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EXPERIMENTAL METHOD OF DEFINING BIAXIAL FATIGUE PROPERTIES OF ELASTIC-PLASTIC CONSTRUCTION MATERIALS. PART 2 – EXPERIMENTAL RESEARCH

Artur Cichański
Janusz Sempruch

Faculty of Mechanical Engineering, University of Technology and Agriculture in Bydgoszcz
e-mail: artur.cichanski@atr.bydgoszcz.pl

The paper is concerned with the authors' own method of defining biaxial fatigue properties of elastic-plastic construction materials, for which a theoretical basis was formulated before (Cichański and Sempruch, 2005). In the present paper, results of experimental investigations (divided into preliminary and main research) appropriate for the proposed method are described. In the preliminary research phase, an experimental verification of major assumptions and numerical verification of conditions on preparation of test specimens were carried out. During the main research phase, model parameters of plastic and fatigue properties were described and the evaluation of the proposed model (Cichański and Sempruch, 2005) was done.

Key words: biaxial fatigue, yield surface, testing methods

1. Introduction

Experimental determinatiion of biaxial fatigue properties requires choosing a research method appropriate for the specified quantity. Numerous research works carried out in the field have not yet led to formulation of adequate normalization acts. A review of research methods of biaxial fatigue in bibliography indicates three commonly-used methods (Cichański and Świtała, 2002) and a number of rarely used ones. Analysis of biaxial fatigue research methods carried out towards tests specimen shaping shows that in the group of most often used methods, the research is done with the use of a cylindrical, thin-walled tubular and cruciform specimen (Cichański and Świtała, 2002). In the group of more rarely used methods, specimens with uniaxial loading can be met, in which the stress state biaxiality results from the specimen geometry. The group includes oval (Schewchuk *et al.*, 1968) and rhombic plate specimens (Zamrik *et al.*, 1997) exposed to cyclic bending, and thick-walled tubular specimens (McDiarmid, 1991) cyclically loaded with altering pressure. Also specimens in the shape of a rotating disc (Findley *et al.*, 1961) can be met as well as specimens whose geometrical shape result from researcher's own experience (Sawert, 1943). In the case of shape specimens, the change of realized stress proportions is obtained by the change of specimen geometrical dimensions. The advantage of methods using shape specimens for biaxial fatigue research is the possibility of conducting the research at a test stand that generates cyclic uniaxial loading. Their disadvantage comes from the limit of their application to the proportional fatigue.

The paper is concerned with the authors' own method of defining biaxial fatigue properties of elastic-plastic construction materials. In the first part of the paper, the theoretical basis of the proposed method is discussed (Cichański and Sempruch, 2005). In the continuing part, the results of experimental research, divided into the preliminary and main research, are presented. During the preliminary research phase, the most important assumptions were verified experimentally, while the conditions for preparing the specimens for tests were verified numerically. At the main research phase, the model parameters describing plastic and fatigue properties are given, and the evaluation of the proposed model (Cichański and Sempruch, 2005) is done.

2. Experimental procedure

The idea of the method for conducting an experimental research was outlined in Cichański and Sempruch (2005). The characteristic feature of the method is the possibility of obtaining controlled plastic straining of the tested material, as it is schematically shown in Fig. 1.

Higher quality constructional steel 45 in the shape of plates of size $450 \times 200 \times 4$ mm was used for the research. The chemical composition of the tested steel is presented in Table 1. The plates from which the specimens were cut out were subject to normalising in order to remove the texture resulting from rolling.

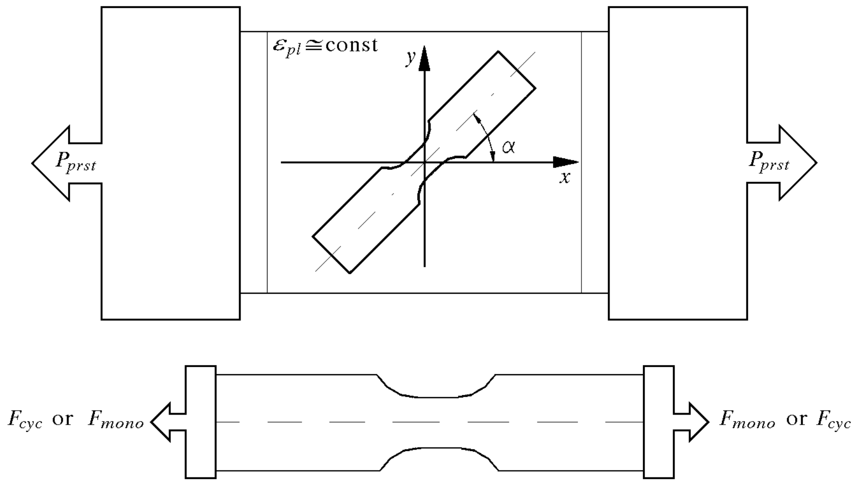

Fig. 1. The method of obtaining specimens for tests (Cichański and Sempruch, 2005)

Table 1. Tested steel chemical composition

Element	C	Mn	Si	P	S
Determined	0.48%	0.74%	0.42%	0.02%	0.01%
Polish Standard	0.42-0.5%	0.5-0.8%	0.1-0.4%	0-0.04%	0-0.04%

The research on plastic properties was conducted in a tension test carried out on flat bars, Fig. 2. The specimen geometric shape and other research conditions were in accordance with the Polish Standard [8].

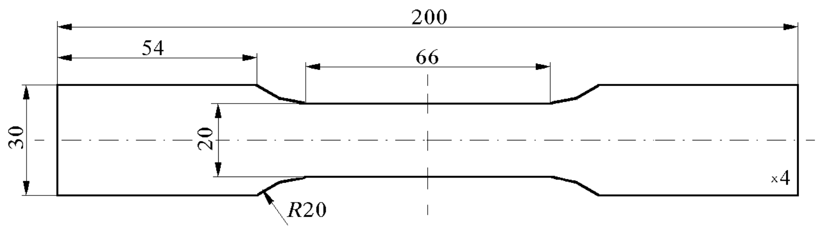

Fig. 2. Geometric shape of the specimen for determination of plastic properties

The fatigue research was carried out on a universal INSTRON 8501 machine in conditions of uniaxial sinusoid symmetrical ($R = -1$) tension-compression loading. For the preliminary research of fatigue properties flat dumbbell specimens, made of the virgin material, were used (Fig. 3) in accordance with the Polish Standard [9].

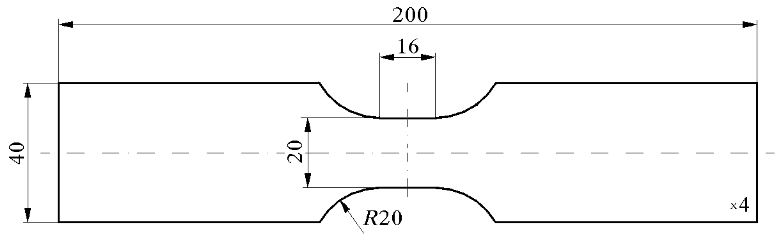

Fig. 3. Geometric shape of the specimen for the preliminary fatigue research

During the preliminary research, it was found that the specimen presented in Fig. 3, made of prestrained material, underwent buckling. A new geometrical specimen shape (Fig. 4), not subject to stability loss, was established experimentally. Eventually, in the main research, specimens made of prestrained material were used with geometry shown in Fig. 4.

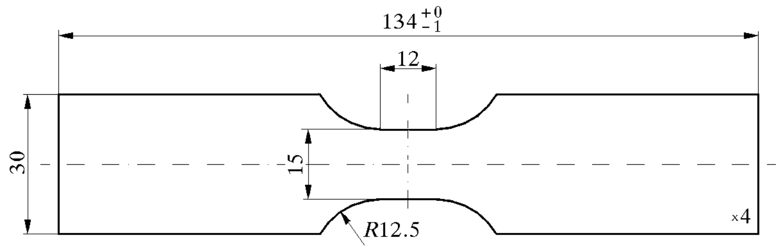

Fig. 4. Geometric shape of the specimen used in the main fatigue research

3. Verification of test conditions

3.1. Verification of the method of specimen preparation for tests

Due to specificity of the method of preparing specimens for tests inducing inhomogeneity of plastic strain distribution in restrained material plates, a numerical verification of the appointed method was carried out. The aim of the analysis was to define the plate area, where plastic strain gradients have negligible influence on the repeatability of properties test specimens. Numerical calculations were done with the use of the finite element method in the ANSYS program environment. The research was a two-dimensional nonlinear analysis, taking into account elastic-plastic material behaviour.

Making use double symmetry of both geometric shape and boundary conditions of the plate, one fourth of the plate was tested. Flat elements with two translatory degrees of freedom in each node, described as PLANE42 in the ANSYS library, were used for automatic meshing of the chosen plate area. The omitted plate parts were modelled by establishing adequate degrees of freedom in nodes lying on plate section edges. The elastic-plastic model with kinematic multi-linear hardening was used for description of material behaviour. Characteristic model points were read on a diagram σ-ε, obtained in a static tensile testing of the virgin material. The force course was controlled during calculations. Its highest value 300 kN was established on the basis of experimental research as a force causing a strain of about 2% in the measurement base of 200 mm. The calculation results in the form of a map of strains remaining in the plate after the initial plastic prestrain process are presented in Fig. 5.

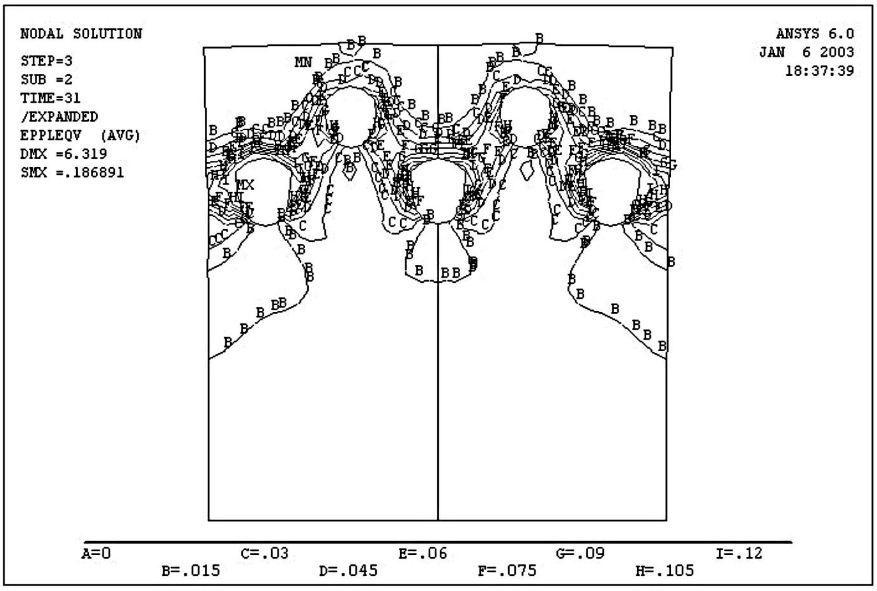

Fig. 5. The distribution of reduced plastic strains in the plate

The reduced plastic strains ε_{pl} take the highest values near the holes, through which the load is transmitted onto the plate. Moreover, the strains ε_{pl} in plate areas surrounding the holes are characterised by significant gradients. The strains are distributed more and more uniformly together with moving away from the hole line in the direction towards the plate centre. The relatively lower uniformity can be observed at the edge of the plate area, where the grip parts of specimens are to be shaped.

On the basis of carried out analysis, it can be stated that plastic strain gradients in the distance of approx. 100 mm from the plate centre have a negligible value from the point of view of the research purpose. Therefore, little variability of other mechanical properties in this area can be assumed. For further needs of the experimental research, the testing specimens cut out from the area of prestrained plates were assumed to be characterised by repeatable properties.

3.2. Research on plastic properties

The aim of this research phase was experimental verification of the assumption about isotropy of plastic properties of the virgin material. Also identification of the influence of plastic prestrain on the course of the yield stress defined on the basis of agreed methodology was carried out.

During preliminary metallographic tests, the preferential structural orientation of the virgin material was observed, which was the remains of the technological process of sheet rolling. Structures occurring near the sheet surface and in the middle of sheet intersection were chosen for metallographic observations. What results from the observations is that the heat treatment of annealing caused full recrystallisation only near the sheet surface (Fig. 6).

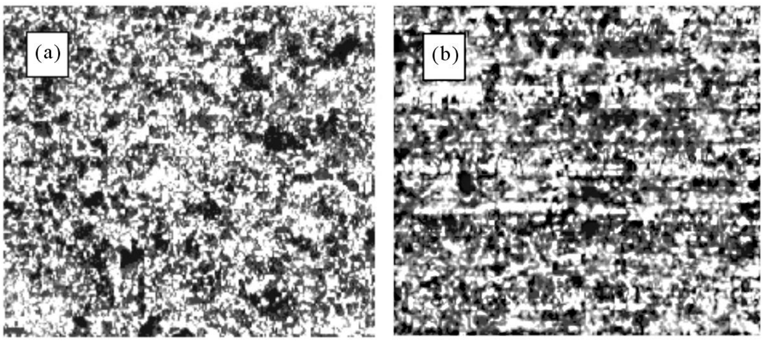

Fig. 6. Sheet metallographic structures: (a) near the surface; (b) in the middle of the intersection (multiplied by 80×)

To evaluate the observed banding influence on mechanical properties of the tested virgin material sheet, a number of tests on specimens cut out at the angles $\alpha \in \{0°, 15°, \ldots, 90°\}$ were carried out. Values R_e and R_m, obtained during the tests, are presented in Fig. 7. Apart from the experimentally determined values, the recommended R_e for tested steels according to Polish Standards is shown in Fig. 7 (dashed line).

To evaluate the method of preparing specimens for tests, a number of tests on specimens cut out from the prestrained material at the angles $\alpha \in \{0°, 15°, \dots, 90°\}$, for which $\varepsilon_{pl} = 2\%$. For this material, not showing a clear yield point, values of $R_{0.2}$, whose course is presented in Fig. 7, were found.

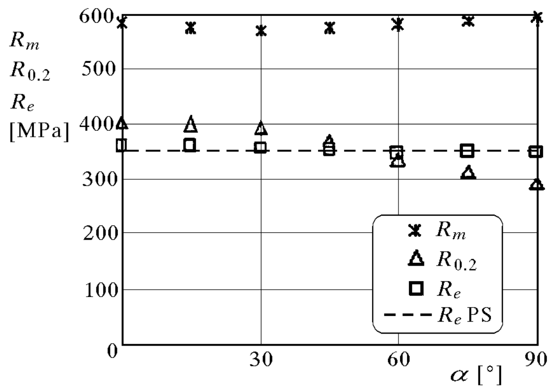

Fig. 7. Research results on chosen monotonic properties

The yield point R_e found for the virgin material reaches the predicted by PS value of 355 MPa with the accuracy of $\pm 3\%$, not showing a clear relation to the angle α. Also the course of R_m does not indicate the correlation of the tested properties with the sheet banding direction. For the needs of testing, it was assumed that for the virgin material, the assumption about the isotropy of plastic properties was fulfilled.

Introduction of material initial strains of $\varepsilon_{pl} = 2\%$ caused a distinct modification of plastic properties, which found its representation in the course of $R_{0.2}$, see Fig. 12. The largest growth of $R_{0.2}$ by $+14.9\%$ occurred in the direction of the initial plastic prestrain $\alpha = 0°$. In the direction perpendicular to the initial plastic prestrain $\alpha = 90°$ one observes the highest fall of $R_{0.2}$ by -16.3%. For needs of the tests, the course of changes of $R_{0.2}$ was assumed to well represent the anisotropy of mechanical properties of the prestrained material

3.3. Estimation of fatigue properties

The first stage of preliminary research on fatigue properties was to estimate the influence of the preferential orientation observed in the metallographic structure on the fatigue strength. Due to their time consumption, the tests were carried out for three directions of cut-out specimens at $\alpha = 0°$, 45° and 90°. The tests were conducted on flat dumbbell specimens made of the virgin material, see Fig. 3. During the tests the amplitude of sinusoid-cycle-variable stress with the frequency of 10 Hz was controlled. The test results in the form of Wöhler's lines are presented in Fig. 8.

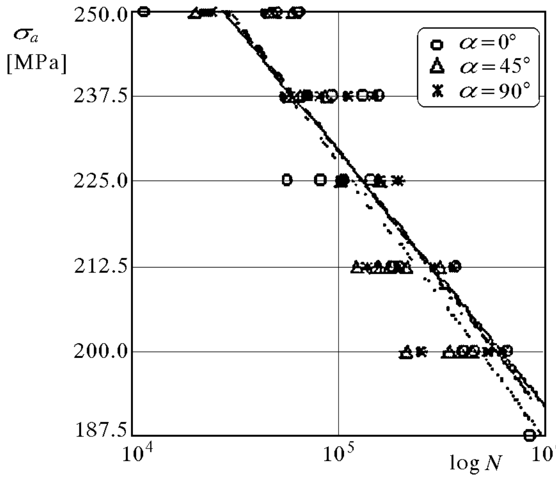

Fig. 8. Wöhler's lines for the virgin material

A slight aberration of Wöhler's lines observed for three angles α indicate that the fatigue properties of the tested material do not show a significant dependence on the test direction. Because of the test aim, an assumption was made that the virgin material was characterised by the isotropy of fatigue properties.

4. The main research

4.1. The aim and scope of research

The main goal of the research was to verify experimentally the allowable stress amplitude presented in the model approach in Cichański and Sempruch (2005). Due to specificity of the model formulated in the mentioned work, which consists in combining the plastic and fatigue properties altogether, the realization of the research goal required realisation of three detailed aims. The detailed aims were as follows: carrying out experimental research on plastic properties, carrying out experimental research on fatigue properties and analyzing the allowable stress amplitude on the basis of test results in both areas of the research.

The first group of the main research concerned the determination of the parameters describing the plastic part of the model. At that phase, the yield stress for seven directions described by angles $\alpha \in \{0°, 15°, \dots, 90°\}$ was determined. The tests were conducted for the virgin material as well as for the prestrained material.

In the second phase of research, the parameters describing the fatigue part of the model were determined. Due to a minor dependence of the fatigue properties of the virgin material on the investigated angle, Wöhler's line was determined for one direction described with the angle $\alpha = 90°$. For the prestrained material, Wöhler's lines were determined for three characteristic directions: $\alpha = 0°$, $45°$ and $90°$.

4.2. Results of the investigation on plastic properties

The results of experimental research on plastic properties for both types of materials are presented in the form of yield lines distinguished on the yield surface. For the virgin material, characterised by a distinct yield point, R_e was assumed as the yield stress value. For the prestrained material, which shows no clear yield point, values of yield stresses corresponding to three different plastic strains: $R_{0.05}$, $R_{0.1}$, $R_{0.2}$ were determined. The obtained results served to establish, on the basis of methodology described in Cichański and Sempruch (2005), experimental shapes of the BEC ellipse (Fig. 10) for the virgin material, and $B'E'C'$ for the prestrained material. Figure 9 presents both ellipses seen in the direction $D0$ (accordingly to designation in Fig. 10), while Fig. 11, shows them in the direction $A0$. In both figures, the BEC ellipse is marked with bold lines, while the remaining lines represent different shapes of the $B'E'C'$ ellipse, depending on plastic strains assumed for the definition of yield stresses.

For the virgin material, the yield line seen in the direction $D0$ (Fig. 9) is a line segment. For the angle $\alpha = 60°$, the line curves. For the prestrained material, the yield line shape depends on the strain ε_{pl} accepted as the material yield measure. In the case of assuming the yield stress as the quantity $R_{0.05}$, the yield line curving can be noticed both angles $\alpha < 15°$ and $\alpha > 45°$; therefore, the condition was omitted in further considerations. In the case of assuming the yield stress as $R_{0.1}$ or $R_{0.2}$, the yield line for angles $\alpha \in < 0°, 45° >$ is a straight line segment. The smallest value of ε_{pl} for which yield line does not curve was taken for further consideration. Yield stresses in the form of $R_{0.1}$ correspond to this case. The yield line curving for angles $\alpha > 45°$ can be observed; however, the effect deepens with increasing angle α. The strongest effect is observed for the angle $\alpha = 90°$.

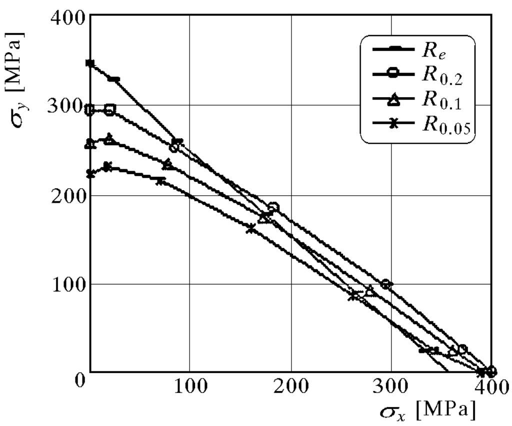

Fig. 9. Ellipses BEC and $B'E'C'$ – view along $D0$ direction

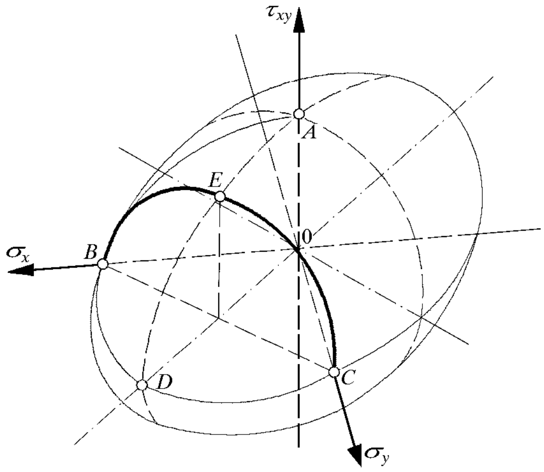

Fig. 10. Huber-Mises-Hencky's ellipsoid

For the virgin material, the yield line seen in the direction $A0$ (Fig. 11) is an ellipse. In the case of the prestrained material, a shift of this ellipse occurs in the direction σ_x – the same direction in which the plastic prestrain was made. All of this agrees with the hypothesis of kinematic hardening.

4.3. Results of the investigation on fatigue properties

The results of experimental tests on fatigue properties in the form of Wöhler's lines are presented in Fig. 12. The lines were determined with the use of specimens (Fig. 4) made of the virgin material and with the use of

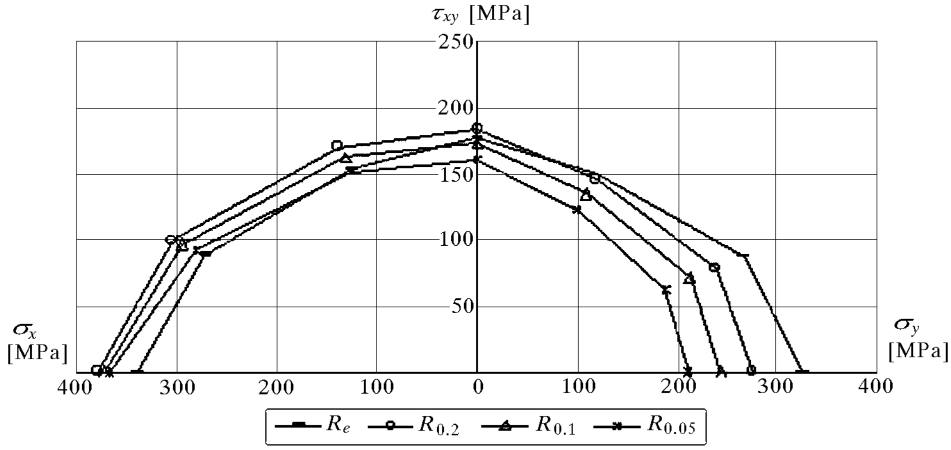

Fig. 11. Ellipses BEC and $B'E'C'$ – view along $A0$ direction

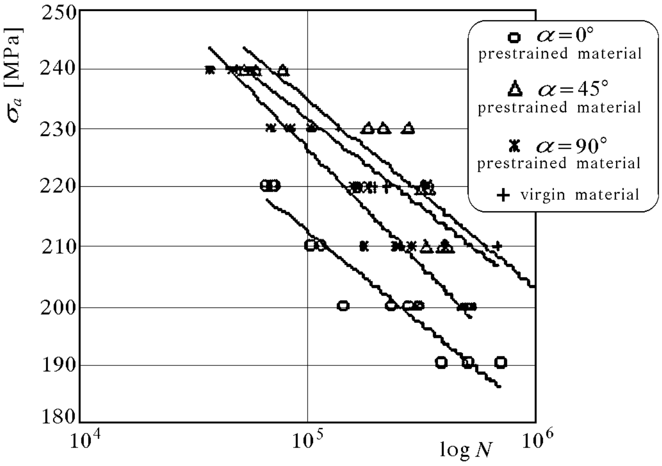

Fig. 12. Wöhler's line for the virgin and prestrained materials

specimens (Fig. 4) made of the prestrained material for three characteristic directions $\alpha = 0°$, $45°$ and $90°$.

During the research conducted for the prestrained material, the highest fatigue lives were achieved for the angle $\alpha = 45°$, lower fatigue lives were determined for the angle $\alpha = 90°$, while the lowest values were obtained for the angle $\alpha = 0°$. Fatigue lives determined for the virgin material are found slightly below the values determined for the prestrained material for the angle $\alpha = 45°$

$$\sigma_a^{virgin}(N) = 382.28 - 13.078 \ln N \tag{4.1}$$
$$\sigma_a^{prestr.0^\circ}(N) = 369.71 - 13.643 \ln N \tag{4.2}$$
$$\sigma_a^{prestr.45^\circ}(N) = 394.15 - 13.832 \ln N \tag{4.3}$$
$$\sigma_a^{prestr.90^\circ}(N) = 425.97 - 17.315 \ln N \tag{4.4}$$

Equations (4.1)-(4.3) characterising the behaviour of the virgin material and the prestrained material tested for the angle $\alpha = 0^\circ$ and 45° are described with parameters slightly different from each other. Due to only minor differences in the direction coefficient, the lines described with these equations are situated in a slightly non-parallel way. The decisive factor for the position of Wöhler's lines in a particular direction is the absolute term in the equations taking the highest value for the prestrained material and angle $\alpha = 45^\circ$, see (4.3), and slightly lower for the virgin material, see (4.1). The lowest value of the absolute term, corresponding to the lowest fatigue lives, was determined for the prestrained material and $\alpha = 0^\circ$, (4.2). The equation describing the behaviour of the prestrained material tested at the angle $\alpha = 90^\circ$, (4.4), both for the directional coefficient and the absolute term, considerably differs from the other three equations. The Wöhler line described with equation (4.4) runs obliquely with respect to Wöhler's lines described with equations (4.1)-(4.3).

5. Achieved results and comparison of proposed models

The results achieved during the experimental research on plastic and fatigue properties served to verify the allowable stress amplitude described in Cichański and Sempruch (2005). In the plastic part, the model parameter was the plastic strain accepted for definition of the yield stress. To verify notation used in the model $R_{0.1}$ was accepted as a definition of yield stress. In the fatigue part, the model parameter was the number of the limiting cycle established for considerations. The verification of notation was made for the considered fatigue life of $N_{lim} = 200\,000$ cycles.

The main model parameter is the coefficient a_N. Its value decides about the relationship between the $B'E'C'$ plastic ellipse and $b'e'c'$ fatigue one, determined for the prestrained material. The coefficient a_N was determined on the basis of experimental results for the virgin material. The yield stress value ε_{pl}^{izo} equalled 346 MPa. The stress value σ_A^{izo} defined on the basis of dependence (4.1) for the considered fatigue life N_{lim} was 222.6 MPa. Eventually, the a_N parameter obeys a following formula

$$a_{200\,000} = \frac{\sigma_A^{izo}}{\sigma_{pl}^{izo}} = \frac{222.6\,\text{MPa}}{346\,\text{MPa}} = 0.64 \tag{5.1}$$

According to the accepted assumption about the effect of allowable amplitude of the fatigue stress on yield stresses, and using the established a_N coefficient as well as plastic properties found from experiments, the model ellipse $b'e'c'$ was determined. Its course is presented in Fig. 14 with lines having triangle markers. For comparison, in the same figure, the plastic ellipse $B'E'C'$ determined for the prestrained materials indicated with square markers.

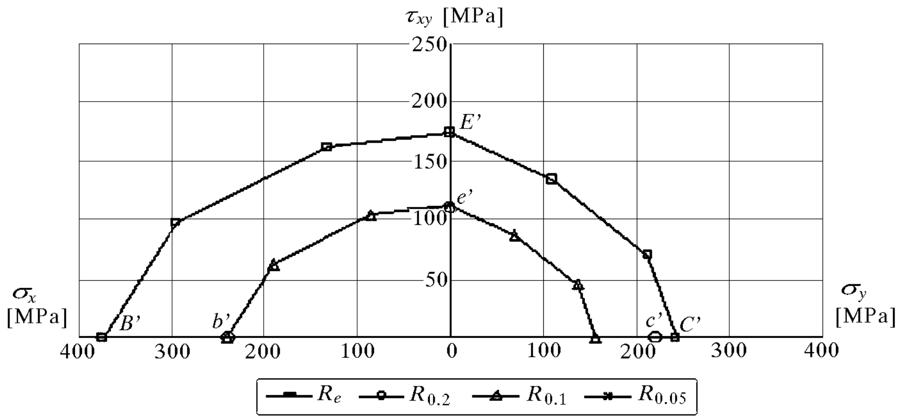

Fig. 13. Results of investigatiions and comparison of proposed models

The correctness of the model ellipse $b'e'c'$ was verified in three points corresponding to three characteristic directions $\alpha = 0°$, $45°$ and $90°$, where the fatigue research was conducted. According to the accepted assumption on the isotropy of fatigue properties, the stress σ_A^{izo} for these directions was established on the basis of dependence (4.1), see Table 2. The σ_A^{anizo} stress, being a modified quantity for particular angles α, and determined for the considered fatigue life N_{lim} on the basis of (4.2)-(4.4), is also presented in Table 2.

Table 2. Fatigue properties

α [°]	σ_A^{izo} [MPa]	σ_A^{anizo} [MPa]
0	222.6	203.2
45	222.6	225.3
90	222.6	214.6

The stress σ_A^{model}, determined for three characteristic directions on the basis of plastic properties and formula (5.1), the stress σ_M^{eksp}, determined on the basis of fatigue properties and the model approach to the allowable amplitude for these directions, are presented in Table 3. The table also presents the relative error resulting from the comparison of σ_M^{eksp} and σ_A^{model}.

Table 3. Numerical results for the proposed model

α [°]	σ_M^{eksp} [MPa]	σ_A^{model} [MPa]	Error [%]
0	252.9	254.5	−0.63
45	218.5	222.8	−1.99
90	235.1	166.3	29.29

The stresses σ_M^{eksp}, calculated for appropriate angles α, after some transformations, are presented in Fig. 14 with round markers. For angles $\alpha = 0°$ and $45°$, the points determined on the basis of the model course are situated on the model ellipse $b'e'c'$. The numerical error determined for these angles is smaller than 2%. For the angle $\alpha = 90°$, the point determined on the basis of the model course lies outside the model ellipse $b'e'c'$. In this case, the numerical error equals approximately 30%.

6. Conclusions

Experimental verification of the research conditions allows one to accept an assumption on the isotropy of plastic and fatigue properties of the virgin material. On the basis of the conducted numerical analysis, it can be assumed that, despite noticeable influence of the manner in which plates are loaded on the plastic strain distribution, there exists such an area on the prestrained material plate from which specimens for tests with repeatable mechanical properties can be cut out.

The agreement between the model and research results for angles $\alpha = 0°$ and $45°$ as well as lack of such agreement for $\alpha = 90°$ were observed. The determined difference is caused by two factors. The yield line, found from experiments on plastic properties, shows curving for the discussed angle. Wöhler's line, resulting from research on fatigue properties in the prestrained material ($\alpha = 90°$) runs obliquely towards Wöhler's line determined for the remaining angles. On this basis, it can be assumed that the allowable stress amplitude in the model approach sufficiently describes the material behaviour as for the non-deformed yield line.

The research method proposed in the paper successfully underwent the experimental verification of most important assumptions and the numerical verification of specimen preparation for testing conditions. The paper indicated the possibility of application of that method for determination of proposed characteristic that describe biaxial fatigue properties of an elastic-plastic material.

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Doświadczalna metoda wyznaczania dwuosiowych własności zmęczeniowych sprężysto-plastycznych materiałów konstrukcyjnych. Część 2 – Badania eksprerymentalne

Streszczenie

Niniejsza praca jest zorientowana na własną metodę wyznaczania dwuosiowych własności zmęczeniowych sprężysto-plastycznych materiałów konstrukcyjnych, dla której podstawy teoretyczne zostały sformułowane wcześniej (Cichański i Sempruch, 2005). W prezentowanej pracy przedstawiono wyniki badań doświadczalnych właściwych dla zaproponowanej metody, podzielonych na badania wstępne i główne. Na etapie badań wstępnych wykonano doświadczalną weryfikację najważniejszych założeń metody oraz numeryczną weryfikację warunków przygotowania próbek do badań. Na etapie badań zasadniczych wyznaczono modelowe parametry opisu własności plastycznych i zmęczeniowych oraz dokonano oceny modelu zaproponowanego w Cichański i Sempruch (2005).

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