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DESCRIPTION OF MOTION OF A MOBILE ROBOT BY MAGGIE'S EQUATIONS

Józef Giergiel
Wiesław Żylski

Rzeszow University of Technology
e-mail: jgiergiel@chello.pl; wzylski@prz.edu.pl

This work analyses problems of dynamics of a given two-wheeled mobile robot. Motion of such a non-holonomic system is described by Lagrange's and Maggie's equations. The received dynamical equations of motion allow for solving the direct and converse of problem dynamics. Maggie's equations give a simpler form of the dynamical equations of motion. While solving the converse problem with the use of these equations, it is possible to determine the driving torque acting on the robot wheel. Values of the torque allow one to create a follow-up movement control algorithm of the robot.

Key words: wheeled mobile robots, Maggie's equations

1. Analytical modelling

The analysis of the problems of dynamics of a two-wheeled mobile robot is mainly made in order to properly solve the problem of control of motion of such systems. Simple models, which do not take into account mass of many mobile elements, are very often assumed for the system description. Authors describing dynamics of such systems use classical equations taken from general mechanics. Most often their studies describe motion of those systems using Lagrange's equation of the second kind.

Two-wheeled mobile robots are an interesting structural solution. The model presented schematically in Fig.1 – Fig. 4 has been accepted for the description of such a robot. The model consists of frame 4, two road driving wheels 1 and 2 and a free rolling castor wheel 3. For dynamical description, a virtual model with virtual driving wheel $1z$ with its centre in point A and radius r, with nodal line described as α, substituting driving wheels 1, 2 and free rolling wheel 3, was assumed. For the model accepted in such a form, assuming that there is one plane of motion, unambiguous formulation requires setting of the coordinates of the point A, angle of instantaneous rotation β, and angle α, that is the following coordinates: x_A, y_A, β, α. The analysed system is non-holonomic, so these coordinates are linked by equations of constraints imposed on the velocity

$$\dot{x}_A - r\dot{\alpha}\cos\beta = 0 \qquad\qquad \dot{y}_A - r\dot{\alpha}\sin\beta = 0 \tag{1.1}$$

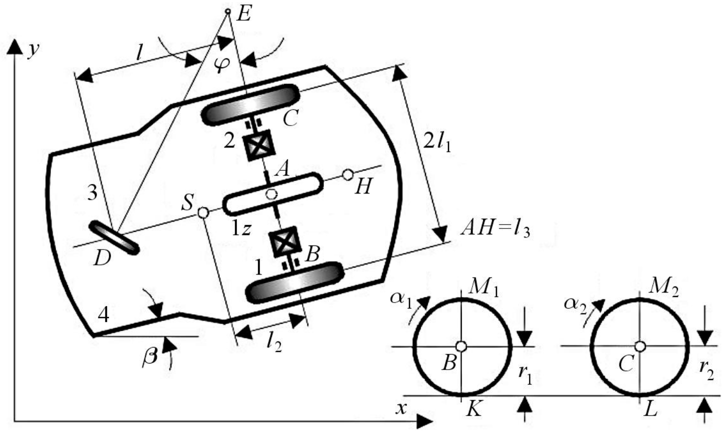

Fig. 1. The mobile robot model

Dependences (1) arise from projections of the point of contact of wheel $1z$ contact with the road onto XY axes. These are equations of classical non-holonomic constraints, which can be written in a matrix form

$$\mathbf{J}(\boldsymbol{q})\dot{\boldsymbol{q}} = \mathbf{0} \tag{1.2}$$

where the Jacobian matrix is defined as

$$\mathbf{J}(\boldsymbol{q}) = \begin{bmatrix} 1 & 0 & 0 & -r\cos\beta \\ 0 & 1 & 0 & -r\sin\beta \end{bmatrix} \tag{1.3}$$

whereas $\dot{\boldsymbol{q}} = [\dot{x}_A, \dot{y}_A, \dot{\beta}, \dot{\alpha}]^\top$.

To describe motion of the model, Lagrange's equations of the second kind are used. For a non-holonomic system they are written in the following vector form

$$\frac{d}{dt}\Big(\frac{\partial E}{\partial \dot{\boldsymbol{q}}}\Big)^\top - \Big(\frac{\partial E}{\partial \boldsymbol{q}}\Big)^\top = \boldsymbol{Q} + \mathbf{J}^\top(\boldsymbol{q})\boldsymbol{\lambda} \tag{1.4}$$

where

$\boldsymbol{q}$	–	vector of generalized coordinates, $\boldsymbol{q} = [x_A, y_A, \beta, \alpha]^\top$
E	–	kinetic energy of the system, $E = E(\boldsymbol{q}, \dot{\boldsymbol{q}})$
$\boldsymbol{Q}$	–	generalized forces vector involving rolling resistance of the wheels and the driving torque of wheels 1 and 2
$\mathbf{J}(\boldsymbol{q})$	–	Jacobian matrix resulting from equations of constraints (1.1)
$\boldsymbol{\lambda}$	–	vector of Lagrange's multipliers.

In the analysed model there are dry friction forces lying in the plane of contact of wheel $1z$ with the road, applied in the point of contact with the road. Dynamical equations of motion of the analysed model are defined by the following system of differential equations (Giergiel *et al.*, 2002; Żylski, 1997, 1999)

$$
\begin{aligned}
&(m_1 + m_2 + m_4)\ddot{x}_A + [(m_1 - m_2)l_1 \cos\beta + m_4 l_2 \sin\beta]\ddot{\beta} \cdot \\
&\cdot[(-m_1 + m_2)l_1 \sin\beta + m_4 l_2 \cos\beta](\dot{\beta})^2 = \lambda_1 \\
&(m_1 + m_2 + m_4)\ddot{y}_A + [(m_1 - m_2)l_1 \sin\beta - m_4 l_2 \cos\beta]\ddot{\beta} \cdot \\
&\cdot[(m_1 - m_2)l_1 \cos\beta + m_4 l_2 \sin\beta](\dot{\beta})^2 = \lambda_2 \\
&[(m_1 - m_2)l_1 \cos\beta + m_4 l_2 \sin\beta]\ddot{x}_A + [(m_1 - m_2)l_1 \sin\beta - m_4 l_2 \cos\beta]\ddot{y}_A + \\
&+[(m_1 + m_2)l_1^2 + m_4 l_2^2 + I_{z4} + I_{x1} + I_{x2} + (I_{z1} + I_{z2})h_1^2]\ddot{\beta} + \\
&+(I_{z1} - I_{z2})h_1\ddot{\alpha} = (M_1 - M_2 - N_1 f_1 + N_2 f_2)h_1 \\
&(I_{z1} + I_{z2})\ddot{\alpha} + (I_{z1} - I_{z2})h_1\ddot{\beta} = \\
&= M_1 + M_2 - N_1 f_1 - N_2 f_2 - \lambda_1 r \cos\beta - \lambda_2 r \sin\beta
\end{aligned}
\tag{1.5}
$$

where

$m_1 = m_2, m_4$	–	virtual masses of wheels 1 and 2 and frame, respectively
$I_{x1} = I_{x2}$	–	mass moments of inertia of wheels 1 and 2 with respect to x_1 and x_2 axis
$I_{z1} = I_{z2}$	–	mass moments of inertia of these wheels with respect to the nodal line
I_{z4}	–	mass moment of inertia of the frame with respect to the z_4 axis connected to the frame.

It has been assumed that the axe of the reference system connected to the i-part are main central axes of inertia, while

N_1, N_2	–	loads applied to corresponding wheels
f_1, f_2	–	coefficients of rolling friction of corresponding wheels
M_1, M_2	–	driving torques
l, l_1, l_2, h_1	–	corresponding distances resulting from the system geometry, $h_1 = r_1 l_1^{-1}$
$r_1 = r_2 = r$	–	radii of corresponding wheels.

The above equations do not take into account the small mass of wheel 3 and rolling friction of this wheel.

While analysing converse the dynamical problem, if the kinematic parameters of motion are known, it is possible to determine the driving torque and the multipliers out of the derived equations of motion. On account of the found form of the right-hand sides of these equations, it is advantageous to introduce a transformation for decoupling the multipliers from the torques (Żylski, 1996). Aiming at this, equations (1.5) should be expressed in a matrix form

$$\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\boldsymbol{q} = \mathbf{B}(\boldsymbol{q})\boldsymbol{\tau} + \mathbf{J}^\top(\boldsymbol{q})\boldsymbol{\lambda} \tag{1.6}$$

The matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{B}$ present in the above equations result from dynamical equations of motion (1.5). The vector of coordinates $\boldsymbol{q}$ can be decomposed as

$$\boldsymbol{q} = [\boldsymbol{q}_1, \boldsymbol{q}_2]^\top \qquad \boldsymbol{q} \in R^n, \quad \boldsymbol{q}_1 \in R^m, \quad \boldsymbol{q}_2 \in R^{n-m} \tag{1.7}$$

In this case, equation of constraints (1.1) is given in the form

$$[\mathbf{J}_1(\boldsymbol{q}), \mathbf{J}_2(\boldsymbol{q})] \begin{bmatrix} \dot{\boldsymbol{q}}_1 \\ \dot{\boldsymbol{q}}_2 \end{bmatrix} = \mathbf{0} \qquad \det \mathbf{J}_1(\boldsymbol{q}) \neq 0 \tag{1.8}$$

The vector $\boldsymbol{q}_2$ should be selected in such a way so that its size would correspond to the number of degrees of freedom and so that $\det \mathbf{J}_1(\boldsymbol{q}) \neq 0$. In such a case

$$\dot{\boldsymbol{q}} = \begin{bmatrix} \mathbf{J}_{12}(\boldsymbol{q}) \\ \mathbf{I}_{n-m} \end{bmatrix} \dot{\boldsymbol{q}}_2 = T(\boldsymbol{q})\dot{\boldsymbol{q}}_2 \qquad \ddot{q} = \dot{T}(\boldsymbol{q})\dot{\boldsymbol{q}}_2 + T(\boldsymbol{q})\ddot{\boldsymbol{q}}_2 \tag{1.9}$$

where $\mathbf{J}_{12} = -\mathbf{J}_1^{-1}(\boldsymbol{q})\mathbf{J}_2(\boldsymbol{q})$, and $\mathbf{I}_{n-m}$ stands for the identity matrix. In this case, dynamical equations of motion (1.6) can be put down in the form

$$\begin{aligned} M_{12}(\boldsymbol{q})\ddot{\boldsymbol{q}}_2 + C_{12}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_2 &= \mathbf{B}_1(\boldsymbol{q})\boldsymbol{\tau} + \mathbf{J}_1^\top(\boldsymbol{q})\boldsymbol{\lambda} \\ M_{22}(\boldsymbol{q}_2)\ddot{\boldsymbol{q}}_2 + C_{22}(\boldsymbol{q}_2\dot{\boldsymbol{q}}_2)\dot{\boldsymbol{q}}_2 &= \mathbf{B}_2(\boldsymbol{q}_2)\boldsymbol{\tau} \end{aligned} \tag{1.10}$$

The structure of matrices present in equations (1.10) is described in the work by Żylski (1996). Obtained equations (1.10) show the so-called reduced form of description of the system with non-holonomic constraint imposed on motion. If we assume that $\boldsymbol{q}_2 = [\beta, \alpha]^\top$, system of equations (1.10) can be written in the form

$$
\begin{aligned}
&m_4 l_2[\ddot{\beta}\sin\beta + (\dot{\beta})^2\cos\beta] + (2m_1 + m_4)r[\ddot{\alpha}\cos\beta - \dot{\alpha}\dot{\beta}\sin\beta] = \lambda_1\\
&m_4 l_2[-\ddot{\beta}\cos\beta + (\dot{\beta})^2\sin\beta] + (2m_1 + m_4)r[\ddot{\alpha}\sin\beta + \dot{\alpha}\dot{\beta}\cos\beta] = \lambda_2\\
&(2m_1 l_1^2 + m_4 l_2^2 + I_{z4} + 2I_{x1} + 2I_{z1}h_1^2)\ddot{\beta} - m_4 l_2 r\ddot{\alpha}\dot{\beta} =\\
&= (M_1 - M_2 - N_1 f_1 + N_2 f_2)h_1[(2m_1 + m_4)r^2 + 2I_{z1}]\ddot{\alpha} + m_4 l_2 r(\dot{\beta})^2 =\\
&= M_1 + M_2 - N_1 f_1 - N_2 f_2
\end{aligned}
\tag{1.11}
$$

In equations (1.11), the right-hand sides are decoupled – such form of equations facilitates their solution. In the case of analysis of dynamical converse problem (1.11), they allow for determination of driving torques a well as Lagrange's multipliers. In the case of analysis of the direct problem, the solution to these equations will allow for determination of kinematic parameters of the analysed model.

2. Maggie's dynamical equations of motion

Motion of non-holonomic systems can be described with the use of another mathematical formalism, e.g. Maggie's equations (Gutowski, 1971). These equations, describing motion of systems in terms of generalized coordinates, are determined in the form

$$
\sum_{j=1}^{n} C_{ij}\Big[\frac{d}{dt}\Big(\frac{\partial E}{\partial \dot{q}_j}\Big) - \frac{\partial E}{\partial q_j}\Big] = \Theta_i \qquad i = 1, \dots, s \tag{2.1}
$$

where s stands for the number of independent parameters of the system expressed in generalized coordinates q_j $(j = 1, \dots, n)$, the number of which corresponds to the number of the system degrees of freedom. All generalized velocities are in this case given by

$$
\dot{q}_j = \sum_{i=1}^{s} C_{ij}\dot{e}_i + G_j \tag{2.2}
$$

The variables $\dot{e}_i$ are called the characteristics or kinetic parameters of the system defined in the generalized coordinates. The right-hand sides of equations

(2.1) are coefficients for variations δe_i. From the principle of virtual work, these coefficients can be defined by the relation (Giergiel *et al.*, 2004)

$$\sum_{i=1}^{s} \Theta_i \delta e_i = \sum_{i=1}^{s} \delta e_i \sum_{j=1}^{n} C_{ij} Q_j \tag{2.3}$$

The given Maggie's equations can be used to describe motion of a two-wheeled mobile robot. Assuming that the mobile robot, whose model is shown in Fig. 1, moves in one plane, the unique formulatiion of the model requires giving the position of the point A, i.e. coordinates of this point x_A and y_A and the frame orientation, i.e. the instantaneous angle of rotation β as well as positions describing nodal lines of driving wheels denoted respectively by α_i and α_2. In this case, vectors of generalized coordinates and generalized velocities have the form

$$\begin{aligned} \boldsymbol{q} &= [x_A, y_A, \beta, \alpha_1, \alpha_2]^\top \\ \dot{\boldsymbol{q}} &= [\dot{x}_A, \dot{y}_A, \dot{\beta}, \dot{\alpha}_1, \dot{\alpha}_2]^\top \end{aligned} \tag{2.4}$$

while determining the external forces effecting the analysed system, one should take into account also the unknown dry friction forces existing in the plane of contact of the driving wheels with the road. Figure 2 shows distribution of these forces, where $\overline{T}_{10}$ and $\overline{T}_{20}$ stand for circumferential dry friction forces, and $\overline{T}_{1P}$ and $\overline{T}_{2P}$ for transverse dry friction forces.

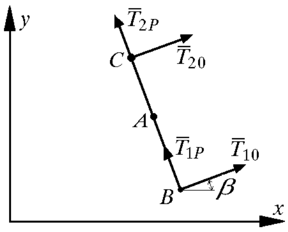

Fig. 2. Friction forces actong on driving wheels

The described system has two degrees of freedom. Assuming that positions of nodal lines of the driving wheels α_1 and α_2 are independent coordinates, the decomposition of velocity vectors of the points A, B and C results in the following relationship to velocities

$$\begin{aligned} \dot{\beta} &= \frac{r}{2l_1}(\dot{\alpha}_1 - \dot{\alpha}_2) \qquad \dot{x}_A = \frac{r}{2}(\dot{\alpha}_1 + \dot{\alpha}_2)\cos\beta \\ \dot{y}_A &= \frac{r}{2}(\dot{\alpha}_1 + \dot{\alpha}_2)\sin\beta \end{aligned} \tag{2.5}$$

All generalized velocities can be noted on the basis of equation (2.2) as

$$
\begin{aligned}
\dot{q}_1 &= \dot{x}_A = \frac{r}{2}(\dot{\alpha}_1 + \dot{\alpha}_2)\cos\beta = \frac{r}{2}(\dot{e}_1 + \dot{e}_2)\cos\beta = C_{11}\dot{e}_1 + C_{21}\dot{e}_2 + G_1\\
\dot{q}_2 &= \dot{y}_A = \frac{r}{2}(\dot{\alpha}_1 + \dot{\alpha}_2)\sin\beta = \frac{r}{2}(\dot{e}_1 + \dot{e}_2)\sin\beta = C_{12}\dot{e}_1 + C_{22}\dot{e}_2 + G_2\\
\dot{q}_3 &= \dot{\beta} = \frac{r}{2l_1}(\dot{\alpha}_1 - \dot{\alpha}_2) = \frac{r}{2l_1}(\dot{e}_1 - \dot{e}_2) = C_{13}\dot{e}_1 + C_{23}\dot{e}_2 + G_3\\
\dot{q}_4 &= \dot{\alpha}_1 = \dot{e}_1 = C_{14}\dot{e}_1 + C_{24}\dot{e}_2 + G_4\\
\dot{q}_5 &= \dot{\alpha}_2 = \dot{e}_2 = C_{15}\dot{e}_1 + C_{25}\dot{e}_2 + G_5
\end{aligned}
\tag{2.6}
$$

That means that the coefficients C_{ij} and G_j assume the following values

$$
\begin{array}{lll}
C_{11} = \dfrac{r}{2}\cos\beta & C_{21} = \dfrac{r}{2}\cos\beta & G_1 = 0\\
C_{12} = \dfrac{r}{2}\sin\beta & C_{22} = \dfrac{r}{2}\sin\beta & G_2 = 0\\
C_{13} = \dfrac{r}{2l_1} & C_{23} = -\dfrac{r}{2l_1} & G_3 = 0\\
C_{14} = 1 & C_{24} = 0 & G_4 = 0\\
C_{15} = 0 & C_{25} = 1 & G_5 = 0
\end{array}
\tag{2.7}
$$

The generalized forces, defined out of equation (2.3) and the coefficients C_{ij} defined by formula (2.7),finally yield

$$
\begin{aligned}
\Theta_1 &= C_{11}Q_1 + C_{12}Q_2 + C_{13}Q_3 + C_{14}Q_4 + C_{15}Q_5 = M_1 - N_1 f_1\\
\Theta_2 &= C_{21}Q_1 + C_{22}Q_2 + C_{23}Q_3 + C_{24}Q_4 + C_{25}Q_5 = M_2 - N_2 f_2
\end{aligned}
\tag{2.8}
$$

In the analysed case, Maggie's equations have the following form

$$
\begin{aligned}
&(2m_1 + m_4)\Big(\frac{r}{2}\Big)^2(\ddot{\alpha}_1 + \ddot{\alpha}_2) + 2m_4\Big(\frac{r}{2l_1}\Big)^2 r l_2(\dot{\alpha}_2 - \dot{\alpha}_1)\dot{\alpha}_2 + I_{z1}\ddot{\alpha}_1 +\\
&+(2m_1 l_1^2 + m_4 l_2^2 + 2I_{x1} + I_{z4})\frac{r}{2l_1}(\ddot{\alpha}_1 - \ddot{\alpha}_2) = M_1 - N_1 f_1\\
&(2m_1 + m_4)\Big(\frac{r}{2}\Big)^2(\ddot{\alpha}_1 + \ddot{\alpha}_2) + 2m_4\Big(\frac{r}{2l_1}\Big)^2 r l_2(\dot{\alpha}_1 - \dot{\alpha}_2)\dot{\alpha}_1 + I_{z2}\ddot{\alpha}_2 +\\
&-(2m_1 l_1^2 + m_4 l_2^2 + 2I_{x1} + I_{z4})\frac{r}{2l_1}(\ddot{\alpha}_1 - \ddot{\alpha}_2) = M_2 - N_2 f_2
\end{aligned}
\tag{2.9}
$$

It should be mentioned that if relationships (2.5) are introduced into equations (1.5), and $\dot{\alpha} = (\dot{\alpha}_1 + \dot{\alpha}_2)/2$ is taken into account, then elimination of the multipliers out of these equations will result in Maggie's equations, i.e. system (2.9). Thus, the Maggie's method allows one to avoid the elimination of the multipliers existing in Lagrange' equations, which in the case of complex systems proves to be really laborious. Maggie's equations give an advantageous form of the equations of motion for solving both direct and converse dynamical problems.

3. Numerical verification

On the basis of the obtained equations in during the analysis of the converse dynamical and kinematical problem of a two-wheeled mobile robot, computer simulation of motion was carried out using the Matlab/Simulink package. Various periods of motion were examined: the start-up running with a constant velocity of the characteristic point A, stop, running along a straight line and a circular arc of the radius R. The following data were used in these simulations Coefficients occurring in the equations

$m_1 = m_2 = 1.5$ kg	$m_4 = 5.67$ kg	$r_1 = r_2 = 0.0825$ m
$l_1 = 0.163$ m	$l_2 = 0.07$ m	$l = 0.217$ m
$I_{z1} = I_{z2} = 0.007\ \mathrm{kg\,m^2}$	$I_{x1} = I_{x2} = 0.003\ \mathrm{kg\,m^2}$	$I_{z4} = 0.154\ \mathrm{kg\,m^2}$
$N_1 = N_2 = 31.25$ N	$f_1 = f_2 = 0.01$ m	

The obtained results of computer simulation for $R = 2$ m and $v_A = 0.3$ m/s are presented in Fig. 3 and Fig, 4.

Figure 3a presents changes (taking place when the robot is running) of the characteristic angular parameters of motion, i.e. angular velocities of the driving wheels $\dot{\alpha}_1$ and $\dot{\alpha}_2$, angular velocity of the virtual wheel nodal line $\dot{\alpha}$ and angular acceleration $\ddot{\alpha}$. Figure 3b presents changes (taking place when the robot is running) of the characteristic angular parameters of the frame; i.e. the angle of rotation β, angular velocity $\dot{\beta}$ and angular acceleration $\ddot{\beta}$. The first two periods of motion are: starting and running with a constant velocity v_A – in this case, the robot frame moves translationally. During these stages of motion, the angular velocities of driving wheels are equal and the rotation sped is zero. The third stage of motion corresponds to the running along a circular arc. The robot frame moves in a plane, the angular velocities of nodal lines of driving wheels 1 and 2 are changing, also the instantaneous angular velocity of the frame is changing. The fourth and fifth stages of motion, presented in the picture, correspond respectively to the running with a constant velocity v_A

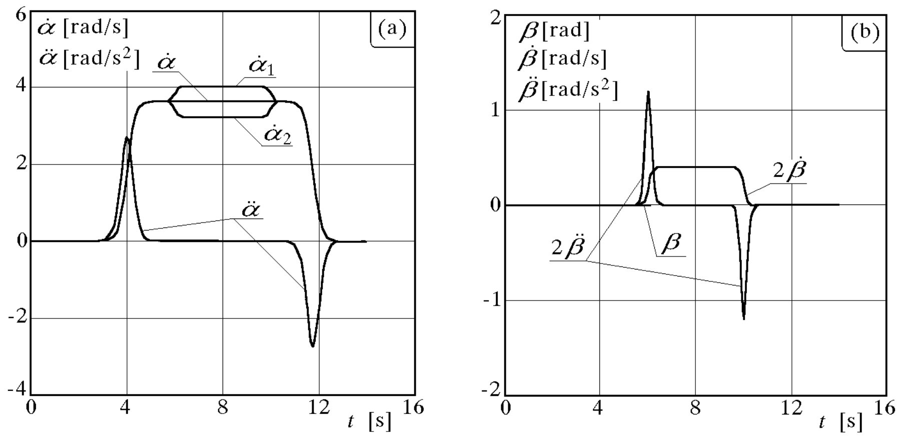

Fig. 3. Parameters of motion

after going out of the circular turn and braking while the mobile robot frame moved translationally. During these stages of motion the values of angular velocity of driving wheels 1 and 2 are equal, and the instantaneous angular velocity is zero. Figure 4 presents changing values of the driving torque of wheel 1 and 2, corresponding to M_1 and M_2 in the above mentioned periods of motion. The values of these torques are determined by equations (2.9), the differences in the torques occur during the running along the angular turn, in the remaining stages of motion these values are the same. During running with a constant velocity v_A, the driving torques are constant.

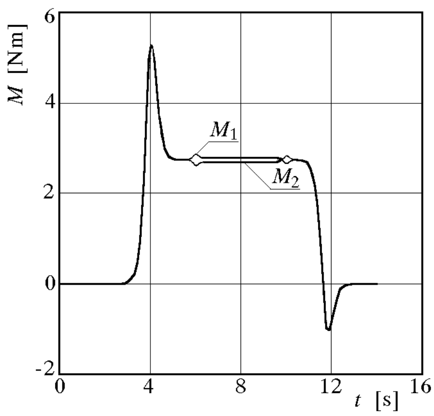

Fig. 4. Driving torques

4. Comments and conclusions

In the problem of dynamics of mobile robots, we are interested in values of driving torques propelling wheels, which means that we analyse the converse problem of dynamics. When motion of the analysed model is described by Lagrange's formalism, dynamical equations of motion are the result; however, because of a very complex form of these equations, their solution is very complicated. It is necessary then to decouple Lagrange's multipliers form torques, in order to reduce the equations of motion to the so-called reduced form describing a system with non-holonomical constrains. The reduced form facilitates the determination of the examined torques, and consistently, determination of Lagrange's multipliers, i.e. dry friction forces acting in the plane of contact between wheel and the road. Knowledge of these values allows one to solve the problem of controlling the follow-up movement of the wheeled mobile robot. It is also possible to describe the dynamics of the wheeled mobile robot using Maggie's equations. In this case, the complicated process of elimination of the multipliers, existing in Lagrange's formalism, is avoided.

This analysis of dynamics of a model of a two-wheeled mobile robot, enables description of dynamics of all other models of such vehicles. The obtained solutions may be used both by designers and operators of such equipment as well as design engineers of systems controlling the follow-up movement of wheeled robots (Żylski, 1997, 1999, 2004).

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Opis ruchu mobilnego robota równaniami Maggiego

Streszczenie

W pracy analizuje się zagadnienia dynamiki wybranego dwukołowego mobilnego robota. Ruch tego nieholonomicznego układu opisano równaniami Lagrange'a oraz równaniami Maggiego. Otrzymane dynamiczne równania ruchu umożliwiają rozwiązywanie zadania prostego i odwrotnego dynamiki. Równania Maggiego dają prostszą formę dynamicznych równań ruchu. Przy rozwiązywaniu zadania odwrotnego dynamiki z równań tych można bezpośrednio określić wartości momentów napędzających koła mobilnego robota. Wartości tych momentów umożliwiają zaprojektowanie algorytmu sterowania ruchem nadążnym robota.

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