatorname{grad} \boldsymbol{u}_A + \operatorname{grad}^\top \boldsymbol{u}_A)
\end{aligned}
\tag{4.10}
$$

where, $\mathbf{E}_A$ is the tensor of elasticity, $\boldsymbol{\varepsilon}_A$ is the (right) Cauchy-Green strain tensor. The contact constraints, are in the form of inequalities (4.7) and (4.8), and they restrict admissible displacements $(\boldsymbol{u}_A)$ and stresses $(\boldsymbol{\sigma}_A)$.

The boundary conditions out of the contact region are following

$$
\begin{aligned}
&\boldsymbol{u}_A = \widehat{\boldsymbol{u}}_A \qquad && \text{on } \Omega_{uA} \\
&\boldsymbol{\sigma}_A \boldsymbol{n}_A = \widehat{\boldsymbol{q}}_A \qquad && \text{on } \Omega_{qA}
\end{aligned}
\tag{4.11}
$$

where, $\boldsymbol{n}_A$ is the unit vector normal to the body boundary and directed outward the contact region, $\widehat{\boldsymbol{u}}_A$ and $\widehat{\boldsymbol{q}}_A$ are prescribed displacements and loads (see Fig. 3).

The contact zone (Ω_{cA}) and the contact forces $(\boldsymbol{p}_A)$ are usually unknown in advance, and they depend on the solution to the contact problem. Contemporary methods used in contact mechanics are mainly based on computational techniques. First conferences on computational methods in contact mechanics were organized by Raous (1988) and Aliabadi and Brebbia (1993). Most contributions in (Raous, 1988) were devoted to applications of finite elements to contact mechanics. Studies presented in (Aliabadi and Brebbia, 1993) dealt with boundary elements used to solve contact problems. First books on computational contact mechanics were published by Kikuchi and Oden (1988), Zhong (1993), Laursen (2002) and Wriggers (2002). In the books, finite elements were applied to solve nonlinear contact problems. Various techniques were used to find contact stresses and to satisfy kinematical contact constraints, i.e. method of Lagrange multipliers and their generalizations: Joo and Kwak (1986), Ju and Taylor (1988), Alart and Curnier (1991), Simo and Laursen (1992), Wriggers (1995), Oancea and Laursen (1997); method of penalty function e.g. Laursen and Simo (1993), Zhong (1993), Wriggers (1995). Incremental-iterative methods have been commonly used, since solutions to the contact problems depend on the loading history, see Cescotto and Charlier (1993), Buczkowski and Kleiber (1997), Stupkiewicz and Mróz (1999). Bajer (1997) used original space-time finite elements in contact mechanics. Zmitrowicz (2000, 2001a, 2001b) included the interfacial layer of wear debris between contacting bodies.

5. Laws of wear

5.1. Archard's law of abrasive wear

Friction and wear depend as much on sliding conditions (the normal pressure and the sliding velocity) as on properties of materials concerned. The normal pressure and the sliding action are necessary for wear, i.e. mechanical wear is a result of the mechanical action. Therefore, the wear process discussed in this study depends first of all on the rubbing process.

The earliest contributions to the wear constitutive equations were made by Holm (1946). Holm established a relationship for the volume of the material removed by wear (W) in the sliding distance (s) and related it to the true area of contact. Archard (1953) formulated the wear equation of the form: the volume of the material removed (W) is directly proportional to the sliding distance (s), the normal pressure (p_n) and the dimensionless wear coefficient (k), and inversely proportional to the hardness of the surface being worn away (H), i.e.

$$W = k\frac{p_n s}{H} \tag{5.1}$$

Nowadays, it is generally recognized that wear is related to the wear coefficient, the pressure and the sliding distance. Pin-on-disc test experiments can be used to determine how wear is affected by the pressure and the sliding distance.

By analogy to Archard's law, in the most simple case, wear velocities (4.2) can be defined as functions of the normal pressure and the sliding velocity, i.e.

$$\begin{aligned} v^+ &= -i_A|p_n^A|\,|\boldsymbol{V}_{AB}| \\ v^- &= -i_B|p_n^B|\,|\boldsymbol{V}_{BA}| \end{aligned} \tag{5.2}$$

where

$$\boldsymbol{p}_n^B = (\boldsymbol{n}^- \otimes \boldsymbol{n}^-)(\boldsymbol{\sigma}_B \boldsymbol{n}^-) \equiv p_n^B \boldsymbol{n}^-$$
$$|p_n^A| = |p_n^B| \qquad\qquad \boldsymbol{V}_{AB} = -\boldsymbol{V}_{BA} \tag{5.3}$$

i_A and i_B are wear intensities of the bodies A and B. They may depend on fields of temperature in the bodies, i.e.

$$\{i_A, i_B\} = f(T_A, T_B, \operatorname{Grad} T_A, \operatorname{Grad} T_B) \tag{5.4}$$

where, T_A and T_B are temperatures in the bodies A and B.

The dimensionless wear coefficient k in Archard's law can be defined in various ways depending on the physical model which is assumed in deriving the wear equation. A simple interpretation is to consider k as a measure of the efficiency of the material removed for the given amount of work done.

Wear equations (5.2) differ from Archard's law (5.1) in the omission of the term representing the inverse proportionality of the surface hardness H. In the tribology literature, the wear equation coefficients i_A and i_B used in (5.2) are called the dimensional wear constants or the specific wear rates. If i_A and i_B are multiplied by the hardness of bodies, then we get dimensionless intensities of wear $i_A H_A$ and $i_B H_B$. Therefore, the hardness can be easily included in the quantitative estimates of the dimensional wear intensity coefficients i_A and i_B.

5.2. Extensions of Archard's law of wear

It can be presumed that anisotropic friction induces anisotropic wear, i.e. the intensity of the removed mass depends on the sliding direction (Zmitrowicz, 1992, 1993a,b). Anisotropy of friction results from roughness anisotropy of contacting surfaces and anisotropy of mechanical properties of materials with microstructures (crystals, composites, polymers, ceramics, biomaterials). Anisotropic wear velocities depend on the normal pressure, the sliding velocity and the sliding direction. Notice that p_n^A and $\boldsymbol{V}_{AB}$ do not define directional properties of the anisotropic wear.

Let us assume that the wear intensity i_A is a function of the sliding direction parameter α_v, i.e.

$$i_A = i_A(\alpha_v) \qquad\qquad \alpha_v \in \langle 0, 2\pi \rangle \tag{5.5}$$

α_v is the measure of the oriented angle between the given reference direction at the contact surface (e.g. Ox axis) and the sliding velocity direction. We postulate that the wear intensity function and the anisotropic friction force component $\mu_\alpha^{\parallel}$ are functions of the same type, i.e.

$$i_A(\alpha_v) \sim \mu_\alpha^{\parallel}(\alpha_v) \tag{5.6}$$

The coefficient of the friction force component tangent to the sliding direction is defined by

$$\mu_\alpha^{\parallel} = -\frac{1}{|p_n^A|}\boldsymbol{p}_t^A \cdot \boldsymbol{v} \tag{5.7}$$

where, the sliding velocity unit vector is as follows

$$\boldsymbol{v} = \frac{\boldsymbol{V}_{AB}}{|\boldsymbol{V}_{AB}|} \tag{5.8}$$

We apply the constitutive equation of the anisotropic friction force (Zmitrowicz, 1991) in form of a trigonometrical polynomial with the second order constant friction tensors ($\mathbf{C}_{1k};\ k = 0,1,2$), i.e.

$$\boldsymbol{p}_t^A = -|p_n^A|[\mathbf{C}_{10} + \mathbf{C}_{11}\cos(n_1\alpha_v) + \mathbf{C}_{12}\sin(m_1\alpha_v)]\boldsymbol{v} \qquad n_1, m_1 = 0,1,2,\ldots \tag{5.9}$$

where, the components of friction force vector (5.9) and unit vector (5.8) are given by

$$\begin{aligned} p_t^i &= -|p_n^A|[C_0^{ij} + C_1^{ij}\cos(n_1\alpha_v) + C_2^{ij}\sin(m_1\alpha_v)]v_j \qquad i,j = 1,2 \\ \boldsymbol{v} &= [\cos\alpha_v, \sin\alpha_v]^\top \end{aligned} \tag{5.10}$$

Depending on the form of the friction tensors, we get descriptions of anisotropic friction and anisotropic wear with a different number of constants and parameters.

Taking the following components of the friction tensors

$$C_k^{11} = \mu_{(k)1} \qquad C_k^{22} = \mu_{(k)2} \qquad k = 0,1,2 \tag{5.11}$$

the friction force component collinear with the sliding direction is given by

$$\begin{aligned} \mu_\alpha^{\parallel}(\alpha_v) = -\{&[\mu_{(0)1} + \mu_{(1)1}\cos(n_1\alpha_v) + \mu_{(2)1}\sin(m_1\alpha_v)]\cos^2\alpha_v + \\ &+[\mu_{(0)2} + \mu_{(1)2}\cos(n_1\alpha_v) + \mu_{(2)2}\sin(m_1\alpha_v)]\sin^2\alpha_v\} \end{aligned} \tag{5.12}$$

According to similarity postulate (5.6), we obtain the wear intensity coefficient, i.e.

$$\begin{aligned} i_A(\alpha_v) = &[i_1 + i_2\cos(n\alpha_v) + i_3\sin(m\alpha_v)]\cos^2\alpha_v + \\ &+[i_4 + i_5\cos(n\alpha_v) + i_6\sin(m\alpha_v)]\sin^2\alpha_v \end{aligned} \tag{5.13}$$

where, $n, m = 0, 1, 2, \ldots$ are two parameters, i_1, i_2,..., i_6 are six constants.

Using three spherical friction tensors

$$C_k^{ij} = \mu_k \delta^{ij} \qquad k = 0, 1, 2 \qquad i, j = 1, 2 \tag{5.14}$$

the friction force coefficient is the following trigonometrical polynomial

$$\mu_\alpha^{\|} = -[\mu_0 + \mu_1 \cos(n_1 \alpha_v) + \mu_2 \sin(m_1 \alpha_v)] \tag{5.15}$$

In this case, the wear intensity coefficient has the following form

$$i_A(\alpha_v) = i_1 + i_2 \cos(n\alpha_v) + i_3 \sin(m\alpha_v) \tag{5.16}$$

where, n, m are two parameters, i_1, i_2, i_3 are three wear constants.

Using the first friction tensor spherical, and the second and third equal to zero, i.e.

$$C_0^{ij} = \mu_0 \delta^{ij} \qquad C_1^{ij} = C_2^{ij} = 0 \qquad i, j = 1, 2 \tag{5.17}$$

the friction force coefficient and the wear intensity are constant

$$\mu_\alpha^{\|} = \mu_0 \qquad i_A(\alpha_v) = i_1 \tag{5.18}$$

where, i_1 is the wear constant.

Different types of anisotropic wear can be distinguished depending on a number of particular sliding directions: (a) neutral directions (if wear is independent of the sense of the sliding direction)

$$\exists \alpha_v \in \langle 0, 2 \rangle : \quad i_A(\alpha_v) = i_A(\alpha_v + \pi) \tag{5.19}$$

and (b) extreme value directions (if it gives extreme values of wear)

$$i_A(\alpha_v) = \min\{i_A(\widetilde{\alpha}_v) : \quad \widetilde{\alpha}_v \in \langle 0, 2\pi \rangle\} \tag{5.20}$$

or

$$i_A(\alpha_v) = \max\{i_A(\widetilde{\alpha}_v) : \quad \widetilde{\alpha}_v \in \langle 0, 2\pi \rangle\} \tag{5.21}$$

With the aid of symmetry groups, the following types of anisotropic wear can be distinguished: isotropic, anisotropic, orthotropic, trigonal anisotropic, tetragonal anisotropic, centrosymmetric anisotropic and non-centrosymmetric anisotropic (Zmitrowicz, 1993a,b). Symmetry properties of anisotropic wear are identified by elements of the symmetry groups i.e.: identity $(+\mathbf{1})$, inversion $(-\mathbf{1})$, rotations $(\boldsymbol{R}_n^\gamma,\ \gamma = 2\pi/n,\ n = 1, 2, \ldots)$ and mirror reflections. The mirror reflections are with respect to neutral directions $(\boldsymbol{J}_u)$ or extreme value directions $(\boldsymbol{J}_s)$. If anisotropic wear has a finite number of neutral directions, then the mirror reflections are with respect to the neutral directions. If there is a finite number of extreme value directions and all sliding directions are neutral, then the mirror reflections are with respect to the extreme value directions.

Heterogenous wear and wear dependent on the sliding path curvature have been investigated by Zmitrowicz (2005a).

It is postulated that for a given normal pressure and sliding velocity at the contact of two surfaces, the resultant rate of mass flowing out is equal to the product of a "composition coefficient" (κ) and a sum of rates of the mass flowing out obtained for each surface taken separately, i.e.

$$m_{AB} = \kappa(m_A + m_B) \tag{5.22}$$

The wear intensity functions of individual surfaces can be determined experimentally by sliding a test third body with isotropic wear properties. The resultant mass flowing out from the contact can be defined by

$$m_{AB} = \rho i_{AB}(\alpha_v^A)|p_n^A|\,|\boldsymbol{V}_{AB}| \tag{5.23}$$

where ρ is the average mass density. After substituting the rates of mass flowing out related to the individual surfaces A and B into (5.22), we obtain the following resultant wear intensity

$$\begin{gathered} i_{AB}(\alpha_v^A) = \kappa_A i_A(\alpha_v^A) + \kappa_B i_B(\alpha_v^A - \varphi) \\ \kappa_A = \frac{\kappa\rho_A}{\rho} \qquad\qquad \kappa_B = \frac{\kappa\rho_B}{\rho} \end{gathered} \tag{5.24}$$

We have assumed different reference directions on both contacting surfaces. Then, the following relation holds between the sliding direction parameters on the surfaces A and B

$$\alpha_v^B = \alpha_v^A - \varphi \tag{5.25}$$

where, φ is the angle of relative position of the contacting surfaces (i.e. the angle between the reference directions). The symmetry group of the resultant wear at the contact of two surfaces with different wear properties is equal to an intersection of the symmetry groups for the surfaces A and B.

5.3. Thermodynamical restrictions

The wear constitutive equations (5.2) satisfy the axiom of material objectivity, i.e. two different observers of the sliding at the contact region recognize the same wear velocities, see Zmitrowicz (1987, 1992, 1993a,b).

The constants in wear constitutive equations are restricted by two thermodynamic requirements (Zmitrowicz, 1987):

5.3.1. Restrictions following from the Second Law of Thermodynamics

The internal energies spent at the wear process are always positive, i.e. they satisfy the following inequalities

$$m_A\epsilon_A \geqslant 0 \qquad\qquad m_B\epsilon_B \geqslant 0 \tag{5.26}$$

where, ϵ_A and ϵ_B are internal energies (or energies consumed by formation of a unit mass of wear debris). Substituting constitutive equation (5.2) and definition (4.1) into thermodynamic inequality (5.26), and taking into account that

$$\rho_A, |p_n^A|, |\boldsymbol{V}_{AB}|, \epsilon_A \geqslant 0 \tag{5.27}$$

we obtain the following restriction on the wear intensity

$$i_A(\alpha_v) \geqslant 0 \qquad\qquad \alpha_v \in \langle 0, 2\pi\rangle \tag{5.28}$$

It means that the wear intensity is positive for any sliding direction (and for any location of the contact point).

5.3.2. Constraints of the energy dissipated at the frictional contact

The principal question is: how mechanical energy of the friction process can be dissipated? In general, the whole energy dissipated in the friction process is converted (in a priori unknown proportion) into frictional heat, the energy spent at the wear process and other forms of energy. Therefore, the following constraint of energy dissipated at the contact must be satisfied

$$\boldsymbol{p}_t^A \cdot \boldsymbol{V}_{AB} = q_f^A + q_f^B + \beta_A + \beta_B \tag{5.29}$$

where, q_f^A and q_f^B are frictional heat fluxes entering into the bodies A and B, β_A and β_B are the energies spent in the wear process, i.e.

$$\beta_A \equiv -m_A\epsilon_A \qquad\qquad \beta_B \equiv -m_B\epsilon_B \tag{5.30}$$

In equation (5.29), the frictional power is converted into frictional heat and energies spent on the wear process (other forms of energy are neglected). Equation (5.29) does not decide which part of the friction force power appears as frictional heat and wear process energy.

5.4. Other models of wear

Bahadur (1978) presented 13 analytical expressions for the prediction of wear published in the years 1937-1974. Earlier relationships connected the volume of the removed material with hardness only. Meng and Ludema (1995) found more than 300 equations of wear published during the period 1955-1995. The great number of independent variables that influence the wear rate was used in those equations, namely 625 variables. Meng and Ludema (1995) classified three general stages of deriving wear equations: empirical equations, phenomenological equations, equations based on materials failure mechaisms. Ludema (1996) classified the variables of wear equations as follows: variables from fluid mechanics, variables from solid mechanics, variables from material sciences, variables from chemistry. In spite of that, at present, Archard's law is a quantitatively simple calculation procedure of wear, see Rabinowicz (1995), Ravikiran (2000), Kato (2002).

Some researchers try to define the so called wear criterion. They define a place in the body boundary where the wear process is initiated. Usually it depends on the existence and development of the sub-surface plastic zones. Kennedy and Ling (1974) assumed that the wear criterion is a modified criterion for plastic flow. If the second invariant J_2 of the stress deviator equals or exceeds the critical value $\overline{W}$ (i.e. the yield stress of the material) then wear will occur

$$J_2 \geqslant \overline{W} \tag{5.31}$$

where, the second invariant of the deviatoric part of the stress tensor and the critical value are given by

$$J_2 = \frac{1}{2}\operatorname{tr}\boldsymbol{S}^2 \qquad \boldsymbol{S} = \boldsymbol{\sigma} - \frac{1}{3}(\operatorname{tr}\boldsymbol{\sigma})\mathbf{1} \qquad \overline{W} = \overline{k}^2 \tag{5.32}$$

where, $\overline{k}^2$ is the yield stress of the softer material. If $J_2 < \overline{W}$ no wear will occur. It is assumed that when the accumulated strain energy reaches the critical value at which the yielding occurs, most of this energy is transformed into production of wear particles. In such a place, the amount of wear is calculated, i.e. the volume of the removed material or the wear depth.

Yang *et al.* (1993) assumed that wear is first expected to take place at the point with the maximum critical stress. The wear equation can be derived from the maximum stress principle criterion (e.g. von Mises criterion) and yield strength. According to Ting and Winer (1988), wear is assumed to result from the yielding of materials. The yielding condition is governed by the temperature dependent yield strength and the stress field of a material. The stress field is combined from the isothermal stress field generated by surface traction and the thermal stress field induced by frictional heating.

Ohmae and Tsukizoe (1980) simulated numerically large wear particle formation (so called delamination wear) using the finite element method and theory of elastic-plastic deformations. Ohmae and Tsukizoe (1980) divided the contact surface into very small finite elements. It was assumed that each element represented a dislocation cell. The typical cell size was found to be $0.5\,\mu$m. When the equivalent stress of the element exceeds the failure stress (i.e. the fracture stress), a void of failure is generated. From these voids, cracks are propagated along directions of maximum plastic stresses. Penetrating into the bulk of the body, the cracks cause generation of wear particles. Ohmae (1987) recognized that large plastic deformations that lead to the void nucleation can only be dealt with properly used theory of large strains and displacements. Furthermore, Ohmae (1987) discussed different wear criteria used by various researchers: stress intensity factor, J-integral (proposed by Rice), etc.

Different approaches to modelling of wear are present in the literature. In the opinion of some researchers, wear laws should not be accepted as postulates, but as a consequence of both a geometry of surface micro-regions (asperities) and elastic-plastic deformations of interlocking surface asperities, see Kopalinsky and Oxely (1995), Torrance (1996).

6. Calculations of the abraded mass and fields of temperatures in the pin-on-disc test rig

The pin-on-disc testing machine is a typical device used to study friction and wear of materials. A stationary pin is pressed against a rotating disc, Fig. 4. After some time of sliding, surface damages can be observed, and wear particles can be seen inside and outside the wear track. Pałżewicz and Jabłoński (1993) investigated a short duration contact between the pin and the steel disc under high pressure conditions. The removed mass and fields of temperature in the pin are calculated in this study.

We assume that the cylindrical pin has small dimensions in the cross-section but it has a finite dimension along its axis. Therefore, the pin can be described by a one-dimensional continuum $y \in \langle 0, +\infty)$, see Fig. 4. In the model, the contact region, the normal pressure and the sliding velocity are known in advance. The normal pressure is given by p_n. The disc moves with the sliding velocity V with respect to the pin. The wear velocity of the pin is defined by v^+. The depth of the material removed u_n^+ in the time of sliding t is defined by equation $(4.6)_1$. Abraded mass from the pin is given by

$$m_{pin} = \rho_p \int\limits_{A_p} u_n^+ \, dS \tag{6.1}$$

where, ρ_p is the mass density of the pin, A_p is the area of contact (the cross-section area of the pin). The law of wear is assumed in the following form

$$v^+ = i_p p_n V \tag{6.2}$$

where, i_p is the wear intensity of the pin.

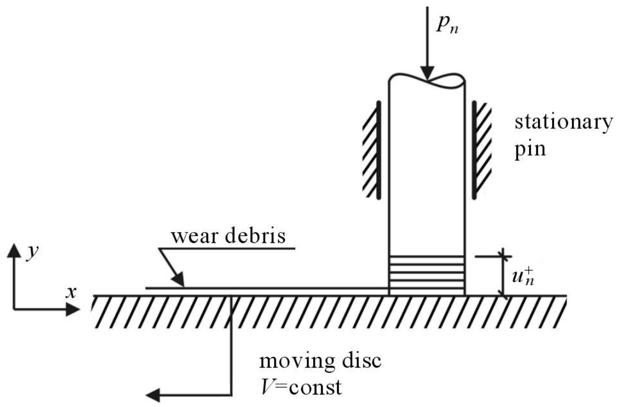

Fig. 4. The wear profile of the pin contacting with the moving disc: u_n^+ – wear profile, p_n – normal pressure, V – disc velocity

When two solids slide one against another, most of the energy dissipated in the friction process appears as heat and energy of the wear process. It is assumed that 10% of the friction power is transformed into the wear process, the rest part of the friction power is converted into heat. The heat generated at or very close to the contact surface is transferred away into the bulk of rubbing solids, and it is conducted into surrounding. Equation of energy dissipated in contact (5.29) reduces to the following relation given in the scalar notation

$$-p_t V = \underbrace{q_f}_{90\%} + \underbrace{\rho_p v^+ \epsilon_p}_{10\%} \tag{6.3}$$

where, p_t is the isotropic friction force (the inclination angle between the friction force and the sliding direction is equal to zero), ϵ_p is the internal energy of the pin, 90% is the fraction of the friction power dissipated into heat, 10% is the fraction of the friction power dissipated into wear of the pin.

Most of the wear occurs only on the pin. Assuming that 90% of the friction power converts into frictional heat and the rest of the power transforms into wear of the pin, we get the following relations

$$-0.9p_tV = q_f \qquad\qquad -0.1p_tV = \rho_p v^+ \epsilon_p \tag{6.4}$$

After substituting the isotropic friction force, i.e.

$$p_t = \mu_p p_n \tag{6.5}$$

and wear velocity (6.2) into $(6.4)_2$, we obtain a quantitative restriction on the wear intensity coefficient

$$i_p = 0.1\frac{\mu_p}{\rho_p \epsilon_p} \tag{6.6}$$

where, μ_p is the friction coefficient of the pin. Rabinowicz (1995, p.162) suggested that in estimations of ϵ_p one should consider the sliding body "loaded to the limit".

The material constants used in the calculations are collected in Table 2. Three types of bearing materials have been investigated: steel, aluminium (Al), tin (Sn). The friction coefficients were obtained by experimental testing. The wear intensities were identified with the aid of wear coefficient tables, see Rabinowicz (1995), i.e.

$$\begin{aligned} &i_{steel} = 0.03 \cdot 10^{-6}\ \mathrm{MPa}^{-1} \qquad\qquad i_{Al} = 0.5 \cdot 10^{-6}\ \mathrm{MPa}^{-1} \\ &i_{Sn} = 2.0 \cdot 10^{-6}\ \mathrm{MPa}^{-1} \end{aligned} \tag{6.7}$$

Table 2. Material constants used in calculations of the pin-on-disc test set-up

Type of material	Coefficient of friction μ_p	Wear intensity i_p [MPa^{-1}]	Mass density ρ_p [kgm^{-3}]	Thermal diffusivity K [$\mathrm{m}^2\mathrm{s}^{-1}$]	Coefficient of heat transfer b [s^{-1}]
Steel	0.1	$0.03 \cdot 10^{-6}$	$7.9 \cdot 10^3$	$13.8 \cdot 10^{-6}$	$2.26 \cdot 10^{-3}$
Al	0.07	$0.5 \cdot 10^{-6}$	$2.7 \cdot 10^3$	$85.9 \cdot 10^{-6}$	$3.46 \cdot 10^{-3}$
Sn	0.026	$2.0 \cdot 10^{-6}$	$7.3 \cdot 10^3$	$39.8 \cdot 10^{-6}$	$4.96 \cdot 10^{-3}$

Equation of the abraded mass from the pin (6.1) reduces to the following form

$$m_{pin} = \rho_p i_p p_n V A_p t \tag{6.8}$$

The cross-section of the pin is equal to $A_p = 200 \cdot 10^{-6}\,\mathrm{m}^2$. The normal pressure and the sliding velocity are shown in Fig. 5 - Fig. 7. The abraded mass from the pin after 4 seconds, is as follows: $2.12 \cdot 10^{-6}$ kg for steel, $12.07 \cdot 10^{-6}$ kg for aluminium, $130.5 \cdot 10^{-6}$ kg for tin. Figure 4 shows the removed mass from the pin with respect to the sliding time. For a short duration contact, the influence of temperature on the wear intensity coefficient i_p can be neglected. Coefficients (6.7) deal with the fixed room temperature 20°C.

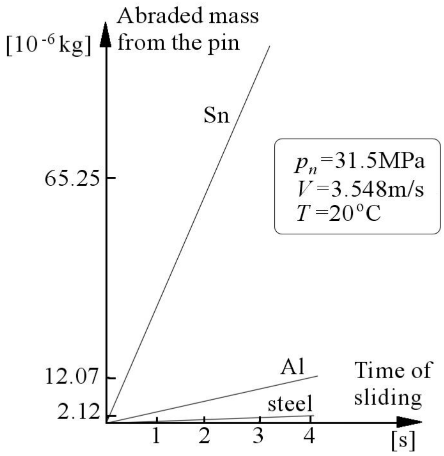

Fig. 5. The progress of the abraded mass from the pin after 0 to 4 seconds for three types of materials: steel, aluminium, tin

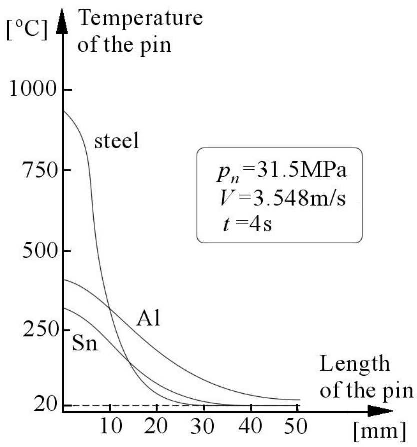

Fig. 6. Temperature distributions in the pin with respect to the distance from contact for three types of materials

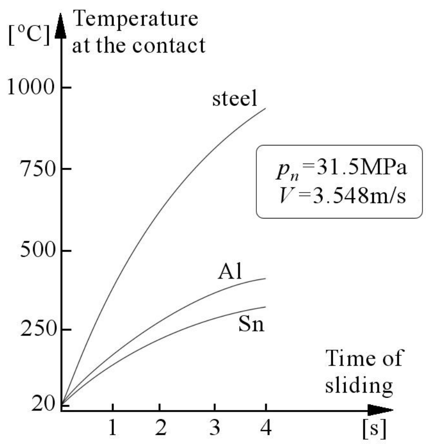

Fig. 7. The progress of the contact temperatures for the pin; the plots correspond to the real time of sliding from 0 to 4 seconds and for three types of materials

The field of temperature induced by friction is inseparable behavior of the contact phenomenon. Therefore, it is important to consider the loss of the material and frictional heat together. Frictional heat is generated at the contact between the pin and the disc. The fields of temperature and contact temperatures were investigated theoretically in numerous papers, e.g. Carslaw and Jaeger (1959), Kennedy (1981), Kikuchi (1986), Barber and Comninou (1989), Vick *et al.* (2000). The equation of heat conduction in a one-dimensional continuum with included heat transfer into the surrounding has the following form

$$\begin{aligned} &\frac{\partial T}{\partial \tau} = K\frac{\partial^2 T}{\partial y^2} - b(T - T_a) \\ &T = T(y,\tau) \qquad y \in \langle 0, \infty) \qquad \tau \in \langle 0, t \rangle \end{aligned} \tag{6.9}$$

where, T is temperature of the pin, y is the coordinate along the pin axis, T_a is room temperature (20°C). K is the thermal diffusivity coefficient, i.e.

$$K = \frac{k}{c\rho_p} \tag{6.10}$$

where, k is the thermal conductivity, c is the specific heat. In equation $(6.9)_1$, the Newton rule of heat transfer into the surrounding is assumed. The coefficient of Newton's heat transfer rule b is as follows

$$b = \frac{hg}{c\rho_p A_p} \tag{6.11}$$

where, h is the coefficient of heat transfer, g is the circumference of the pin surface $(g = 50 \cdot 10^{-3}\,\mathrm{m})$.

The initial and boundary conditions of the heat conduction problem are given by

$$T(\tau = 0) = T_a \qquad k\frac{\partial T}{\partial y}(y = 0) = q_f \qquad \lim_{y\to\infty} T(y, \tau) = T_a \tag{6.12}$$

The frictional heat flux vector is defined by

$$q_f = wp_nV \tag{6.13}$$

where, w is the frictional heat intensity coefficient. The frictional heat q_f is divided into two parts, a part of heat which is transferred into the pin (γ), and a part of heat transferred into the disc $(\gamma - 1)$, i.e.

$$q_f = \gamma q_f + (1 - \gamma)q_f \tag{6.14}$$

where, γ is the coefficient of heat partition between the pin and the disc. It is assumed that the 90% is divided into two parts, i.e. 37% (it is the friction power converted into heat and transferred into the pin) and 53% (it is the friction power converted into heat and transferred into the disc). The temperature distribution along the pin axis and the contact temperature are given by

$$\begin{aligned} T_{pin} &= T(y, t) = T_a + \frac{wp_nV}{k}\sqrt{\frac{K}{\pi}}\int\limits_0^t \frac{1}{\sqrt{\tau}}\mathrm{e}^{-\left(b\tau + \frac{y^2}{4K\tau}\right)}\,d\tau \\ T_{contact} &= T(y = 0, t) \end{aligned} \tag{6.15}$$

Calculated temperatures in the pin and at the contact surface are shown in Fig. 6 and Fig. 7. Figure 6 presents temperature plots in the pin with respect to the length coordinate of the pin, i.e. with respect to the distance from contact. The temperatures strongly decrease along the pin axis since the heat is conducted into the surrounding. The highest temperatures occur at the contact surface. Figure 7 illustrates contact temperatures for the pin sliding in the disc. The contact temperatures increase with respect to time of sliding. Notice, the highest contact temperature is for the steel pin and the least abraded mass is for the steel pin. All contact temperature plots correspond to the real time of sliding. We do not consider cooling effects.

7. Conclusions

- Contact surface damages can have various patterns: abrasion, fatigue, ploughing, corrugation, erosion and cavitation. Irreversible changes in body shapes and the increase of gaps between contacting solids are principal results of abrasive wear. The sizes of the wearing out bodies are gradually reduced in the process of abrasion.
- The amount of the removed material can be estimated with the aid of the wear depth and the wear profile. They are useful measures of the wear process, and they can be described with the aid of the wear laws. The definition of the gap includes deformations of bodies and evolutions of wear profiles.
- The measures of wear have non-zero values as long as an observable amount of the material is removed. Usually, but not always, a significant period of the sliding time must be taken into account. Due to this, rather long simulation times are needed.
- The constitutive equations of anisotropic wear describe abrasion of materials with microstructures. The equations are derived in the frame of Archard's law of wear. In the present time, Archard's law is commonly used in engineering calculations.

The proposed theoretical descriptions of wear can be used in numerical codes for strength calculations of structures and materials. The calculations can be useful for improvement of wear-resistance of materials and durability of rubbing and wearing out machine parts and tools.

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Formy zużycia i prawa zużycia: przegląd

Streszczenie

Zużycie jest to stopniowe usuwanie materiału z powierzchni stykających i ślizgających się ciał stałych. Następstwem zużycia są uszkodzenia powierzchni styku. Mogą one przyjmować różną formę (ścierania, zmęczenia, bruzdowania, korugacji, erozji i kawitacji). Konsekwencjami procesu zużycia ściernego są nieodwracalne zmiany kształtów ciał oraz powiększanie luzów (szczelin) między stykającymi się ciałami. Użyteczną miarą usuniętego materiału jest profil głębokości zużycia powierzchni. Definicja luzu między stykającymi się ciałami uwzględnia odkształcenia ciał oraz ewolucję profili zużycia. Głębokość zużycia może być oszacowana z pomocą praw zużycia. Wyprowadzone w niniejszej pracy równania konstytutywne anizotropowego zużycia są rozszerzoniem prawa zużycia Archarda. Opisują one ścieranie materiałów z mikrostrukturą. W przykładzie ilustracyjnym obliczono ubytek masy i temperaturę w stanowisku doświadczalnym typu pin-on-disc.

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