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A FATIGUE FAILURE CRITERION FOR MULTIAXIAL LOADING WITH PHASE SHIFT AND MEAN VALUE

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A new criterion based on the critical plane approach has been developed for multiaxial non-proportional fatigue failure. The criterion correctly takes into account the influence of phase shift and mean values under combined bending and torsion loading. From a certain point of view, the criterion with such a defined non-proportionality measure can be understood as a combination of the two approaches: critical plane and integral approach. The criterion has the following form

$$\tau_{\alpha^*(eqnp)} = (\tau_{\alpha^*(a)} + c_1\sigma_{\alpha^*(a)} + c_2\sigma_{\alpha^*(m)})\Big(1 + \frac{t_{-1}}{b_{-1}}H^n\Big) \leqslant c_3$$

where the multiplicand of the equivalent shear stress $\tau_{\alpha^*(eqnp)}$ contains the amplitude of the shear stress $\tau_{\alpha^*(a)}$, the amplitude $\sigma_{\alpha^*(a)}$ and mean value $\sigma_{\alpha^*(m)}$ of the normal stress acting in the critical plane. The multiplier contains the loading non-proportionality measure H. Taking into account the fact of different sensitivity of various materials to loading non-proportionality, the equation also includes the material data: t_{-1} – fatigue limit in torsion, b_{-1} – fatigue limit in bending.
The predictive capability of the criterion was demonstrated by analyzing 67 experimental results from the literature. The predicted results are generally in good agreement with the experimental ones.

Key words: high cycle fatigue, multiaxial fatigue, non-proportional loading, out-of-phase loading, mean value

Notations

b_{-1}	–	fatigue limit under bending for $R = -1$
b_0	–	fatigue limit under bending for $R = 0$

t_{-1}	–	fatigue limit under torsion for $R = -1$
σ_f	–	ultimate tensile strength
$\sigma_{x(a)}$	–	bending stress amplitude
$\tau_{xy(a)}$	–	torsional shear stress amplitude
$\sigma_{x(m)}$	–	bending stress mean value
$\tau_{xy(m)}$	–	torsional shear stress mean value
α	–	plane at an angle measured from the bending stress plane
α^*	–	critical plane
τ_α	–	shear stress on the plane α
σ_α	–	normal stress on the plane α
$\tau_{\alpha(a)}$	–	shear stress amplitude on the plane α
$\tau_{\alpha(m)}$	–	shear stress mean value on the plane α
$\sigma_{\alpha(a)}$	–	normal stress amplitude on the plane α
$\sigma_{\alpha(m)}$	–	normal stress mean value on the plane α
$\sigma_{\alpha(max)}$	–	maximum value of normal stress on the plane α
$\tau_{t(max)}$	–	maximum value of shear stress for a given moment of time of the loading cycle on an arbitrary plane α
$\widehat{\tau}_{t(\max)}$	–	maximum value of shear stress in the loading cycle on an arbitrary plane α, the maximum of $\tau_{t(max)}$
$\tau_{\alpha(eq)}$	–	proportional equivalent shear stress amplitude on an arbitrary plane α
$\tau_{\alpha^*(eq)}$	–	proportional equivalent shear stress amplitude calculated for the plane α^*
$\tau_{\alpha^*(eqnp)}$	–	non-proportional equivalent shear stress amplitude calculated for the plane α^*
H	–	loading non-proportionality measure
W	–	weight function
R	–	stress ratio $\sigma_{x(min)}/\sigma_{x(max)}$ or $\tau_{xy(min)}/\tau_{xy(max)}$
λ	–	stress amplitudes ratio $\sigma_{x(a)}/\tau_{xy(a)}$
φ	–	phase shift angle between σ_x and τ_{xy}
β	–	phase shift angle between σ_α and τ_α

1. Effect of loading non-proportionality on fatigue properties

In many cases of operational conditions one deals with non-proportional loadings. The most significant feature of that state is the turn of the principal stress/strain axes due to fatigue. For many structural materials, such loading conditions have major influence on their fatigue life and strength.

A group of materials for which, under such conditions, a considerable decrease of the fatigue life is observed is ductile structural steels. The fatigue strength decrease is usually strongly correlated with extra hardening. For example, the results of Fatemi's (from Socie, 1987) research showed that increase in the extra hardening by 10-15% caused reduction of the life by half. When the extra hardening was larger by 100%, the life was ten times shorter. Also Socie (1987), on the basis of his study, estimated that the life in non-proportional loading conditions may be even ten times smaller.

There also exists substantial influence of the loading non-proportionality on the fatigue limit value. Although, the smaller the loading, the smaller the decrease of life, and the life curves for proportional and non-proportional loadings converge asymptotically on the level of the fatigue limit (Ellyin *et al.*, 1991), in the range of unlimited life, a fall of the fatigue limit is still visible. Extensive research concerning the topic was conducted by Nisihara and Kawamoto (1945).

The causes for such fatigue scenarios are seen in the behaviour of the maximum shear stress vector, which changes its location in time. According to Sakane *et al.* (1997), when the biggest shear stress vector comprises all α planes, as a result of the non-proportional loading, many slip systems are activated. As a consequence, dislocations moving on the planes initiate additional interactions. Fatigue damage accumulation intensifies.

The effect of this are characteristic changes in the dislocation structure picture. Even on the basis of only Jiao's *et al.* (1996), Rios's *et al.* (1989) and Sakane's *et al.* (1997) research, it may be shown that in comparison to structures obtained in proportional loading conditions these are: greater dislocation density, larger wall misorientation and perfection, smaller cell sizes, remaining of the dislocation within the cells and thus, more homogeneous dislocation distribution. Non-proportional dislocation structures remain in the same kind of relation to proportional dislocation structures as, under the conditions of proportional loadings, the structures corresponding to a greater number of cycles to the structures that come into being at a smaller number of cycles. The only exception is the remaining of the dislocation in the cell inside. It can be remarked that the dislocation structures obtained under the conditions of non-proportional loading exhibit higher values of cumulated fatigue damage. What results from this is that the non-proportional loading is more destructive in relation to the proportional one.

What should also be noticed is that for other types of materials, the non-proportional loading influence is different. An increase in the fatigue life is observed under a non-proportional loading for low-ductile (so-called brittle) materials like cast aluminium, cast iron, sintered steels. There are also semiductile materials, which reveal no difference between the in- and out-of-phase multiaxial loading, e.g. cast steels and wrought aluminium alloys, Sonsino and Maddox (2001).

2. Existing fatigue criteria

Since 1935, when Gough (1950) suggested ellipse quadrant and ellipse arc equations, there has come into existence a great number of multiaxial fatigue criteria. At the same time, many propositions of their division have been suggested. According to Weber *et al.* (2001), fatigue criteria can be divided into three groups:

- Empirical criteria
- Critical plane approach criteria
- Global approach criteria.

Although, as the author emphasizes, this classification is not precise as some criteria could belong to one category from certain aspects and to another one considered from the point of view of other aspects, this division makes the analysis of some interesting criteria characteristics easier. Belonging to particular groups it decides to some extent, about possible manners of including the loading non-proportionality influence in the criterion record.

One of the first multiaxial fatigue criteria, like the equations of Gough (1950) and Nishihara and Kawamoto (1945), as well as some later ones, like S.B. Lee's (1989) and Y.L. Lee's (1985), fall into the group of the Empirical criteria. Apart from their undoubted advantages, such as simplicity and engineer-friendliness, these criteria have significant weaknesses. The most obvious drawback is that they are dedicated to model the fatigue behaviour of material under particular types of conditions only, and are generally restricted to these applications.

A very important group of criteria are those based on the idea of the Critical Plane. The concept of these criteria is established on the statement that the fatigue behaviour of the material is a result of action of stresses that are acting on the so-called critical plane. Application of this criterion requires, firstly, defining the orientation of the critical plane and next, determining the criterion stress quantities relative to this plane for determining the severity of the multiaxial cycle for the given material. The choices both of the critical plane definition and the stresses components that are involved in the criteria formulation depend on authors. To illustrate, Findley (1959), McDiarmid (1981) and Dang Van *et al.* (1989) proposed some criteria of that category.

According to Weber's *et al.* (2001) division, in the group of the Global Approach Criteria three sub-categories may be distinguished. Some criteria use invariants of the stress or deviatoric stress tensors, and these are the criteria by Sines (Sines and Ohgi, 1981) and Crossland (from Dietrich *et al.*, 1972). The next subgroup constitute the Energetic criteria as those proposed by Froustey (from Weber *et al.* 2001). Finally, the criteria which utilize contributions of all possible material planes passing through the material point where the fatigue assessment is realised for predicting the material fatigue behaviour. This concept termed as the Integral Approach, requires calculating of a damage indicator to take into account the quantities connected with all possible planes through the point where the fatigue, e.g. life, is assessed. Papadopoulos (1995) proposed such a criterion.

Depending on the group, the criteria differently deal with the problem of the influence of the loading non-proportionality on the fatigue properties.

The easiest way to make the results of forecast fatigue properties in non-proportional loadings conditions real is to introduce the phase shift φ to the criterion records. The idea was used in S.B. Lee's (1989) and Y.L. Lee's (1985) criteria, where the non-proportional parts were functions of the out-of-phase angle. This solution is of course restricted to a particular loading case and sometimes is deprived of physical interpretation. Moreover, the phase shift angle is not an appropriate non-proportionality parameter because the loading non-proportionality degree depends also on other nominal stress parameters, like for example the amplitude relation λ or average stress values.

The criterion group based on the critical plane idea possesses a strong physical interpretation. As Kanazawa *et al.* (1979) emphasise on the basis of physical observation, fatigue processes (crack development) and properties (fatigue limit) depend on quantities connected with the selected plane.

To achieve conformity with experimental results in the non-proportional range, specially defined non-proportionality functions are used for this group of criteria. The non-proportional factor of the non-proportional strain parameter defined by Itoh *et al.* (1997), the Duprat (1997) model for predicting fatigue life, Morel's *et al.* (1997) phase-difference coefficient or the rotation factor introduced by Kanazawa *et al.* (1979) may serve as examples.

The question about correct, possessing physical interpretation and in agreement with experimental results considering the loading non-proportionality, concerns also the criteria from the Integral Approach group. It is believed (e.g. Witt *et al.*, 2001) that this approach appears particularly useful in the case of a non-proportional loading, as the volume element is uniformly loaded in all planes with the direction of principal stress changing with time. However, according to the author, this approach does not always allow one to describe the stress state correctly either. For example, it does not make it possible to tell the difference when the stresses acting on the α planes result from the principle axis rotation, and when they are the result of turning transformation. From this model perspective, no difference is seen between the proportional and non-proportional loading state.

Taking into account both group properties, Sonsino and Maddox (2001) made a division of the criteria usefulness. For materials not showing the non-proportional loading sensitiveness (semi-ductile), they suggest the Critical Plane, while for materials sensitive to principle axis turns (ductile), they suggest using a criterion form the Integral Approach group.

There arises a question if the imperfectness of the critical plane concept really forces us in the case of ductile materials to give up this idea for the benefit of the Integral Approach. Irrespectively of the material, fatigue processes are characterised by directivity. Together with the material change, only the critical plane discriminant changes, but the idea seems to remain always right.

On the other hand, the idea of taking into consideration the influence of other planes in the state of a non-proportional loading, proposed by the Integral Approach, is also very promising.

3. Assumptions concerning the new criterion formulation

It is assumed that the influence of the loading non-proportionality on the fatigue strength and life may be described by means of the critical plane model completed with the loading non-proportionality measure being a quantity described on the basis of stresses acting outside the critical plane. The correctness of the critical plane idea is assumed even in non-proportional loading conditions; however, for executing the correct results it is necessary to take into account additional interactive influence of the stresses on other planes, the planes which are embraced by the turning vector of the maximum shear stress.

4. Criterion formulation

Formulation of the newly proposed criterion requires deciding about the following questions:

- Selecting the critical plane position
- Indicating other planes (outside the critical one), on which the acting stresses should be taken into account in the proposed description
- Deciding on the stress state components participating in the fatigue process and deciding on the criterion quality, i.e. a mathematical form of the relation between the indicated components
- Formulating the loading non-proportionality function.

As a model of the non-proportional loading, bending with torsion with the phase shift and the nominal stress mean value was accepted.

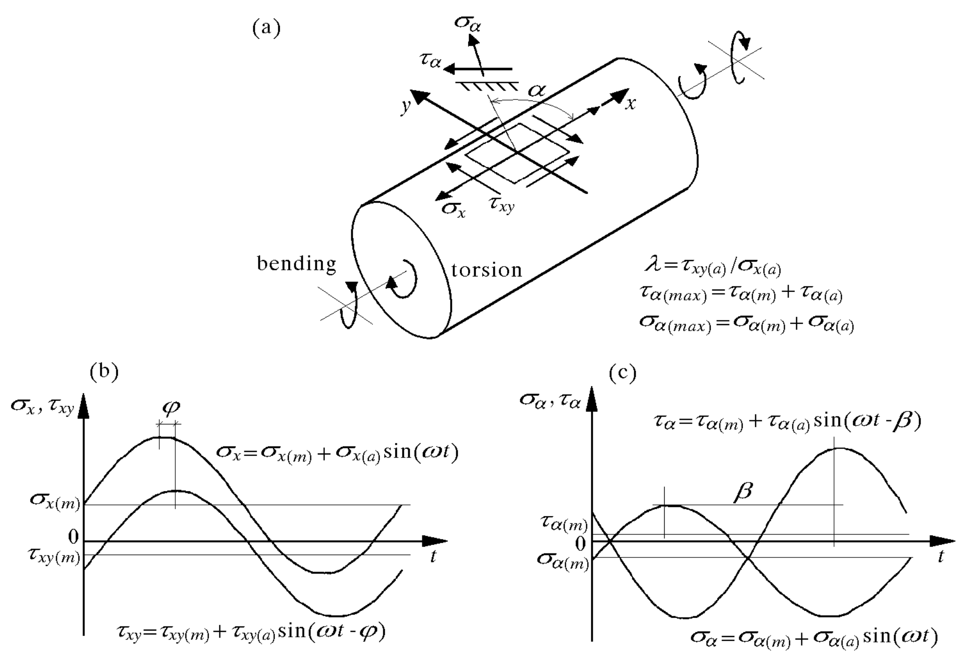

Fig. 1. Non-proportional stress histories; (a) definition of the directions x and α, (b) stress histories in the direction x, (c) stress histories in the direction α

For the above mentioned model loadings, due to the maximum shear stress vector behaviour, the following possible states were distinguished:

- If the mean values and the phase shift angle equal zero, during the fatigue cycle only the maximum shear stress value changes while the direction does not – situation (a)

- If the phase shift angle differs form zero and the mean values equal zero, the maximum shear stress vector is fully rotated, and this stress hodograph is a closed curve – situation (b)
- If the mean values differ form zero and the phase shift angle equals zero, the principle axis turn is executed only to a certain extent of the angle, and the stress hodograph is an open curve – situation (c)
- If both the phase shift angle and the mean values differ form zero, the maximum shear stress vector behaviour depends on these values interrelation, i.e. at the appropriately low value of the mean stresses it executes the full turn – situation (b), while when these values are appropriately high the turn is partial – situation (c).

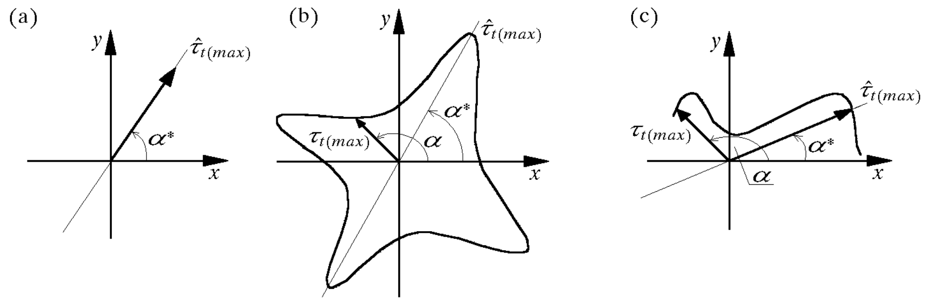

Fig. 2. The distinguished states of the maximum shear stress vector; (a) $\varphi = 0$, $\sigma_{x(m)} = \tau_{xy(m)} = 0$, (b) $\varphi \neq 0$, $\sigma_{x(m)} = \tau_{xy(m)} = 0$, (c) $\sigma_{x(m)} \neq 0$, $\tau_{xy(m)} \neq 0$

On the basis of this analysis, the questions necessary to describe the criterion shape were concluded.

4.1. Critical plane selection

In each of the above mentioned nominal stress states, there exists the critical plane. In Fig. 2 this is the plane described by the cycle maximum shear stress vector $\widehat{\tau}_{t(\max)}$. In the case of the proposed criterion, a solution identical with Findley's (1959) is accepted. It is assumed that, taking into account the analysed material type (ductile), what should be taken for the critical plane α^* is the plain with the critical combination of the shear and normal stresses.

4.2. Selection of other planes considered in the proposed description

During the non-proportional loading, the direction of action of the maximum shear stress vector changes. The turning maximum shear stress vector embraces with its action many planes, potentially initiating many slide systems. Therefore, these plane sets should be considered which are embraced with the maximum shear stress $\tau_{t(max)}$ action during a cycle. In situation (b) these are all planes, while in situation (c) only those which are located within a certain range of the angle of rotation.

4.3. Defining the stress state components and the relation between them

For the considered fatigue quantity, i.e. the fatigue limit, the fatigue crack initiation life is a large portion of the total fatigue life (Susmel and Lazzarin, 2002). Under such conditions, the maximum fatigue damage is produced along the direction of and governed by shear stress amplitude $\tau_{\alpha(a)}$. However, the fatigue crack initiation as well as fatigue crack growth are conditioned by the stress normal to the initiation plane $\sigma_{\alpha(a)}$. The compression component inhibits the persistent slip band laminar flow, whereas the traction component favours their flow, Susmel and Lazzarin (2002).

The considered loading case takes also into account the occurrence of the mean value. Papadopoulos (1995) shows that for a very high fatigue life of the order of one million cycles or more, the limiting shear stress amplitude is independent of the mean shear stress. But when it comes to the normal stress influence, the fatigue limit in bending strongly depends on the superimposed mean (static) normal stress $\sigma_{\alpha(m)}$ in such a way that the tensile mean normal stress reduces the fatigue limit, whereas the compressive mean stress leads to a net increase.

On the basis of the above mentioned, the general dependence of the equivalent shear stress amplitude may be formulated

$$\tau_{\alpha^*(eq)} = \tau_{\alpha^*(a)} + c_1\sigma_{\alpha^*(a)} + c_2\sigma_{\alpha^*(m)} \tag{4.1}$$

When the equivalent stresses are calculated on an arbitrary plane α, different from the critical plane α^*, the symbol $\tau_{\alpha^*(eq)}$ should be replaced with $\tau_{\alpha(eq)}$.

Dependance of the shape (4.1) is used in the case of e.g. McDiarmid's criterion (1985). Sometimes, the influence of the amplitude and the normal stress mean value is combined through the highest stress value $\sigma_{\alpha(max)}$, as in the case of Findley (1959) or Fatemi and Socie (1988). In this paper, the solution with normal stress components separation was accepted.

Afterwards, the coefficients c_1 and c_2 were defined.

From Gough's criteria (1950) it results that the coefficient c_1, with the assumption that the critical plane position is described with the vector $\widehat{\tau}_{t(\max)}$, has the form of $2(t_{-1}/b_{-1}) - 1$. The normal stress influence depends therefore on the t_{-1}/b_{-1} relation and changes from 1 (which is equivalent with accepting the maximum principle stress hypothesis) to 0.5 (as in the case of the maximum shear stress hypothesis 0.5). The lower the relation, the lower the influence of the stress $\sigma_{\alpha(a)}$.

In the proposed solution, the best agreement between the calculation and experimental results was obtained when the following $c_1 = 1.9(t_{-1}/b_{-1}) - 1$ was accepted.

In the case of criteria describing similar loading conditions, the coefficient c_2 is usually a function of t_{-1} and b_{-1} or b_0, see e.g. Sines and Ohgi (1981) and Kakuno and Kawada (from Papadopoulos, 1997). Due to the fact that b_0 is seldom available, b_0 may be expressed by means of the Googman line in the function of σ_f. In both cases the coefficient c_2 is expressed with the following dependence $(3t_{-1}/b_0) - \sqrt{3}$ or $\sqrt{3}b_{-1}/\sigma_f$.

In McDiarmid'a (1985) the value c_2 is expressed with the dependance $0.5b_{-1}/(0.5\sigma_f)^2$, however the normal stress appears in the 1.5 power, and in the later criterion of his (1990), c_2 equals $t_{-1}/(2\sigma_f)$.

In this paper, the best conformity of the calculations with the experimental data was obtained when $c_2 = 0.5b_{-1}/\sigma_f$.

4.4. Formulation of the loading non-proportionality function

The moment-maximum shear stress vector embraces with its action a number of planes. Depending on the plane initiated by the maximum shear stress vector and on the stress conditions on the plane, the loading non-proportionality extent is different. In this paper, the non-proportionality range is described with a non-proportionality function. Similarly as in the Integral Approach, the influence of the stress acting on the selected planes on the fatigue damage cumulating process is summed.

The measure of non-proportionality is defined on the basis of observation of geometry behaviour of stress hodographs drawn for cases of different degrees of the loading non-proportionality. The measure is described in a detailed way in the works of Skibicki and Sempruch (2001, 2002a,b). Below, the eventual notation of the function is given

$$H = \frac{\int\limits_{\alpha} (\tau_{\alpha(eq)} W)}{\frac{\pi}{2} (\tau_{\alpha^*(eq)})^2} \tag{4.2}$$

The participation of stresses acting outside the critical plane was described with a filling coefficient, defined as a quotient of the area within the hodograph and the area of the circle described on the hodograph. The quantity W appearing in the formula is a weight function, and takes the following form

$$W = \sin[2(\alpha - \alpha^*)]^k \tag{4.3}$$

The function W becomes zero on the critical plane, and for the direction most remote from the critical plane (similarly as in Kanazawa's rotation factor (Kanazawa *et al.*, 1979), this is the direction rotated by $45°$), it assumes the value of one.

The taking into account the loading non-proportionality function criterion in the notation requires also considering the material sensitiveness (answer) to the loading non-proportionality, which is a function of the quotient t_{-1}/b_{-1}. As a result, the following is obtained

$$\tau_{\alpha^*(eqnp)} = \tau_{\alpha^*(eq)}\Big(1 + \frac{t_{-1}}{b_{-1}}H^n\Big) \leqslant c_3 \tag{4.4}$$

In this way, for $\varphi = 0$ and $\tau_{xy(m)} = \sigma_{x(m)} = 0$, which hold in the case of proportional loading. The value H equals zero, and the equation describes the proportional loadings state $\tau_{\alpha^*(eqnp)} = \tau_{\alpha^*(eq)}$. For $\varphi \neq 0$ or/and $\tau_{xy(m)} \neq 0$, $\sigma_{x(m)} \neq 0$, the proportional part (multiplicand) value decreases but the non-proportionality measure value H increases and the multiplier increases. Owing to this, the calculation value of the fatigue limit $\tau_{\alpha^*(eqnp)}$ is constantly close to the experimental t_{-1}.

The detailed form of the criterion has been obtained by means of approximation of the experimental data with equation (4.4)

$$\tau_{\alpha^*(eqnp)} = \tau_{\alpha^*(eq)}\Big(1 + \frac{t_{-1}}{b_{-1}}H(W)^3\Big) \leqslant t_{-1} \tag{4.5}$$

where

$$\begin{aligned}
&\tau_{\alpha^*(eq)} = \Big[\tau_{\alpha^*(a)} + \Big(1.9\frac{t_{-1}}{b_{-1}} - 1\Big)\sigma_{\alpha^*(a)} + \frac{b_{-1}}{2\sigma_f}\sigma_{\alpha^*(m)}\Big]\\
&W = \sin[2(\alpha - \alpha^*)]^5
\end{aligned}$$

5. Calculation results

By means of the criterion, the fatigue limit values were calculated for 67 cases of fatigue loadings, taken from literature (Table 1). The literature data that was used concern examining the influence on the fatigue limit value: firstly, of the loading nominal parameters (like phase shift angle φ, amplitude λ ratio and the mean values) and secondly, of a wide range of materials characterized by the coefficient t_{-1}/b_{-1}. The data was divided into 10 groups (Column 1). A group consists of the results obtained by one author during the same type of research, concerning one type of material. The number of the data is put in the next column. The data with numbers from 1 to 18 are taken from Nisihara and Kawamoto (1945), from 19 to 40 – from Lemmp (1997), from 41 to 46 – from Sonsino (1983), from 47 to 50 – Neugebauer (from McDiarmid, 1987), from 51 to 58 – from Lemmp (from Weber *et al.*, 2001), and from 59 to 67 – from Froustey and Lasserre (1989).

The calculation results are presented in Columns 13 and 14, where Column 13 presents the equivalent stress value, and Column 14 – the relative error, i.e. $\tau_{\alpha^*(eqnp)}/t_{-1}$.

Table 1. Literature data and calculation results

group	data index	t_{-1}/b_{-1}	t_{-1}	b_{-1}	σ_f	λ	$\sigma_{x(a)}$	$\tau_{xy(a)}$	$\sigma_{x(m)}$	$\tau_{xy(m)}$	φ	$\tau_{\alpha^*(eqnp)}$	$\tau_{\alpha^*(eqnp)}/t_{-1}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	Nisihara and				1.21	99.9	120.9	0	0	0	136.2	0.99
	2	Kawamoto (1945)				1.21	103.6	125.4	0	0	60	139.1	1.01
	3	0.583	137.3	235.4	–	1.21	108.9	131.8	0	0	90	144.7	1.05
	4					0.5	180.3	90.2	0	0	0	137.3	1.00
	5					0.5	191.4	95.7	0	0	60	135.4	0.99
	6					0.5	201.1	100.6	0	0	90	138.3	1.01
	7					0.21	213.2	44.8	0	0	0	127.2	0.93
	8					0.21	230.2	48.3	0	0	90	130.2	0.95
2	9	Nisihara and				1.21	138.1	167.1	0	0	0	193.8	0.99
	10	Kawamoto (1945)				1.21	140.4	169.9	0	0	30	200.5	1.02
	11	0.625	196.2	313.9	–	1.21	145.7	176.3	0	0	60	206.8	1.05
	12					1.21	150.2	181.7	0	0	90	212.4	1.08
	13					0.5	245.3	122.7	0	0	0	196.5	1.00
	14					0.5	249.7	124.9	0	0	30	201.0	1.02
	15					0.5	252.4	126.2	0	0	60	195.9	1.00
	16					0.5	258.0	129.0	0	0	90	198.0	1.01
	17					0.21	299.1	62.8	0	0	0	190.3	0.97
	18					0.21	304.5	63.9	0	0	90	188.0	0.96
3	19	Lemmp (1997)				0.48	140	67.2	0	0	0	102.6	1.03
	20	0.568	100	176	–	0.48	168	80.6	0	0	90	107.1	1.07

4	21	Lemmp (1997)				0.48	201	96.5	0	0	0	148.6	1.02
	22	0.575	146	254	–	0.48	234	112.3	0	0	90	152.4	1.04
5	23	Lemmp (1997)				0.21	226	47.5	0	0	0	133.8	0.98
	24	0.58	136	235	–	0.21	233	48.9	0	0	90	130.6	0.96
	25					0.5	180	90	0	0	0	136.2	1.00
	26					0.5	187	93.5	0	0	60	131.0	0.96
	27					0.5	201	100.5	0	0	90	136.5	1.00
	28					1.21	98	118.6	0	0	0	133.2	0.98
	29					1.21	101	122.2	0	0	60	134.7	0.99
	30					1.21	109	131.9	0	0	90	143.8	1.06
6	31	Lemmp (1997)				0.21	299	62.8	0	0	0	191.8	0.97
	32	0.63	198	314	–	0.21	299	62.8	0	0	90	186.8	0.94
	33					0.5	245	122.5	0	0	0	197.5	1.00
	34					0.5	245	122.5	0	0	60	192.4	0.97
	35					0.5	245	122.5	0	0	90	190.7	0.96
	36					0.5	255	127.5	0	0	90	198.5	1.00
	37					1.21	137	165.8	0	0	0	193.0	0.97
	38					1.21	137	165.8	0	0	0	193.0	0.97
	39					1.21	142	171.8	0	0	60	203.1	1.03
	40					1.21	147	177.9	0	0	90	209.6	1.06
7	41	Sonsino (1983)				0.58	135	78.3	0	0	0	112.8	0.94
	42	0.6	120	200	–	0.58	152	88.2	0	0	90	117.6	0.98
	43					0.58	160	92.8	0	0	0	132.3	1.02
	44					0.58	168	97.4	0	0	90	126.8	0.98
	45					0.58	185	107.3	0	0	0	155.3	0.97
	46					0.58	207	120.1	0	0	90	161.7	1.01
8	47	Neugebauer				0.57	183	104.3	0	0	0	168.9	0.97
	48	(from McDiarmid, 1987)				0.57	195	111.2	0	0	90	189.0	1.08
	49	0.7	175	250	–	1	135	135	0	0	0	173.2	0.99
	50					1	150	150	0	0	90	205.1	1.17
9	51	Lemmp				0.48	328	157	0	0	0	266.6	1.03
	52	(from Weber, 2001)				0.48	286	137	0	0	90	231.4	0.89
	53	0.65	260	398	1025	0.96	233	224	0	0	0	280.6	1.08
	54					0.96	213	205	0	0	90	262.2	1.01
	55					0.48	266	128	0	128	0	204.2	0.79
	56					0.48	283	136	0	136	90	215.6	0.83
	57					0.48	280	134	280	0	0	268.6	1.03
	58					0.48	271	130	271	0	90	237.6	0.91
10	59	Froustey and				0.58	485	280	0	0	0	414.1	1.01
	60	Lasserre (1989)				0.58	480	277	0	0	90	390.8	0.95
	61	0.62	410	660	1880	0.58	480	277	300	0	0	463.2	1.13
	62					0.58	480	277	300	0	45	454.4	1.11
	63					0.58	470	270	300	0	60	437.9	1.07
	64					0.58	473	273	300	0	90	433.2	1.06

65	0.25	590	148	300	0	0	435.8	1.06
66	0.25	565	141	300	0	45	405.2	0.99
67	0.25	540	135	300	0	90	370.6	0.90

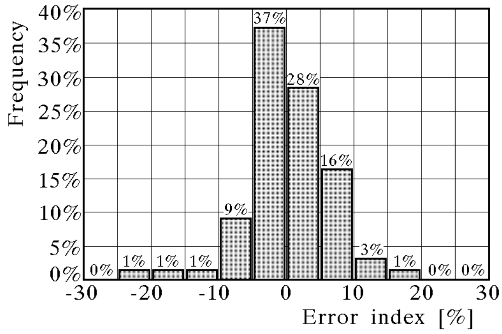

Fig. 3. Histogram of calculation results

For all 67 result groups, the mean value and the relative error standard deviation were calculated. These quantities values were respectively 1 and 0.06. Apart from figure parameters, the results were characterised with a frequency histogram, Fig. 3. What is worth noticing is the fact that more than 90% of the results falls into the range between $\pm 10\%$.

The results of all calculations are also presented in Fig. 4. What can be seen in this diagram is the dependence between the obtained results and the non-proportionality degree expressed with the phase shift angle (designation – $\square$, Fig. 4). To illustrate the non-proportionality function influence, the results of calculations for the criterion without the non-proportionality function were presented in the same diagram (designation – $\triangle$, Fig. 4). The greater the phase shift angle is, the worse the results are. However, the non-proportionality function improves the conformity of the obtained results with the experimental results, see e.g. points 6 and 16.

What is worth mentioning is the fact that the non-proportionality function does not cause an increase in the equivalent stress value in the situations where, despite a big phase shift angle, the loading non-proportionality, due to the other loading parameters value (e.g. $\lambda = 1.21$), is small and does not have any influence on the obtained fatigue limit value (points 11 and 12). Criteria based on the phase shift angle, as the only non-proportionality measure, are in these cases burdened with a considerable error. The non-proportionality also results from the existence of the nominal loading mean values. In these cases, however, the principle axis turn is limited to a minor range (Fig. 2c), therefore, the

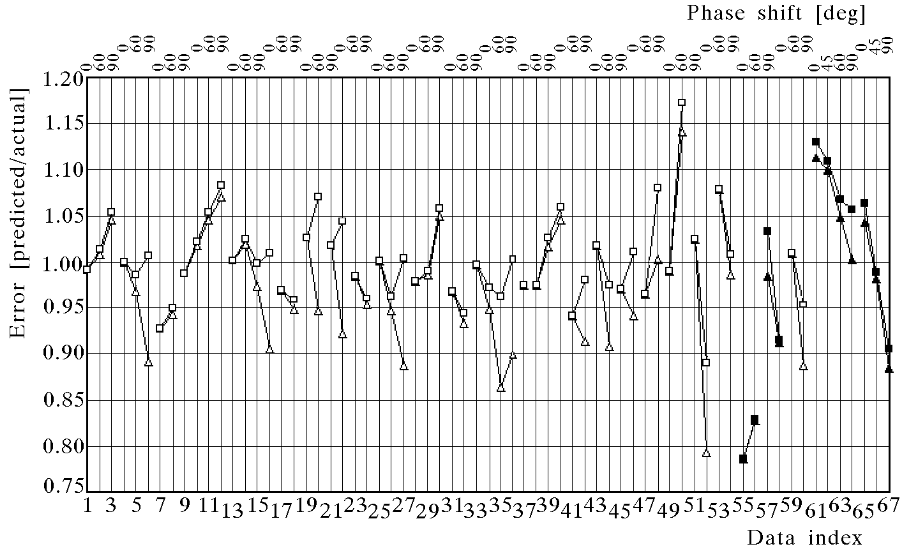

Fig. 4. Calculation results, □ – $\tau_{\alpha^*(eqnp)}$, △ – $\tau_{\alpha^*(eq)}$

non-proportionality influence is in these cases insignificant (Fig. 4, blackened symbols ▲, ■). It is most clearly seen in points 61 and 65, for which $\varphi = 0$, and the calculation error equals 2% of the result value.

In general, the calculation results in the case where the mean value influence is taken into account (points 55-58 and 61-67), although relatively the worst of all obtained, are still satisfactory – they fall into the scope of -21% to $+13\%$.

What is also worth carrying out is the comparison of the above results with the results obtained from other criteria for the same literature data.

The comparative analysis of different criteria (Crossland, Sines, Matake, McDiarmid, Dietman, Papadopoulos) was conducted by Papaudoulos (1997). A number of calculations were done for the same literature data as in this paper (groups 2, 9 and 10). The results obtained on the basis of the proposed criterion as well as Papadopoulos's analysis results are contrasted in Table 2 and in Fig. 4, where the mean values and index standard deviations are compared.

As a rule, the proposed criterion gives more correct results, i.e. the mean error values are closer to one, and the standard deviations in comparison with the first five criteria (indexes: 2, 3, 4, 5 and 6) are smaller. Only the results obtained on the basis of Papadopoulos's criterion (1997), (index 7) are more conforming with the experimental data than the proposed criterion results.

Table 2. Compatative analysis of the results obtained with different criteria

group	data index		$\tau_{\alpha^*(eqnp)}/t_{-1}$	Crossland	Sines	Matake	McDiarmid	Dietman	Papadopoulos
			1	2	3	4	5	6	7
2	9-18	mean	1.01	0.95	0.90	1.03	0.96	0.99	1.03
		std. dev.	0.04	0.06	0.06	0.04	0.06	0.05	0.03
9	51-58	mean	0.95	0.87	0.83	1.00	0.83	0.91	0.96
		std. dev.	0.11	0.14	0.16	0.13	0.14	0.14	0.07
10	59-67	mean	1.03	0.90	0.94	1.10	0.92	1.02	1.01
		std. dev.	0.07	0.11	0.14	0.08	0.07	0.13	0.03

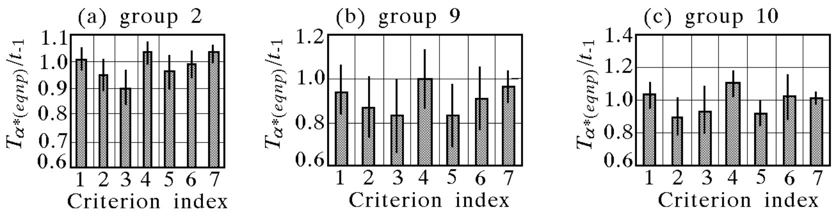

Fig. 5. Comparative analysis of the applied criteria

6. Conclusions

The new fatigue failure criterion has been developed for the multiaxial non-proportional loading. The criterion is an attempt to combine two known but alternatively used models. The main assumptions on which the criterion is based result from the Critical Plane idea, while the non-proportionality function used in the criterion applies the Integral Approach postulates.

The criterion correctly describes the influence of the mean nominal loading and the influence of the non-proportionality of loading (caused by non-zero values of the phase shift and mean values) on the fatigue limit value. The predictive capability of the criterion was demonstrated by analysing 67 experimental results from the literature. The predicted results were generally in good agreement with the experimental ones.

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Kryterium wytrzymałości zmęczeniowej w warunkach wieloosiowych obciążeń z udziałem przesunięcia fazowego i wartości średnich

Streszczenie

Opracowano nowe, oparte o koncepcje płaszczyzny krytycznej kryterium dla wieloosiowej wytrzymałości zmęczeniowej. Kryterium poprawnie ujmuje wpływ przesunięcia fazowego i wartości średnich w warunkach kombinacji zginania i skręcania. Z pewnego punktu widzenia, tak zdefiniowane kryterium może być rozumiane jako kombinacja dwóch modeli: płaszczyzny krytycznej i podejścia całkowego (nielokalnego). Kryterium ma następującą postać

$$\tau_{\alpha^*(eqnp)} = (\tau_{\alpha^*(a)} + c_1\sigma_{\alpha*(a)} + c_2\sigma_{\alpha^*(m)})\Big(1 + \frac{t_{-1}}{b_{-1}}H^n\Big) \leqslant c_3$$

gdzie mnożna naprężenia zredukowanego $\tau_{\alpha^*(eqnp)}$ zawiera amplitudę naprężenia stycznego $\tau_{\alpha^*(a)}$, amplitudę $\sigma_{\alpha^*(a)}$ i wartość średnią $\sigma_{\alpha^*(m)}$ naprężenia normalnego działających w płaszczyźnie krytycznej. Mnożnik zawiera miarę nieproporcjonalności obciążenia H. Biorąc pod uwagę fakt różnej wrażliwości materiałów na nieproporcjonalność obciążenia, równanie zawiera również dane materiałowe: t_{-1} – granicę zmęczenia na skręcanie, b_{-1} – granicę zmęczenia na zginanie. Zgodność wyników obliczeń z wynikami uzyskanymi eksperymentalnie została zweryfikowana na 67 danych zaczerpniętych z literatury. Zgodność ta w większości przypadków jest satysfakcjonująca.

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