JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
42, 2, pp. 269-283, Warsaw 2004

THE EFFECT OF THE METHOD OF DETERMINATION OF YOUNG'S MODULUS ON THE ESTIMATION OF FATIGUE LIFE OF STRUCTURAL ELEMENTS

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Cyclic loading of a material entails modifications of its properties. In the paper, the problem of the influence of determination of the elasticity modulus E on calculations of the fatigue life of structural elements was presented. Different values of the modulus of elasticity obtained by different methods were used for modelling of cyclic stress-strain curves and for analysis of local stresses and strains. In the calculations the strain-life and energy-life approach was applied.

Key words: fatigue, modulus of elasticity, material properties

1. Introduction

The divergence between fatigue lives of structural elements determined analytically and experimentally is the cause for continuous search for the ways of modification of the existing methods of fatigue life calculation or creating new ones. The research conducted at the Department of Machine Design at the University of Technology and Agriculture in Bydgoszcz (UTA) (Szala and Boroński, 1995; Szala *et al.*, 1994, 1998) has shown possible error sources in the prediction of the fatigue life, such as extrapolating life diagrams onto areas not included in fatigue tests or making use Young's modulus E determined in a monotonic tension test in further calculations.

Analysis of bibliographical data (e.g. *Fatigue Design Handbook*, 1988; Kocańda and, Szala 1991; Roessle and Fatemi, 2000; Schijve, 2001; Smith *et al.*, 1970; Topper and Lam, 1997) indicates common application of Young's modulus E with values assigned for monotonic loadings. The recently realised research has given the possibility to check the variability of the modulus E

according to the type of the material loading. This, in turn, has allowed one to define the differences in the life estimation in the strain- and energy-based calculation methods, which are caused by alterations of the value E.

2. Theoretical basis

When carrying out calculations using both the strain-based method and the energy-based one (Szala, 1998), low-cycle material properties are used, which are the basis for the estimation of the fatigue life diagrams (Fig. 1) and material-describing ones, e.g. stress-strain curves (Fig. 2). The modulus E appears in descriptions of these diagrams as a proportionality coefficient for elastic strain components.

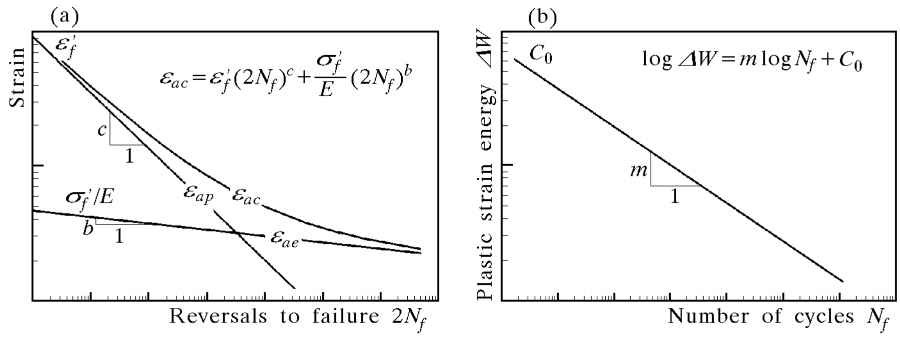

Fig. 1. Fatigue life diagrams

In the case of fatigue life diagram description in the strain approach using Manson-Coffin's equation (Fig. 1a)

$$\varepsilon_{ac} = \varepsilon_{ap} + \varepsilon_{ae} = \varepsilon'_f(2N_f)^c + \frac{\sigma'_f}{E}(2N_f)^b \tag{2.1}$$

the modulus E does not affect the strain or life value. This is because fatigue tests according to standards (PN 84/H04334; ASTM 606-92) make use of the quotient σ'_f/E, and changes in the modulus E cause only changes in σ'_f with a constant value of σ'_f/E maintained, and thus the fatigue diagram remains unchanged.

The situation becomes different in the case of stress-strain curves describing a material being loaded under variable conditions, described with

Ramberg-Osgood's dependence

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \Big(\frac{\sigma}{K'}\Big)^{\frac{1}{n'}} \tag{2.2}$$

A standard research conducted to find the discussed characteristics gives only values describing the plastic element of the strain in dependence (2.2), i.e. K' and n'. Therefore, the assumed modulus E may change the material description quantitatively.

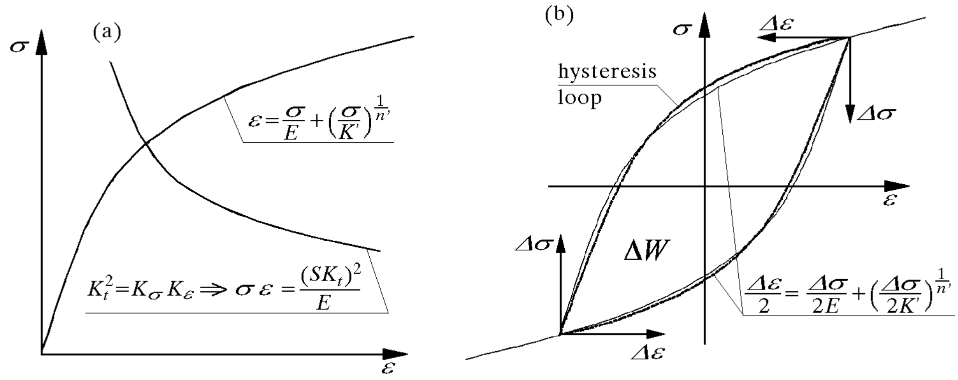

Fig. 2. Material properties (description in text)

A stress-strain diagram is used in the strain-based calculation method to determine the local strain and stress, e.g. using Neuber's method (Neuber, 1961), see Fig. 2a, while in the energy-based calculation method, it may be used to describe the hysteresis loop (Fig. 2b).

Observation of the first and next load cycles (Fig. 3) allows one to notice that a change in the material stiffness takes place, which causes a change in the modulus E. Hence, it may be agreed that there exist at least two E values: "static" (monotonic) and "dynamic". Moreover, a precise analysis of the hysteresis loop shows that it actually does not possess a linear-elastic segment, which increases the number of the possible-to-use E values by the static and linear approximation-based ones.

The ambiguity connected with the determination of the modulus of elasticity was presented among others in the work by Kandil (1999). The analysis of the methods of plastic strain determination using Young's modulus E based on Standards BS 7270:1990; ASTM E606-92; ISO/DIS 12106; prEN 3988 for specimens made of the Nimonic 101 nickel based superalloy showed differences of over 30%. Moreover, different values of elasticity modulus for tension and compression semicycles and its decrement together with an increase in the

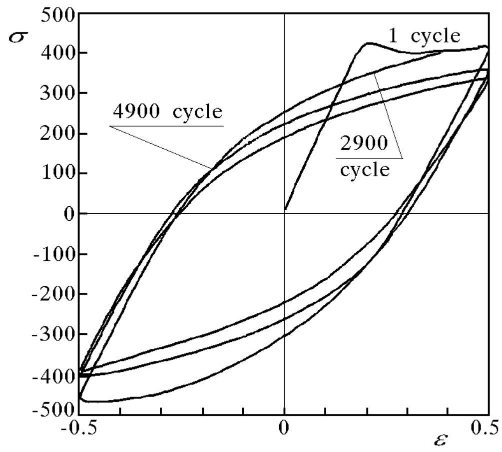

Fig. 3. Hysteresis loops for 45 steel

strain were observed. The latter phenomena were described also in the work by Morestin and Boivin (1996) on investigations of specimens made of a plain steel and alloys A33, XC 38 and AU4G with a large cross-section and upon XE280D sheet metal type with a small cross-section.

The aim of the paper is a quantitative analysis of the Young modulus E that depends on loading conditions (monotonic or cyclic variable), and in the case of cyclic variable loading, depends also on the loading amplitude. Moreover, differences in the estimated fatigue lives resulting from application of different moduli E are to be determined. The tests have been carried out for three different materials. Additionally, changes in the modulus E during the fatigue tests have been observed.

3. Experimental data

The determination of the longitudinal elasticity modulus requires, according to ASTM E111 – 82/88 Standard, carrying out a monotonic tension test. The thus obtained stress dependence (force by cross-section) in function of strain is the basis for defining Young's modulus. In the case of linear elastic materials, the modulus is a directional coefficient of a straight line describing the range of linear (proportional) material properties.

In the case of non-linear elastic materials, the tangent and secant moduli are introduced. The methods for their determination are shown in Fig. 4.

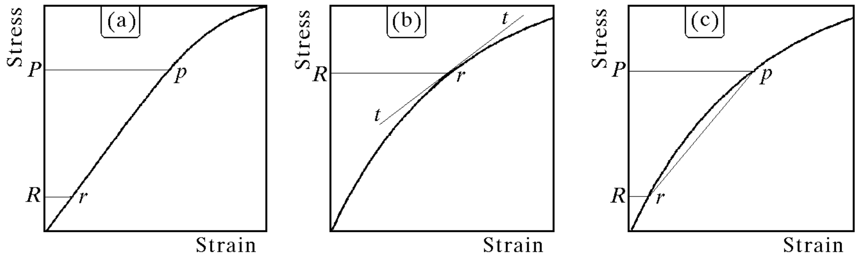

Fig. 4. Methods of determination of the elasticity modulus: (a) linear, (b) tangent, (c) secant

Three types of materials have been considered in the described research: Steel 45, Steel 30HGSA and Aluminium PA6.

The investigations were carried out in the Laboratory of Department of Machine Design at the University of Technology and Agriculture in Bydgoszcz using the INSTRON 8501 servohydraulic fatigue system with digital control system 8500. In the tests, standard specimens with round cross-sections were applied. The main dimensions of the specimens are shown in Fig. 5.

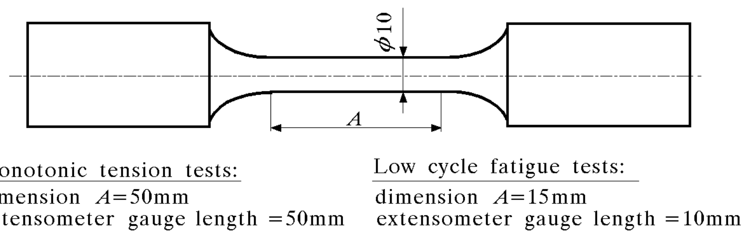

Fig. 5. Specimens used in the investigations

The INSTRON axial extensometers (collaborating with the strain channel of the loading control system) with two values of gauge length were used for strain measurements: 50 mm in the case of monotonic tension tests and 10 mm in the case of low cycle fatigue testing.

During the tests, the specimens were fastened by standard INSTRON hydraulic grips.

The monotonic tension tests for all three materials revealed in the elastic range nearly linear courses. The coefficients of the straight line correlation describing the range were $r^2 = 0.99993$ for Steel 45, $r^2 \in \langle 0.9999, 1\rangle$ for 30HGSA and $r^2 \in \langle 0.9983, 0.9988\rangle$ for Aluminium PA6. The obtained courses were also described by a second degree polynomial, and tangent moduli were determined. Under those circumstances, the correlation coefficients were as follows: $r^2 \in \langle 0.9999, 1\rangle$ for Steel 45, $r^2 = 1$ for 30HGSA and $r^2 \in \langle 0.9997, 0.9999\rangle$ for Aluminium PA6.

The obtained average moduli E are presented in Table 1. The variability coefficients determined according to ASTM and calculated on the basis of the dependence

$$V_1 = 100\sqrt{\frac{\frac{1}{r^2} - 1}{K - 2}} \tag{3.1}$$

where: K is the number of data pairs and r^2 is the correlation coefficient, are given there as well.

Table 1

Material	Young's modulus, linear approxim., the average value	V_1	Tangent modulus, polynomial approxim., the average value	V_1
45 Steel	222164 MPa	0.5%	226189 MPa	< 1.2%
30HGSA Steel	208463 MPa	< 0.5%	212920 MPa	0%
PA6 Aluminium	74237 MPa	< 1.58%	83726 MPa	< 1%

The hysteresis loops recorded during the fatigue tests in the low-cycle range allowed one to determine the elasticity modulus, further called the dynamic modulus E_d. Due to the fact that the hysteresis loop branches (Fig. 6) do not possess the linear elastic part, the modulus of elasticity was decided to be described according to the methodology shown in Fig. 4.

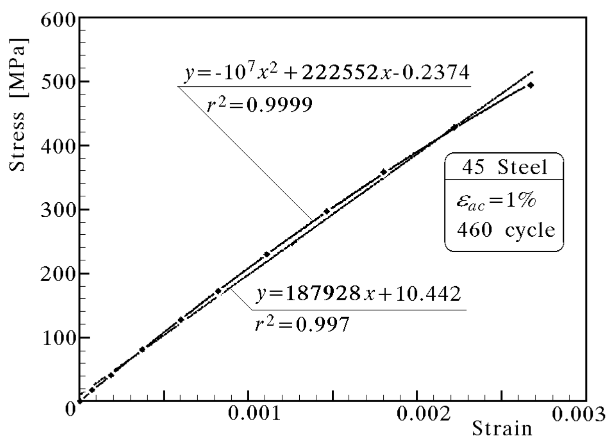

Fig. 6. Analysis of the hysteresis loop branch

In determining the tangent modulus, the elastic part of the hysteresis loop was approximated by a second degree polynomial with choosing such a fragment of the loop that the correlation coefficient was $r^2 = 0.9999$. Due to the fact that the secant value of the modulus E does not have a physical sense in the description of a cyclic strain diagram (the total strain must be on the right side of the linear-elastic part), the directional coefficient of the straight line approximating the initial segment of the hysteresis loop was agreed to be the value of the second modulus E (the same one as in the polynomial approximation), obtaining $r^2 > 0.9999$. The thus obtained moduli E (average ones) for different strain levels and for different materials are presented in Table 2.

Table 2

Material	Dynamic modulus E_d, linear approximation					
	Strain ε_{ac}%					Average value
	0.35	0.5	0.8	1.0	2.0	
45 Steel	165896	176302	180356	181431	174707	175738
30HGSA Steel	198774	190304	192406	192082	189422	192598
PA6 Aluminium	67304	73995	73330	70799	69660	71018

Material	Dynamic modulus E_d, linear approximation					
	Strain ε_{ac}%					Average value
	0.35	0.5	0.8	1.0	2.0	
45 Steel	209516	205587	209674	209369	201509	207131
30HGSA Steel	213649	208898	209842	209888	216278	211711
PA6 Aluminium	68337	76057	80365	74707	72727	74439

The obtained moduli are also presented in Fig. 7.

4. Analysis of tests on the modulus E

The analysis of the obtained values shows that the so-called dynamic modulus is lower than its static counterpart found in monotonic tests. Moreover, the modulus changes depending upon the strain which is applied to the specimen. The changes are irregular, and the realisation of the fatigue tests on a considerably greater number of strain levels would be required to determine a possible function describing the dependence of the modulus E on loading.

The modulus E_d differs with respect to E, at the maximum, by about 21% for Steel 45, about 8% for Steel 30HGSA and about 11% for Aluminium PA6.

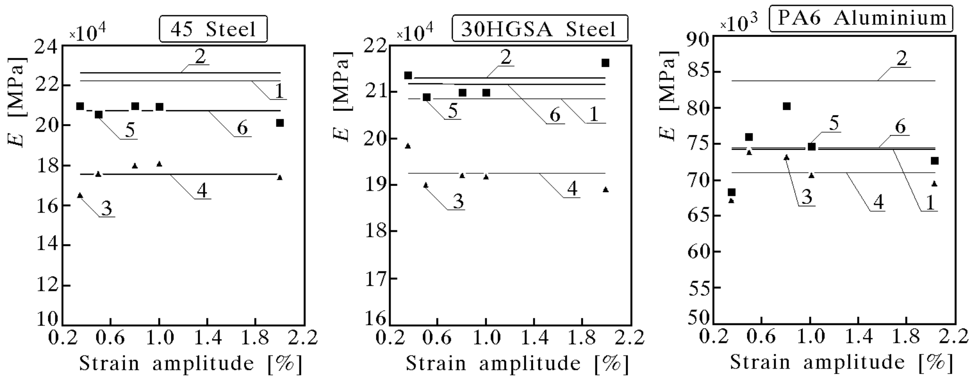

Fig. 7. Values of the modulus E; 1 – E (static) linear approximation, 2 – E (static) tangent, 3 – E_d (dynamic) linear, 4 – E_d linear-mean, 5 – E_d (dynamic) tangent, 6 – E_d tangent-mean

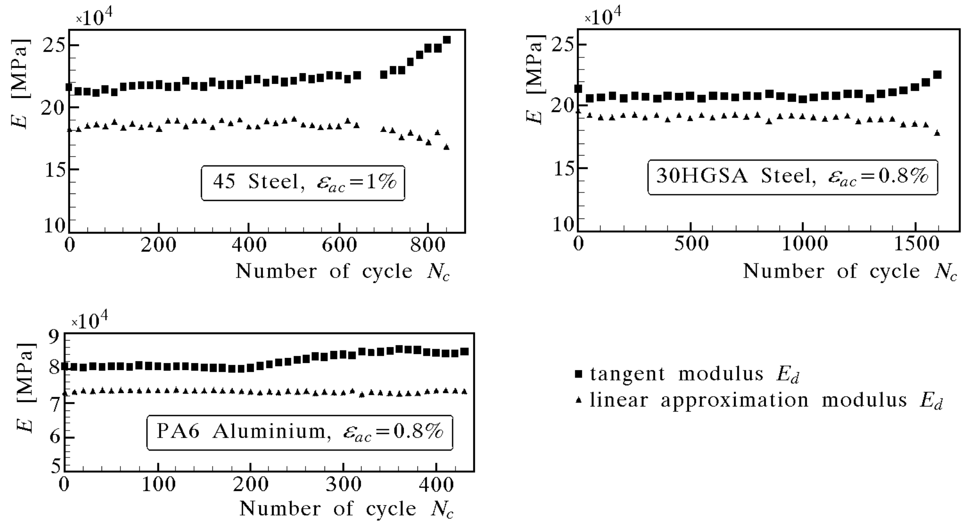

Fig. 8. Exemplary courses of the modulus E for specimens made of the three tested materials

During the research, variability of the modulus E in function of the number of realised load cycles were observed as well. Figure 8 presents examples of moduli E for specimens made of the three tested materials, loaded in symmetric cycles of a constant strain value ε_{ac}. The comparison of the observed values of the tangent modulus and of the modulus resulting from linear approximation in the case of steel specimens indicates increasingly weaker linearity of the elastic hysteresis loop segment (an increase in the tangent modulus with a simultaneous decrease in the "linear" modulus), and a decrease in the material stiffness. In the case of aluminium specimens, a minor increase in the modulus values occurred, which may indicate a growth in the material stiffness. The differences between the initial and final moduli were growing together with the intensification of the load applied to the specimens. For all the materials, the differences did not exceed 10%.

5. Analysis of fatigue life calculations

The determined, on the basis of tests, static and dynamic, values of the modulus E were used for calculation of the fatigue life of notched structural elements by means of the strain-based method (for the three materials) and energy-based method (for 45 Steel).

In the case of the strain-based calculation method, the notched element was modelled according to Neuber's hypothesis (Fig. 2a), assuming in the material model description a stress-strain curve of different values of the modulus E. In the case of the energy-based calculation method, the plastic strain energy was calculated by describing the hysteresis loop branch by means of a doubled stress-strain curve (Fig. 2b) and Neuber's local strain and stress analysis

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \Big(\frac{\Delta\sigma}{2K'}\Big)^{\frac{1}{n'}} \tag{5.1}$$

assuming different values of E.

Low-cycle properties of the three tested materials, discussed in the works by Szala *et al.* (1998), were used in the calculations. These properties are presented in Table 3.

Table 3

Material	c	b	ε'_f	σ'_f [MPa]	K' [MPa]	n'	C_0	m
45 Steel	−0.43915	−0.11668	0.165836	1304	1436	0.226796	2.68604	−0.5933
30HGSA Steel	−0.81030	−0.08716	2.139042	1660	1068	0.066426		
PA6 Alumin.	−0.84613	−0.10016	0.117573	894	797	0.085848		

The stress-strain curves used in both methods and defined for different values of E are shown in Fig. 9. The diagram analysis indicates fairly low

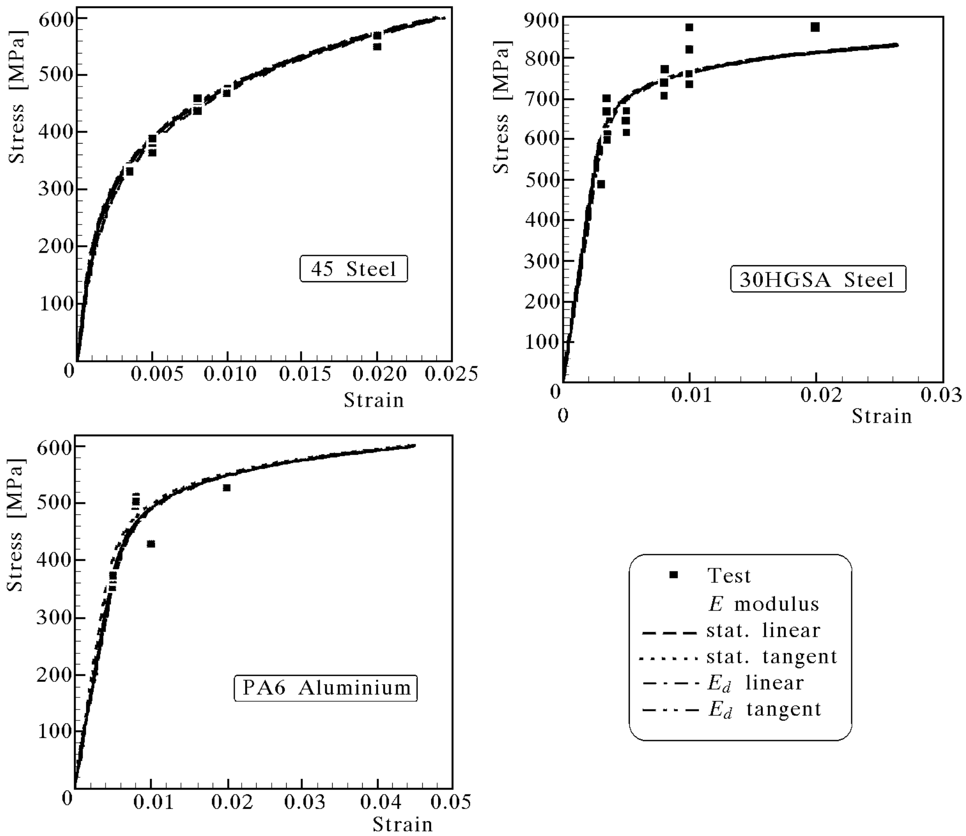

Fig. 9. Stress-strain curves for different values of the modulus E

influence of variability of the modulus E on the qualitative course of the material modelling curve.

For 45 Steel, the greatest stress differences equal about $\Delta\sigma = 40$ MPa for the strain $\varepsilon = 0.1\%$, and in the case of strain of about $\Delta\varepsilon = 0.08\%$ for stresses from the range of $\sigma \in \langle 400, 440\rangle$ MPa. This means that the maximum difference in the case of strain-based stress determination is about 20%, while reciprocally, i.e. determining the strain on the grounds of stress the result may differ by about 14.5%.

For 30HGSA Steel, the greatest differences equal about $\Delta\sigma = 40$ MPa for the strain $\varepsilon = 0.27\%$ (8% difference), and in the case of strain of about $\Delta\varepsilon = 0.035\%$ for stresses $\sigma \in \langle 620, 660\rangle$ MPa (12% difference).

For PA6 Aluminium the greatest stress differences are about $\Delta\sigma = 50$ MPa for the strain range $\varepsilon \in \langle 0.3, 0.5\rangle\%$ (which gives the difference of about 24%), and in the case of strain of about $\Delta\varepsilon = 0.1\%$ for stresses $\sigma \in \langle 460, 520\rangle$ MPa (difference by about 14.3%).

The changes of the analytically determined local strain and plastic strain energy (the hysteresis energy) for the given range of variability of the modulus E in a structural element made of 45 Steel are presented in Fig. 10. Three values of the nominal stress S (200, 300 and 400 MPa) and two values of the stress concentration factor K_t (1.5, 2.5) were used in the calculations. The differences in the local strains (Fig. 10a) were about 20%. In the case of fatigue life calculations (Fig. 10b) it caused differences from 57 up to 100%.

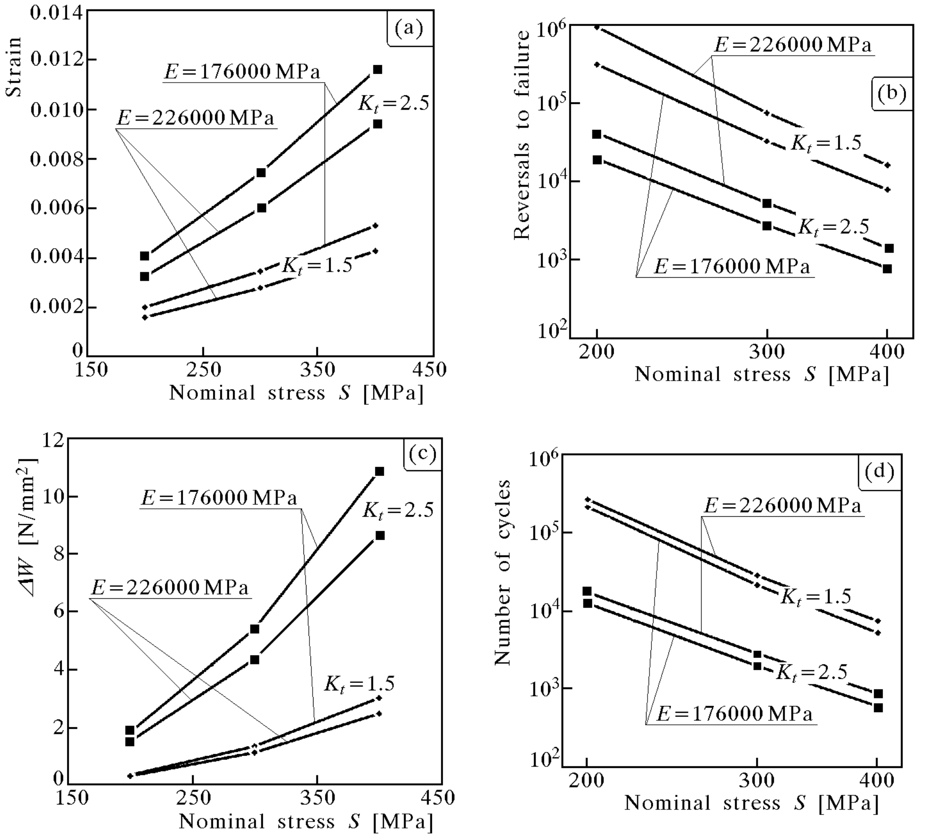

Fig. 10. Changes of the local strain (a), plastic strain energy (c), fatigue life for strain-based calculations (b) and energy based calculations (d) for 45 Steel

Similarly, in the case of the energy based approach, the maximal differences of the plastic strain energy ΔW (Fig. 10c) for the analysed range of variability of the modulus E were 13-23%, which entailed differences in the fatigue life (Fig. 10d) from 21 up to 38%.

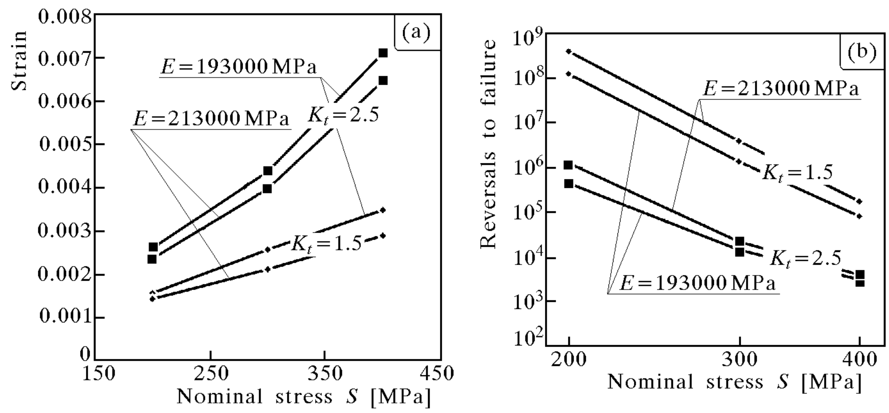

Fig. 11. Changes of the local strain (a), fatigue life for strain-based calculations (b) for 30HGSA Steel

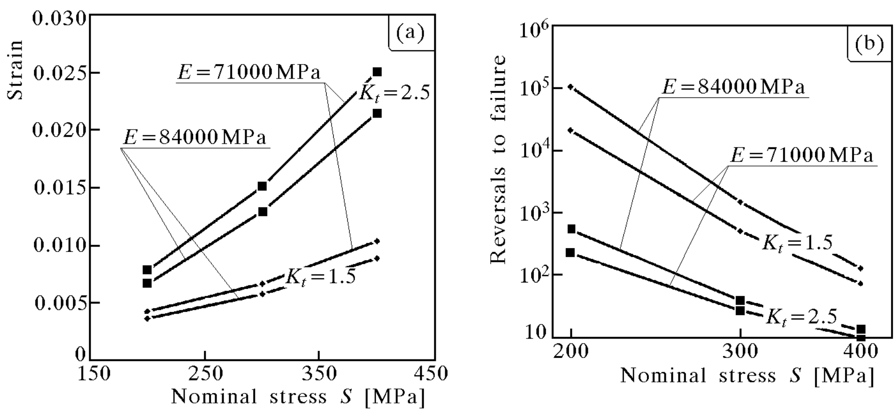

Fig. 12. Changes of the local strain (a), fatigue life for strain-based calculations (b) for PA6 Aluminium

The results of analogous strain-based calculations made for 30HGSA Steel and PA6 Aluminium are shown in Fig. 11 and Fig. 12. The differences in the prediction of the local strains and fatigue life for the three analysed materials are collected in Table 4.

Table 4

Material	Differences of			
	ε	$2N_f$	ΔW	N_c
	$\varepsilon_r/\varepsilon_m$ [%]	$(2N_f)_r/(2N_f)_m$ [%]	$\Delta W_r/\Delta W_m$ [%]	$(N_c)_r/(N_c)_m$ [%]
45 Steel	21-23	57-100	13-23	21-38
30HGSA Steel	9-10	23-104		
PA6 Aluminium	15-17	27-130		

where subscript $r = range$, $m = mean$.

6. Conclusions

On the basis of the conducted tests and calculations, several conclusions concerning the sensitivity of Young's modulus to variable loads can be drawn:

- cyclic loading in the range of loads producing plastic strains decreases Young's modulus; in the conducted tests, the phenomenon was least visible in the case of Steel 30HGSA (tangent modulus), and most visible in the case of Steel 45 (linear approximation modulus),
- the value of modulus obtained by the linear approximation decreases with the number of cycles which, with somewhat growing tendency of the tangent modulus, indicates increasingly weaker linearity of the elastic part of the hysteresis loop,
- a too small number of the realised loading levels does not allow one to determine the influence of the loading amplitude on Young's modulus.

The subsequently made simulations, in which different moduli E were applied, allowed one to notice that the determination method did not considerably affect the calculated local strains and plastic strain energy. However, even a small variation of the local strain may cause big differences in the fatigue life when the strain-life approach is applied, especially for small values of strain.

For the least advantageous case, for PA6 Aluminium, with the coefficient $K_t = 1.5$ and the nominal stress value $S = 200\,\text{MPa}$, the difference in the estimation of the fatigue life was $\Delta = 130\%$.

In spite of this, it can be stated that making use of moduli E defined on the basis of the literature data (including handbooks) should not produce significant errors in engineering calculations on the fatigue life of structural elements.

The obtained calculation results indicate also the necessity of further search on the sources of errors that appear in various methods of determination of the fatigue life.

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Wpływ sposobu określania wartości modułu E na obliczenia trwałości zmęczeniowej elementów konstrukcyjnych

Streszczenie

W pracy przedstawiono analizę ilościowej zmienności modułu E w zależności od rodzaju obciążenia (monotonicznie i cyklicznie zmienne), a w przypadku obciążenia cyklicznie zmiennego także od wartości amplitudy obciążenia. Ponadto wyznaczono różnicę szacowanych trwałości wynikającą z przyjęcia różnych wartości modułu E. Badania przeprowadzono dla trzech różnych materiałów. Dodatkowo obserwowano zmianę wartości modułu E w różnych okresach trwałości zmęczeniowej.

Manuscript received November 19, 2003; accepted for print December 22, 2003