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EXPERIMENTAL METHOD OF DEFINING BIAXIAL FATIGUE PROPERTIES OF ELASTIC-PLASTIC CONSTRUCTION MATERIALS. PART 1 – MODEL FORMULATION

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The paper presents formulation of a method of biaxial fatigue testing in conditions of cyclic uniaxial loads with the use of specimens made of a material characterised by controlled mechanical properties. The origin for the paper was a successful attempt to adapt the plastic potential for describing biaxial fatigue strength made by Troost and El.-Magd (1974) in relation to the "stress anisotropy" effect, pointed out by him. On the basis of the carried-out reasoning, a model formulation of the allowable stress amplitude was introduced, which is a formalised record of the relations between the fatigue stress amplitudes and the yield stresses. The article suggests an experimental method of defining the appointed model parameters, which is a development of Szczepiński's method (Szczepiński, 1963).

Key words: biaxial fatigue, yield surface, testing methods

1. Introduction

The fatigue research is carried out in order to define the fatigue life or strength. Both the features may be assigned in tests treated as material or constructional ones. The material research is conducted on standardised specimens and in standardised conditions [13,15]. The constructional research is realised on actual construction elements, e.g. Jachimowicz *et al.* (2002). One of the significant elements determining the research conditions is a time-variable load defined for each particular test separately [12]. The basic type of the time-variable load, defined as standard for the use of fatigue tests is a uniaxial load, sinusoid-cycle variable with a defined value of the realised amplitude of stress, strain or load [13].

The cycle position in relation to zero value is defined by the mean level. There exist branch legal documents, which define the load model corresponding to operational loads in the form of a spectrum (Berger *et al.*, 2002). The spectrum describes the distribution of conventional variations of amplitude values and the mean level of cycles assigned during the courses of randomly changing operational loads (Szala and Kocańda, 1997).

The above-mentioned considerations relate univocally to a uniaxial cyclic load of the analysed specimen or construction element [14]. Due to geometrical complexity of actual constructional elements and relations between geometrical forms and appearing loads occurring in them, leaving cyclic bi- or triaxial loads arising in the area of weak construction elements out of account is frequently too far-reaching simplification of the loading model (Bogdaniuk, 1998).

The phenomenological relation (fatigue criterion) of the fatigue stress or strain state constituents, occurring in such a situation, allows one to assign the reduced value and to write down the strength condition with the allowable value assigned during a uniaxial fatigue test taken into account. The amount of such relations, their limited range of application and in most cases not satisfying the verification level cause that this approach in the engineering practice has a simplified form and is only limited to the best-known hypothesis, e.g. Garud (1981). This approach does not take into account many characteristic features of the currently verified state.

The alternative approach, i.e. assigning an appropriate material or constructional features directly to experimental testing requires specialised testing machines realising multi-axial cyclic load and defining a typical testing methodology. Numerous research designs connected with the problem have not led to the standardisation of the testing methods (Cichański and Świtała, 2002).

This paper has undertaken such an attempt, assuming that the suggested experimental testing method is to result in defining the constructional material sensitivity to the occurrence of a complex fatigue stress state. To describe the sensitivity, a model formulation of the allowable stress amplitude was derived. Another significant assumption was that the tests should be realised with the use of a standard strength machine. On the basis of bibliography studies and the authors' own preliminary tests, a thesis has been formulated that it is possible to carry out fatigue research in a uniaxial load state, with the use of a specimen made of a material characterised by mechanical properties purposefully modified by direction-oriented plastic strain, in such a way that the results will one allow to conclude about the fatigue strength corresponding to the biaxial load state (Cichański and Sempruch, 2001).

2. Theoretical background

Let us consider the distribution of fatigue biaxial stresses in an elementary area (Fig. 1). In this area, the co-ordinate system $0x^*y^*$ has been agreed in such a way that its axis directions overlap with the directions of load operation. The values σ_x^*, σ_y^* and τ_{xy}^* are nominal stresses resulting from the applied load. Together with a change in the angle ϑ, indicating the plane where the stresses are being considered, the rotation of the system $0xy$ takes place, accompanied by the rotation of related stresses σ_x, σ_y and τ_{xy}.

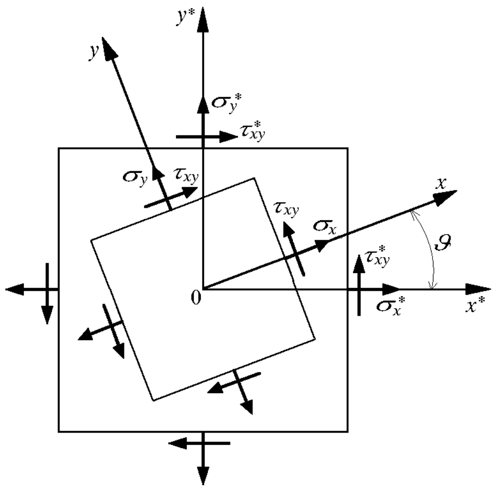

Fig. 1. The fatigue stress distribution in the elementary area

One of the ways for realising biaxial fatigue is such an application of biaxial sinusoid loads that the nominal stresses σ_x^*, σ_y^* and τ_{xy}^* resulting from them may be expressed by means of the following dependences

$$\begin{aligned}
\sigma_x^*(t) &= \sigma_{xm}^* + \sigma_{xa}^* \sin(\omega_x t) \\
\sigma_y^*(t) &= \sigma_{ym}^* + \sigma_{ya}^* \sin(\omega_y t - \phi_y) \\
\tau_{xy}^*(t) &= \tau_{xym}^* + \tau_{xya}^* \sin(\omega_{xy} t - \phi_{xy})
\end{aligned} \tag{2.1}$$

In a general notation, the components of the stress state can be put down as $\sigma_i(t)$, where the index $i \in (x, y, xy)$ describes the direction and character of a given component

$$\sigma_i(t) = \sigma_{im} + \sigma_{ia} \sin(\omega_i t - \phi_i) \tag{2.2}$$

The thus defined biaxial cyclic stresses are called proportional when for all i directions the mean level σ_{im} and the phase shift ϕ_i equal zero, and the frequencies ω_i are the same. The biaxial stresses are called non-proportional when the above-mentioned condition is not fulfilled.

In the case when stress non-proportionality occurs, the fatigue strength decreases. The fatigue strength in the i direction, in conditions of the mean level different from zero, is described by the allowable stress amplitude σ_{iA}. If the realised amplitude σ_{ia} exceeds the value σ_{iA} depending on σ_{im}, fatigue damage occurs on the plane ϑ_{kr}, called the critical plane. Bibliography gives a number of suggestions on how to write down the relation of σ_{iA} to σ_{im} (Troost and El.-Magd, 1981). One of the commonly approved approaches assumes that the value of the allowable stress amplitude σ_{xA}, corresponding to Z_{rc}, changes together with the angle ϑ as it is described below

$$\begin{aligned}\sigma_{xA}(\vartheta) &= Z_{rc} - \frac{2Z_{rc} - Z_{rj}}{Z_{rj}}\sigma_{xm}(\vartheta) \\ \sigma_{xm}(\vartheta) &= (1 + \cos 2\vartheta)\frac{\sigma^*_{xm}}{2} + (1 - \cos 2\vartheta)\frac{\sigma^*_{ym}}{2} - \tau^*_{xym}\sin 2\vartheta\end{aligned} \tag{2.3}$$

A diagram presenting dependences of the allowable amplitudes σ_{xA} on the angle ϑ for different cases, where a non-zero mean cycle level occurs, is presented in Fig. 2.

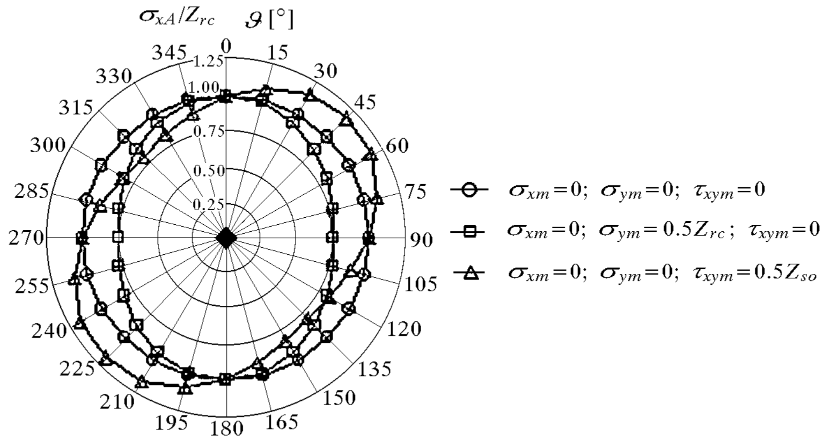

Fig. 2. Dependence of σ_{xA} on the angle ϑ

The picture of the allowable amplitudes distribution in polar coordinates for loads showing the mean level equal to zero is a circle. This indicates the independence of σ_{xA} of the angle ϑ. In the case of introducing non-zero mean levels, the circle takes an oval-like shape (Fig. 2). The analysis of this stress state leads to definition of the angles, for which σ_{xA} achieves extreme values. This indicates the dependence of the allowable amplitudes distribution on the direction described by the angle ϑ. This phenomenon was described by Troost as anisotropic fatigue behaviour of an isotropic material (Troost and El.-Magd, 1974), and the QVH hypothesis was introduced for the description of the fatigue strength in such conditions. The plasticity potential formulated to describe plasticity conditions of the anisotropic material (Troost and El.-Magd, 1981) was the basis of the QVH hypothesis.

One of the ways of causing anisotropy of material mechanical properties, among other fatigue properties, is initiating plastic changes in the material. Plastic changes cause modification of material characteristics, including the yield surface. Thin-walled tubular specimens are widely used for experimental material yield surface testing in the initial state and after preliminary plastic prestrain. A unique method of such tests with the use of flat specimens was introduced by Szczepiński (1963). This method is schematically presented in Fig. 3.

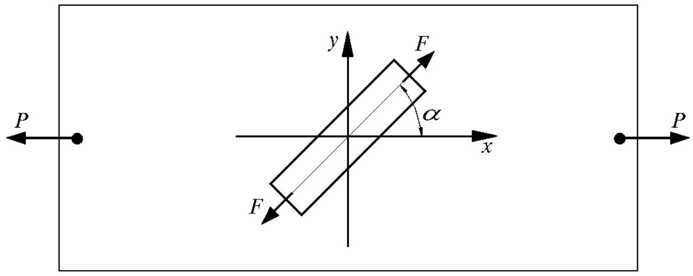

Fig. 3. Methods for preliminary and secondary specimen loading (Szczepiński, 1963)

The initial loading P, causing plastic strains of a predefined value, is applied to large thin-plates made of a material in the initial state. Small specimens are cut out of thus prepared large plates with a certain angle with respect to the initial loading operation direction. Under conditions of secondary loads F, the tested plastic properties are defined using these specimens.

3. Experimental procedure

To fulfil the aim of the work it was necessary to formulate such a method of conducting the experimental tests, which would take into account the possibility of controlling the plastic strain of the tested material. Owing to this, a specific method of specimen preparation, which is a development of Szczepiński's method (Szczepiński, 1963), was proposed. The material in the form of a thin-plate is prestrained by forces P_{prst} in a specially designed grip (Fig. 4).

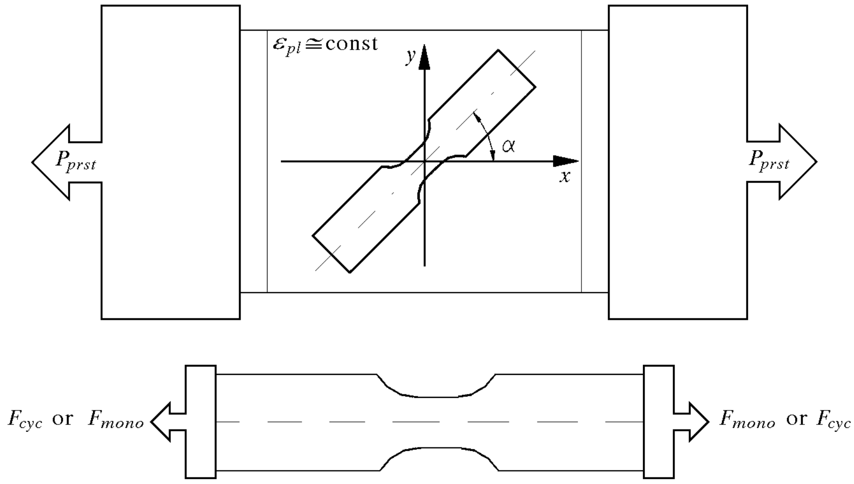

Fig. 4. The specimen acquisition method

The forces P_{prst} have such values so as to cause plastic strains ε_{pl} in the plate central part. Flat dumbbell specimens for further research are cut out of the strained plates at the angle α (Fig. 4). These specimens are tested in the conditions of uniaxial monotonic loads F_{mono} with the aim of the material plastic properties assignment. Material fatigue properties are defined under the conditions of cyclic uniaxial loads F_{cyc}.

The load P_{prst} is transmitted from the grip onto the stretched plate through a series of five pins placed in specially prepared holes. The arrangement of holes is expected to affect the shape of plastic strain fields occurring in the plate. The stipulated area, where the gradients ε_{pl} have negligible influence on the repeatability of properties of the test specimens, is presented in Fig. 4 with a dotted line.

4. Model for determination of allowable stress amplitude

4.1. Assumptions

Correspondingly to the agreed experimental testing method, the material is used in the initial state and after a preliminary plastic prestrain. For the material in the initial state, further called the virgin material, an assumption on the mechanical properties isotropy was made. The yield surface of the material is described by the Huber-Mises-Hencky (H-M-H) condition. The virgin material fatigue strength is proportional to the yield stresses. An assumption about kinematic hardening was made for describing the yield surface of the preliminarily plastically prestrained material, further called the prestrained material. For this material, an assumption that the initial plastic strain caused a decrease in the fatigue strength was made. The considerations were limited to plane stress, which appeared on free surfaces, where the main changes deciding about the fatigue strength occured.

4.2. Description of plastic properties

The following dependence presents the yield H-M-H condition, expressed by means of nominal stresses in the approach typical for a plane stress

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2 = \sigma_{pl}^2 \tag{4.1}$$

The yield condition has its graphic representation in the form of a yield surface situated in the stress space described with the co-ordinate system $0\sigma_x\sigma_y\tau_{xy}$. An ellipsoid in Fig. 5 depicts relationship (4.1). The ellipsoid is situated in such a way that one of its axis overlaps with the axis $0\tau_{xy}$, and the other two constitute bisectors between the axis $0\sigma_x$ and $0\sigma_y$.

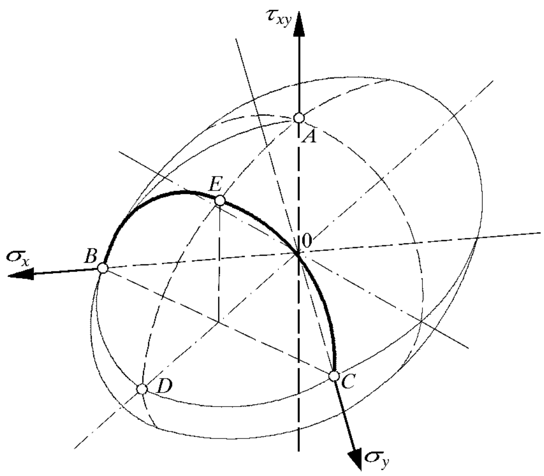

Fig. 5. Huber-Mises-Hencky ellipsoid

On the H-M-H ellipsoid, a number of ellipses (Fig. 5) may be seen, which results from surface intersection with planes describing different stress states realised in the course of experimental tests with the use of specimens of various geometrical shapes. For further consideration, the ellipse BEC was chosen, created by H-M-H ellipsoid intersection with a plane described by the following dependence

$$\sigma_x + \sigma_y = \sigma_{pl} \tag{4.2}$$

Analysis of the stress state meeting the conditions described by (4.1) and (4.2) leads to the conclusion that one of the main stresses in this state must be equal to zero. The ellipse BEC is a geometrical picture of the appointed stress state and its points correspond to yield stresses assigned to uniaxial load conditions for the virgin material. Coordinates of the ellipses BEC in the stress space are described by means of transforming dependences (4.3), where the angle φ defines the direction for appointing ε_{pl} in the agreed coordinate system

$$\begin{aligned}\sigma_x &= \frac{1}{2}\sigma_{pl}(1+\cos 2\varphi)\\ \sigma_y &= \frac{1}{2}\sigma_{pl}(1-\cos 2\varphi)\\ \tau_{xy} &= \frac{1}{2}\sigma_{pl}\sin 2\varphi\end{aligned} \tag{4.3}$$

Ellipses BEC, seen in the direction $D0$, according to the system in Fig. 5, are presented in Fig. 6 by means of a line with the square marker.

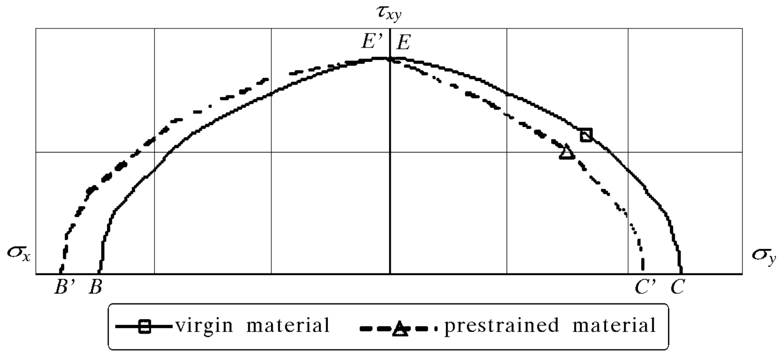

Fig. 6. Ellipses BEC and $B'E'C'$ – $D0$ direction view (assumption)

The plastic strain causes modification of the yield surface. According to the assumption about the kinematic nature of material hardening, the ellipsoid BEC moves in the direction defined by the initial plastic load. What depicts it is the change of the ellipses BEC corresoinding to the virgin material. The new ellipse marked as $B'E'C'$ corresoinding to the prestrained material, is presented in Fig. 6 by means of a line with the triangle marker. The co-ordinate system $0\sigma_x\sigma_y\tau_{xy}$ was selected in such a way that the ellipse $B'E'C'$ moved in the direction $0\sigma_x$, which corresponds to the angle $\varphi = 0°$. Changes in ellipses BEC and $B'E'C'$ seen in Fig. 6, indicate an increase in yield stresses in the direction of $\varphi = 0°$ (point B') and a decrease in yield stresses in the direction of $\varphi = 90°$ (point C').

4.3. Description of fatigue properties

For most materials, the fatigue strength σ_{iA} is proportional to the yield stress σ_{pl}, as it is symbolically presented by the relation

$$\sigma_{iA} \stackrel{def}{=} a_N \sigma_{pl} \tag{4.4}$$

The proportionality coefficient a_N, appearing in dependence (4.4), is defined on the basis of tests on the virgin material as a relation of appropriate stresses, as follows

$$a_N = \frac{\sigma_A^{izo}}{\sigma_{pl}^{izo}} \tag{4.5}$$

On the basis of results of tests regarding plastic properties of the material related to the angle φ as well as equation (4.4) and the coefficient a_N, theoretical values of the allowable stress amplitudes σ_A can be assigned. The values, after transformation by means of relation (4.3), create a curve in the space $0\sigma_{xa}\sigma_{ya}\tau_{xya}$, called a fatigue ellipse, which is shown and marked bec (Fig. 7).

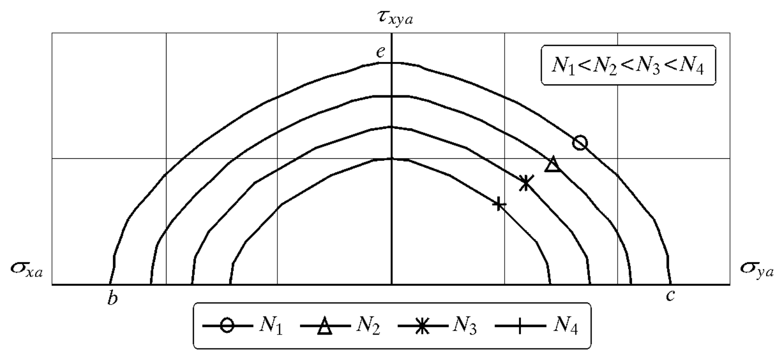

Fig. 7. Ellipses bec for different fatigue life – $D0$ direction view (assumption)

The theoretical values of allowable stress amplitudes depend on the number of cycles N, assumed in the definition of the coefficient a_N. The greater is the cycle number N, the lower remains the value of the appointed amplitude, which is depicted by lessening of the ellipses bec (Fig. 7).

Through equation (4.4), the fatigue ellipse bec is connected with the ellipse BEC, assigned for the virgin material. For the prestrained material, the plastic ellipse takes the shape of $B'E'C'$ (Fig. 6), which has the corresponding fatigue ellipse $b'e'c'$ presented in Fig. 8.

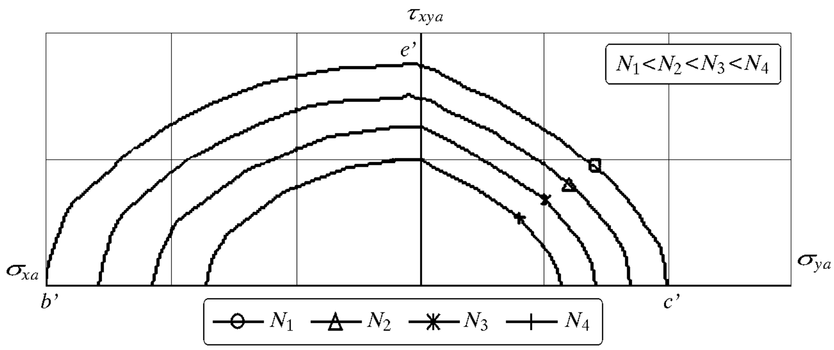

Fig. 8. Ellipses $b'e'c'$ for different fatigue life – $D0$ direction view (assumption)

4.4. Allowable stress amplitude model approach formulation

Plastic strain causes that the material is given a preferential orientation. This orientation is assumed to be described with an angle α measured with respect to direction of the initial load operation (Fig. 4). The thesis about the possibility of using the material with controlled mechanical properties for biaxial fatigue modelling is expressed by the postulate about the angles φ and α equality.

According to the hypothesis of kinematic hardening in the direction of plastic load operation, yield stresses become higher. Due to the fact that the fatigue ellipse $b'e'c'$ is determined on the basis of the plastic ellipse $B'E'C'$, also quantities constituting the ellipse $b'e'c'$ grow in the direction of plastic load operation. One of the assumptions for the model is a claim that the initial plastic prestrain causes a decrease of the fatigue strength. The tests results indicate that this assumption is valid (Ingerma and Ranna, 1974). To assure the conformity of behaviour of the fatigue ellipses with experimental results, σ_M is introduced, which is a model formulation of the allowable stress amplitude (Cichański and Sempruch, 2001). The amplitude σ_M is a quantity to be compared with the amplitudes creating the fatigue ellipses $b'e'c'$.

The model formulation of the allowable stress amplitude σ_M $(4.6)_1$ constitutes the base part σ_{Mb} $(4.6)_2$ and the modifying part σ_{Mo} $(4.6)_3$ of the stress. The base part is independent of the direction α, while the modifying part changes together with alteration of α

$$\begin{aligned} \sigma_M &= \sigma_{Mb} + \sigma_{Mo} \\ \sigma_{Mb} &= \sigma_A^{izo} \\ \sigma_{Mo} &= \frac{1}{a_N}(\sigma_A^{izo} - \sigma_A^{anizo}) \end{aligned} \tag{4.6}$$

where:

a_N	–	model scaling coefficient
σ_A^{izo}	–	allowable stress amplitude for the virgin material
σ_A^{anizo}	–	allowable stress amplitude for the prestrained material
σ_M	–	allowable stress amplitude in the model formulation
σ_{Mb}	–	base model stresses
σ_{Mo}	–	modifying model stresses.

The values of σ_A^{izo} and σ_A^{anizo} are defined on the basis of Wöhler's lines, assigned for the virgin and prestrained materials. To describe the fatigue properties, three characteristic directions $\alpha = 0°$, $45°$ and $90°$ were selected. Wöhler's lines for the virgin material, according to the assumption on the isotropy of mechanical properties, should have a similar course for three characteristic directions (Fig. 9a).

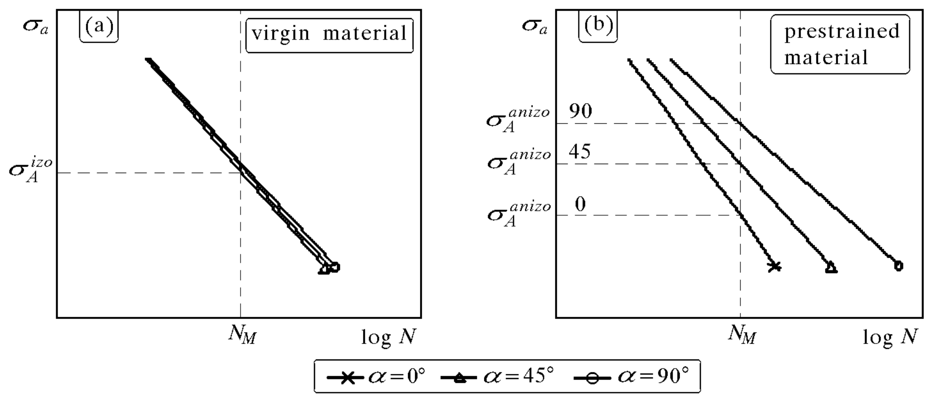

Fig. 9. Wöhler's lines: (a) virgin material; (b) prestrained material (assumption)

There exist fatigue properties variability together with the angle α for the prestrained material. According to the assumption that the initial plastic strain causes the decrease of fatigue strength, Wöhler's line determined in the direction of the plastic load operation (for $\alpha = 0°$) should be placed in the lowest position (Fig. 9b). The influence of the initial prestrains on the fatigue strength should descend together with deviation from the direction of plastic load operation. It should be reflected by higher and higher positions of Wöhler's lines for rising values of the angle α (Fig. 9b).

On the basis of Wöhler's diagram for the virgin material (Fig. 9a), the values of σ_A^{izo} are defined. On the basis of a diagram corresponding to the considered angle α for the prestrained material (Fig. 9b), the values of σ_A^{anizo} are defined. The amplitudes read on suitable Wöhler's diagrams, for the applied strength N, are introduced in equations (4.6). For the virgin material $\sigma_A^{anizo} = \sigma_A^{izo}$, which causes that for this material the modifying part equals zero. The thus determined values of σ_M and dependent on the cycle number N, after transformation by means of (4.3), lead to generation of ellipses bec and $b'e'c'$. In the variability interval of the N-th cycle, the limited cycle number N_{lim} can be defined for which the value σ_M, found from the fatigue research, is comparable with the value σ_A, found from the rescaling of the plastic properties. For this case, equation (4.4) describes the relation between the ellipses BEC and bec for the virgin material, and $B'E'C'$ and $b'e'c'$ for the prestrained material.

5. Conclusions

The paper presents an idea of a method of investigating biaxial material fatigue properties. The characteristic of the material, called a model formulation of the allowable stress amplitude, corresponds to the proposed testing method. A graphic representation of this characteristic is a fatigue ellipse, which is a curve lying in the stress space $0\sigma_x\sigma_y\tau_{xy}$ and describing a flat fatigue stress state.

The testing method formulated in the paper and resulting material characteristics are an attempt to describe the constructional material sensitivity to a complex fatigue stress state. Using the suggested testing method and the model, such tests can be carried out on specimens made of the plastically prestrained material in conditions of cyclic simple loads. The determined tests results will allow one to conclude about the biaxial material fatigue strength.

The presented notation of the model formulation of the allowable stress amplitude is characterised by two parameters. The obvious parameter of the model is the relation of the allowable stress amplitude to yield stresses found for the virgin material. A non-evident model parameter is the number of cycles suitable for defining the allowable stress amplitude.

To estimate the correctness of the model formulation of the allowable stress amplitude, basic experimental tests need to be made with the use of the suggested method.

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Doświadczalna metoda wyznaczania dwuosiowych własności zmęczeniowych sprężysto-plastycznych materiałów konstrukcyjnych. Część 1 – Sformułowanie modelu

Streszczenie

W artykule przedstawiono sformułowanie metody badania dwuosiowego zmęczenia w warunkach cyklicznych obciążeń jednoosiowych z wykorzystaniem próbek wykonanych z materiału o kontrolowanych własnościach mechanicznych. Jako genezę pracy wskazano udaną próbę adaptacji potencjału plastycznego do opisu dwuosiowego zmęczenia wykonaną przez Troosta Troost i El.-Magd, 1974) w odniesieniu do wskazanego przez niego efektu „anizotropii naprężeniowej". Na podstawie przeprowadzonego rozumowania wyprowadzono modelowe ujęcie dopuszczalnej amplitudy naprężeń będące sformalizowanym zapisem zależności amplitudy naprężeń zmęczeniowych od naprężeń uplastyczniających. W pracy zaproponowano doświadczalną metodę wyznaczania parametrów wskazanego modelu, będącą rozwinięciem metody Szczepińskiego (Szczepiński, 1963).

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